

# Physics 623

## Johnson Noise

April 2021

*Answer the questions 1 through 14 in this writeup **before** coming to the lab. They will be collected at the **beginning** of the lab. **Read** the sections of the text referenced here, if you haven't already.*

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### 1 Purpose

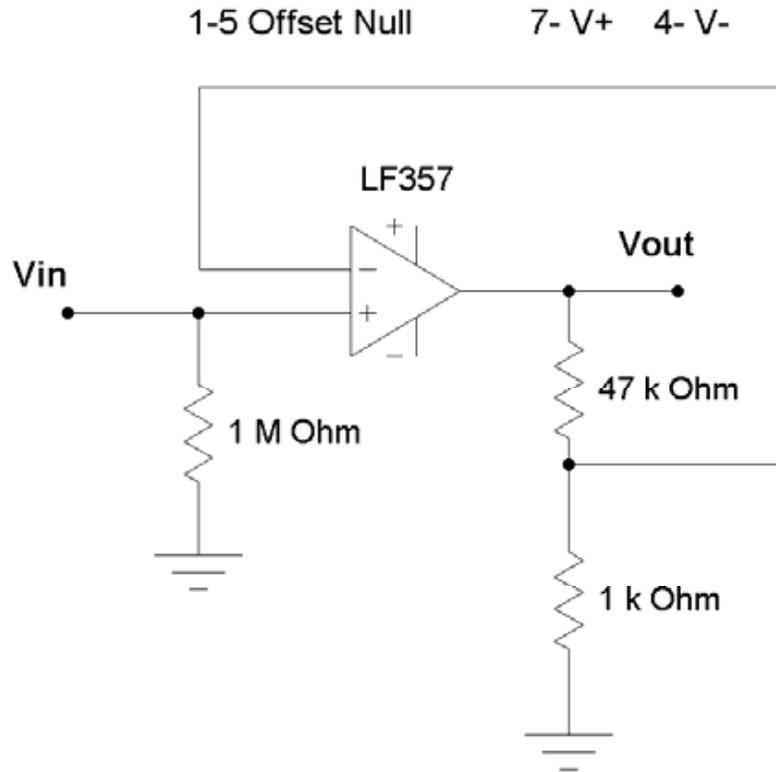
- To look at one type of fundamental noise: Johnson noise or the thermal noise voltage that appears across every resistor or any other device that can dissipate power.
- In the next lab implement one powerful technique for measuring small signals in the presence of noise: the phase detector, or lock in amplifier.

### 2 Introduction

The Johnson noise voltage “spectral density” that appears across a resistor is  $e_n = \sqrt{4k_BTR}$ . (See sections 7.11-7.14; pp 430-443 in *Horowitz – Hill 2<sup>nd</sup>* edition.) For a 1M resistor at room temperature (300 K)  $e_n = 1.3 \times 10^{-7} \text{ V}/\sqrt{\text{Hz}}$ , or  $\sim 4 \mu\text{V}$  rms in a 1 KHz bandwidth. We will, therefore, need a high gain amplifier whose intrinsic noise is small so that it does not cover up the thermal noise from the resistor.

### 3 Procedure

For the first amplifier stage, start near the left edge of the upper breadboard block (to the left of your tuned amplifier) and wire the following circuit:



Clip the component leads short and bend them so the component bodies lie flat against the breadboard – this minimizes stray capacitances. Use a single bus on the socket for all the grounds (probably the inner strip below the op-amp). It is a good idea to put  $0.1\ \mu\text{F}$  capacitors from the  $+15\text{V}$  and the  $-15\text{V}$  supply pins of each op-amp to ground to bypass the power supply.

The LF357 is an inexpensive op-amp with JFET input transistors and reasonably low noise. Use information from the spec sheet to find the following.

**Question 1** What is the gain of this stage (measured from the + input)?

**Question 2** The **voltage noise spectral density** ( $e_n$ ) at the input due to the op-amp voltage noise? Use values for  $f \sim 1\text{ kHz}$ .

**Question 3** The **voltage** noise spectral density at the input due to the op-amp current noise ( $i_n$ ) (for the circuit with  $R_{in} = 1\text{M}$ )?

**Question 4** The **total** voltage noise at the input from these two amplifier-related sources (remember that independent random voltages add in quadrature!)?

**Question 5** The **noise temperature** of this op-amp with  $R_{in} = 1\text{M}$ ?

**Question 6** The **noise figure** for the op-amp with  $R_{in} = 1\text{M}$ ? (See Section 7.12 in *Horowitz–Hill, 2nd edition*.)

