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BETA DECAY

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Abstract

The beta decay spectrum of Cs^{137} is measured using a magnetic spectrometer and a Geiger counter detector for the electrons. A Kurie plot analysis is made to extract the beta decay parameters and compare to theory. The internal conversion line resulting from the decay scheme of Cs^{137} is also studied.

Theory

Beta decay refers to the process in which a nucleus spontaneously breaks down by emitting an electron (a "Beta ray") and an antineutrino. The process is seen in the decay of a free neutron to a proton, electron and an antineutrino, releasing 785 keV with a half life of 10.5 minutes:

$$\mathrm{n} \rightarrow \mathrm{p} + e^- + \bar{\nu}_e$$

The process is also seen in the decay of nucleii which have too many neutrons to be stable. The process always increases the atomic number Z of the nucleus due to the extra proton which is created. The example used in this experiment is:

$$\mathrm{Cs}^{137} \rightarrow \mathrm{Ba}^{137} + e^- + \bar{\nu}_e$$

The neutrino was postulated in 1931 by Pauli to explain how in beta decay the following could be conserved:

- energy – if no neutrino, then energy disappears.
- linear momentum – the p and e^- do not recoil in opposite directions.
- angular momentum – if no neutrino, then $1/2\ \hbar$ of angular momentum disappears.

There are several features which should be noted:

1. The antineutrino is also required to balance the "e-lepton number." It is possible to assign such a quantum number to particles; for example:

e^-	electron	e-lepton number	=	+1
e^+	positron		=	−1
ν_e	e-neutrino		=	+1
$\bar{\nu}_e$	e-antineutrino		=	−1
	muon, neutron, proton, γ etc.		=	0

   The conservation of "e-lepton number" expresses the fact that we have never observed electrons or e-neutrinos being created or destroyed except in pairs.

   Although not needed in this experiment, you should know of a similar μ-lepton number which is also conserved.

μ^-	+ muon	μ lepton number	=	+1
μ^+	−muon		=	−1
ν_μ	μ neutrino		=	+1
$\bar{\nu}_\mu$	μ antineutrino		=	−1
	electron, neutron, proton, γ etc.		=	0

   For example, the decay of a muon must obey both conservation laws (check this):

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

2. The neutrino has unusual properties:

   (a) It must move at or about the velocity of light because it has a very small if not exactly zero rest mass.

(b) It has a very small interaction with normal matter. A neutrino can easily pass through a light-day of lead.

(c) It has a spin of $\frac{1}{2}$ (similar to that of an electron rather than the spin 1 of a photon).

(d) The spin of a neutrino is about the direction of motion. The spin vector of a neutrino is opposite to its momentum and that of an antineutrino is parallel to its momentum.

(e) It has zero charge.

3. Some nucleii which are proton-rich spontaneously emit positrons by changing a proton to a neutron.

$$\mathrm{p} \rightarrow \ \mathrm{n} \ + e^{+} + \nu_e$$

This occurs only within a nucleus since a proton has a smaller rest mass than a neutron and so cannot decay by itself.

4. Energy and momentum must be conserved in the decay. Hence, depending upon the angles of the ejected particles, various energy values are found for the electrons. It was this distribution of observed electron energies as well as the imbalance of spin angular momentum which led to the prediction of the neutrino as a particle with no mass or charge.

5. Beta Decay is an example of the weak interaction. Remember that all interactions can be classified as either strong, electromagnetic, weak, or gravitational.

6. An electron can be knocked out of the electron "cloud" of an atom by a photon. This is the Photo-Electric effect or the Compton Effect (not Beta Decay). If the photon originates in the nucleus and knocks out an atomic electron, then that phenomenon is called "Internal Conversion". In this experiment both beta decay electrons and internal conversion electrons are detected.

