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MILLIKAN OIL-DROP EXPERIMENT

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Abstract

The charge of the electron is measured using the classic technique of Millikan. Measurements are made of the rise and fall times of oil drops illuminated by light from a Helium–Neon laser. A radioactive source is used to enhance the probability that a given drop will change its charge during observation. The emphasis of the experiment is to make an accurate measurement with a full analysis of statistical and systematic errors.

Introduction

Robert A. Millikan performed a set of experiments which gave two important results:

(1) Electric charge is quantized. All electric charges are integral multiples of a unique elementary charge e.

(2) The elementary charge was measured and found to have the value $e = 1.60 \times 10^{-19}$ Coulombs.

Of these two results, the first is the most significant since it makes an absolute assertion about the nature of matter. We now recognize e as the elementary charge carried by the electron and other elementary particles. More precise measurements have given the value

$$e = (1.60217733 \pm 0.00000049) \times 10^{-19} \text{ Coulombs}$$

The electric charge carried by a particle may be calculated by measuring the force experienced by the particle in an electric field of known strength. Although it is relatively easy to produce a known electric field, the force exerted by such a field on a particle carrying only one or several excess electrons is very small. For example, a field of 1000 volts per cm would exert a force of only 1.6×10^{-14} N on a particle bearing one excess electron. This is a force comparable to the weight of 10^{-12} grams.

The success of the Millikan Oil-Drop experiment depends on the ability to measure small forces. The behavior of small charged droplets of oil, weighing only 10^{-12} gram or less, is observed in a gravitational and electric field. Measuring the velocity of fall of the drop in air enables, with the use of Stokes' Law, the calculation of the mass of the drop. The observation of the velocity of the drop rising in an electric field then permits a calculation of the force on, and hence the charge, carried by the oil drop.

Although this experiment will allow one to measure the total charge q on a drop, it is only through an analysis of the data obtained and a certain degree of experimental skill that the charges can be shown to be quantized. By selecting droplets which rise and fall slowly, one can be certain that the drop has a fairly small charge. A number of such drops should be observed and their respective charges q calculated. If the charges q on these drops are integral multiples of a certain smallest charge e, then this is an observation that charge is quantized.

Question

Some students measure a medium size charge q and then divide it by whatever large integer, n, which will give

$$\frac{q}{n} = 1.60 \times 10^{-19} \text{ coulombs!}$$

What is wrong with this?

Since a different droplet has been used for measuring each charge, there remains the question as to the effect of the drop itself on the charge. This uncertainty can be eliminated by changing the charge on a single drop while the drop is under observation. An ionization source placed near the drop will accomplish this. In fact, it is possible to change the charge on the same drop several times. If the results of measurements on the same drop then yield charges q which are integral multiples of some smallest charge e, then this strongly suggests that charge is quantized.

The measurement of the charge of the electron also permits the calculation of Avogadro's number. The charge F required to electrodeposit one gram equivalent of an element on an electrode (the Faraday) is equal to the charge of the electron multiplied by the number of molecules in a mole. Through electrolysis experiments, the Faraday has been found to be F $= 9.625 \times 10^7$ coulombs per kilogram equivalent weight.

$$\begin{aligned}\text{Hence Avogadro's Number } N = F/e &= \frac{9.625 \times 10^7 \text{ coulombs/kg equiv. wt.}}{1.60 \times 10^{-19} \text{ coulombs/molecule}} \\ &= 6.02 \times 10^{26} \text{ molecules/kg equiv. wt.}\end{aligned}$$

Equations for calculating the charge on a drop

An analysis of the forces acting on the oil drop let will yield the equations for the determination of the charge carried by the droplet.

Figure 1 shows the forces acting on the drop when it is falling in air and has reached its terminal velocity (terminal velocity is reached in a few milliseconds for the droplets used in this experiment). In Fig. 1, v_f is the velocity of fall, k is the coefficient of friction between the air and the drop, m_1 is the mass of the drop, m_2 is the mass of air displaced by the drop and g is the acceleration due to gravity.

