PHYSICAL REVIEW A **111**, 012402 (2025)

Conditional acceleration of entanglement generation enabled by dissipation

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(Received 9 October 2024; accepted 13 December 2024; published 2 January 2025)

Dissipation is usually considered a negative factor for observing quantum effects and for harnessing them for quantum technologies. Here, we propose a scheme for conditionally speeding up the generation of quantum entanglement between two coupled qubits by introducing a strong dissipation channel to one of these qubits. The maximal entanglement is established by evenly distributing a single excitation between these two qubits in a probabilistic manner. When the excitation is initially held by the qubit with stronger dissipation, the dissipation accelerates the excitation redistribution process for the quantum state trajectory without quantum jumps. Our results show that the time needed to conditionally attain the maximal entanglement is monotonously decreased as the dissipative rate is increased, at the price of a quickly decreasing success probability. We further show that this scheme can be generalized to accelerate the production of the W state for the three-qubit system, where one qubit with strong dissipation is symmetrically coupled to two qubits with weak dissipation.

DOI: 10.1103/PhysRevA.111.012402

I. INTRODUCTION

One of the most fundamental postulations of quantum mechanics is that the state of an isolated quantum system is described by a wave function, whose evolution is governed by a Hermitian Hamiltonian according to the Schrödinger equation. However, any quantum system is inevitably coupled to its surrounding environment, which acts as a meter that continuously acquires information about the system [1]. At zero temperature, the environment can be regarded as a reservoir, composed of infinitely many bosonic modes, each initially in its vacuum state [2–4]. The system's information is gradually leaked into the reservoir by exchanging energy quanta. The most remarkable feature of this process is that the system's state evolution trajectory is significantly modified even if no quanta are leaked into the environment, as a consequence of measurement backaction.

For the no-jump trajectory, the system's evolution still can be described by the Schrödinger equation with a non-Hermitian (NH) Hamiltonian, featuring the competition between Hermitian and dissipative terms [5]. Such environment-induced non-Hermiticity endows the Hamiltonian's eigenstates and eigenenergies with striking features that are otherwise inaccessible, such as real-to-complex spectral phase transitions and NH topology [6–9]. So far, these NH effects have been experimentally demonstrated in distinct physical systems [10–30]. Recently, exceptional entanglement phenomena have been investigated in both theory and experiment in a NH spin-boson system [31]. In a very recent work [32], it was demonstrated that the entanglement between two interacting NH qubits can be generated within a time significantly shorter than that for two Hermitian ones. In the scheme, the two-qubit entanglement appears as a consequence of the competition between the intraqubit coupling and individual driving of the qubits, and the entanglement speedup is enabled by proximity to a higher-order exceptional point. Each NH qubit is realized with a three-level natural or artificial atom, where the highest and intermediate levels are used to encode the quantum information, and the non-Hermiticity is manifested by the decay from the intermediate to the lowest levels. This scheme is valid only when the decaying channel from the highest to intermediate levels is negligible. However, such a condition cannot be satisfied for the general case. For example, the decaying rate of the levels of a normal nonlinearly oscillator scales with the quantum number.

We here propose an alternative scheme for exploiting non-Hermiticity to speed up entanglement generation between two qubits. The theoretical model involves two qubits with distinct dissipative rates interacting with each other by swapping coupling, which conserves the total excitation number. The entanglement is generated in the single-excitation subspace. The excitation, initially possessed by the qubit with the larger dissipative rate, is distributed between the two qubits by energy exchange for the no-jump case. The maximal entanglement is attained when the two qubits are equally populated. We find that the non-Hermiticity can significantly accelerate this excitation distribution process in a probabilistic manner, enabling the two qubits to be entangled within a time shorter than the dynamical timescale of the interqubit interaction. In distinct contrast with the scheme of Ref. [32], our approach does not require encoding the qubits in the highest and intermediate levels of a three-level system, so that the state evolution is not subjected to the error coming from the decay

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of the third level. Furthermore, individual driving of the qubits is unnecessary, which plays a critical role in the dynamics of Ref. [32], where the entanglement evolution strongly depends upon the amplitude of the individual drives. Consequently, the effects of the amplitude fluctuations of the drives, which are one of the main error sources of the previous protocol, are avoided in our scheme. The idea can be directly extended to the three-qubit system, where a strongly dissipative qubit is symmetrically coupled to two weakly dissipative ones. When the excitation is initially held by the strongly dissipative qubit, the dissipation helps to speed up generation of the three-qubit W-type maximally entangled state in a probabilistic manner.

II. CONDITIONAL TWO-QUBIT ENTANGLEMENT SPEEDUP

We here consider the system involving two qubits (Q_1 and Q_2) with the same frequency interacting with each other by swapping coupling, as shown in Fig. 1(a). The system dynamics is described by the master equation

$$\frac{d\rho}{dt} = -i[\mathcal{H}_{\rm NH}, \rho] + \sum_{j=1,2} \kappa_j \sigma_j^- \rho \sigma_j^+, \quad (1)$$

where κ_j represents the dissipative rate for the jth qubit, and $\mathcal{H}_{\rm NH}$ represents the two-qubit NH Hamiltonian, defined as

$$\mathcal{H}_{\rm NH} = \lambda(\sigma_1^+\sigma_2^- + \text{H.c.}) - \frac{i}{2}\sum_{j=1,2}\kappa_j|1\rangle_j\langle 1|. \quad (2)$$

Here, $\sigma_j^+ = |1\rangle_j\langle 0|$ and $\sigma_j^- = |0\rangle_j\langle 1|$ with $|1\rangle_j$ ($|0\rangle_j$) denoting the upper (lower) level of Q_j, and λ is the interqubit coupling strength. The system evolution is a weighted mixture of infinitely many trajectories governed by the NH Hermiltonian but interrupted by randomly occurring quantum jumps. For the trajectory without a quantum jump, the system state evolution is governed by $\mathcal{H}_{\rm NH}$.