The following is extracted from "Atomic and Nuclear Physics" by D. L. Livesey (Blaisdell Publishing Co., 1966)

In present-day theory, a nucleus is regarded as an assembly of A particles, each with baryon number +1 and similar mass; of these Z are protons and $(A - Z)$ are neutrons. For many purposes it is convenient to treat the neutron and the proton as two states of the same particle, the *nucleon*, which always interacts strongly with other nucleons, whatever their charges may be. This model avoids many of the difficulties encountered by the proton-electron model. For instance, it is possible to measure the total angular momentum, often called the "nuclear spin," of nuclei in their ground states. It is found that all nuclides with even A have zero or integral spin (measured in terms of the fundamental unit $\hbar$).

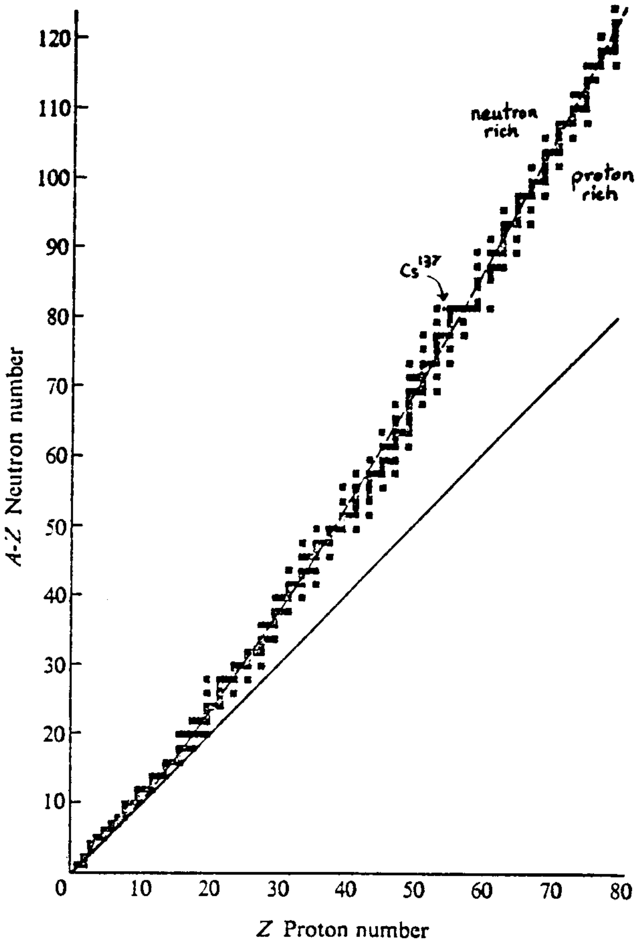


FIGURE 9.19 *Chart of the stable nuclides up to $Z = 80$, each being plotted as a small square against the proton number (Z) and neutron number $(A - Z)$. The full line is at 45° to each axis while the dotted line follows the general trend for stability, showing that heavy nuclei require more neutrons than protons for this condition.*

Some information about the forces binding nuclei together can be obtained by direct study of the systematics of stable nuclides. If, for example, we plot the neutron number $(A - Z)$ against the proton number (Z) for stable isotopes from $Z = 1$ to $Z = 82$, we find the pattern shown in Figure 9.19. Certain details of this pattern require considerable attention later on, but it is clear that the general trend is for the points to be grouped about a line which starts off at 45° to each axis. Thus in light nuclides stability is achieved when the neutron and proton numbers are roughly equal. However, the line curves upwards, indicating that the stable heavy nuclides always contain many more neutrons than protons. This result may be explained on the supposition that extra Coulomb repulsion between the protons at high Z values must be overcome by extra binding, which is provided by an increase in the ratio of neutrons to protons.

Momentum Spectrometer

We first review some relativistic equations. The relativistic momentum is defined $\vec{p} = \gamma m_o \vec{v}$ where $\gamma = (1 - \frac{v^2}{c^2})^{-1/2}$. Force is equal to the time rate of change of momentum:

