# Compton Scattering 

Advanced Laboratory, Physics 407

University of Wisconsin
Madison, Wisconsin 53706


#### Abstract

The Compton scattering of the 662 keV gamma rays from the decay of $\mathrm{Cs}^{137}$ is measured using a Sodium Iodide detector. The scattered energy and the differential cross section are both measured as a function of scattering angle, and the results are compared to the full relativistic quantum theory of radiation.


## Theory

The Compton effect is the elastic scattering of photons from electrons. As a reaction, the process is:

$$
\gamma+e^{-} \rightarrow \gamma+e^{-}
$$

Since this is a two body elastic scattering process, the angle of the scattered photon is completely correlated with the energy of the scattered photon by energy and momentum conservation. This relation is usually written as:

$$
\begin{equation*}
\Delta \lambda=\lambda^{\prime}-\lambda=\frac{h}{m c}(1-\cos \theta) \tag{1}
\end{equation*}
$$

where $\lambda$ and $\lambda^{\prime}$ are the wavelengths of the incident and scattered photon respectively, and $\theta$ is the photon scattering angle.

The energy of a photon is related to its frequency and wavelength as:

$$
\begin{equation*}
E=h \nu=h c / \lambda \tag{2}
\end{equation*}
$$

where $c$ is the velocity of light. Combining Eq. (1) and (2) the energy of the scattered photon is:

$$
\begin{equation*}
E^{\prime}=\frac{E}{1+\frac{E}{m c^{2}}(1-\cos \theta)} \tag{3}
\end{equation*}
$$

The kinetic energy of the recoil electron is:

$$
\begin{equation*}
T_{e}=E-E^{\prime}=E \frac{\gamma(1-\cos \theta)}{1+\gamma(1-\cos \theta)} \tag{4}
\end{equation*}
$$

where $\gamma=h \nu / m c^{2}$.
The quantity $h / m c$ in Eq. (1) is called the Compton wavelength and has the value:

$$
h / m c=2.426 \times 10^{-10} \mathrm{~cm}=.02426 \AA .
$$

For low energy photons with $\lambda \gg .02 \AA$, the Compton shift is very small, whereas for high energy photons with $\lambda \ll 0.02 \AA$, the wavelength of the scattered radiation is always of the order of $0.02 \AA$, the Compton wavelength.

As an example, in this experiment gamma rays from $\mathrm{Cs}^{137}$ are scattered from an aluminum target; since $E=0.662 \mathrm{MeV}$, we have $\gamma=1.29$ so that gamma rays scattered at $\theta=180^{\circ}$ will have $E^{\prime}=E / 3.6$, which is less
than $1 / 3$ of their original energy. It thus becomes quite easy to observe the Compton energy shift. This would not be the case for X-ray energies.

Another useful kinematic relation is the electron scattering angle in terms of the photon scattering angle:

$$
\cot \varphi=(1+\gamma) \tan \theta / 2
$$

where $\varphi$ is the electron scattering angle relative to the incident photon direction.


Figure 1
The above kinematic relations as well as the following discussion on cross sections may be found in Melissimos pp. 252-265 which is required reading for this experiment.

When $h \nu \ll m c^{2}$ the probability for Compton scattering can be regarded as a classical process and is given by the Thompson cross section which is the classical limit of the exact Compton scattering cross section formula.

$$
\begin{equation*}
\left.\frac{d \sigma}{d \Omega}\right|_{\text {Thompson }}=r_{0}^{2}\left(\frac{1+\cos ^{2} \theta}{2}\right) \tag{5}
\end{equation*}
$$

where $r_{0}=\frac{e^{2}}{4 \pi \epsilon_{0} m c^{2}}$ is the "classical electron radius" and has the value $r_{0}=2.818 \times 10^{-13} \mathrm{~cm}$. When integrated over all scattering angles, Eq. (5)
yields the total Thompson cross section:

$$
\begin{equation*}
\sigma_{T}=\frac{8}{3} \pi r_{0}^{2} \tag{6}
\end{equation*}
$$

This simple cross section has several failings:

1. It does not depend on the photon energy, a fact not supported by experiment;
2. the electron, although free, is assumed not to recoil;
3. the treatment is nonrelativistic;
4. quantum effects are not taken into account.

