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Abstract

When a magnetic field is applied to atoms, some of the atomic energy levels may be changed and some levels which had identical energies may be split into levels with different energies. This was discovered by Zeeman in 1896 and is called the Zeeman Effect.

In this experiment you will measure the energy level splitting and polarizations due to the Zeeman Effect and will apply the theory for the Zeeman effect to calculate the Bohr magneton. You will also study and use a Fabry-Perot Etalon.

You may perform this experiment before you discuss the full theory in your Quantum Mechanics class. For that reason, this description is more detailed than the descriptions of most other experiments in the Advanced Laboratory.

Theory

In 1896 Zeeman discovered a broadening effect in the yellow lines of sodium when the light source was placed in a strong magnetic field. Soon afterward, Lorentz predicted from classical electromagnetic theory that each line should split into 3 components.

We now understand the splitting of lines by a magnetic field in terms of the Dirac theory for the electron. If an atom has an eigenstate with the angular momenta and spins of all the electrons giving vector totals:

- (Σ angular momenta) has magnitude $L' = \sqrt{\ell'(\ell'+1)}\,\hbar$
- (Σ angular momenta) in a given direction $= m'_\ell \hbar \qquad -\ell \le m'_\ell \le \ell$
- (Σ intrinsic spins) has magnitude $S' = \sqrt{s'(s'+1)}\,\hbar$
- (Σ intrinsic spins) in a given direction $= m'_s \hbar$

(We use a notation with primes indicating totals for all electrons and lower case referring to quantum numbers.)

Then that state is split by the magnetic field B which causes an additional potential energy term:

$$\Delta E = -\vec{\mu} \cdot \vec{B}$$

where $\vec{\mu}$ is the effective magnetic moment of the state.

$$\begin{aligned} \Delta E &= -\left[\Sigma_i \left(-g_\ell \frac{e}{2m_e}\vec{L}_i\right) + \Sigma_i \left(-g_s \frac{e}{2m_e}\vec{S}_i\right)\right] \cdot \vec{B} \\ &= \frac{e}{2m_e}\left[g_\ell \vec{L}' + g_s \vec{S}'\right] \cdot \vec{B}. \end{aligned}$$

Here $\vec{L}_i$ and $\vec{S}_i$ are the angular momentum and spin of the ith electron. The charge and mass of an electron are designated by e and m_e respectively.

The factors g_ℓ and g_s from the Dirac and QED Theory are:

$$g_\ell = 1 \text{ exactly} \quad \text{and} \quad g_s = 2.00232 \quad .$$

Often the 2.00232 is approximated by 2 and the equation is written

$$\Delta E = \frac{\mu_B}{\hbar}\left[\vec{L}' + 2\vec{S}'\right] \cdot \vec{B} \quad ,$$

where $\mu_B = \frac{e\hbar}{2m_e}$ called the Bohr Magneton.

The total angular momentum

$$\vec{J'} = \vec{L'} + \vec{S'}$$

has a z component which is quantized into $(2j'+1)$ discrete values:

$$J'_z = m'_j \hbar \qquad m'_j = (-j'), (-j'+1) \ldots (j'-1), (j').$$

However the magnetic moment

$$\vec{\mu} = \frac{\mu_B}{\hbar} \left[\vec{L'} + 2\vec{S'} \right]$$

is not parallel to $\vec{J'}$ and so the z component of μ depends upon both $\vec{L'}$ and $\vec{S'}$.

The z component can be derived either easily by using the "vector model" which is semi-classical or by using full quantum mechanics with perturbation calculations. The student should read and understand the vector model method which is given in most standard texts.

Both methods give the z component of the magnetic moment as

$$\mu_z = \frac{\mu_B}{\hbar} [g \; m'_g \; \hbar] = g \; m'_j \; \mu_B$$

and $\Delta E = g \; m'_j \; \mu_B \; B$.

The g is called the Landé g factor. We would expect $g = 1$ for a simple orbital angular momentum but the intrinsic spin causes:

$$g = 1 + \frac{j'(j'+1) + s'(s'+1) - \ell'(\ell'+1)}{2j'(j'+1)} \quad .$$

The energy of the photons emitted then depends upon both the splitting of the initial state and the splitting of the final state. Further, there are restrictions upon the changes which can occur in the various quantum numbers.

Selection Rules

The most likely radiation to be emitted is that due to "electric dipole" transitions of single electrons.

(a) Induced Transitions

If an atom is placed in an oscillating electric field so that the electric potential V is a function of x and t; $V(x,t)$ then the transition rate from states n to state m can be shown to be

$$\frac{da_m}{dt} = \frac{-i}{\hbar} \sum_n a_n e^{-i(E_n - E_m)t/\hbar} \, V_{mn} \quad ,$$

where V_{mn} is the matrix element depending upon the electric potential V, and the wave functions of the initial and final states:

$$V_{mn} = \int \psi_m^* \, V(x,t) \psi_n \quad .$$

If

(i) the perturbation is small,

(ii) the electric field has only one frequency f with electric field $\vec{E}$,

(iii) the wave length of the electromagnetic field $\lambda = c/f$ is large compared with the diameter of the atom,

(iv) the energy difference $(E_m - E_n) = hf$,

(v) most of the atoms are in one state with energy E_n, then the transition rate can be shown to be:

$$\frac{d}{dt}(a_m^* a_m) = \frac{1}{\hbar^2} \mu_{mn}^* \mu_{mn} E^2 \quad ,$$

where μ_{mn} has been derived from V_{mn} by assuming that $V(x,t) = E\, ex$.

$$\mu_{mn} = \int \psi_m^* e \, x \psi_n \, dx \quad .$$

Hence an electric field with an electric vector in the x direction will cause transitions from state n to state m only if the integral is non zero. Remember that if $m \neq n$, then the states are orthogonal and so:

$$\int \psi_m^* \psi_n = 0 \quad .$$

The integral we are interested in now is different because it includes the coordinate x:

$$\int \psi_m^* x \psi_n = ?$$

but is also often zero. The non-zero conditions depend upon the quantum numbers of the initial and final states and can be derived by integrating the initial and final Associated Legendre functions of $\cos\theta$ and the Associated Laguerre functions of the radius.