$$\begin{aligned}
\vec{F} = \frac{d\vec{p}}{dt} &= \frac{d}{dt}(\gamma m_o \vec{v}) \\
&= \gamma m_o \frac{d\vec{v}}{dt} + \frac{d\gamma}{dt} m_o \vec{v} \\
&= \gamma m_o \frac{d\vec{v}}{dt} + \frac{d}{dt}(1 - \frac{\vec{v}.\vec{v}}{c^2})^{-1/2} m_o \vec{v} \\
&= \gamma m_o \frac{d\vec{v}}{dt} - 1/2\ m_o \vec{v}(1 - \frac{\vec{v}.\vec{v}}{c^2})^{-3/2}(-\frac{2}{c^2}\vec{v}.\frac{d\vec{v}}{dt}) \\
&= \gamma m_o \vec{a} + \gamma^3 m_o \vec{v}(\frac{\vec{v}.\vec{a}}{c^2}).
\end{aligned}$$

where $\vec{a}$ is the acceleration. Check that this agrees with $\vec{F} = m\vec{a}$ if $v << c$.

In this magnetic spectrometer, the force F is caused by the field B:

$$\vec{F} = e(\vec{v} \times \vec{B}).$$

The acceleration may be found from the two equations above:

$$e(\vec{v} \times \vec{B}) = \gamma m_o \vec{a} + \gamma^3 m_o \vec{v}\ (\frac{\vec{v}.\vec{a}}{c^2}).$$

This is a vector equation. Let $\vec{a} = a_\perp + a_{||}$ where $a_\perp$ is perpendicular to $\vec{v}$ and $a_{||}$ to $\vec{v}$.

The components parallel to v are:

$$\begin{aligned}
0 &= \gamma m_o a_{||} + \gamma^3 m_o v\ (\frac{v a_{||}}{c^2}) \\
&= \gamma^3 m_o a_{||}, \qquad \text{hence } a_{||} = 0.
\end{aligned}$$

The components perpendicular to v are:

$$e(\vec{v} \times \vec{B}) = \gamma m_o a_\perp,$$

hence $a_\perp = (\frac{evB}{\gamma m_0})$

In this spectrometer, the field B is perpendicular to v. The acceleration causes a circular orbit of radius r:

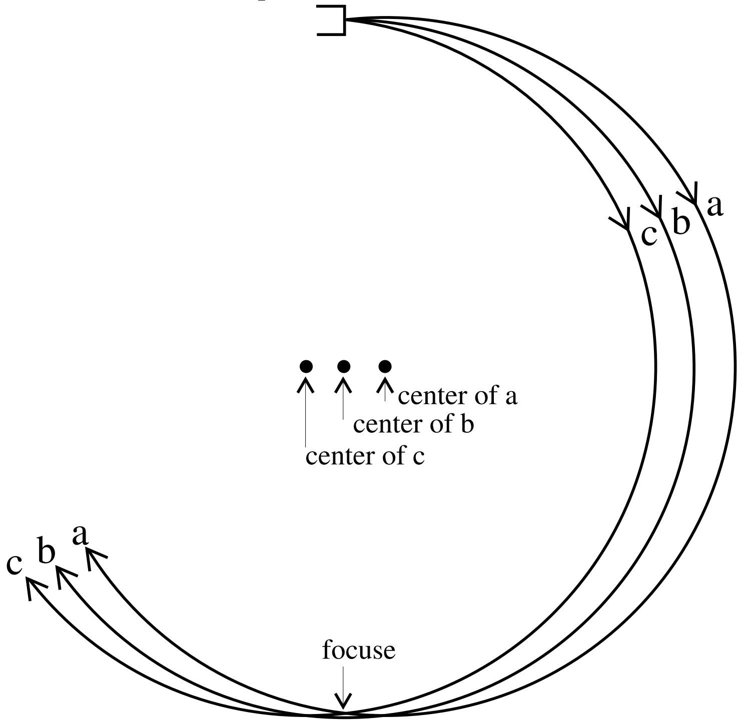
$$a = \frac{v^2}{r}.$$

Hence,

$$\begin{aligned}
\frac{v^2}{r} &= \frac{evB}{\gamma m_o} \\
\gamma m_o v &= Ber \\
p &= Ber.
\end{aligned}$$

In this experiment, the field direction is chosen so that negative charges have a trajectory leading to the detector. The source emits electrons with various angles and various momenta.