The problem was solved by Klein and Nishina in 1928 giving the correct quantum-mechanical calculation for Compton scattering, the so called KleinNishima formula:

$$
\begin{align*}
\frac{d \sigma}{d \Omega}= & r_{0}^{2}\left(\frac{1+\cos ^{2} \theta}{2}\right) \frac{1}{(1+\gamma(1-\cos \theta))^{2}} \\
& \times\left[\frac{\gamma^{2}(1-\cos \theta)^{2}}{\left(1+\cos ^{2} \theta\right)(1+\gamma(1-\cos \theta))}+1\right] \tag{7}
\end{align*}
$$

This result is for the cross section averaged over all incoming photon polarizations. By integrating Eq. (7) over all angles, the total cross section can be obtained. While the expression for the total cross section is a lengthy formula, two asymptotic expressions for the total cross section $\sigma_{c}$ in the low energy and high energy case are more simple.

$$
\begin{align*}
\text { Low energies }(\gamma \ll 1) \quad \sigma_{c} & =\sigma_{T}\left\{1-2 \gamma+\frac{26}{5} \gamma^{2}+\cdots\right\} \\
\text { and High energies }(\gamma \gg 1) \quad \sigma_{c} & =\frac{3}{8} \sigma_{T} \frac{1}{\gamma}\left(\ln 2 \gamma+\frac{1}{2}\right) . \tag{8}
\end{align*}
$$

Note that Eq. (5) for the Thompson cross section gives a symmetric angular distribution of scattered photons (i.e., the angular distribution is symmetric about $90^{\circ}$ ). The Klein-Nishina formula (Eq. (7)) on the other hand predicts a strongly forward peaked cross section as $\gamma$ increases.

The following table and graph show the Klein-Nishina cross section as a function of photon scattering angle. The table is for the .662 MeV gamma
ray energy of $\mathrm{Cs}^{137}$, while Fig. 2 shows the angular distribution for a range of incident photon energies.


Figure 2: Angular distribution of the Compton scattering as a function of the angle of scattering for various primary frequencies $\gamma=h \nu / m c^{2}$

## Radiation Units

Milliroentgen per hour ( $\mathrm{mR} / \mathrm{h}$ ) are units of radiation exposure. Exposure indicates the production of ions in a material by radiation, and it is defined as the amount of ionization produced in a unit mass of dry air at standard temperature and pressure. The roentgen is the conventional unit for exposure, where
radiation exposure unit: 1 roentgen $=1 \mathrm{R}=2.58 \times 10^{-4}$ coulomb per kilogram.
Thus, 1 R of radiation produces $2.58 \times 10^{-4} \mathrm{C}$ of positive ions in a kilogram of air at standard temperature and pressure, and an equal charge of negative ions.

Radiation safety standards are expressed in units of roentgen equivalent mammal per year (rem/yr). We now relate roentgens to rems via the rad unit and the $R B E$ or $Q F$, discussed below.