**Question 7** The **noise resistance** of the op-amp?

**Questions 8** Repeat 2) through 7) for an OP-27 ultra-low noise **bipolar** input op-amp:

$$e_n = 3 \text{ nV}/\sqrt{\text{Hz}}, i_n = 0.4 \text{ pA}/\sqrt{\text{Hz}}.$$

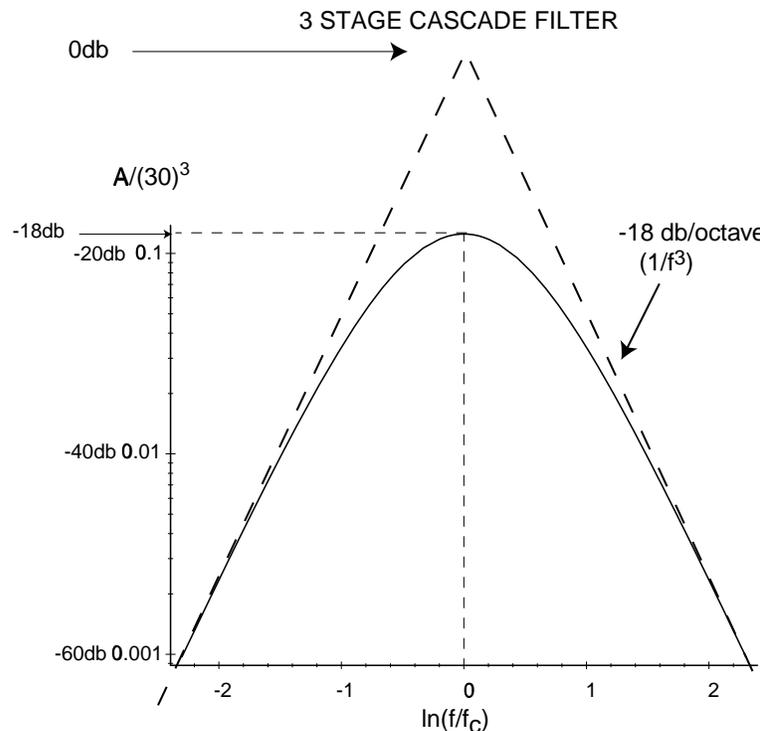
**Question 9** Which is the better op-amp for this application?

Since we will use the rms function on the scope to measure an *rms* **voltage**,  $v_{rms}$ , we need to define a bandwidth so that this voltage can be predicted from the Johnson noise **spectral density**,  $e_n$ .

Now refer to your previous laboratory (Operational Amplifiers) and consider the three stage tuned amplifier that you previously constructed on the right side of the top block of the breadboard.

**Question 10** What is  $f_o$ , the center of the bandpass? (As usual, remember the difference between  $f(\text{Hz})$  and  $\omega(\text{rad/sec})$ )

The use of the three stages results in a better definition of the bandwidth and reduces the gain at 60 Hz ( $\ll f_o$ ) to the point where we have some hope of operating a completely unshielded high gain circuit like this in the presence of the large line-frequency fields in the laboratory. The overall gain is sketched below:



**Question 11** What is the gain of the three stage amplifier at  $f=60$  Hz?

**Question 12** How many db down from the peak gain is this?

[Power levels, such as 30 watt or 40 watt, are often compared by taking the logarithm of the ratio of the Power levels. When comparing two power levels  $P_a$  and  $P_b$ , we might generate

$$n = \log\left(\frac{P_a}{P_b}\right)$$

and say the “Power difference is n bels”. More usually, we generate

$$m = 10.0 \log\left(\frac{P_a}{P_b}\right)$$

and we say the “Power difference is m decibels” or “a is m db above b”.

When comparing voltages, across identical resistors  $R$ , then the ratio of the powers in the resistors is related to the ratio of the square of the voltages.

$$m = 10.0 \log\left(\frac{P_a}{P_b}\right) = 10.0 \log\left(\frac{V_a^2/R}{V_b^2/R}\right) = 20.0 \log\left(\frac{V_a}{V_b}\right) \text{ (since } P \sim V^2, 10 \log(P_2/P_1) \sim 20 \log(V_2/V_1) \text{)}$$

For example, if the voltage  $V_a$  is **twice** the voltage  $V_b$ , then

$$m = 20.0 \log\left(\frac{2}{1}\right) = 20.0 \times (0.3010) = 6.02$$

and we say that “the voltage at a is **6 decibels or 6 db above** the voltage at b”. One can compare voltage gains in the same way as comparing voltages.

For example, if a voltage gain is increased two-fold, then it has been raised by 6 db.]

Confirm that the three tuned stages are connected together, but for the moment don't connect the output of the LM357 stage to the input of the first tuned stage.