The downward force due to gravity is $m_1g - m_2g$. The viscous retarding force is kv_f. Then

$$m_1g - m_2g = kv_f \quad .$$

Let $m = (m_1 - m_2)$, then

$$mg = kv_f \quad . \tag{1}$$

Figure 1

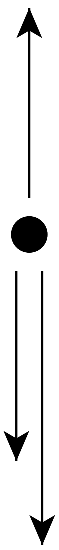


Figure 2

Figure 2 shows the forces acting on the drop when it is rising under the influence of an electric field. In Fig. 2, E is the electric intensity, q is the charge carried by the drop and v_r is the velocity of rise. Adding the forces vectorially yields:

$$Eq = mg + kv_r \quad . \tag{2}$$

Eliminating k from equations (1) and (2) and solving for q yields:

$$q = \frac{mg(v_f + v_r)}{Ev_f} \quad . \tag{3}$$

To eliminate m from equation (3), one uses the expression for the volume of a sphere:

$$\begin{aligned} m &= (m_1 - m_2) \\ m &= (4/3)\pi a^3(\sigma_1 - \sigma_2) \end{aligned} \tag{4}$$

where a is the radius of the droplet, σ_1 is the density of the oil and σ_2 is the density of the air. Equations (3) and (4) are combined:

$$q = \frac{4\pi}{3}\frac{g}{E}(\sigma_1 - \sigma_2)(1 + \frac{v_r}{v_f})a^3 \quad . \tag{5}$$

To calculate a, one employs Stokes' Law, relating the radius a of any spherical body to its velocity of fall in a viscous medium (with the coefficient of viscosity, η):

$$\text{velocity falling} = \frac{2}{9}\frac{ga^2}{\eta}(\sigma_1 - \sigma_2)$$

In the theoretical derivation of Stokes' Law the following five assumptions are made:

(1) that the inhomogeneities in the medium are small in comparison with the size of the sphere;

(2) that the sphere falls as it would in a medium of unlimited extent;

(3) that the sphere is smooth and rigid;

(4) that there is no slipping of the medium over the surface of the sphere;

(5) that the velocity with which the sphere is moving is so small that the resistance to the motion is all due to the viscosity of the medium and not at all due to the inertia of such portion of the medium as is being pushed forward by the motion of the sphere.

In the case of our small drops, the assumptions (2), (3), (4) and (5) are valid. However, the assumption (1) is not completely valid since the drop radii are about 1 or 2 microns and not much greater than the mean free path of the air molecules.

The drop will tend to fall more quickly in the "holes" between the air molecules. Kinetic theory indicates that a correction must be made to the formula for the falling velocity

$$v_f = \frac{2}{9}\frac{ga^2}{\eta}(\sigma_1 - \sigma_2)\left(1 + A\frac{\ell}{a}\right) \quad ,$$

where ℓ is the mean free path of the air molecules and A is a dimensionless correction factor.

The mean free path ℓ is dependent upon the air pressure P and so we use a more convenient form

$$v_f = \frac{2}{9}\frac{ga^2}{\eta}(\sigma_1 - \sigma_2)\left(1 + \frac{b}{Pa}\right) \tag{6}$$

where b is a correction factor, and P is the pressure.

To calculate the radius a we must solve this equation. First rearrange.

$$\frac{9\eta v_f}{2g(\sigma_1 - \sigma_2)} = a^2 + \frac{b}{P}a \quad .$$

Let

$$\theta = b/2P \quad , \tag{7}$$

and

$$\phi = \frac{9\eta v_f}{2g(\sigma_1 - \sigma_2)} \quad . \tag{8}$$

Then $a^2 + 2\theta a - \phi = 0$. The solution must be the positive root

$$a = -\theta + \sqrt{\theta^2 + \phi} \quad . \tag{9}$$

The electric intensity is given by $E = V/d$, where V is the potential difference across the parallel plates separated by a distance d. E, V and d are all expressed in the mks system of units and so E is measured in volts/meter.

The charge q may be obtained by calculating θ and ϕ with equations (7) and (8) then calculating the radius a with equation (9) and finally q with equation (5).

Calculations

It is suggested that you lay out the results of your calculations for each drop in the form of a table so that errors may be found more easily. Use MKS units and the following columns:

average fall time t_f	average rise time t_r	$\left(1+\frac{t_f}{t_r}\right) = \left(1+\frac{v_r}{v_f}\right)$	fall velocity v_f	ϕ	$(\theta^2+\phi)$	$\sqrt{\theta^2+\phi}$	a	a^3	q

(1) The fall velocity $v_f = s/t_f$ where s is distance between the images of the graticule lines. You will have to determine s experimentally.