| $\theta$ <br> (Degrees) | cos $\theta$ | Classical <br> $(\mathrm{keV})$ | Relativistic $E$ <br> $(\mathrm{keV})$ | Klein-Nishima <br> $\left(10^{-30} \mathrm{~m}^{2}\right)$ |
| :---: | ---: | :---: | :---: | :---: |
| 0 | 1.00000 | 662.00 | 662.00 | 7.941 |
| 5 | .99619 | 658.74 | 658.75 | 7.834 |
| 10 | .98481 | 649.06 | 649.22 | 7.524 |
| 15 | .96593 | 633.24 | 634.01 | 7.047 |
| 20 | .93969 | 611.73 | 614.03 | 6.452 |
| 25 | .90631 | 585.20 | 590.35 | 5.793 |
| 30 | .86603 | 554.45 | 564.09 | 5.119 |
| 35 | .81915 | 520.46 | 536.34 | 4.471 |
| 40 | .76604 | 484.32 | 508.02 | 3.875 |
| 45 | .70711 | 447.17 | 479.90 | 3.348 |
| 50 | .64279 | 410.13 | 452.57 | 2.894 |
| 55 | .57358 | 374.23 | 426.43 | 2.513 |
| 60 | .50000 | 340.31 | 401.76 | 2.200 |
| 65 | .42262 | 308.95 | 378.72 | 1.948 |
| 70 | .34202 | 280.49 | 357.37 | 1.746 |
| 75 | .25882 | 255.06 | 337.72 | 1.589 |
| 80 | .17365 | 232.58 | 319.72 | 1.467 |
| 85 | .08716 | 212.88 | 303.31 | 1.374 |
| 90 | .00000 | 195.71 | 288.39 | 1.304 |
| 95 | -.08716 | 180.79 | 274.87 | 1.254 |
| 100 | -.17365 | 167.86 | 262.65 | 1.217 |
| 105 | -.25882 | 156.65 | 251.63 | 1.192 |
| 110 | -.34202 | 146.95 | 241.73 | 1.176 |
| 115 | -.42262 | 138.55 | 232.85 | 1.166 |
| 120 | -.50000 | 131.29 | 224.92 | 1.161 |
| 125 | -.57358 | 125.01 | 217.87 | 1.160 |
| 130 | -.64279 | 119.60 | 211.62 | 1.161 |
| 135 | -.70711 | 114.95 | 206.13 | 1.164 |
| 140 | -.76604 | 110.99 | 201.34 | 1.168 |
| 145 | -.81915 | 107.63 | 197.22 | 1.172 |
| 150 | -.86603 | 104.83 | 193.71 | 1.176 |
| 155 | -.90631 | 102.53 | 190.80 | 1.181 |
| 160 | -.93969 | 100.69 | 188.45 | 1.184 |
| 165 | -.96593 | 99.29 | 186.64 | 1.187 |
| 170 | -.98481 | 98.31 | 185.37 | 1.190 |
| 175 | -.99619 | 97.72 | 184.60 | 1.191 |
| 180 | -1.00000 | 97.53 | 184.35 | 1.191 |
|  |  |  |  |  |

Table 1

The absorbed dose is the radiation energy absorbed per kilogram of absorbing material. It is measured in rads, where

$$
\text { absorbed dose unit: } 1 \mathrm{rad}=0.01 \text { joule per kilogram. }
$$

In animal tissue it takes about 30 eV or $4.8 \times 10^{-18} \mathrm{~J}$ to produce an ion pair, and assuming the magnitude of the charge of each ion is $1.60 \times 10^{-19} \mathrm{C}$, then for animal tissue

$$
1 \mathrm{R} \times \frac{4.8 \times 10^{-18} \mathrm{~J}}{1.6 \times 10^{-19} \mathrm{C}}=2.58 \times 10^{-4} \frac{\mathrm{C}}{\mathrm{~kg}} \times 30 \frac{\mathrm{~J}}{\mathrm{C}}=0.008 \frac{\mathrm{~J}}{\mathrm{~kg}} \simeq 1 \mathrm{rad}
$$

Hence, a 1-R exposure to x rays or $\gamma$ rays produces an animal tissue absorbed dose of approximately 1 rad .

The effects of radiation on biological systems depend on the type of radiation and its energy. The relative biological effectiveness (RBE) or quality factor (QF) of a particular radiation is defined by comparing its effects to those of a standard kind of radiation, which is usually taken to be $200-\mathrm{keV}$ x rays. The RBE or QF is the ratio of the dose in rads of a particular kind of radiation to a 1 -rad dose of $200-\mathrm{keV}$ x rays, where the particular radiation produces the same biological effect as the x rays. Note that RBE or QF is dimensionless.

$$
\text { RBE or QF: } \frac{\text { Number of rads of a particular kind of radiation }}{1 \mathrm{rad} \text { of } 200-\mathrm{keV} \mathrm{x} \text { rays }}
$$

In animal tissue the RBE is about 1.0 for x rays, $\gamma$ rays, and $\beta$ rays, and it ranges from about 2 to 20 for protons, neutrons, and $\alpha$ particles.

The rem is defined as

$$
\text { rem } \equiv \text { dose in rads } \times \mathrm{RBE} .
$$

For animal tissue a 1-rad dose of $\gamma$ rays is equivalent to an exposure of 1 R , and the RBE is about 1 for $\gamma$ rays in animal tissue; therefore, the dose in rems and the exposure in roentgens are equivalent.