The transitions occur only if <u>all</u> of the following are true:

(i) $\Delta\ell = \pm 1$ for the electron making the transition. This means that the parity of the electron wavefunction must change.

A change in ℓ is necessary (otherwise $\int \psi_m^* x \psi_n = 0$) but changes in ℓ greater than ± 1 require the electric field to be nonuniform within the atom and so are very unlikely if the wavelength > atomic diameter.

(ii) $\Delta S' = 0$ for the whole atom. The intrinsic spin and magnetic moment of an electron interacts only with a magnetic field gradient (as in the Stern Gerlach Expt). We are considering electromagnetic radiation with long wavelengths and so the gradient over the atom will be very small.

These unlikely transitions which change S would be called "spin-flip" transitions.

(iii) $\Delta\ell' = 0$ or ± 1 for the whole atom. Changes of ℓ can occur in the separate electron wavefunctions without causing the total ℓ to change. However, the total ℓ cannot change more than the ℓ of the electron making the transition.

(iv) $\Delta j' = 0$ or ± 1 for the whole atom (but $j' = 0$ to $j' = 0$ is not allowed). The total angular momentum cannot change by more than 1 since the spin of a photon is 1 and angular momentum must be conserved.

(v) $\Delta m_j' = 0$ or ± 1 for the whole atom (but if $\Delta j = 0$ then $m_j = 0$ to $m_j = 0$ is not allowed).

(b) <u>Spontaneous Transitions</u>

The laws of Relativistic Quantum Mechanics predict that a state will decay to a lower state only in the presence of an external alternating electromagnetic field. Hence Relativistic Quantum Mechanics will <u>not</u> allow "spontaneous transitions."

However, the theory of Quantum Electrodynamics (QED) goes further by allowing for quantization of the electric field and predicts that a state may decay due to quantum fluctuations in its own electric field. These fluctuations arise from the "zero-point energy" of the electromagnetic field which is similar to the "zero-point energy" of a particle in the lowest energy level of a potential well. The fluctuations can be regarded as "virtual photons" being continuously emitted and absorbed. The fluctuations are of all frequencies and can cause spontaneous transitions in the same way as external fields induce transitions.

Although the transition rates can only be calculated from QED, the selection rules for spontaneous transitions are identical to those of induced transitions since the rules are derived from the same integrals of the initial and final wave functions.

Polarization of Radiation

The polarization of the emitted light depends upon the change Δm_j. Assume the magnetic field B is along the z axis. Here are three explanations of the same phenomena:

Explanation 1

The same theory which gave the Δm restrictions also makes predictions for the electric field.

(a) If $\Delta m'_j = 0$, then the alternating $\vec{E}$ is parallel to the magnetic field. This light is then linearly polarized and mostly emitted perpendicularly to the magnetic field. An observer in the x direction would see $\vec{E} \parallel \vec{z}$, an observer in the z axis would see no radiation.

(b) If $\Delta m'_j = \pm 1$, then the alternating $\vec{E}$ is perpendicular to the magnetic field (z axis).

- An observer on the x axis will see $\vec{E} \parallel \vec{y}$
- An observer on the z axis will see $\vec{E}$ in both $\vec{y}$ and $\vec{x}$ directions A careful analysis of the integrals shows that the observer on the z axis will see:

right hand circular polarization if $\Delta m'_j = +1$ and
left hand circular polarization if $\Delta m'_j = -1$.

Explanation 2

This can be partly understood by remembering that photons have a spin angular momentum of one unit of $\hbar$ and that angular momentum must be conserved.

(a) If $\Delta m'_j = 0$, then no angular momentum about the z axis can be carried off by a photon and so an observer on the z axis will see no light.

An observer on the x axis will see photons due to $\Delta m'_j = 0$ and $\Delta m'_j = \pm 1$. The linearly polarized light is equivalent to a superposition of equal amplitudes of oppositely circularly polarized light.

(b) If $\Delta m'_j = +1$, then the angular momentum about the z axis must be conserved by the photon taking off with an $s = -1$. Hence an observer on the z axis will see 100% right hand circularly polarized light.

Similarly with $\Delta m'_j = -1$, the photons will have an $s = +1$ and an observer on the z axis will see 100% left hand circularly polarized light.

Explanation 3

From G.R. Fowles "Introduction to Modern Optics":

Coherent States Consider a system that is in the process of changing from one eigenstate Ψ_1 to another Ψ_2. During the transition the state function is given by a linear combination of the two state functions involved, namely

$$\Psi = c_1\psi_1 e^{-iE_1 t/\pi} + c_2\psi_2 e^{-iE_2 t/\pi} \tag{1.1}$$

Here c_1 and c_2 are parameters whose variation with time is slow in comparison with that of the exponential factors. A state of the above type is known as a *coherent state*. One essential difference between a coherent state and a stationary state is that the energy of a coherent state is not well defined, whereas that of a stationary state is.