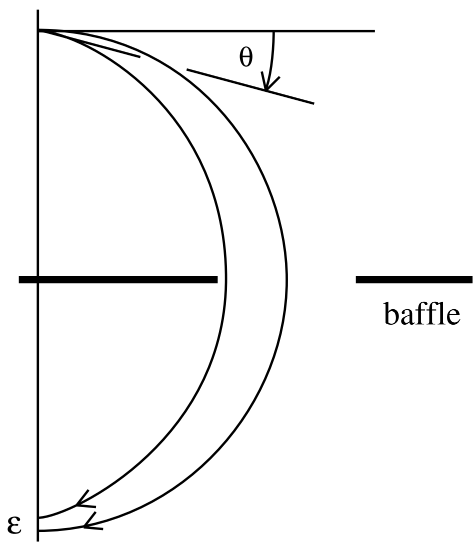
1. Variation of Angle. Consider several electrons emitted with the same momenta but at slightly different angles. Each particle will follow a circular path with a slightly different center point.



Note that the particles will pass through a focus point one diameter away from the source. Of course, if the angles are very large then the particle will pass near the focus. You may like to show that the particle will miss the focus by a distance ϵ.

$$\epsilon \simeq r \ \sin^2 \theta$$

where θ is the angle shown.

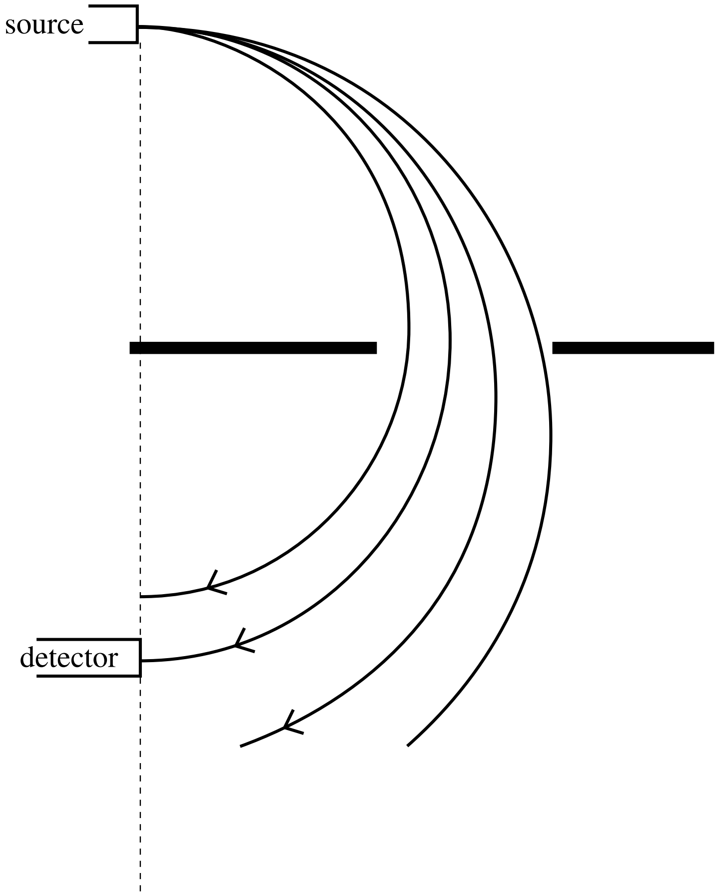


Because $\sin^2 \theta$ changes slowly near $\theta = 0$, ϵ is very small. A baffle with a slit is used to restrict the detected particles to those with θ small. In this spectrometer the restriction is:

$$\theta < 1/4 \text{ radian} \qquad (14.3°)$$

$$\sin^2\theta < 0.0612$$

2. Variation of Momenta.



The particles of different momenta will pass through different focus points. By using a small source and a small detector only those in a small range from p to $p + dp$ will be accepted.

