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Quantum mysteries revisited again

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This paper describes a device, consisting of a central source and two widely separated detectors with six switch settings each, that provides a simple *gedanken* demonstration of the nonclassical correlations that are the subject of Bell's theorem without relying on either statistical effects or the occurrence of rare events. The mechanism underlying the operation of the device is revealed for readers with a knowledge of quantum mechanics.
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I. INTRODUCTION

This paper presents a *gedanken* experiment involving a source and two widely separated detectors/observers that conveys the essence of the nonclassical correlations that are the subject of Bell's theorem[1] to a lay audience without relying on either statistical correlations or the occurrence of rare events. The device on which this demonstration is based was suggested by the work of David Mermin and Asher Peres (see Sec. IV for a more detailed statement of credits).

The present demonstration is set within the same general framework as several of Mermin's earlier nontechnical demonstrations[2–4] of Bell's theorem. In Mermin's demonstrations, a central source emits several particles that fly off toward an equal number of widely separated detectors/observers. Each particle enters a detector, whose switch can be set to one of a small number of positions, and causes a light next to the chosen switch position to flash red or green. A complete demonstration with such a setup consists of a large number of repetitions of the following two basic steps: (1) a button is pressed on the source, releasing a set of particles that speed off toward their respective detectors; and (2) an observer at each detector randomly sets its switch to one of the allowed positions and notes the color of the light that flashes when the particle enters his/her detector. At the end of all these runs the observers get together to compare their records of detector settings and light flashings. It is then that they discover that they have come face to face with the spookiness of quantum entanglement, which amounts to an informal appreciation of the central point of Bell's theorem.

Table I lists the salient features of Mermin's three earlier demonstrations of Bell's theorem, with the corresponding features of the present scheme listed underneath. One conspicuous difference between the present scheme and the earlier ones is that two particles now go to each detector, rather than just one. However the more significant differences are the numbers in the third and fourth columns. The present scheme involves only two detectors (a simplification compared to the GHZ–Mermin scheme) but each detector now has six switch settings (a complication relative to all the other schemes). An advantage of the present scheme over the Bell–Mermin scheme is that it does not rely on statistical features of the data to demonstrate its nonclassical effects, while an advantage over the Hardy–Mermin scheme is that it does not rely on the occurrence of rare events. However a disadvantage compared to the earlier schemes is that the technology needed to implement the present scheme in the laboratory is more complex.

II. THE *GEDANKEN* EXPERIMENT

A source S emits four particles, two of which fly off to the left toward Alice and the other two to the right toward Bob (see Fig. 1). Each pair of particles enters a detector which performs a measurement on it and displays the results on a screen segmented into nine square panels arranged in the form of a 3×3 array, as shown in Fig. 1. The measurement that is performed depends on the switch settings chosen by Alice and Bob. Each detector's switch can be set to one of six positions, each of which causes an entire row or column of panels on it to light up in response to the incoming particles. Each panel that lights up upon receipt of the particles lights up either red or green. (Figures in the online edition of the journal are in color; in the print edition red and green show up as black and gray, respectively.) The six switch settings on each detector will be denoted R1, R2, and R3 (for the three rows of panels, from top to bottom) and C1, C2, and C3 (for the three columns of panels, from left to right). Figure 1 shows the results of a run of this experiment in which Alice chooses the detector setting R1 and Bob the setting C2, and their panels light up as shown.

A complete demonstration with the above device consists of a large number of repetitions of the following two steps: (1) a button is pressed on the source, releasing four particles, two of which proceed toward Alice and the other two toward Bob, and (2) Alice and Bob each independently and randomly set their detectors to one of its six possible settings and note the colors of the panels that light up upon entry of the particles. A very important feature of this demonstration is that the switch settings and measurements at the two detectors in any run are always made within a very short time interval, which is too short to allow the transfer of any information from one detector to the other; in other words, the

Table I. Salient features of several nontechnical demonstrations of Bell's theorem.