The probability distribution of the coherent state represented by Eq. 1.1 is given by the following expression:

$$\Psi^*\Psi = {c_1}^* c_1 {\psi_1}^* \psi_1 + {c_2}^* c_2 {\psi_2}^* \psi_2 + {c_1}^* c_2 {\psi_1}^* \psi_2 e^{i\omega t} + {c_2}^* c_1 {\psi_2}^* \psi_1 e^{-i\omega t} \tag{1.2}$$

where

$$\omega = \frac{E_1 - E_2}{\hbar} \tag{1.3}$$

or, equivalently

$$f = \frac{E_1 - E_2}{h}$$

The above result shows that *the probability density of a coherent state undergoes a sinusoidal oscillation with time. The frequency of this oscillation is precisely that given by the Bohr frequency condition.*

The quantum-mechanical description of a radiating atom may be stated as follows. During the change from one quantum state to another, the probability distribution of the electron becomes coherent and oscillates sinusoidally. This sinusoidal oscillation is accompanied by an oscillating electromagnetic field that constitutes the radiation.

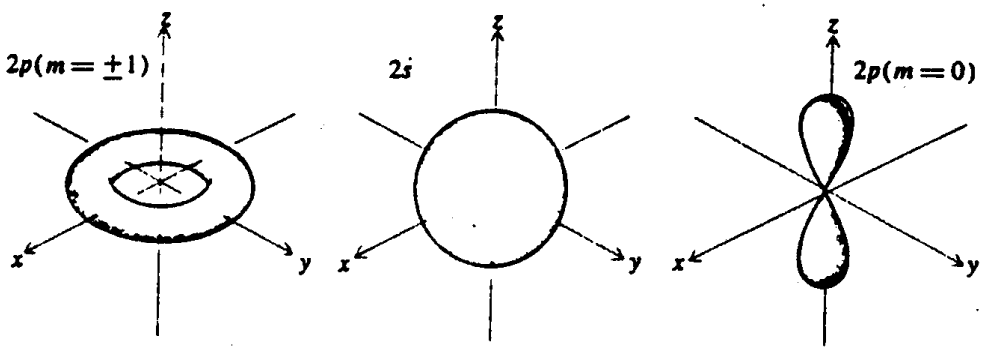


Fig. 1.1 Probability density for the first excited state ($n = 2$) of the hydrogen atom.

Radiative Transitions and Selection Rules

When an atom is in the process of changing from one eigenstate to another, the probability density of the electronic charge becomes coherent and oscillates sinusoidally with a frequency given by the Bohr frequency condition. The way in which the charge cloud oscillates depends on the particular eigenstates involved. In the case of a so-called *dipole transition*, the centroid of the negative charge of the electron cloud oscillates about the positively charged nucleus. The atom thereby becomes an *oscillating electric dipole*.

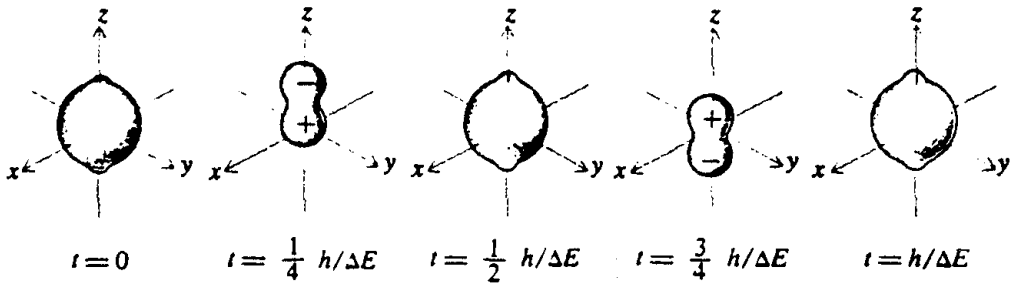


Fig. 1.2 Charge distribution in the coherent state $1s + 2p_0$ as a function of time. The atom is an oscillating dipole.

Figure 1.2 is a diagram showing the time variation of the charge distribution for the hydrogen atom when it is in the coherent state represented by the combination $1s + 2p(m = 0)$. It is seen that the centroid of the charge moves back and forth along the z axis. The associated electromagnetic field has a directional distribution that is the same as that of a simple dipole antenna lying along the z axis. Thus, the radiation is maximum in the xy plane and zero along the z axis. The radiation field, in this case, is linearly polarized with its plane of polarization parallel to the dipole axis.

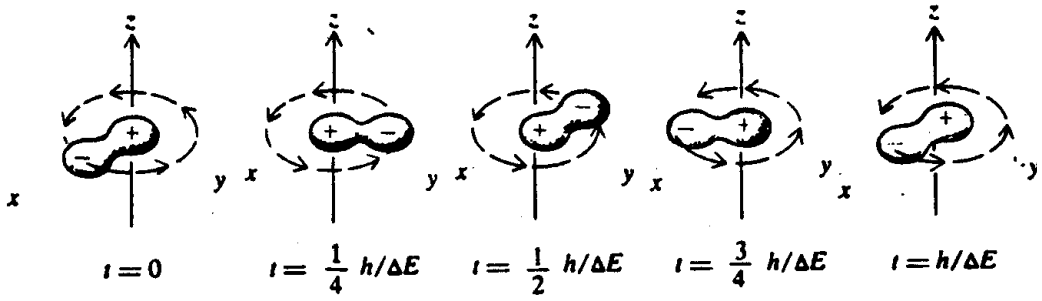


Fig. 1.3 Charge distribution in the coherent state $1s + 2p_1$ as a function of time. The atom is a rotating dipole.

A different case is shown in Figure 1.3. Here the coherent state is the combination $1s + 2p(m = +1)$. The centroid of the electronic charge now moves in the circular path around the z axis. The angular frequency of the motion is also that given by the Bohr frequency formula $\omega = \Delta E/\hbar$.

Instead of an oscillating dipole, the atom is now a *rotating dipole*. The associated radiation field is such that the polarization is circular for radiation traveling the direction of the z axis and linear for radiation traveling in a direction perpendicular to the z axis. For intermediate directions, the polarization is elliptical. The cases are illustrated in Figure 1.4. The coherent state $1s + 2p(m = -1)$ is just the same as the state $1s + 2p(m = +1)$ except that the direction of rotation of the electronic charge is reversed. Consequently, the sense of rotation of the associated circularly polarized radiation is also reversed.