Radiation standards adopted by the United States Government are the following:

1. For workers employed around nuclear facilities: $5 \mathrm{rem} / \mathrm{yr}$, which would be $2.5 \mathrm{mrem} / \mathrm{h}$ continuously while at work for those on a $40-\mathrm{h}$ week. For comparison, 300 to 600 rem of acute whole-body radiation is fatal to humans.
2. For the general population living near a nuclear facility: 0.5 rem/yr.
3. For worldwide population: 5 rem total up to age $30(0.17 \mathrm{rem} / \mathrm{yr})$, in addition to natural background radiation, which is about the same intensity. Primary concern is for genetic damage. It is estimated, rather uncertainly, that $0.17 \mathrm{rem} / \mathrm{yr}$ may produce 5000 extra deaths and 5000 birth defects in the United States per year.

## Shielding Calculations

The calculations consist of two parts. First we calculate the attenuation of gammas which is desirable. Then we calculate the thickness of lead to give this attenuation.

1. A A source of 100 millicuries $(100 \mathrm{mCi})$ of $\mathrm{Cs}^{137}$ gives an exposure of 0.039 Roentgen/hour ( $\mathrm{R} / \mathrm{hr}$ ) at a distance of 1 meter. By the inverse square law, the exposure at the distance of 0.3 meters is

$$
x=0.43 \mathrm{R} / \mathrm{hr} \quad \text { (with no shielding) }
$$

B This corresponds to a dose equivalent $D$ for tissue of $.415 \mathrm{rads} / \mathrm{hr}$.
C Converting this to the Dose Equivalent in rems.

$$
\mathrm{DE}=D \times \mathrm{QF}
$$

where $\mathrm{QF}=1$ for gamma rays.
Hence we have

$$
\mathrm{DE}=0.415 \mathrm{rems} / \mathrm{hr} \quad \text { (if no shielding) }
$$

D Assume that a student may spend 16 hours/month standing at this short distance ( 0.3 meters) from the source. The Dose equivalent/year with this occupancy factor is then

$$
79.7 \mathrm{rem} / \text { year (if no shielding) }
$$

E Radiation standards adopted by the U.S. (1990) for worldwide population: 5 rem total up to age 30 ( $0.17 \mathrm{rem} / \mathrm{yr}$ ), in addition to natural background radiation, which is about the same intensity. We will assume that $1 \times 10^{-3} \mathrm{rem} / \mathrm{yr}$ is satisfactory for the
shielding of our $\mathrm{Cs}^{137}$ source. We have therefore designed the lead shielding to give an attenutation factor of:

$$
f=\frac{1 \times 10^{-3} \mathrm{rem} / \mathrm{yr}}{79.7 \mathrm{rem} / \mathrm{yr}}=12.5 \times 10^{-6} .
$$

A shielding calculation (not shown here) for 662 keV gamma rays gives the result that 10.5 cm of lead will accomplish the above attenuation factor.

## Apparatus

## Gamma Source

The gamma source is $\mathrm{Cs}^{137}$. The present source had a nominal strength of 100 millicuries when it was purchased in 1975, although the actual strength appears to have been somewhat smaller. A careful measurement of the source strength in May, 1991, yielded a value of $42 \pm 2$ millicuries.

This is a strong source and so there are 3 very important rules. The decay scheme of $\mathrm{Cs}^{137}$ is shown in Fig. 3. The details of the source are shown in Fig. 4.

1. Do not attempt to remove the source from inside its lead container.
2. The container barrel points towards a concrete wall. Do not try to move the container or table.
3. Do not place any part of your body within $15^{\circ}$ of the barrel axis unless the stopper is in the position to block the barrel.

The lead container is designed conservatively so that the dose you would receive if you leaned against the container for 16 hours each month would be still less than $10 \%$ of the allowable dose for students and $2 \%$ of the allowable dose for industrial workers. The dose is approximately $1 \%$ of the natural background dose. As a result, the lead container weighs about 540 lbs . or 250 kg .

In practice, of course, you will spend less time than 16 hours and will be at a distance which reduces the background intensity 50 fold.


Figure 3


Isotope Products Inc. Model HEG-137-100
Figure 4

## Source Container and Barrel

The container is made of lead and so the 662 KeV gammas have a nearly equal probability of interacting via the Photo-Electric effect and via the Compton Effect. The most important angle of scattering is in the forward $\pm 45^{\circ}$ cone because
(a) The differential cross section is larger in the forward direction, and
(b) The minimum energy is lost by the gamma when it is scattered in the forward direction.

