Physics 623
Transmission Lines and Characteristic Impedance
January 16, 2023

1 Prelab

In this lab, we are going to build the circuit of Figure 1 to investigate propagation characteristics of transmission lines. From reflection measurements, we will determine the propagation speed of waves inside a transmission line, and measure its impedance. Noting Figure 1, consider a transmission line of length L, impedance Z0. Let's denote the propagation speed of electromagnetic waves inside the line as u0. We supply a voltage pulse with time duration of T at point C of the circuit. When the pulse hits point B, due to impedance mismatch, part of the wave is reflected. This reflected wave encounters a similar phenomenon when it reaches point A. As a result, multiple reflections can be observed at points **A** and **B**.

1) Assume that $T \ll L/u_0$, and Z_B , $Z_i \gg Z_0$. Plot what you would expect to see on points **A** and **B** (if we look on a scope, for example).

2) Repeat (1), but now assume that $Z_T = 0$ (short circuit). What would you expect to see on points **A** and **B**.

3) Repeat (1), but now let's introduce some loss to our system (in reality every circuit is lossy). Let's assume that, while propagating from A to B, 10% of the wave is attenuated. What would you expect to see on points **A** and **B**.

4) In the lecture, we have derived the telegraph equations for a transmission line. The plane wave solution for these equations is of the form

$$V(z,t) = V_0 cos(\omega_0 t - k_0 z) \tag{1}$$

where ω_0 is the angular frequency of the wave. As we have also derived in the lecture, the phase velocity for a monochromatic wave is $u_0 = \omega_0/k_0 = 1(\sqrt{C_L L_L})$, where L_L is the inductance per unit length, and C_L is the capacitance per unit length. When the wave contains multiple frequencies, its velocity can be different than the phase velocity. For the simple case of a two-frequency wave, one has:

$$V(z,t) = V_0 cos(\omega_0 t - k_0 z) + V_0 cos(\omega_1 t - k_1 z). \tag{2}$$

Show that the wave will propagate with a velocity

$$u_g = \frac{\omega_1 - \omega_0}{k_1 - k_0}. \tag{3}$$

The quantity, u_g is called the group velocity of the wave. In the laboratory, we are going to measure the velocity of a square pulse, by measuring the time it takes for the pulse to travel from point A to point B. Are we measuring the phase velocity or the group velocity? (In our laboratory experiment these two velocities are almost identical ug u0, and we will not worry about the difference between the two velocities).

(Optional) Rigorously prove your answer to question (5), by taking the Fourier transform

of a time domain square pulse, propagating each Fourier component and then reconstructing the pulse. What is the condition that the pulse doesn't change its shape while propagating
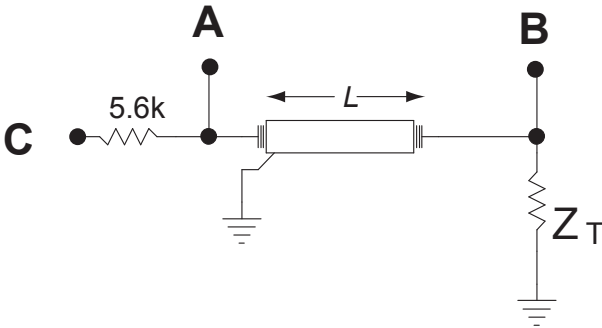
through the transmission line?

2 Lab Instructions

2.1 Purpose

- To experimentally determine the speed of signal propagation and the characteristic impedance of transmission lines.

- To interpret these measurements in terms of the capacitance and inductance per unit length of the cable.

- To observe termination effects in cables and interpret them in terms of virtual running waves.

- To observe clipping, capacitive charging and resonance in cables and interpret the observations in terms of the previously measured characteristics.

2.2 Procedure

At low frequencies, we can approximate the wires and cables that are used to connect the separate elements of the circuit as ideal connectors which transmit voltage and current unchanged in magnitude. However, in high frequencies ($\sim$ 1 MHz.) and in pulse applications, this approximation breaks down and the cable itself must be considered an integral part of the circuit with its own characteristic properties.

The line you will use for these measurements is a coil of coaxial cable (RG-58 or a similar RG-223/U which is a double-shielded version of the same Z_0 and u_0). The length of the cable L is indicated on the attached tag. Two of the measurable parameters associated with the line are: Z_0 = Characteristic Impedance and u_0 = Speed of Transmission.