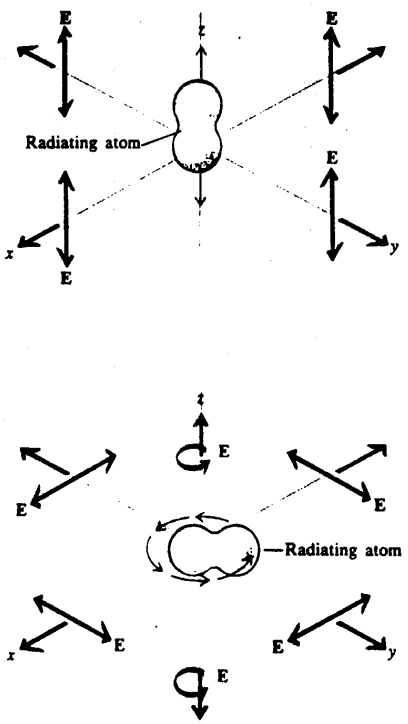


Fig. 1.4 Polarization of the associated electromagnetic radiation (**E** vector) for (a) an oscillating dipole and (b) a rotating dipole.

The Spectra

We will study the 546.1 nm green line of mercury. Each atom has 80 electrons and in the ground state these have the distribution:

n = 1	shell	filled		2
n = 2	shell	filled		8
n = 3	shell	filled		18
n = 4	shell	filled		32
n = 5	shell	$\ell = 0$	subshell	2
		$\ell = 1$	subshell	6
		$\ell = 2$	subshell	10
		$\ell = 3$	subshell	empty
		$\ell = 4$	subshell	empty
n = 6	shell	$\ell = 0$	subshell	2

The excited states have one electron lifted from the n=6, ℓ=0 subshell and are shown in the following diagram from Herzberg.

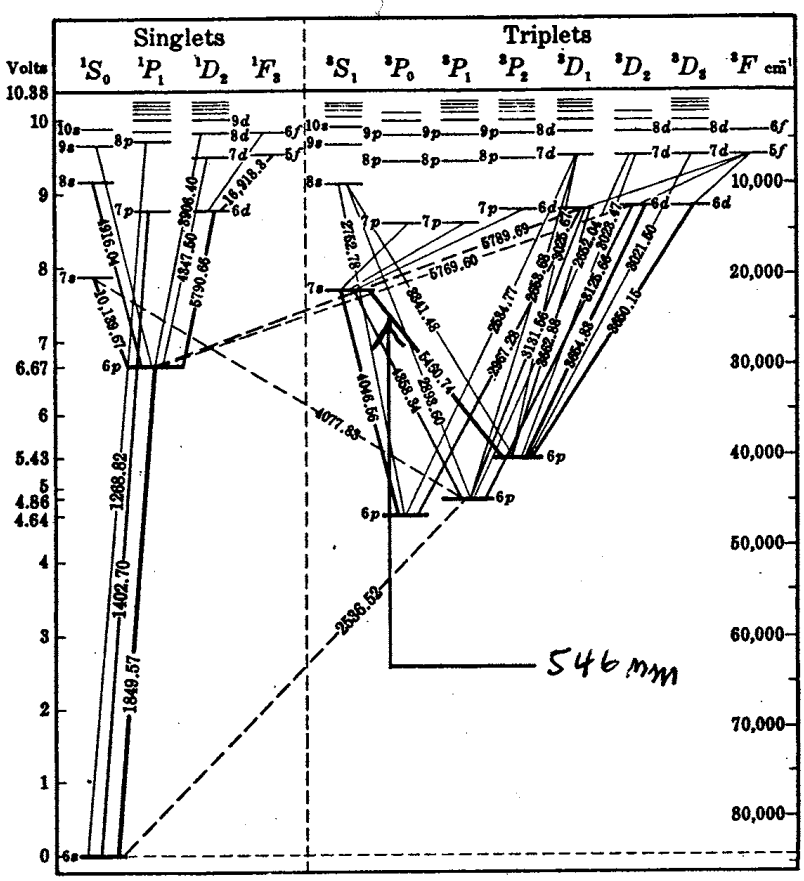


Fig. 2.1 Energy Level Diagram of Hg I [Grotrian (8)]. The wavelengths of the more intense Hg Lines are given (cf. Fig. 6, p. 6). The symbols $6p$, etc., written near each level, indicate the true principal quantum number and the ℓ value of the emission electron.

The levels are labelled by the s', ℓ', and j' of the total atom. The full shells and subshells have zero contributions and so the net s', ℓ' and j' are due to one electron left in the n = 6, $\ell = 0$ subshell and to the excited electron.

Remember the traditional spectroscopic notation:

- (superscript $= 2s' + 1$)
- (S, P, D, F etc. for $L = 0$, 1, 2, 3, etc.)
- (Subscript $= j'$)

The notation sometimes includes the n and ℓ of the excited electron. For example:

- $7s\ ^3S_1$ has the excited electron with n = 7, $\ell = 0$ with the totals $s' = 1$, $\ell' = 0$, $j' = 1$.

Notice that most transitions have no change in s'. The levels with $s' = 0$ are called singlets (no spin-orbit splitting) and the levels with $s' = 1$ are called triplets (spin-orbit interactions give 3 close levels).