(2) From v_f, calculate ϕ by multiplying by a factor $9\eta/2g(\sigma_1 - \sigma_2)$ where:

$\eta = ?$ N sec m^2 (Use graph on page 9: 1 N sec m^2 = 1 kg m^{-1} sec^{-1}).

g = 9.81 meters/sec^2.

$\sigma_1 = ?$ kg/meters^3 (density of oil to be measured).

$\sigma_2 = 1.192$ kg/meters^3 (density of air at 1 atmosphere and 22°C).

(3) Calculate θ from the barometric pressure P measured in cm of mercury and use $b = 6.17 \times 10^{-6}$ (cm of Hg-meter).

(4) Add $\theta^2 + \phi$. The θ^2 will be small.

(5) Take the square root $\sqrt{\theta^2 + \phi}$.

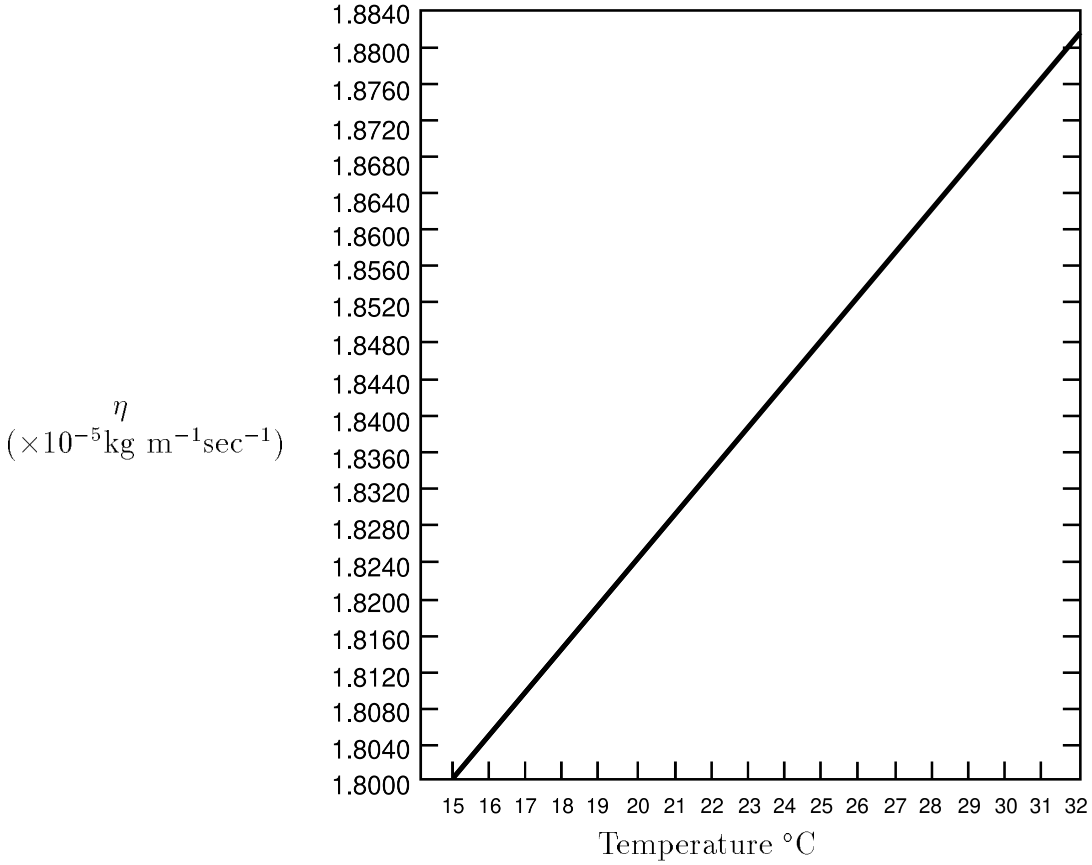
(6) Subtract θ to obtain a.

(7) At this stage, check that the radius a is reasonable.

(8) Obtain a^3.

(9) Compute a multiplier $\frac{4\pi}{3}\frac{gd(\sigma_1-\sigma_2)}{V}$. (You will have to measure d.)

(10) Combine the multiplier, $(1+\frac{v_r}{v_f})$ and a^3 to obtain q.



Viscosity of dry air as a function of temperature

Apparatus

The experiment uses:

1. A PASCO SCIENTIFIC CORP apparatus.

2. A 4 milliwatt He-Ne laser for illumination of the drops. The laser is fastened to the supporting plate so that its light cannot enter your eyes accidentally. However, be careful, and do not thoughtlessly unfasten the laser or introduce shiny or reflecting objects into the laser beam.