To measure these quantities, you must use an extremely short timing pulse. First, use the coaxial cable to connect the Pulse Generator output to the scope input (say to "CH 1").

1. Adjust the pulse generator to emit a pulse of approximate width about 10 to 30 ns (1 ns = 10^{-9} s).

2. Adjust the scope sensitivity, time/cm and trigger so you can verify the pulse width.

Now make the following circuit to study the reflected pulses and measure the cable attributes.

1. Uncouple the cable you have just used.

2. Attach a BNC "Tee" to the scope and connect the far end of the cable (B) to one end of the "Tee".

3. Attach a small shielded box which has, within, a $\sim$ 5.6k resistor, to the pulse generator (C) using a short cable. It is very important in constructing the circuit that the signals be shielded as much as possible and that the leads be as short as possible. This minimizes the noise pick-up and is a general rule to be observed at all times in the laboratory.

4. Using another BNC "Tee", attach the near end of the cable (A) to the other side of the 5.6k resistor box and connect this end (A) to the other scope channel.

5. Verify that your circuit will bring the signal from the pulse generator and feed it through the 5.6k resistor and though the cable to the scope input.

6. *Ensure that the pulses are well separated.* In other words, ensure that the the pulse period or time between pulses is large compared with the pulse width.

7. The high impedance input at the scope and 5.6k resistor lead to repeated reflections of the pulses at the two ends of the cable.

8. With the scope, examine the pulse on the near end with the far end open ($Z_T = \infty$) and shorted ($Z_T = 0$). In the open ended case, you may be interested in examining the signal at the far end on the second scope channel.

9. Interpret your observations.

10. Measure, as carefully as you can, the length of time required for the signal to make 10-20 round trips on the cable. Using the length of the cable, calculate the speed of propagation u_0 of the signal. Remember that, between any repeated shape upon the scope, the signal must travel **twice** the length of the cable!

11. To measure Z_0, the characteristic impedance of the cable, connect a variable resistor (not wire wound) to the far end of your cable and vary it until you obtain no reflection. *Because of stray capacitance, you may see a small differentiated signal. Try to minimize the algebraic average of the residual signal.*

12. Then with the digital VOM, measure the value of the resistance, which is then Z_0.

13. From these measurements determine the inductance per meter and the capacitance per meter using the relations:
$$u_0 = \frac{1.}{\sqrt{L_L C_L}}$$
and
$$Z_0 = \sqrt{\frac{L_L}{C_L}}$$

14. Qualitatively describe the reflections obtained by varying the termination resistor from $R_T = 0$ to $R_T = \infty$.

15. Next terminate the far end with a capacitor of $C \sim 500$ pf and insert an incident pulse of width $T \approx \frac{1}{2}\tau$, where τ is the round trip time.

16. For the capacitor termination determine which features of the pulse are related by Fourier transformation to high and low frequencies and analyze how these extreme frequencies "see" the capacitor. (Or think of the impedance of the capacitor when the voltage across it is changing rapidly or slowly.)

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17. Next feed a pulse $T \gg 100\,\tau$ into the cable (still through the 5.6k resistor); and observe the results at the near end with the far end open. This shows the superposition of running waves each delayed a time τ with respect to one another and all **in phase**. The result is that you see a "stairstep" wave which corresponds to the capacitive charging of your transmission line. It is clear that the effective capacitance is $C_{eff} = LC_L$, where L is the length of the cable.

    Draw the low frequency equivalent circuit showing component values and calculate the RC time constant of the exponential charging.

18. Measure the time constant (of the "discharge", so you have a well defined asymptote at zero) and verify your expectation.

    The results obtained with the far end shorted illustrate the use of "shorting stubs".

19. Now short the far end and terminate the <u>near</u> end with Z_0. This illustrates the use of "shorting stubs" in pulse shaping. For input pulses of any width we get an output pulse of width $2L/u_0$ where L is the length of the shorted cable, which can be chosen at will. The negative going portion of the wave can be removed in practical situations with a diode clamp.

20. If time permits, eliminate the 5.6k resistor and feed the signal directly into the cable. Note and explain the differences from your previous observations.