We will study the bright 546.1 nm green line which is easily distinguished from the fainter 579.0 and 577.0 nm yellow and the bright 435.8 nm blue. The 546.1 nm is between two states with different Landé g factors:

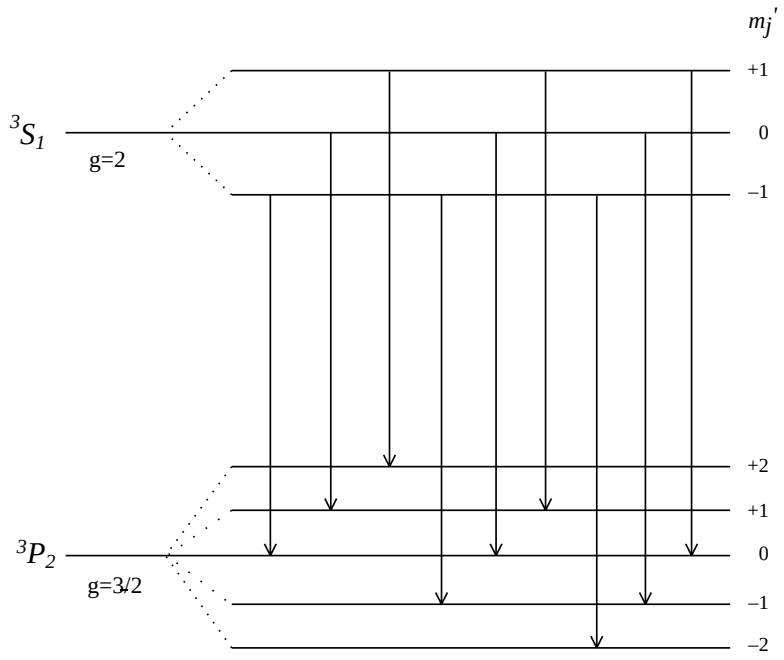
- $7s\ ^3S_1 \quad s' = 1 \quad \ell' = 0 \quad j' = 1 \quad g = 2 \quad$ 3 values of m'_j
- $6p\ ^3P_2 \quad s' = 1 \quad \ell' = 1 \quad j' = 2 \quad g = 3/2$ 5 values of m'_j

The restriction that $\Delta m'_j = 0, \pm 1$ gives nine transitions which are split by the Zeeman Effect. Let $\delta = (g\ m'_j)_{\text{initial}} - (g\ m'_j)_{\text{final}}$.

The two unequal g factors cause the 9 possible transitions to have 9 different wave numbers $\nu \equiv \frac{1}{\lambda} = \Delta E/hc$:

$$\nu = \nu_0 + \delta \frac{\mu_B B}{hc} \quad . \tag{2.1}$$

δ	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
initial m'_j	-1	0	$+1$	-1	0	$+1$	-1	0	$+1$
final m'_j	0	$+1$	$+2$	-1	0	$+1$	-2	-1	0
$\Delta m'_j$	$+1$	$+1$	$+1$	0	0	0	-1	-1	-1
Not seen on axis				π	π	π			
Seen on axis as circularly polarized	σ	σ	σ				σ	σ	σ



Modified Section from Melissinos:

Reduction of the Data Obtained from a Fabry-Perot

Constructive interference occurs for a Fabry-Perot etalon of spacing t when:

$$n\lambda = 2t\cos\theta \ . \tag{3.1}$$

When parallel rays of angle θ are brought to a focus by the use of a lens of focal length f and Eq. 3.1 is satisfied, bright rings will appear in the focal plane. Their radius given by:

$$r = f\tan\theta \simeq f\theta \ . \tag{3.2}$$

Thus the interference rings formed in the focal plane have radii:

$$r_n = f\theta_n \tag{3.3}$$

where μ is the index of refraction and the angle θ_n is given by:

$$n = \frac{2\mu t}{\lambda}\cos\theta_n = n_0\cos\theta_n = n_0\left(1 - 2\sin^2\frac{\theta_n}{2}\right) \ . \tag{3.4}$$

Since θ is always small, we obtain:

$$n = n_0\left(1 - \frac{\theta_n^2}{2}\right) \quad \text{or} \quad \theta_n = \sqrt{\frac{2(n_0 - n)}{n_0}} \ . \tag{3.5}$$

Now if θ_n is to correspond to a bright fringe, n must be an integer; however, n_0, which gives the interference at the center ($\cos\theta = 1$ or $\theta = 0$ in Eq. 1.1), is in general *not* an integer:

$$n_0 = \frac{2\mu t}{\lambda} \ . \tag{3.6}$$

(There is no bright spot in the center of the pattern, in general.) If n_1 is the interference order of the first ring, clearly $n_1 < n_0$ since $n_1 = n_0\cos\theta_1$. We then let $n_1 = n_0 - \epsilon$, with $0 < \epsilon < 1$ where n_1 is the closest integer to n_0 (smaller than n_0). Thus, we have in general for the pth ring of the pattern, as measured from the center out,

$$n_p = (n_0 - \epsilon) - (p - 1) \ . \tag{3.7}$$

Combining Eq. 3.7 with Eqs. 3.5 and 3.3, we obtain for the radii of the rings

$$r_p = \sqrt{\frac{2f^2}{n_0}}\sqrt{(p-1)+\epsilon}\ . \tag{3.8}$$

We note (a) that the difference between the squares of the radii of adjacent rings is a constant

$$r_{p+1}^2 - r_p^2 = \frac{2f^2}{n_0} \tag{3.9}$$

and (b) that the fraction of an order ϵ can be found by extrapolating to $r_p^2 = 0$ (according to the slope $2f^2/n_0$)*