Scheme	No. particles	No. detectors or observers	No. detector settings
Bell–Mermin[a]	2	2	3
GHZ–Mermin[b]	3	3	2
Hardy–Mermin[c]	2	2	2
Present scheme	4	2	6

[a]References 1 and 2.
[b]References 3 and 5.
[c]References 4 and 6.

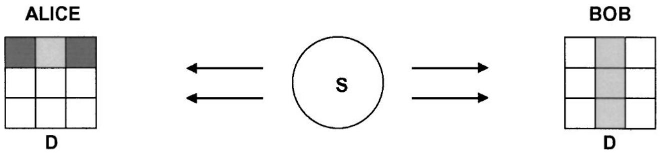

Fig. 1. The *gedanken* experiment. A source *S* emits four particles, two of which move off to the left and the other two to the right. Each pair of particles enters a detector *D*, adjusted to one of six switch settings, and causes an entire row or column of panels on it to light up. In the run above, Alice chooses the setting R1 on her detector and causes the panels in the first row to light up red, green, red (from left to right), while Bob chooses the setting C2 and causes all the panels in the second column to light up green.

conditions are such that neither detector can influence the outcome of the other in any run as a result of either its switch setting or its registered response.

After a large number of such runs, Alice and Bob get together to compare their records of detector settings and light flashings. When they do this they find that all their observations, without exception, can be summarized in the form of the following two rules.

Rule 1 (the "parity" rule): For any of the detector settings R1, R2, R3, C1 or C2, an even number of panels lights up red and an odd number lights up green. However for the setting C3 an odd number of panels lights up red and an even number lights up green. Further, the four possible outcomes for each detector setting occur randomly (i.e., each occurs a fourth of the time). Figure 2 illustrates this rule by showing the four ways in which the panels can light up for each of the six detector settings.

Rule 2 (the "correlation" rule): In those runs in which Alice's and Bob's detector settings cause one or more common (i.e., similarly placed) panels on their detectors to light up, the common panels always light up the same colors.

This rule is illustrated in Fig. 3, which shows Alice's and Bob's detector responses alongside each other for a number of runs. If one looks at the first, second, and fourth of the runs listed, one sees that the common panels that light up on both detectors always have the same colors. However no common panels light up in the third run, and Rule 2 does not apply in this case. Note that Rule 1 is always obeyed by both the detectors in every run.

The above demonstration presents us with an interesting puzzle: how can the source and detectors be constructed so that, if the experiment is carried out as described, only results in conformity with both the ''parity'' and ''correlation'' rules are ever observed?

Let us begin by focusing on the correlation rule. The only reasonable explanation for this rule in accordance with common sense notions would seem to be that the particles carry ''instruction sets'' to their detectors telling them how to respond for each of their switch settings. Indeed, in the absence of any exchange of signals between the detectors, it is difficult to see how else the common panels on both detectors can always light up the same colors no matter what switch settings are chosen by Alice and Bob. The instruction sets must clearly be such that (a) identical instructions are provided to both detectors in every run (otherwise identical switch settings would lead to violations of the correlation rule), and (b) any panel on either detector always lights up the same color no matter whether a row or column switch setting is used to activate it (otherwise one of these settings would lead to a violation of the correlation rule). An alternative way of phrasing the conclusion in (b) is to say that the properties of the particles revealed by the detector measurements are ''elements of reality''[7] in the sense that they can be determined without disturbing the particles, or the detectors with which they interact, in any way. To understand this point, suppose that one wishes, in a particular run, to determine the property of Bob's particles revealed by the color of the panel at the top left corner of his detector. One can do this by having Alice use either the setting R1 or C1 and observe the color of the top left panel on her detector, and then conclude, from the correlation rule, that Bob's panel too must have that color in this run. (Recall, from the conditions of the experiment, that enlisting Alice's cooperation in this way causes no disturbance to Bob's particles.) One can in fact extrapolate from this example and assert that the properties of Bob's particles revealed by all nine of his detector panels must be elements of reality in every run, because there is no telling which of them could be forced to reveal themselves as a result of Alice's and Bob's random choices. And it follows, by symmetry, that the same statement must hold true of Alice's particles as well.

