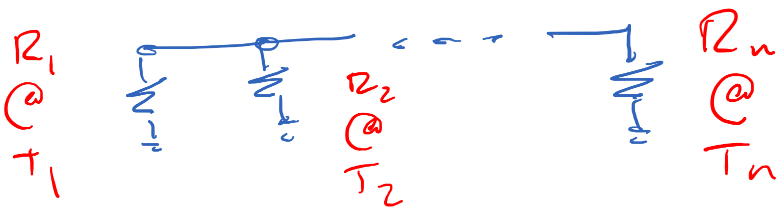
PS 5 SOLUTIONS

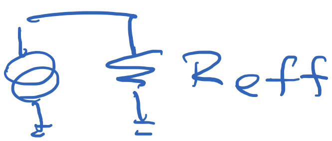
(1)



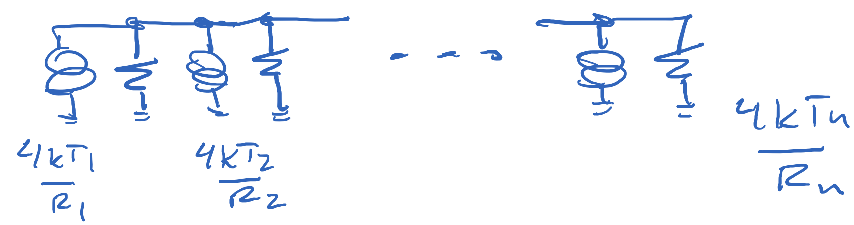
$$R_{eff} = \left[\sum_i \frac{1}{R_i} \right]^{-1}$$, AS USUAL

TO DETERMINE T_{eff}, CONSIDER

$$S_I' = \frac{4kT_{eff}}{R_{eff}}$$

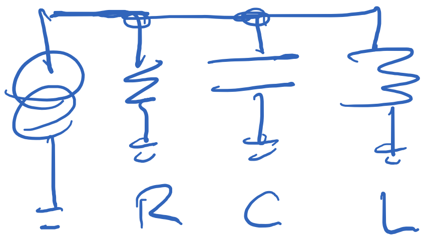


$\Longleftrightarrow$



$$\therefore \quad \frac{T_{eff}}{R_{eff}} = \sum_i \frac{T_i}{R_i} \implies T_{eff} = \left[\sum_i \frac{T_i}{R_i} \right] \left[\sum_i \frac{1}{R_i} \right]^{-1}$$

②



$$S_I(\omega) = \frac{2kT}{\pi R}$$

NOTE: I'M USING ANGULAR FREQUENCY

LET $\omega = \omega_0 + \Delta$; $\omega_0 = \frac{1}{\sqrt{LC}}$

USE $Z_0 \equiv \frac{1}{\omega_0 C} = \omega_0 L$

$Q = R/Z_0$

TAKE $Q \gg 1$

DEFINE $X \equiv \omega C - \frac{1}{\omega L}$

IN HIGH-Q LIMIT, $X \approx \frac{2}{Z_0} \Delta/\omega_0$

ADMITTANCE $Y(\omega) = \frac{1}{R} + jX$

IMPEDANCE $$Z(\omega) = \frac{\frac{1}{R} - jX}{\frac{1}{R^2} + X^2}$$

OVER $\longrightarrow$

IN HIGH-Q LIMIT, FIND

$$Z(\omega) = Q Z_0 \frac{1 - j\, 2Q\left(\Delta/\omega_0\right)}{1 + 4Q^2\left(\Delta/\omega_0\right)^2}$$

$$|Z(\omega)|^2 = \frac{Q^2 Z_0^2}{1 + 4Q^2\left(\Delta/\omega_0\right)^2}$$

SPECTRAL DENSITY OF VOLTAGE NOISE ACROSS THE TANK CIRCUIT IS THEREFORE

$$S_V'(\Delta) = \frac{2}{\pi} \frac{k_B T}{R} \frac{Q^2 Z_0^2}{1 + 4Q^2\left(\Delta/\omega_0\right)^2}$$

$\longrightarrow$

$$\langle v^2 \rangle = \frac{2}{\pi}\frac{k_B T}{R} Q^2 Z_0^2 \underbrace{\int_{-\infty}^{\infty} d\Delta \frac{1}{\left(1+4Q^2\left(\frac{\Delta}{\omega_0}\right)^2\right)^2}}_{\text{w/ TRIG. SUBSTITUTION, FIND } \int d\Delta = \frac{\pi}{2}\frac{\omega_0}{Q}}$$

$$\therefore \langle v^2 \rangle = \cancel{\frac{2}{\pi}} \frac{k_B T}{\cancel{R}} \cancel{Q}^{\cancel{2}} Z_0^{\cancel{2}} \cdot \cancel{\frac{\pi}{2}} \frac{\omega_0}{\cancel{Q}} = k_B T\, \omega_0 Z_0$$

$$\therefore \langle v^2 \rangle = \frac{k_B T}{C}$$

$$\left\{ \text{EQUIPARTITION SAYS } \frac{1}{2} C \langle v^2 \rangle = \frac{1}{2} k_B T \right\}$$

③ FROM AD797 DATASHEET:

TAKE $e_n = 1.0 \text{ uV}/\sqrt{Hz}$
$\quad i_n = 2.0 \text{ pA}/\sqrt{Hz}$ $\Longrightarrow R_N = \frac{1.0 \text{nV}}{2.0 \text{pA}} = \boxed{500\,\Omega}$

@ $R_S = 500\,\Omega$, WE FIND

$$T_N = \frac{e_n^2 + i_n^2 R_N^2}{4 k_B R_N} = \boxed{72\text{K}}$$

$$NF = 10 \log_{10}\left(\frac{372\text{K}}{300\text{K}}\right) = \boxed{0.93 \text{ dB}}$$