Now,† if there are two components of a spectral line with wavelengths λ_1 and λ_2, very close to one another, they will have fractional orders at the center ϵ_1 and ϵ_2:

$$\begin{aligned}\epsilon_1 &= \frac{2t}{\lambda_1} - n_1(1) = 2t\bar{\nu}_1 - n_1(1)\\ \epsilon_2 &= \frac{2t}{\lambda_2} - n_1(2) = 2t\bar{\nu}_2 - n_1(2)\end{aligned}$$

where $n_1(1)$, $n_1(2)$ is the order of the first ring. Hence, if the rings do not overlap by a whole order ($n_1(1) = n_1(2)$), the difference in wave numbers between the two components is simply

$$\bar{\nu}_1 - \bar{\nu}_2 = \frac{\epsilon_1 - \epsilon_2}{2t}\ . \tag{3.10}$$

If the orders are overlapped x times,

$$\bar{\nu}_1 - \bar{\nu}_2 = \frac{x + \epsilon_1 - \epsilon_2}{2t}\ . \tag{3.11}$$

From Eq. 3.10 we see that we do not need to know t much more accurately than $\epsilon_1 - \epsilon_2$. The fractional order $\epsilon_1, \epsilon_2, \ldots$, can hardly be measured to 1/1000; therefore knowledge of t to this accuracy of 1/1000 is amply adequate; this can be easily achieved with a micrometer or a microscope.

The resolution of the Fabry-Perot etalon can be obtained from Eq. 3.1:

$$\bar{\nu} = \frac{1}{\lambda} = \frac{n}{2t\cos\theta}$$

*See figure on page 18.

†From here on we set $\mu = 1$.

and by differentiation

$$\Delta\bar{\nu} = \frac{\Delta n}{2t}\left[\frac{1}{\cos\theta} - \frac{n\sin\theta}{\cos^2\theta}\right] \ . \tag{3.12}$$

Since θ is always small

$$\Delta\bar{\nu} \simeq \frac{1}{2t}\Delta n \tag{3.13}$$

where Δn is the *fraction of order* by which one ring pattern is shifted with respect to another. (Note that when this fraction of order is measured at the center, Eq. 3.13 becomes exact, and since $\Delta n(\theta = 0) \equiv \epsilon_1 - \epsilon_2$ we get back Eq. 3.10)

The fraction of order Δn that can be measured experimentally depends on the quality of the plates, on the proper alignment and *focusing* of the optical system, and on the width and relative intensity of the components that are being measured. Values of $\Delta n \approx 1/100$ are common, and with some care this value can be exceeded.

Using $\Delta n = 1/100$, we then find for the resolving power of the Fabry-Perot at a wavelength of $\lambda = 5000$ Å and a spacing $t = 5.0$ mm

$$\frac{\Delta\bar{\nu}}{\bar{\nu}} = \Delta n\left(\frac{\lambda}{2t}\right) = 10^{-2} \times 5 \times 10^{-5} = 5 \times 10^{-7} \tag{3.14}$$

which is quite satisfactory.

In reducing the data our aim is to obtain the orders of fractional interference $\epsilon_1, \epsilon_2, \ldots$, for all the components of the line, and also to know if any of the comonents overlap in order, and in that case by how many orders.

Let R_p be the radius of the pth ring as measured on the photographic plate. Note that it is possible to measure R_p only if the center of the pattern is included on the plate. From Eq. 3.8 it follows that:

$$R_{p+1}^2 = (R_{p+1}^2 - R_p^2) \times (p + \epsilon) = \frac{2f^2}{n_0} \times (p + \epsilon) \tag{3.15}$$

and taking into account the magnification m of the camera we have:

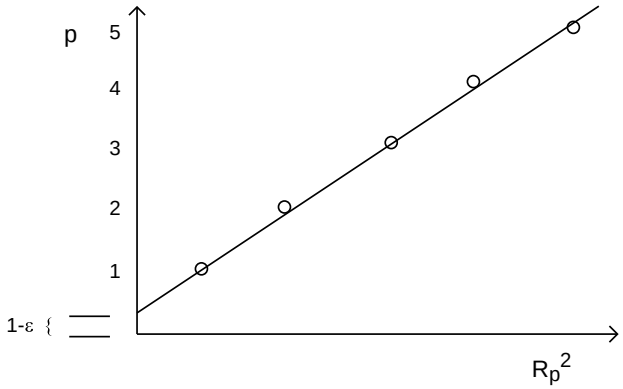
$$R_p^2 = \frac{2f^2}{n_0}[p - (1 - \epsilon)]m^2 \tag{3.16}$$

and

$$p = \left(\frac{n_o}{2f^2m^2}\right)R_p^2 + (1 - \epsilon) \ . \tag{3.17}$$

We note that since $n_0/2f^2m^2$ is a constant, any adjacent pair of rings can yield a value for ϵ. However, since the squares of the radii of successive rings are linearly related (they form an arithmetic progression), in order to utilize all available information a least squares fit to Eq. 3.17 will considerably improve the accuracy in the determination of ϵ.

Alternately, a plot of the integer p against R_p^2 gives an intercept of $(1-\epsilon)$.



Apparatus

The equipment is similar to that described in Melissinos, page 320.

- Discharge tube. The tube contains a low pressure of the mercury isotope Hg^{198} and 3 mTorr of Argon buffer gas. The spectral lines are sharp because:

  (a) the nuclear spin I of Hg^{198} is zero and so there is no hyperfine structure.

  (b) All of the atoms have the same mass (unlike those of natural mercury) and so all the atoms will give the same spectra.

  (c) The discharge is maintained by RF and the tube is cool so the Doppler broadening is less than in an arc discharge.

  The tube is expensive and shockmounted. Do not attempt to move the tube.