The two effects above limit the resolution which is defined by:

$$\begin{aligned} R &= \frac{\Delta p}{p} \\ &= \frac{1}{2r}\left[s + w + r(\phi^2 + \phi\psi^2)\right], \end{aligned}$$

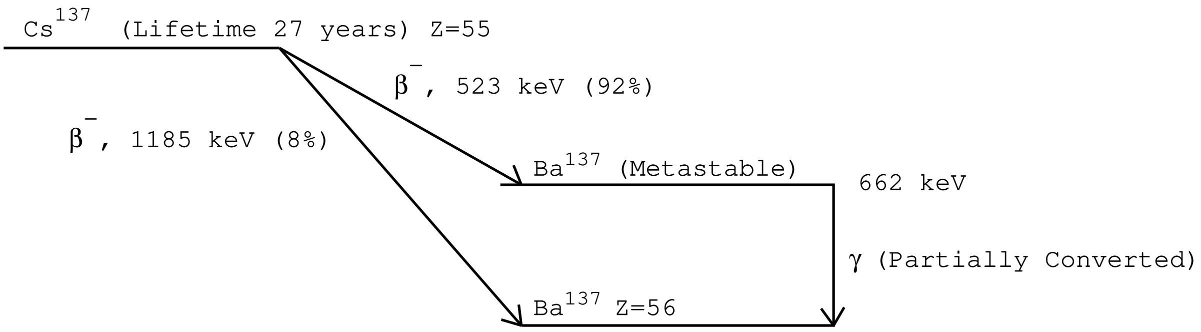
where:

- s and w are the source and detector widths respectively.
- $\phi = d/2r$ is the horizontal acceptance angle where d is the baffle opening.
- $\psi = h/2\pi r$ is the vertical angle subtended between the source and detector, h being the source height.

The fractional transmission of the spectrometer is given to be $f = \frac{\phi\psi}{4\pi}$.

Beta Decay of Cs^{137}

The figure shows that two different decays are possible. 92% of the electrons from Cs^{137} are associated with a transition having an $E_{\max}$ of 523 keV and 8% with an $E_{\max}$ of 1185 keV. Some of the 662 keV Gammas are internally converted.



Intensity

It is possible to calculate the probability that an emitted electron has a momentum between p and $p + dp$. This probability is proportional to dp (if dp is small enough) and is a function of p. Therefore, we label this probability: $I(p)dp$.

The theory for Beta Decay was developed by Enrico Fermi in 1934. This theory assumes that the neutrino has zero mass and predicts:

$$I(p)dp = CF(Z,p)\ (E_{\max} - E)^2\ p^2\ dp$$

where:

- $I(p)dp$ is the probability of a momentum between p and p + dp
- C is a constant for a particular beta decay transition.
- $F(Z,p)$, the "Coulomb factor", takes account of the effect of the nuclear electrostatic field. It may be found in tables in the references and for energetic betas $F \approx 1$.
- Z is the atomic number of the nucleus.
- $E_{\max}$ is the maximum energy of the spectrum considered, i.e. the total kinetic energy of the beta plus that of the neutrino.
- E is the kinetic energy of the beta particle which has momentum p. Since the energies involved are of the order of the rest energy m_0c^2, the correct expression to use is:

  $$E = (m_0^2c^4 + p^2c^2)^{1/2} - m_0c^2$$

  where m_0 is the rest mass of the electron and c is the velocity of light.

A convenient way of plotting experimental results for comparison with the Fermi theory was suggested by Kurie in 1936 and is called a Kurie plot.

Define y:

$$y^2 = \frac{I(p)}{Cp^2F(Z,p)},$$

then $\quad y = E_{\max} - E.$

A plot of y against $(x = E)$ will give a straight line with negative slope which intercepts the x axis at $E_{\max}$.

The x Axis of the Kurie Plot

An expression for the energy as a function of the magnetic field is needed. By means of the previous calibration of the field, the energy may be expressed as a function of the current.

- The rest energy is $E_0 = m_0 c^2$.
- The total energy is $E_T = \sqrt{E_0^2 + p^2c^2} \quad$ (also $E_T = \gamma m_0 c^2$).
- The kinetic energy is $E = E_T - E_0$

$$\begin{aligned} E &= E_T - E_0 \\ &= \sqrt{E_0^2 + p^2c^2} - E_0 \\ &= \sqrt{E_0^2 + (Berc)^2} - E_0. \end{aligned}$$

- If the energies are expressed in electron volt units:

$$\begin{aligned} E &= \sqrt{E_0^2 + (Brc)^2} - E_0 \\ \text{where } E_0 &= 511003.4 \pm 1.4 \text{ eV}. \end{aligned}$$

From each magnet current I we can obtain B in Tesla(webers/meter^2) and then by using r and c (in mks units) we obtain the kinetic energy E for the x axis.