To summarize, the idea that the particles carry ''instruction sets'' to their detectors[2] or that they possess ''elements of reality'' that can be determined without disturbing them in any way[7] appears to be an unavoidable consequence of the existence of the correlation rule.

The solution to our puzzle therefore reduces to the task of designing instruction sets for both detectors in every run in such a way that both the parity and correlation rules are satisfied. As already mentioned, the correlation rule can be taken care of by ensuring that a common instruction set is provided to both detectors in every run. Keeping property (b) of two paragraphs earlier in mind, the task of designing an instruction set reduces to the following: assign a definite color, red or green, to each detector panel in such a way that the parity rule is always satisfied. However this is immediately seen to be impossible in even a single instance if one

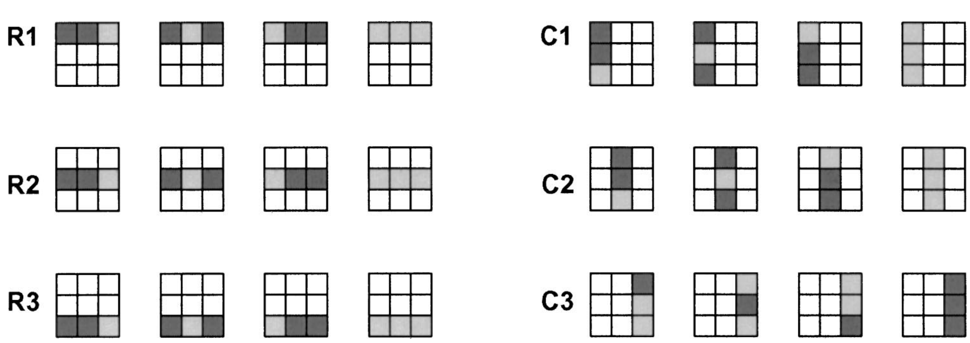

Fig. 2. Illustrating Rule 1 (the ''parity'' rule). The four possible outcomes for each of the six detector settings R1, R2, R3, C1, C2, and C3 are shown.

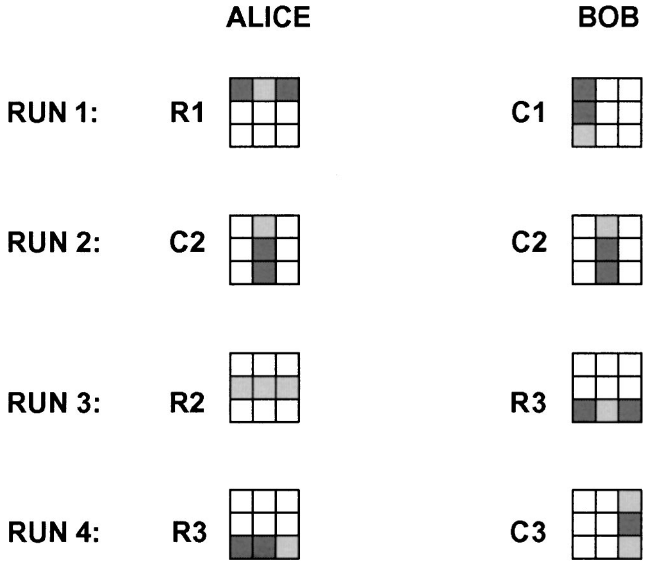

Fig. 3. Illustrating Rules 1 and 2 in a series of runs carried out by Alice and Bob. Each row shows the responses of the detectors when their switches are set to the positions shown. Note that the outcome of each run always conforms to both the ''parity'' and ''correlation'' rules.