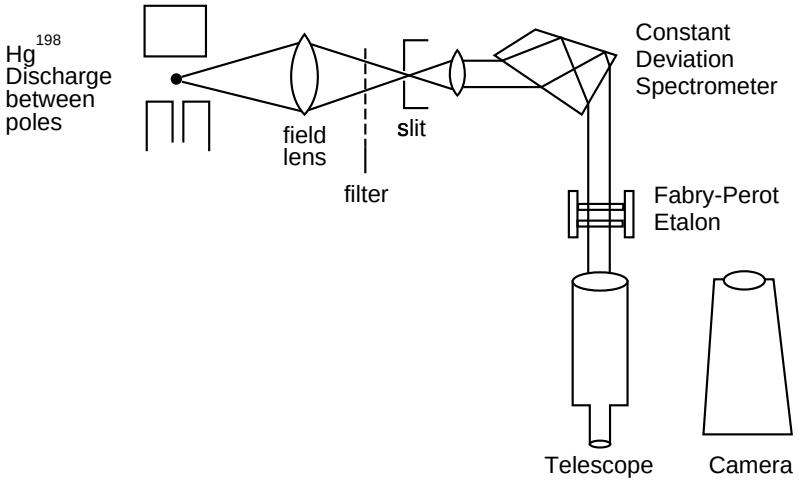
- Tesla Coil. The Tesla Coil should be used only for ionizing the Hg so that the RF can maintain the discharge. This is done by moving the tesla coil near the glass Hg tube and then turning the tesla coil on for a moment. The tesla coil is a high voltage transformer and the tip is at high voltage so do not place the tesla coil near the Hall probe or the Hall probe circuitry may be damaged. Do not touch the tip.

- Tuned RF Oscillator. The oscillator has a frequency of about 100 MHz. It is used to drive the Hg discharge tube. If the discharge cannot be maintained or occupies less than 2/3 of the tube, ask the instructor to check the oscillator tuning.

- Magnet. The magnet supply is stabilized. The control must be turned to zero (anti-clockwise) before the supply is turned on or off. The magnet is mounted on a ball bearing turntable so that it may be rotated to view the discharge tube via a hole in one pole face.

- Hall Probe. The Hall probe is fragile and should be handled with care. The probe is used to measure the magnetic field. The RF oscillator should be turned off during these measurements.

- Lens. The lens focuses the light onto the slit of the constant deviation spectrometer. A linear polarizing or circularly polarizing filter may be placed before or after the lens.



- Constant Deviation Spectrometer. The spectrometer contains a lens system to make a virtual image of the slit at infinity and a Pellin-Broca prism. The prism provides dispersion to separate the spectral lines and deflects the beam through exactly 90°. The wavelength is chosen by rotating the prism.

- Fabry-Perot Etalon. The parallel light from the spectrometer is sent through the Fabry-Perot etalon. The gap width is 4.600 ± 0.005 mm. The Etalon should be covered to provide some protection from air currents and to keep the refractive index of its air gap constant. *The etalon looks small and cheap - it is delicate and expensive.*

- Viewing Telescope. The telescope is used for visually inspecting the spectra. It can be dismounted by unscrewing the clamp screws.

- Camera. The camera is a Santa Barbara Instrument Group Model ST-7 digital camera with 765×510 pixels of size 9×9 μ forming a 6.9×4.6 mm array. The camera lens has a 130 mm focal length and an aperture variable to f/2.8. The camera is linked to a PC running the CCDOPS software which controls the camera.

- Mercury lamp with green filter and diffuser. This lamp is used to check the Fabry-Perot pattern.

- Polarizers. Linear polarizers and a quarter wave plate are used to check polarizations.

Procedure

1. Understand the theory and check the g factors for $^3S_1$ and $^3P_2$ levels.
2. Read the section on apparatus and note the precautions. Read the section on the Fabry-Perot etalon.
3. Start the Mercury Light Source. Be sure to wear the eye protection goggles. The tube emits a considerable amount of UV.
   (a) Turn on the radio frequency (RF) coil power supply.
   (b) Bring the tesla coil near the discharge tube. Turn the tesla coil on for a few seconds and then off.
   (c) The tube of low pressure Hg^{198} should glow a whitish blue. If the glow is weak, ask the instructor to adjust the tuning of the RF system.
4. Align the source and Spectrometer.

(a) The magnet has two viewing conditions: perpendicular and parallel to the magnetic field. The configuration parallel to the magnetic field requires a rotation of the magnet so that light from the source passes through the axial hole in the pole piece. Be careful with the oscillator connecting wires.

(b) Adjust the lens position so that the source is imaged onto the collimator slit.

(c) Gently rotate the magnet and check that light emitted from the source continues to be focussed on the slit.

(d) Adjust the collimator slit width such that the entire source image just passes through the slit.

(e) Mount the viewing telescope. At this point, the Faby-Perot etalon is not yet in the optical path.

(f) After focussing the telescope, rotate the constant deviation dial to center the green spectral line.(This is the Mercury 546.1 nm green line. In the vicinity of this line, a yellow line should be observed.)

5. Check of the Fabry-Perot Etalon.

   (a) Install the Fabry-Perot Etalon on the spectrometer platform. (Consult with the instructor for doing this.)

   (b) With the magnet off, look through the telescope for the ring pattern. The pattern should be sharp. There are two adjustments on the Fabry-Perot platform: a screw underneath the platform to make vertical adjustments, and a horizontal screw on the side to adjust the left-right positioning of the etalon. You will have to adjust these screws to center the Fabry-Perot image.

6. Setting up the magnet

   (a) Turn on the magnet and increase the current until the split lines in different orders no longer overlap.

   (b) Monitor and record the magnet voltage with a multimeter since the ammeter is not accurate enough to reproduce settings.

7. Test the polarization of the spectra

(a) Inspect the light emitted perpendicularly to the magnetic field. Increase the field until the splitting of the rings almost causes them to overlap. You should see each original line split into 9 lines.

(b) Insert a linear polarizing filter. Watch the intensity as the filter is rotated. The outer six lines have $\Delta m'_j = \pm 1$ and so have E vertical when seen perpendicularly to the field, while the inner 3 lines have $\Delta m'_j = 0$ and so have E horizontal (parallel to the field).