The y Axis of the Kurie Plot

If the counting rate is A, the range of momentum dp, I the intensity, and $\Delta p/p$ the momentum acceptance of the spectrometer, then:

$$\begin{aligned} dA &= I(p)\, dp \\ A &\approx I(p)p(\Delta p/p). \end{aligned}$$

The y axis variable for the plot is:

$$\begin{aligned} y &= \sqrt{\frac{I(p)}{CF(Z,p)\, p^2}} \\ &= \sqrt{\frac{A}{CF(z,p)p^3(\Delta p/p)}} \\ &= \sqrt{\frac{A}{CF(z,p)B^3e^3r^3(\Delta p/p)}}. \end{aligned}$$

The momentum acceptance $(\Delta p/p)$ is a geometrical factor and so is a constant for a particular spectrometer. Also $CF(z,p) \simeq 1$.

We need not worry about the value of the constants because we want the intercept on the x axis and so will use:

$$y = \sqrt{\frac{A}{B^3}}.$$

Equipment

Spectrometer

The Atomic Laboratories Beta Ray Spectrometer deflects an electron through an angle of 180° between two points (source and detector) spaced a distance 2r apart (r is nominally 1.5 in). The 180° point on the trajectory is called the "first-order focus", because electrons emitted with small angular deviations from the central trajectory tend to be focussed at 180°.

The principle of the instrument is discussed in the theory section. The dimensions are:

source width	s	$\simeq \frac{1}{8}$ in
detector width	w	$= \frac{1}{16}$ in
radius	r	$= 1.5$ in
gap in baffle	d	$= 0.75$ in
source height	h	$\simeq \frac{1}{8}$ in

From these parameters, the momentum acceptance $\Delta p/p$ may be calculated:

$$\Delta p/p = 0.10 \ (10\%).$$

The fractional transmission which determines the counting rate is:

$$f = 1.05 \times 10^{-3}.$$

In order to use the spectrometer with weak sources, which do not require special safety rules, it was necessary to accept a relatively poor resolution so that the transmission would not be too low. Obviously a stronger source would allow us to improve the resolution by decreasing s, w and d.

Source

Students should not normally remove the source which is on the upper plug of the spectrometer. The Cs^{137} source (approximately 10 μCi) is mounted on a thin aluminized mylar film, which in turn is mounted on a small brass ring. The ring and the source are removable and are held in place by an internal spring clip. Simply grasp the ring firmly and pull it away after the source holder has been removed from the Beta Ray Spectrometer. (Cenco sells an optional Sodium-22 source on the same size ring and this ring in turn can be mounted on the same source holder merely by pushing it in over the spring clips.) During insertion or removal of the source, care has to be taken not to touch the source itself which is in the center of the mylar film. It is unprotected and will scrape off at the least touch.

If it were protected, it would absorb so many beta rays that the experiment could not be conducted properly.

CAUTION: WHEN INSERTING OR REMOVING THE CESIUM, SODIUM, OR ANY OTHER RADIOACTIVE SOURCE, GLOVES SHOULD BE WORN AND HANDS SHOULD BE WASHED BOTH BEFORE AND AFTER THE OPERATION. IT IS QUITE POSSIBLE THAT THE SOURCE, IF TOUCHED DIRECTLY, WILL COME OFF ON THE HAND. GREAT CARE SHOULD BE EXERCISED TO SEE THAT STUDENTS OR UNQUALIFIED PERSONNEL ARE NOT ALLOWED TO TOUCH THESE SOURCES DIRECTLY.

Baffle

This is mounted inside the center plug. Students should not remove the baffle since it can easily be damaged. It is a horizontal plate (1/16 in thick) with a rectangular hole.