$1\otimes\sigma_z$	$\sigma_z\otimes 1$	$\sigma_z\otimes\sigma_z$
$\sigma_x\otimes 1$	$1\otimes\sigma_x$	$\sigma_x\otimes\sigma_x$
$\sigma_x\otimes\sigma_z$	$\sigma_z\otimes\sigma_x$	$\sigma_y\otimes\sigma_y$

Fig. 4. The Mermin–Peres ''magic square'' (Refs. 8 and 9). Each entry in the square is an observable for a pair of qubits, with 1, σ_x, σ_y, and σ_z being the identity and Pauli operators. The observables in each row or column of the square form a mutually commuting set. When a particular switch setting on a detector is selected, the detector carries out a measurement of the observables in the corresponding row or column of the square on the qubits entering it and displays the eigenvalues in the form of colored lights (+1=green, −1=red) on its panels. In the context of Eq. (1), the first and second halves of each observable in the square refer to qubits 1 and 3 of Alice or to qubits 2 and 4 of Bob.

enquires about the total number of red panels on a detector: on the one hand, Rule 1 requires this number to be even (if one sums the red panels over the rows) but, on the other, it requires it to be odd (if one sums the red panels over the columns). This contradiction shows that there is no solution to our puzzle based on instruction sets. A willingness to accept the notion of instruction sets (or ''elements of reality'') to begin with, followed by the recognition that they cannot provide a solution to our puzzle, amounts to an informal appreciation of the central point of Bell's theorem.

What, then, is the ''real'' solution to our puzzle? In other words, what is the inner mechanism of the source and detectors in our demonstration, and how can we understand the results that are obtained? A clue to the inner mechanism is that it involves ''entanglement'' between the source particles that travel toward Alice and Bob. Entanglement is a peculiar property of the quantum world that has no classical analog and cannot be understood in everyday terms. Bell's theorem, more than anything else, has led to a widespread appreciation of the truly paradoxical features that lie at the heart of entanglement. The reader thoroughly familiar with quantum mechanics, and who has also had a previous brush with entanglement, may wish to pause at this point to try and figure out the inner mechanism of the device in Fig. 1 and how it performs its trick. (Warning: this is really hard!) The solution is given in the next section.

III. HOW THE TRICK IS DONE

When the button is pressed on the source, it emits four spin-1/2 particles (''qubits'') in the state

$$|\Psi\rangle=\frac{1}{\sqrt{2}}(|0\rangle_1|0\rangle_2+|1\rangle_1|1\rangle_2)\otimes\frac{1}{\sqrt{2}}(|0\rangle_3|0\rangle_4+|1\rangle_3|1\rangle_4), \tag{1}$$

where $|0\rangle_i$ and $|1\rangle_i$ are eigenstates, with eigenvalues +1 and −1, of the Pauli operator σ_z of qubit i $(i=1,...,4)$. Qubits 1 and 3 of this state go to Alice, and qubits 2 and 4 to Bob. In other words, the source emits a pair of ''Bell'' states, with one member of each Bell state going to Alice and the other to Bob.

Figure 4 shows nine observables for a pair of qubits arranged in the form of a 3×3 array, with the observables in each row or column forming a mutually commuting set. Each observable has only the eigenvalues ± 1 and, further, the product of the observables in any row or column is $+I$, with the exception of the last column for which the product is $-I$ (I being the identity operator).

When any of the settings R1, R2,..., or C3 is chosen on a detector, the detector carries out a measurement of the (commuting) observables in that row or column of Fig. 4 on its qubits and displays the observed eigenvalues as colored lights on its panels according to the convention that $a+1$ is a green and $a-1$ a red (it should be mentioned, in this context, that the first and second parts of each observable in Fig. 4 refer to qubits 1 and 3 of Alice or to qubits 2 and 4 of Bob). Rule 1 then follows as an immediate consequence of the fact[8] that if several mutually commuting observables obey a certain functional relationship, their measured eigenvalues in an arbitrary state also obey a similar relationship; in the present case this implies that the product of the observed eigenvalues of the observables in each row or column of Fig. 4 is +1, with the exception of the last column for which this product is −1. The last statement, when translated into the language of the red and green lights, is nothing but the parity rule.