(c) Now insert a the quarter wave plate and check for circular polarization. The spectra should show no signs of circular polarization.

(d) Now gently rotate the magnet and look down the hole in the axis of the magnet. The inner 3 lines ($\Delta m'_j = 0$) should be missing. The outer 6 lines ($\Delta m'_j = \pm 1$) should be tested with polarizers to verify that 3 have left-handed circular polarization and 3 have right (see page 14).

8. Photograph the spectra

(a) Remove the telescope from its mount.

(b) Set up and align the camera close to the Fabry-Perot etalon. (There is no way to do this except by eye.)

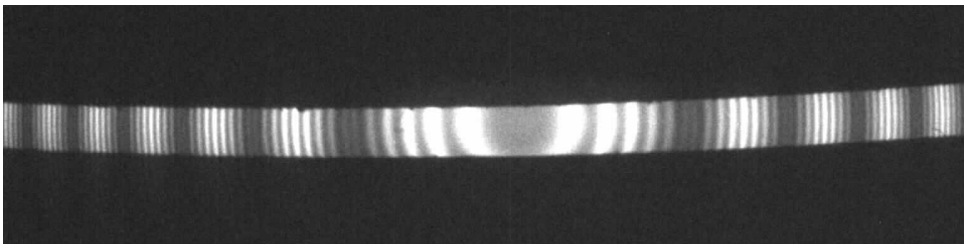
(c) Since the image is at infinity, set the camera focus to infinity. Using the **Focus** command, center the image and determine the optimum focus and exposure time. The **Focus** command takes images and displays them one after another. Subsequently, the **Grab** command is used to take a single image.

(d) Align the camera and adjust the Fabry-Perot so that you see the green ring pattern with an equal number of orders on each side of the center. The ring pattern should be centered in the field of view. (The room lights need to be turned off.)

(e) Adjust the constant deviation prism slightly to make the ring pattern vertical when one looks through the camera.

(f) Take at least 1 exposure with no field and a series of exposures with transverse field, no filters. Below is a sample image taken

with 3.0 A in the magnet, 0.12 sec exposure and aperture f/11. The focus is set to infinity.



(g) Make sure to measure the magnetic field with the Hall probe before the end of the session. The instructor will help you remove the discharge tube.

9. Measurements

   (a) Measure the radii of each of the split rings using the **Cross Hairs** option of the software. You can also **Zoom** the image. It is of course best to measure the diameter directly as opposed to estimating the location of the center and measuring the radius. Any scale can be used. (Why?)

   (b) You should now have a set of radii for each of the 9 values of δ $(-2, -1.5, \ldots + 1.5, +2)$ (see page 14). In each set, label the radii with $p = 1, 2, 3 \ldots$

   (c) Tabulate the squares of the radii for all 9 sets.

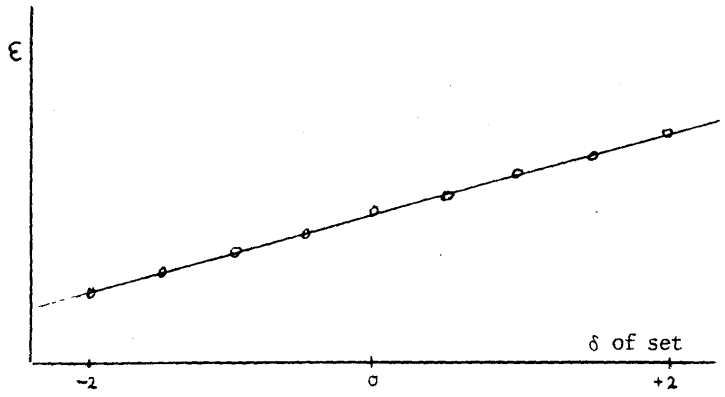
10. Analyze the data

   (a) Plot the ring number p(y)(as measured from the center out) against R_p^2(x) for the set of rings with $\delta = 0$ (no field) as on page 18 and fit to a straight line.

   (b) Measure the intercept $(1 - \epsilon)$ on the p axis. The quantity ϵ is the fractional order for that set of rings and $\delta = 0$.

   (c) Repeat a) and b) for the other 8 sets with δ nonzero (field on).

   (d) Plot the fractional order ϵ(y) obtained for each of the 9 sets against the value of δ(x). Because the g values are 3/2 and 2, the splitting and ϵ are equally spaced.



(e) Fit a straight line through the various fractional orders ϵ. The slope of this line is proportional to the product of the electron magnetic moment times the magnetic field. For example, the expression for the wave numbers for the two outer rings of the nine is: (See Eq. 2.1)

$$\nu = \nu_0 - 2\frac{\mu_B B}{hc} \qquad \Delta m_j' = +1,\ \sigma \text{ transition}$$

$$\nu = \nu_0 + 2\frac{\mu_B B}{hc} \qquad \Delta m_j' = -1,\ \sigma \text{ transition}$$

(f) Derive an expression for μ_BB in terms of the measured slope and the etalon spacing t.

(g) Now use the results from above to calculate the ratio (e/m_e).

$$\frac{e}{m_e} = \frac{2\mu_B}{\hbar}$$

(h) Also calculate the Bohr Magneton.

$$\mu_B = \frac{e\hbar}{2m_e}$$

The best value for μ_B is

$$\begin{aligned} \mu_B = \frac{e\hbar}{2m_e} &= (9.274078 \pm 0.000036) \times 10^{-24} \text{ Joule/Tesla} \\ &= (5.788378 \pm 0.000009) \times 10^{-9} \text{ eV/Gauss.} \end{aligned}$$

(i) Be prepared to answer questions on all aspects of the experiment.

References

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