THERMAL NOISE

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Abstract

The aim of this experiment is to observe the thermal noise in a resistor, to verify that the mean square noise voltage is proportional to the absolute temperature, and to obtain an an experimental value for the Boltzmann constant.

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Theory

J.B. Johnson discovered that any resistor exhibits a small random alternating e.m.f. (now called Johnson noise) and that the noise is dependent on the temperature. Nyquist assumed that the noise was due to the thermal agitation of the electrons in the resistor and, using thermodynamics, developed the expression[1]

$$\bar{V}^2 = 4kT \int_{f_1}^{f_2} \text{Real} \, [Z(f)] df \tag{1}$$

where

- $\bar{V}^2$ is the mean square noise voltage.

- k is Boltzmann's constant

- T is the absolute temperature

- $Z(f)$ is the impedance of the device (a resistor) at the frequency f

- f_1 and f_2 are the frequency limits between which the noise is accepted by the measuring device.

If the device is a simple resistor, this becomes

$$\bar{V}^2 = 4kTR(f_2 - f_1). \tag{2}$$

In this experiment, we will not measure $\bar{V}^2$ directly but will use an amplifier. The amplifier output V_1 is related to the input V by

$$V_1 = A(f)V$$

where A is the frequency dependent amplification. Hence

$$\bar{V}^2 = 4kTR \int_{f_1}^{f_2} A^2(f) df. \tag{3}$$

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Apparatus

Resistor

The resistor assembly is mounted in a copper tube (for good thermal conduction) on the end of a thin stainless steel tube (for lows thermal conduction). Two 500 kΩ metal film resistors in series are used and so R=1 MΩ. The nominal precision of the resistors is 1%.

Amplifier (Stanford Research Systems Model SR560)

This solid state amplifier has an input impedance of 100 MΩ shunted by 25 pF and can be run either DC or AC coupled. There is a wide range of gain and band-width settings. The noise figure for the amplifier is $< 4nV/\sqrt{Hz}$ at 1 kHz. This is to be compared to the Johnson noise value of 129 nV$/\sqrt{Hz}$ at T $= 300°$ K for a 1 MΩ resistor.

RMS Voltmeter (Schlumberger Model SI 7061)

The Schlumberger has a true RMS AC voltage function. The voltmeter will be used on the most sensitive scale (100 mV). This voltmeter also has a General Purpose Interface Bus (GPIB) computer interface with a GBIP connector on the back of the voltmeter. One of the laboratory computers has a GPIB interface card which can be used to read the voltmeter. There is a computer program called "C:\QB45\THERMAL.EXE" which is used to read the meter and perform an average over N readings. The program first prompts for the desired number of readings from which to form an average. You would typically use N$= 5 - 10$. The program then asks for the temperature reading which is then typed in on the command line. Each time the program is run to perform a sequence of readings at a given temperature, the result is added to a sequential file called "RESULT.DAT". This file can be edited and printed if desired.

Function Generator (Stanford Research Systems Model Model DS345)

The function generator is used to produce a sinusoidal output of variable frequency and amplitude for measuring the frequency response of the combined preamp and voltmeter system. It is used together with an in-line attenuator

of 10^4 to produce amplifier inputs that are comparable to the thermal noise voltage level.

Oven (Leybold 200 watt)

The oven is operated from a variable autotransformer (General Radio "Variac") and is used to heat the resistor up to 150° Higher temperatures will damage the thermocouple.

Thermocouple

An iron-constantan thermocouple junction is mounted next to the resistor and is used to measure the temperature of the resistor. A Keithley Model 610C Electrometer is used to measure the thermocouple voltage. The Handbook of Chemistry and Physics contains thermocouple tables.

Procedure

The resistor output connection is the two BNC connectors on the resistor assembly box, corresponding to the two ends of the resistor. These outputs are connected to the A and B inputs of the preamplifier which is run in the A–B differential mode to eliminate common mode noise. The suggested band width settings are 100 to 1000 Hz, and the suggested gain is 100. Use the preamp in the "low noise" setting.

1. Connect the resistor assembly to the preamp as described above. Look at the output of the preamp on the scope. The signal should be pure white noise with no extraneous 60 Hz pickup. The actual RMS voltage measurements are made using the Schlumberger 7061. Calculate the thermal noise expected at the output of the preamplifier at room temperature. For a simple calculation assume that the amplifier frequency response is simply $(f_{low} - f_{high})$ where A is the amplifier gain and f_{low} and f_{high} are the the 6 db low and high roll-off frequencies respectively. Is the actual measured voltage reasonable?

2. Take data above room temperature using the variac controlled oven. Do not exceed 150° C. The experimental fluctuations in the measured

voltage will give you guidance as to how to space the data in temperature. Your data will be used to determine the slope and intercept of the V^2 vs. T curve. The intercept at $0°$ K will be a measure of the noise contributions from other than thermal noise. The slope is used to determine the Boltzmann constant.

3. On the basis of the high temperature data decide if the result is improved by taking data well below room temperature. In principle we can use liquid nitrogen to cool the apparatus, but this is a difficult measurement.

4. You may later find it necessary to directly measure the frequency response of the combined preamp-voltmeter system. This is most conveniently done by inserting the attenuator pad between the function generator and the input to the preamp. The purpose of the attenuator is to be able to work at the same approximate voltage level as the actual noise signal since the voltmeter response may be somewhat voltage dependent.

5. There are several checks you should make. The resistor value should be measured at the highest and lowest temperatures used during the experiment to check that the resistor maintains the same value. the value should be 1 MΩ to 1%. Also, at some point, the amplifier noise should be measured directly by shorting the A and B inputs and reading the output voltage. This noise voltage should be significantly smaller than the thermal noise voltages you are measuring from the resistor.

6. For analysis, first calculate the band-width integral analytically or numerically, using exact analytic expressions for 6 db/octave RC roll-off filters. This procedure will overestimate the bandwidth integral, since the system input capacitance effectively cuts off the bandwidth integral at high frequencies. You can cut off the analytical or numerical integral at some effective high frequency cut-off or you can actually measure the frequency response of the whole system with a sine wave function generator by measuring Vin/Vout as a function of frequency. You can then compute k using the measured value of the slope of the V^2 vs. T curve together with eqn. (3). Your final value should include a full error analysis.

References

1. R.E. Simpson, "Introductory Electronics for Scientists and Engineers",
2nd ed., (Allyn and Bacon, Inc., 1974), Sec. 8.3.