As we require a jet of well collimated 662 KeV gammas with as little contamination as possible of gammas with lower energies, the container barrel has been designed to greatly reduce the chance of gammas escaping after a forward scatter $\left(\theta<90^{\circ}\right)$.

If a 662 KeV photon is scattered through more than $45^{\circ}$ then it is likely to be absorbed by a small thickness of lead.

$$
N=N_{0} e^{-x / \lambda}
$$

| Angle of Scattering | Energy | $\lambda$ in Lead |
| :---: | :---: | :---: |
| $0^{\circ}$ | 662 keV | 0.88 cm |
| $45^{\circ}$ | 479 keV | 0.57 cm |
| $90^{\circ}$ | 288 keV | 0.22 cm |
| $135^{\circ}$ | 206 keV | 0.11 cm |
| $180^{\circ}$ | 184 keV | 0.09 cm |



Figure 5

The gammas which do not pass out through the aperture are incident upon the surfaces A, B, C, D. (See Fig. 5.)

A and B A few gammas will Compton Scatter to give 184 MeV gammas which scatter to the right. However, the 662 gammas will have penetrated about 0.88 cm into the lead and so only a few of the 184 MeV gammas will escape from inside the AB surfaces. Some gammas will interact via the photoelectric effect. Both will cause 75 keV X-rays (Lead K shell) but again only a few X-rays will escape from inside the AB surfaces.

C The aperture is designed so that scattered gammas and X-rays cannot escape directly from C.

D Again scattered gammas and X-rays cannot escape directly. Although the Compton scatters of $90^{\circ}$ to $45^{\circ}$ may have enough energy for a second Compton scatter on the opposite surface E, the distance they must travel from the interaction point in D to leave D is greater than the penetration into D. Note that a few gammas will Compton Scatter near the DE corner and will escape into the main beam. Obviously if we had a ring of dense HIGH $Z$ material such as uranium, at the corner, this background could be reduced. See Fig. 6.


Figure 6
$\underline{\mathbf{E}}$ This is a conical surface designed so that it cannot be directly illuminated by gammas from the source.

F The forward outer surface is positioned so that 662 keV gammas leaving the source must travel through a distance of $12 \lambda$, or 10.56 cm . The attenuation factor is then approximately $e^{-12}=6 \times 10^{-6}$. The total gammas intensity is greater since the scattered gammas are not all absorbed. The "Build Up Factor" for gammas of $\frac{1}{2} \mathrm{MeV}$ in lead is about 2 and so the overall attentuation is about $12 \times 10^{-6}$.

G The backward outer surface has an extra $2 \lambda$ added as an extra (and
unnecessary) safety factor.

$$
\begin{aligned}
H & =14 \times 0.88 \mathrm{~cm} \\
& =12.32 \mathrm{~cm} \\
& =4.85 \text { inches }
\end{aligned}
$$

The overall attenuation factor is then approximately

$$
2 \times e^{-14}=1.6 \times 10^{-6}
$$

The jet of gammas is intended to be
(a) wider than the scatterer so that calculations of the cross section may assume the scatterer was uniformly illuminated, and


Figure 7
(b) sufficiently narrow that the detector may measure scattered intensifies at small angles from the scatterer. See Fig. 7.

The approximate angles are:
Angle of scatterer subtended at the source $\pm 5^{\circ}$
Angle of Uniform $\gamma$ jet from the source $\pm 10^{\circ}$
Angle of outside of $\gamma$ jet from the source $\pm 20^{\circ}$
Angle of scintillator subtended at the scatterer $\pm 8^{\circ}$

Minimum angle of scattering which is outside the jet: $15^{\circ}$.

## Tapered Plug

This is intended to block the beam. The taper is made from brass and so must be longer than if it had been made from lead. We except a few scattered gammas to sneak along the taper since the fit cannot be perfect. For this reason an extra 3.8 cm of steel is used to block those escaping along the taper.
Detector (Harshaw Chemical Type 858 Serial 6V230 with voltage divider)
This is a NaI crystal scintillation cylinder with a diameter of 2 inches and a thickness of 2 inches. The crystal is hermetically sealed and in good optical contact with the photocathode of an RCA 6342 A phototube with 10 stages. The Harshaw Quality Assurance Report states that the detector has a resolution when measuring 662 keV gammas of

$$
\frac{\text { full width at } 1 / 2 \text { maximum counting rate }}{\text { pulse height }}=8.4 \% .
$$

Read the description of the experiment on Interactions of Gammas for further information.