The origin of Rule 2 can be understood as follows. Let $|\psi_i\rangle$ $(i=1,...,4)$ be an arbitrary set of orthonormal states in the joint space of qubits 1 and 3 and suppose that they can be expanded as $|\psi_i\rangle=a_i|0\rangle_1|0\rangle_3+b_i|0\rangle_1|1\rangle_3+c_i|1\rangle_1|0\rangle_3+d_i|1\rangle_1|1\rangle_3$, where $a_i,\ldots,d_i$ are complex coefficients. Then it follows that $|\phi_i\rangle=a_i^*|0\rangle_2|0\rangle_4+b_i^*|0\rangle_2|1\rangle_4+c_i^*|1\rangle_2|0\rangle_4+d_i^*|1\rangle_2|1\rangle_4$ $(i=1,...,4)$ is an orthonormal set of states in the joint space of qubits 2 and 4. It can be verified that the state given in Eq. (1) can be expressed in terms of the $|\psi_i\rangle$ and $|\phi_i\rangle$ as

$$|\Psi\rangle = \tfrac{1}{2}[|\psi_1\rangle|\phi_1\rangle + |\psi_2\rangle|\phi_2\rangle + |\psi_3\rangle|\phi_3\rangle + |\psi_4\rangle|\phi_4\rangle]. \tag{2}$$

When Alice carries out a measurement of one of the sets of commuting observables in Fig. 4, Eq. (2) shows that if she projects her qubits into one of the eigenstates $|\psi_i\rangle$ of this set, she projects Bob's qubits into the associated state $|\phi_i\rangle$. It can be shown that the coefficients $a_i,\ldots,d_i$ are always real for the eigenstates $|\psi_i\rangle$ defined by Fig. 4 (see Exercise 1 in the Appendix), and hence that each $|\phi_i\rangle$ is identical in form to the corresponding $|\psi_i\rangle$ when expressed in terms of its own standard basis. It follows from this that if Bob measures one or more of the same observables as Alice, he always obtains the same eigenvalues as she does for these common observables (see Exercise 2 in the Appendix), which is just the correlation rule. Equation (2) also explains the fact, mentioned at the end of Rule 1, that all four outcomes for each detector setting occur with the same probability (of $\frac{1}{4}$).

IV. CREDITS FOR THE DEMONSTRATION

The "magic square" of Fig. 4, which lies at the heart of the present demonstration, is due to Mermin[8] and Peres.[9] Mermin[8,10] used this array of observables to prove the Bell–Kochen–Specker (BKS) theorem,[11] a close relative of the more famous Bell's theorem. Peres[12] also used this array to give a related, but different, proof of the BKS theorem. The fact that the Mermin–Peres proof of the BKS theorem could be converted into a proof of Bell's theorem was pointed out by Cabello[13] and the author[14] who showed, in slightly different ways, how this could be done by distributing one member each of a pair of Bell states to two observers and having them carry out certain measurements. It is the author's version of this proof of Bell's theorem "without inequalities" that has been turned into the nontechnical demonstration presented here. This very brief survey of the literature makes no attempt at completeness but merely highlight the works that directly influenced this demonstration.

After an earlier version of this paper was posted on the eprint archive, Richard Cleve informed me that David Mermin and he had come up with a similar scheme in which each detector had only three switch settings. This is easily accomplished, within our framework, by allowing Alice to use only the row settings R1, R2, and R3 on her detector and Bob to use only the column settings C1, C2, and C3 on his. If, at the same time, a negative sign is affixed to the second and third observables in the last row of Fig. 4, Rule 1 can be restated in the form that Alice only observes an even number of red squares in any of the rows she activates and Bob an odd number of red squares in any of the columns he activates. Rule 2 is unchanged, and the impossibility of instruction sets follows from the same argument as before.

