Physics 623 — Problem Set 6

NOISE:
Which is better for low-noise applications, bipolar or field-effect (FET) transistors? Answer the following questions.

1. Input noise specifications for two high-performance op-amps (both advertised as "Low-noise") are given as follows.

OP-27: Ultra-low noise bipolar op amp
$e_{n\text{-}A}$ = 3 nV/sqrt(Hz) $i_{n\text{-}A}$ = 0.4 pA/sqrt(Hz)

LF-347: Low-noise FET-input op amp
$e_{n\text{-}A}$ = 18 nV/sqrt(Hz) $i_{n\text{-}A}$ = 0.01 pA/sqrt(Hz)

miscellaneous useful data: Room temperature = 300 K, $k_B = 1.38 \times 10^{-23}$ J/K.

a) for each of these op amps, calculate the *Noise Resistance*, R_N, which is the value of the source resistance for which the *Noise Temperature*, T_N, is a minimum. (T_N is the physical temperature the source resitance would need to have for its Johnson noise to equal the total noise due to the amplifier, giving a *Noise Figure* of 3 dB. (Noise Figure, NF, is the ratio of total noise at the amplifier output to what it would be with an ideal amplifier with $e_{n\text{-}A} = i_{n\text{-}A} = 0$. This is usually expressed in dB.)
Show your work!

OP-27: R_N = _____________ ohms

LF-347: R_N = _____________ ohms

b) Give the minimum value for the noise temperature, T_N, that can be achieved with each amplifier:

OP-27: T_N = ___________ K for $R_S = R_N$

LF-347: T_N = ___________ K for $R_S = R_N$

c) Which op amp would be better (lower Noise Figure) for:

i) measuring the potential of a pH electrode with a source impedance of 5 Megohms?

best amplifier: ____________

ii) measuring the voltage across a thermocouple with a resistance of 3 ohms?

best amplifier: ____________

d) At what source impedance does the room-temperature Johnson noise of the source equal the *voltage* noise of the OP-27?

_____________ ohms

Is the current noise of the op-amp significant for this R_S? ___yes / no____

Explain:

e) Using an LF-347, what bandwidth must be used to measure a 1 μV r.m.s. signal to 1% rms precision if the source resistance is 10^6 ohms?

_____________ Hz

About how long would it take to make one measurement with this bandwidth? _____________ seconds

Could the measurement be made significantly faster with a better amplifier? ___yes / no____

Explain:

<u>FOURIER TRANSFORMS</u>:

2. Use the convolution theorem to prove the trigonometric identity:

$$\cos(\omega_0 t) \bullet \cos(\omega_1 t) = \frac{1}{2}\left(\cos((\omega_0 + \omega_1)t) + \cos((\omega_0 - \omega_1)t)\right)$$

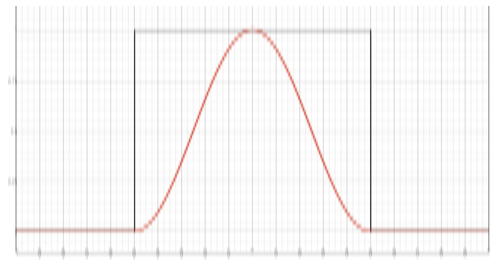
(This is easier to sketch and think about if you make $|\omega_0 - \omega_1|/\omega_0 << 1$.)

The phase detector for the lab we will do in a few weeks is effectively a multiplier that takes advantage of this to convert two frequencies into their sum and difference. The technique, called heterodyne, is also widely used in radio receivers and other instruments.

3. Use the convolution theorem to find the apparent frequency spectrum derived from a 1-second observation of the sum of two cosine waves, one at f = 9 Hz and one at f = 11 Hz. Both have amplitudes of 1 V peak. Sketch the spectrum that would be obtained. (Note that you can mathematically reproduce a one-second observation by multiplying the infinite time sequence by a rectangular function that is 1.0 between t = –0.5 s and t = +0.5 s and zero elsewhere.)

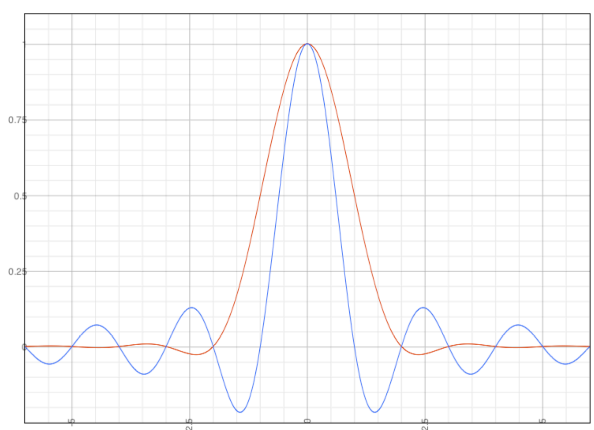
4. The sinc function that smears the spectrum observed for a finite length of time is shown in blue at the left below (compare problem 3). The oscillations can create confusing features in a spectrum that has both strong and weak sharp lines in it. The problem can be alleviated by multiplying the sampled time function by another "window" function that falls off more gently before doing the Fourier transform. One common function that is used is called a "Hanning window". If the time sequence is observed for a time T, the Hanning window function is:

$$\begin{array}{ll} \frac{1}{2}\left(1+\cos\frac{2\pi t}{T}\right) & \text{for } -\frac{T}{2} < t < \frac{T}{2} \\ 0 & \text{otherwise.} \end{array}$$



Hanning window (red) — time domain

This goes smoothly to zero as the ends of the observing interval are approached. The new smearing function will be the Fourier transform of this Hanning window. You can use the convolution theorem again to find this transform without doing any integrals if you construct the Hanning window as a continuous 1+cos multiplied by a rectangle function of length T. (Since FTs are linear, the FT of a sum is just the sum of the FTs. You need to know that the FT of a constant is a delta function at zero frequency to do the "1" part. Both terms have unit amplitude, but the cosine amplitude is split half and half between positive and negative frequencies.) The result is the red curve on the left, although this won't be obvious unless you plot your solution. You can do this, or just give the formula and plug in a couple of key points to check against red plot on the left below.) Applying this type of window to a data sample is called "apodization", or "removing the feet". Note the tradeoff — the Hanning window greatly reduces the extraneous features far from the main response, but there is significant loss of resolution in the main peak (it is broader). The squares of these smearing functions are shown on the right in blue and red and the absolute values in green and orange. These are what would appear in power spectra (V^2) and $V_{\text{r.m.s.}}$ spectra respectively.



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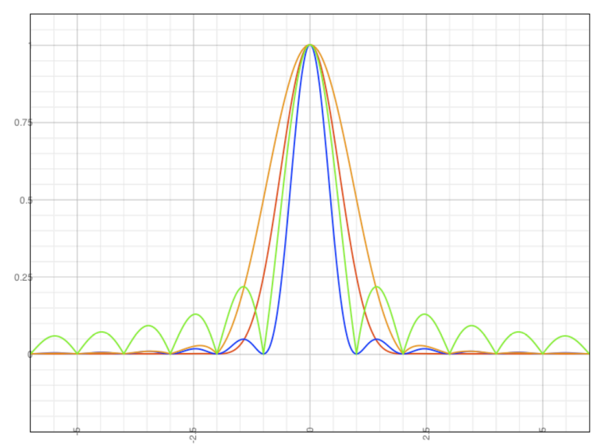

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