3. The high voltage is generated and stabilized within the PASCO apparatus and is measured precisely with a Data Precision Model 1450 multimeter. A correction for the finite input impedance of the meter may be required.

4. The time is measured by a small counter controlled by a microswitch and a 100 kHz crystal oscillator. The count is visible on LEDs and is precise to the least count of 0.01 seconds. The counter is zeroed when the microswitch is pressed and the count starts. The count stops when the switch is released.

5. The apparatus and laser are mounted on one plate (for safety) which can be adjusted so that the electric field E is vertical.

6. An atomizer contains a non-volatile oil of known density.

The apparatus controls are:

PLATE CHARGING CONTROL SWITCH - When the three way lever plate control switch is in the OFF position the condenser plates are disconnected from the high voltage supply and grounded. When the switch is in the TOP PLATE +, position the top plate is positive with respect to the bottom plate. When the switch is in the TOP PLATE –, position the top plate is negative with respect to the bottom plate.

RADIATION SOURCE - when the lever is at the OUT position the radiation source is shielded on all sides by plastic, so that virtually no radiation enters the area of the drops. At the IN position the plastic shielding is removed and the drop area is exposed to the radiation source. NOTE: move the radiation source lever gently to avoid jarring the condenser assembly and knocking the droplet from the viewing area.

The radiation source, initial strength and date of initial strength are all specified on the radiation tag to the left of the radiation source lever. The power Supply specifications are:

Power Input: 110/130 VAC, 50/60 cps.

Range: 300–400 VDC, continuously variable.

Regulation: 1% for 10% line variation.

Ripple: less than 0.1 volt.

Stability: within 1% after warm up.

Procedure

Two people are advisable since one person must follow the drops while the other records the times.

1. Darken the room. A small desk lamp can be used to illuminate your data book.

2. Turn the POWER switch ON, the PLATE CONTROL switch to OFF, the RADIATION SOURCE LEVER to IN.

3. The RETICULE ILLUMINATION control should be set so that the reticule lines are just bright enough to be easily visible. Excessive illumination of these lines may make it difficult to observe very small droplets.

4. Introduce some oil drops into the condenser by placing the nozzle of the atomizer into the hole of the condenser housing cover. A few quick "squirts" of oil will fill the upper chamber of the condenser with drops and begin to force some drops into the viewing area. If no drops are seen, squeeze the atomizer bulb gently until drops appear in the viewing area. If repeated "squirts" of the atomizer fail to produce any drops in the viewing area, but rather a cloudy brightening of the field, the hole in the top plate is probably clogged, and should be cleaned.

   The exact technique of introducing drops will have to be developed by the experimenter. The object is to get a small number of drops, not a large, bright cloud, from which a single drop can be chosen. It is important to remember that the drops are being forced into the viewing area by the pressure of the atomizer. Therefore, excessive use of the atomizer can cause too many drops to be forced into the viewing area and, more important, into the area between the condenser wall and the focal point of the scope. Drops in this area prevent observation of drops at the focal point of the scope.

   NOTE: If the entire viewing area becomes filled with drops, so that no one drop can be isolated, either wait three or four minutes until the drops settle out of view, or disassemble the condenser, thus removing the drops. When the amount of oil on the condenser parts becomes excessive, clean the assembly.

5. Select drops which have a mass which can be accurately measured. If the mass is too large the drop will fall quickly and your percentage error in timing will be poor.

If the mass is too small, the drop will bounce around due to random collisions with air molecules (Brownian motion) and it will be difficult to estimate when it crosses a line. A very small drop may cross a line 10 or 20 times! The laser shows the Brownian motion quite clearly.

We recommend using drops which fall with times between 10 and 40 seconds.