The photomultiplier requires a positive high voltage via the white cable and High Voltage BNC connector. The dynode potentials are controlled by the built in voltage divider with a total resistance of 6.2 Mohms . The anode is at the positive high voltage and so a 1 nf capacitor is used to pick off the $1 \mu \sec 100$ millivolt negative signals. The maximum HV rating is +1500 V but the experiment needs only $\sim 1100-1200 \mathrm{~V}$.

The NaI-PM assembly is mounted inside a tapered lead cylinder. The lead weighs about 23 kg and is intended to serve as a collimator so that only gamma rays from the target region are detected. See Fig. 8.


Figure 8

The NaI crystal diameter is 2 inches but only $13 / 4$ inches are exposed to the gammas to give the greatest probability of full absorption of each gamma entering the NaI.

The lead cylinder is mounted on rollers and rotates about a center which can support a scatterer.

## Scatterer

The scatterer is an aluminum disc ( 1.27 cm thick, 3 cm diameter). The Compton effect depends only on the electrons but does assume that the electrons are loosely held. For this reason, the Z of the scatterer should not be too high. The K shell electrons in aluminum have a level of -1560 eV and the L shell electrons have levels of $-118,-74$ and -73 eV .

The scatterer was chosen as a disc so that the probability of multiple scattering may be minimized. The scatterer is mounted on rigid foam plastic so that the support will scatter few gammas.

## High Voltage Supply

This is regulated and can supply up to 2.5 mA at +2.5 kV . This experiment requires about $190 \mu \mathrm{~A}$ at +1200 V . The gain $G$ of the photomultiplier depends on the voltage, but the gain variation with voltage is approximately $\Delta G / G=n \Delta V / V$ where $n=10$ is the number of dynodes. A typical gain for our operating conditions is $G \simeq 10^{4}$. Consequently if we wish to measure the energy of scattered gammas to $\pm 1 \%$, then the HV must be regulated to better than $\pm(1 / 4) \%$.

Amplifier
An ORTEC Model 575 Amplifier is used to amplify the output of the sodium-iodide detector to match the $0-10 \mathrm{~V}$ full scale range of the multichannel analyzer. The gain is normally set so that the $\mathrm{Cs}^{137}$ peak is in about channel 800.

Multi-Channel Analyzer (Ortec Model 916)
This is a 2048 channel ADC unit mounted inside the IBM PC with the input connector on the back of the PC. The program to run it is called by typing "MCA" from anywhere inside the DOS operating system. Consult the instructor for details on running the associated software.

## Procedure

1. (a) Read the section on the design of the shielding for this experiment.
(b) Use the Radiation Monitor to make a survey of the gamma intensity around the experiment with the plug on the source container both open and shut. We have included a considerable safety factor in the design of this experiment. However, if you are not completely reassured by both of the checks (a) and (b) above, then discuss your numbers with the instructor. You should not, of course, lean into the jet of the gammas.
(c) Connect the equipment and take care that the cables will not be caught.
(d) Make sure the plug is back in the source.
2. Rotate the detector out of the beam and use sources to calibrate the linearity of the analyzing system. Start with $\mathrm{Cs}^{137}$ and keep the spectrum on the display after stopping a short run. Change to $\mathrm{Ba}^{133}$ and accumulate another spectrum on the same display. Note the peak channels for both spectra and plot peak channel \# vs energy. The plot should come fairly close to going through the origin. More sources can be used to check the overall linearity with some redundancy. A sample calibration using eight different photon energies is shown in Fig. 9. Your calibration curve will allow you to convert from peak channel number to energy. Set the amplifier gain so that the $\mathrm{Cs}^{137}$ peak is about $80 \%$ of full scale (use 512 channels full-scale).


Figure 9
3. Mount the Al scatterer, unplug the source and record the spectra due to the scattered gammas at $15^{\circ}$ intervals starting at a scattering angle of $15^{\circ}$. The scatterer should be rotated so as to approximately bisect the angle between the source and the detector. (Why? The gammas are not reflected like light-why not?) Use your knowledge of counting statistics to decide how long to count the spectra.
At each angle take a run with and without the Al target. These runs do not have to be for equal times if they are correctly normalized.
The MCA software may be used to subtract the appropriately normalized background as you go along and to place regions of interest (ROI) about the detected gamma ray peaks. For each ROI, the software can calculate the center of the peak, the full width half maximum (FWHM) and the net area under the peak.