APPENDIX

Below are two exercises (and their solutions) that could aid the reader in understanding some of the points made in Sec. III.

Exercise 1: Calculate the simultaneous eigenstates of the commuting observables in each of the rows and columns of Fig. 4 and verify that they all have real expansion coefficients in terms of the standard basis for a pair of qubits.

Solution: A straightforward calculation shows that the eigenstates of the observables in the three rows (R1, R2, and R3) and three columns (C1, C2, and C3) of Fig. 4 are as follows:

R1:	(1,0,0,0)	(0,0,1,0)	(0,1,0,0)	(0,0,0,1)
R2:	(1,1,1,1)	(1,−1,1,−1)	(1,1,−1,−1)	(1,−1,−1,1)
R3:	(1,1,1,−1)	(1,−1,1,1)	(1,1,−1,1)	(1,−1,−1,−1)
C1:	(1,0,1,0)	(1,0,−1,0)	(0,1,0,1)	(0,1,0,−1)
C2:	(1,1,0,0)	(1,−1,0,0)	(0,0,1,1)	(0,0,1,−1)
C3:	(1,0,0,1)	(1,0,0,−1)	(0,1,1,0)	(0,−1,1,0)

The shorthand notation (a,b,c,d) has been used for the (unnormalized) state $a|0\rangle|0\rangle + b|0\rangle|1\rangle + c|1\rangle|0\rangle + d|1\rangle|1\rangle$ and the eigenstates in each row have been arranged so that they have the eigenvalues $(+1,+1)$, $(+1,-1)$, $(-1,+1)$, and $(-1,-1)$ with respect to the first two observables in the row or column that define them (the eigenvalue of the third observable can be inferred from those of the first two, and so is omitted). The reality of all the numbers in this table shows that the states $|\psi_i\rangle$ for Alice derived from these eigenstates (by taking $|0\rangle|0\rangle$ to be $|0\rangle_1|0\rangle_3$, etc.) are identical in form to the associated states $|\phi_i\rangle$ for Bob. This observation will prove of use in the next exercise.

Exercise 2: Suppose Alice carries out a measurement of the observables in the third row of Fig. 4 and obtains the eigenvalues +1, −1, −1 (in that order). (a) Show that if Bob carries out the same measurement as Alice, he obtains the same eigenvalues as she does. (b) Show that if Bob carries out a measurement of the observables in the third column of Fig. 4, he gets the same eigenvalue as Alice for the one observable they measure in common and also calculate the probabilities with which he obtains the various outcomes for the other two observables.

Solution: The state given in Eq. (1) can be expanded in the form of Eq. (2), with $|\psi_i\rangle$ and $|\phi_i\rangle$ $(i=1,...,4)$ both being given by the eigenstates in the third row (R3) of Fig. 4. A measurement by Alice of the observables in the third row that yields the eigenvalues +1, −1, −1 projects her qubits into the state (1,−1,1,1) and Bob's qubits into this same state as well. Thus, a measurement by Bob of the same observables as Alice leads to the same eigenvalues as she obtains, as was to be shown in part (a). To do part (b), note that the state Bob is left with after Alice's measurement, namely (1,−1,1,1), can be expressed as an equally weighted superposition (with coefficients of $1/\sqrt{2}$ each) of the states (1,0,0,1) and (0,−1,1,0). This shows that if Bob measures the observables $\sigma_z\sigma_z$, $\sigma_x\sigma_x$, $\sigma_y\sigma_y$ he obtains the eigenvalues +1, +1, −1 with a probability of $\frac{1}{2}$ and the eigenvalues −1, −1, −1 with a probability of $\frac{1}{2}$; in either case he obtains the same eigenvalue (namely, −1) for the single observable (namely, $\sigma_y\sigma_y$) that he measures in common with Alice.

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