Detector

This is mounted on the lower plug. It is an end-window Geiger counter. You may dismount it to inspect it but you should be very careful. The Tube has a mica end window with a thickness of only 1.4 - 2.0 mg/cm^2 and is quite fragile. It should not be touched and the tube should be capped when not in use. The tube should not be used above 700 volts.

Probe Port

A Hall probe may be inserted near the baffle by removing the baffle plug. The magnetic field must be measured with no vacuum in the system. The field should be unchanged when the chamber is evacuated.

Vacuum

The electrons are far more energetic (up to 660 keV) and so do not need as good a vacuum as electronic vacuum tubes (0.1 to 0.3 keV) and television tubes (up to 21 keV). The system will operate with a pressure as high as 1 mm of Mercury (1 Torr) but better results are obtained if the pressure is kept less than 100 microns (10^{-1} Torr). The normal vacuum pump on the apparatus will maintain 10 to 20 microns (10^{-2} to 2×10^{-2} Torr). A thermocouple gauge is used to measure the pressure.

Detector High Voltage

A positive regulated supply set to about +500 V is connected to the Geiger counter via a RC decoupling network. The pulse output from the Geiger counter is capacitively decoupled from the Geiger counter HV and is connected from the decoupling network box to the preamplifier input.

Preamplifier

ORTEC Model 435A. Should be set for NEG input pulses and UNIPOLAR output pulses. The gain should be set to drive the ORTEC scaler. A setting of coarse gain = 16 should be about right.

Scalar

ORTEC Model 484. The Scaler records single counts directly, provided that the input pulses (positive) are above the threshold setting. You will have to time the runs. A clock is provided on the bench.

Magnet

This is a simple electromagnet which has 2 coils connected in series. The current is supplied by a variable regulated voltage source (0 to 50 VDC) and is monitored by a series ammeter.

Procedure

1. Read the theory and equipment sections.

2. Pump down the spectrometer. The pressure should be less than 100 microns after a minute. Check that no radioactive sources (from adjacent experiments) are nearby.

3. Predict the direction of the magnetic field to bend electrons from the source to the detector and connect the power leads accordingly. Set the current to about 0.25 amp.

4. Calibrate the Hall Probe by using the internal calibration adjustment. (see Instructor) Insert the Hall probe into the probe port and plot the field B in Tesla against the current I. Take care not to strain the probe—clamp it in position—don't let it hang by its delicate probe. Remember to rotate the probe to find the maximum reading and also to allow for magnet hysteresis by always increasing I towards its measured value. (1 Tesla = 10,000 Gauss)

5. Set the current to about 0.25 Amp and measure the counting rate for various high voltages from 400 volts up to 700 volts (but not beyond). This data should show a plateau. Select a high voltage at the center of the plateau and use it for all following measurements.

6. Measure the counting rate A(I) for various currents from 0 up to 0.90 Amps. Remember that the standard deviation expected on N counts is $\sqrt{N}$.

7. Plot the counting rates A(I) against the current I.

8. Identify the critical regions of E_{max} and the conversion line. Take additional measurements in these regions in order to define them more accurately.

9. Measure the counting rate at 0.9 amps carefully, regard it as a background and subtract it from the other rates.

10. Assume the Hall Probe is linear, $c = 2.9979 \times 10^8$ m/sec and use the given value of r. Calculate $pc/e = Brc$ for each current I.

11. Calculate the energy (in keV) of the internal conversion peak from the observed momentum p by using the value of Brc for the peak and the electron rest mass E_0 = 511003 eV. Then calculate:

$$E = \sqrt{E_0^2 + (Brc)^2} - E_0.$$

How does your result compare with the accepted value?

12. Calculate the x axis and y axis points of the Kurie plot for each current I.

13. Tabulate I, A, B, Brc, the x value, and the y value.

14. Make the Kurie plot. Draw a straight line on the part expected to be linear and determine E_{max} of the Beta Decay. Compare your result with the accepted value.

15. You may want to make another Kurie plot for the data at energies greater than internal conversion peak. This plot should give an $\mathrm{E}_{\mathrm{max}}$ of 1176 keV.