Spectra may be stored on the computer's hard disk (but please delete
them sometime after you have completed all of your data analysis). You also have the option of printing out the spectrum as seen on the screen (PRT SCRN) or printing out the counts vs channel number by printing the file from within the MCA program.
For each spectrum, identify the channel number of the peak of the Compton scattering. Use the plot of (2) above to compute a scattered energy $E_{2}$ for each angle. For each spectrum, obtain the yield of scattered photons. This may be done either of two ways: (a) accept all counts above noise level and apply a correction for the efficiency of the crystal, or (b) integrate the counts in the photopeak only and apply a correction for the "photofraction" as well as for the crystal efficiency. For the latter method, it is probably best to sum the channels that fall within the FWHM of the detected peak and correct back to the full area. For a Gaussian shaped peak, the counts within the FWHM region correspond approximately to $\frac{1}{1.314}$ of the total area. Efficiency and photofraction data for a $2^{\prime \prime}$ diam. $\times 2^{\prime \prime}$ thick NaI crystal are included as figures 10 and 11 .
4. The Compton theory predicts

$$
\frac{1}{E_{2}}=\frac{1}{E_{1}}+\frac{1}{m_{0} c^{2}}(1-\cos \theta) .
$$

Plot $1 / E_{2}$ from your measurments against $1-(\cos \theta)$.
5. The non-relativistic theory suggests a different dependence of $E_{2}$ on $\cos \theta$. The values have been calculated and tabulated for you. Plot these on the same plot as your data.
6. Check and comment upon:
(a) The agreement of your data and nonrelativistic theory.
(b) Would you expect good agreement for small $\theta$ ?
(c) Is the experimental data in the form predicted by relativistic theory?
(d) From the slope, find the rest energy of an electron.
7. Derive the differential cross sections by performing a series of calculations:
$\binom{$ Gamma intensity at }{ scatterer }$=I=\frac{42 \mathrm{mCuries} \times .94 \times .90 \times 3.7 \times 10^{10} \mathrm{dis} / \mathrm{sec}}{4 \pi(\text { source to scatterer })^{2}}$
$\binom{$ Number of electrons }{ in scatterer }$=N=\frac{\text { volume } \times \text { density }}{\text { molecular wt }} \times$ Avogadro's No. $\times Z$
Number of scattered gammas passing into the $\mathrm{NaI} /$ second at angle $\theta=$
$G(\theta)=\frac{\text { counts in photopeak }}{\text { time }} \times \frac{1}{\text { efficiency of NaI at energy } E^{\prime}}\left(\times \frac{1}{\text { photfraction at } E^{\prime}}\right)$

Use the plot below (Fig. 10) for the efficiency. The differential cross section at each angle $\theta$ is then

$$
\frac{d \sigma}{d \Omega}=\left(\frac{G(\theta)}{I N}\right)\left(\frac{1}{\text { solid angle of NaI subtended at scatterer }}\right)
$$



Figure 10
8. Plot your differential cross sections with error bars as a function of $\theta$. On the same plot place points calculated from the Klein-Nishina formula (see the table). Compare the experimental and theoretical predictions.
9. Be prepared to answer questions on all aspects of the experiment such as:
(a) How does a multi-channel analyzer work?
(b) Are you sure the HV did not drift?
(c) What fraction of the gammas would suffer multiple scattering?
(d) Why do the energy peaks have a gaussian shape?
(e) What was the radiation level where you stood?
(f) At what angle $\theta$, is the kinetic energy given to an electron insufficient to knock it free from the K-shell of aluminum?
(g) Could you distinguish between the energy of a photon which suffered a single $\theta=90^{\circ}$ scatter and the energy of a photon which suffered two $\theta=45^{\circ}$ scatters?


Figure 11: Calculated Efficiencies and Photofractions for Various Size Thallium Activated Sodium Iodide Crystals, W.F. Miller, John Reynolds, W.J. Snow, Rev. Sci. Instrum. 28, 717 (1957)