Now we need to calculate the *rms* output voltage we expect for a given voltage spectral density at the input. We will assume “white noise”, where the spectral density,  $e_n$  is independent of frequency. The *rms* input voltage in a bandwidth  $df$  at any frequency  $f$  is then  $e_n \sqrt{df}$ , and the output voltage is  $A(f)e_n \sqrt{df}$ . We must add these voltages for all frequencies where we have any gain, but since the phases are random and independent, we must add their *squares*:

$$\begin{aligned} v_{rms}^2 &= \langle v^2 \rangle = \int_0^\infty ([\text{gain}(f)] e_n)^2 df \\ &= \int_0^\infty e_n^2 [\text{gain}(f)]^2 df = \int_0^\infty e_n^2 [1^{\text{st}} \text{stage gain}]^2 \left[ \frac{(30)^3 (f/f_0)^3}{(1 + (f/f_0)^2)^3} \right]^2 df \end{aligned}$$

**Question 13** Show that:

$$v_{rms} = e_n \left[ (1^{\text{st}} \text{ stage gain}) (30^3) (\sqrt{f_0}) \left( \int_0^\infty \left[ \frac{x}{1+x^2} \right]^6 dx \right)^{1/2} \right]$$

**Question 14** Given that

$$\int_0^\infty \left( \frac{x}{1+x^2} \right)^6 dx = \frac{3\pi}{512}$$

find the numerical value for  $k$  such that  $v_{rms-out} = k e_n$ .

The peak gain of the tuned stages is quite sensitive to the exact values of the capacitors. If you want accurate results, it would be a good idea to **remeasure** the net gain of the three tuned stages **at the frequency where it is maximum**, and scale your calculated  $k$  to the extent that the measure value differs from  $30^3/8$ . To do this, we use a sine wave from the signal generator as a source, inserting the  $\div 100$  and  $\div 10$  attenuators in order to set to a sufficiently low voltage. It is best to use your scope to measure the input and output voltages. Remember to measure to the middle (average value) of the noise.

- Adjust the frequency for maximum output (note the frequency, but don't worry if it differs slightly from the calculated  $f_o$ . If you **must** use the DVM to measure the input and output voltage to get the gain, be sure to properly subtract the input and output noise readings when the attenuators are disconnected from the generator.
- Just to check the noise contribution from the 741 stages, temporarily ground the input of the first tuned stage (make sure it is disconnected from the low-impedance LM357 stage output!!) and measure the *rms* output voltage of the last stage using the rms amplitude function on your scope (the average is only over the width of the screen, so slow the sweep until this is about 1 s) — record your values.
- Now connect the first tuned stage input to the output of the LM357. Temporarily short the 1M resistor to ground the input. Measure the output voltage. Subtract the contribution from the 741 stages (in quadrature) to get the noise due to the 357 stage only.
- Is the correction significant? Use the result of Ques. 14 to convert your *rms* voltage to a spectral density at the input. How well does this agree with  $e_n$  from the data sheet?
- Now remove the short from the resistor and again measure the output voltage. You can subtract the total (input shorted) amplifier noise. Is the correction significant? This does **not** correct for the amplifier current noise, — (Why not?) — but the current noise is unlikely to be significant for  $R_{in} = 1\text{M}$  or less. Convert this to  $e_n$  for the resistor. How does it compare to  $\sqrt{4kTR}$ ? (300 K is close enough to room temperature.)
- Be sure to look at the output on the scope at different sweep rates. It should look like random noise. If it doesn't seem right, check with the instructor.
- Get a 200 K metal film resistor mounted on a short shielded tube (from the instructor, if it is not on your bench!) and substitute it for the 1M resistor. Which side of the resistor should be connected to ground? What value do you get for  $e_n$  noise? Does it scale as you expect with  $R$ ?
- The resistance of metal alloys, such as the nichrome film in these resistors, is essentially independent of temperature at room temperature and below. We can therefore cool the resistor and check the expected T dependence of Johnson noise. Nest two Styrofoam coffee cups and fill the inner one with liquid nitrogen ( $LN_2, T = 77\text{ K}$ ) from the dewar in the lab. Don't spill it in your shoes! (If you have never handled  $LN_2$  before, you may want to ask the instructor to do it for you!) Immerse  $\sim 2$  cm of the end of the tube

in the  $LN_2$ . (The resistance of a carbon composition resistor would almost double, if you used one here.)

- After things settle down, check the scope to see if the “popcorn” due to mechanical contraction in the resistor has pretty much stopped. When it has, take a reading from the rms calculation on the scope and calculate  $e_n$ . Don’t forget to correct for amplifier noise – it’s starting to get important at these low levels.)

**Question 15** By what factor did you expect it to drop? Put your answer in the notebook.

**Note: save your breadboard in the cabinet again – you will use it for the Phase Detector Lab next week.**