6. Now move the RADIATION SOURCE lever to the OUT position (so that the charges will be unlikely to change.)

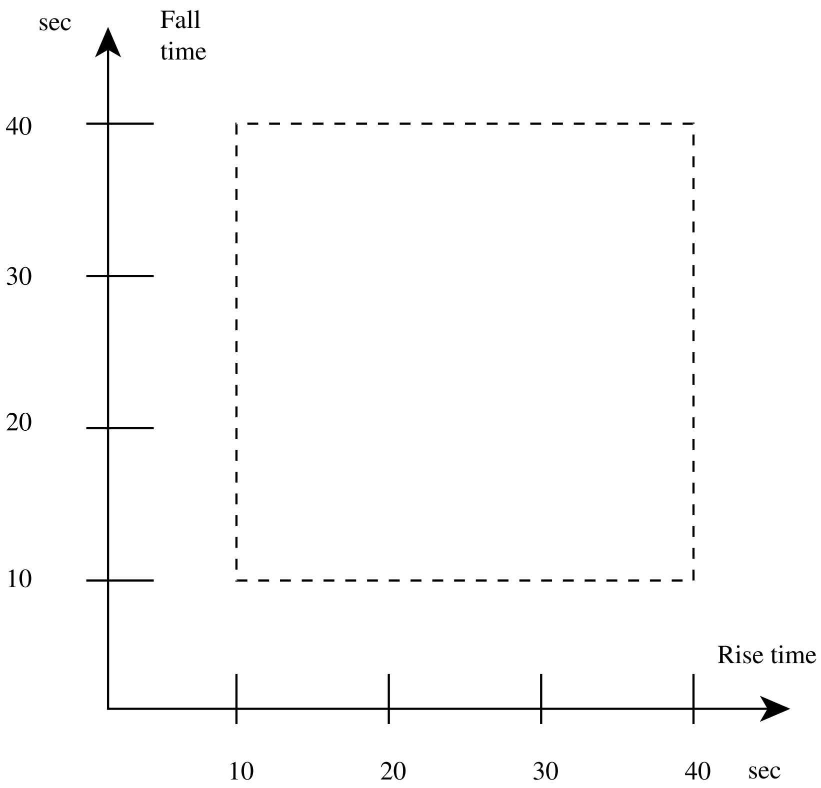
7. Immediately select those drops with measurable charges. If the drop rises quickly then it has a large charge q and so will not be much use in finding the elementary charge.

   Look for drops which take 10 to 40 seconds to rise.

8. If you still have many drops in sight, repeat 5 and 7 to concentrate on a useful drop.

9. Take about 10 measurements of the fall time and of the rise time of a particular drop. Do not jar the apparatus or you may lose your drop.

10. Plot the fall time and rise time of this drop **before** measuring the next drop. Write the drop number beside its point.



11. Calculate the q for the first drop **before** you make any further measurements. If the drop is outside the range of 1 to 10×10^{-19} coulombs then find your procedural or arithmetic error before continuing.

12. Repeat 4 through 10 for about **30 drops**. Choose the fall and rise times so that you have a fairly uniform scatter within the 10–40 sec square on your fall time, rise time plot. This will prevent you from selecting too many identical charges. Include a few points outside the square if you wish.

13. Try a variation on one or two drops by changing the charge. Drops are again introduced into the viewing area and a new drop is selected. After about 20 measurements on this drop have been made, the drop is brought to the top of the field of view and allowed to fall with the RADIATION SOURCE lever at the IN position. A few seconds later the plates should be charged, and, if the rising velocity has changed, the RADIATION SOURCE is moved to the OUT position and a new series of measurements taken. If, however, the charge has

not changed, then turn the Plate Control Switch to OFF and allow the drop to continue falling. After a few seconds, again check for a change in the rising velocity. Continue this procedure until the drop has captured an ion.

If the drop captures an ion such that the drop moves rapidly downward, then reverse the polarity of the plates so that the drop can be made to rise.

Make about 20 measurements of the rising and falling velocity of the drop, and, if possible, change the charge again and repeat the measurement procedure.

14. Record the barometric pressure and the voltage on the condenser for each measurement set.

15. At this time (before doing the calculations) decide which of your observations may be in error. It is very important that you **not** reject any data after you see the result of the calculations. Why?

16. Now calculate q for each of your drops.

17. Make a histogram of your charges q. Use a bin width of 0.2×10^{-19} coulombs. Label each square in the histogram with the drop number.

18. Does your evidence indicate that all charges are integral multiples of an elementary charge?

19. Choose a charge Q such that all charges less than Q, fall in groups which are obvious multiples of an elementary charge. (The errors of the charges larger than Q are too large for clean grouping).

20. Tabulate **all** charges q less than Q (it is now too late to reject a charge), divide each q by the integer appropriate for its group. The final step is an average charge determined from the average charge of each of the integer subgroups. The final result should include an error resulting from correct error propagation of individual error contributions. Be sure to separate systematic and statistical errors.