Under the NH Hamiltonian, the total excitation number of the qubits is conserved. Consequently, the qubit Hilbert space can be divided into three uncoupled subspaces $\mathcal{S}_0 = \{|0_1, 0_2\rangle\}$, $\mathcal{S}_1 = \{|1, 0\rangle, |0, 1\rangle\}$, and $\mathcal{S}_2 = \{|1, 1\rangle\}$. In $\mathcal{S}_1$, the system has a pair of eigenenergies, given by

$$E_\pm = -i\gamma/4 \pm \Omega, \quad (3)$$

where $\gamma = \kappa_1 + \kappa_2$ and $\Omega = \sqrt{\lambda^2 - \kappa^2/16}$ with $\kappa = \kappa_1 - \kappa_2$. Without loss of generality, we here set $\kappa_1 > \kappa_2$. The corresponding eigenstates are

$$|\Phi_\pm\rangle = \mathcal{N}_\pm(E_\pm|1_1, 0_2\rangle + \lambda|0_1, 1_2\rangle), \quad (4)$$

where $\mathcal{N}_\pm = (|E_\pm|^2 + \lambda^2)^{-1/2}$. Suppose that the system is initially in the state $|\psi(0)\rangle = |1_1, 0_2\rangle$. Then the conditional quantum state evolution, associated with the no-jump trajectory, can be written as

$$|\psi_c(t)\rangle = \mathcal{A}(t)|1, 0\rangle - i\mathcal{B}(t)|0, 1\rangle. \quad (5)$$

When $R = \kappa/(4\lambda) < 1$, the coefficients $\mathcal{A}(t)$ and $\mathcal{B}(t)$ are given by

$$\mathcal{A}(t) = e^{-\gamma t/4}\Big[\cos(\Omega t) - \frac{\kappa}{4\Omega}\sin(\Omega t)\Big],$$
$$\mathcal{B}(t) = \frac{\lambda}{\Omega}e^{-\gamma t/4}\sin(\Omega t). \quad (6)$$

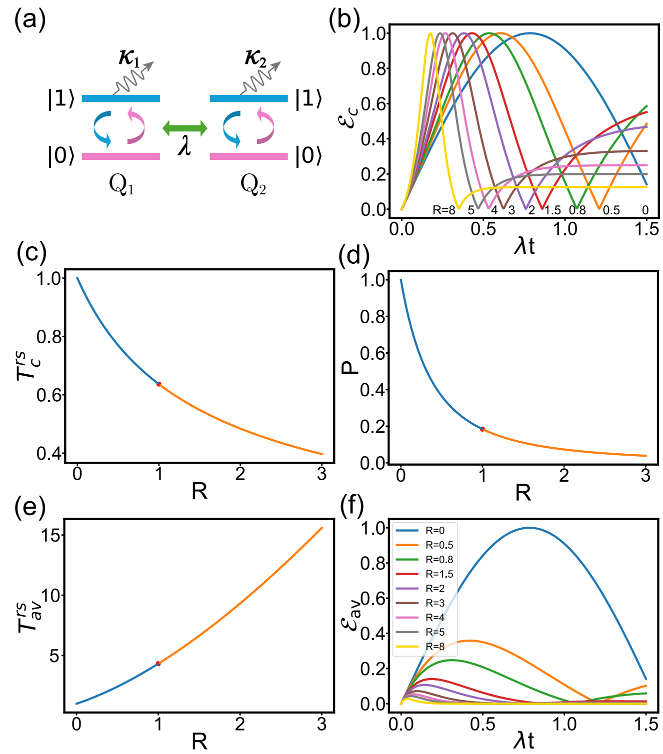

FIG. 1. (a) Sketch of the two-qubit system. The two qubits Q_1 and Q_2 are resonantly coupled to each other with the strength λ. The dissipative rates for Q_1 and Q_2 are κ_1 and κ_2, respectively. (b) Conditional concurrences ($\mathcal{E}_c$) as functions of λt for different values of $R = \kappa/4\lambda$, where $\kappa = \kappa_1 - \kappa_2$. (c) Conditional rescaled entangling time ($T_c^{\rm rs}$) vs R. Here, $T_c^{\rm rs}$ is defined as the ratio between the times needed to reach maximal entanglement under the NH and Hermitian Hamiltonians with the same interqubit coupling λ. The dot denotes the exceptional point $R = 1$, where the entanglement time cannot be well defined. (d) Success probability ($\mathcal{P}$) for obtaining the maximally entangled state as a function of R. (e) Average rescaled entangling time vs R, which is equal to $T_{\rm av}^{\rm rs} = T_c^{\rm rs}/\mathcal{P}$. (f) Average concurrence, defined as the concurrence averaged over all evolution trajectories, vs λt for different values of R. In all the numerical simulations, the system evolves from the initial state $|1, 0\rangle$ under the NH Hamiltonian of Eq. (2). In the simulation, κ_2 is set to be $0.1\kappa_1$.

When $\kappa_1 \gg \kappa_2$, R is approximately proportional to κ_1, and thus can be approximately regarded as the rescaled dissipation rate of Q_1. Here, $|\psi_c(t)\rangle$ is not normalized, and $\langle\psi_c(t)|\psi_c(t)\rangle$ corresponds to the probability for the case that no quantum jump occurs during the interval from 0 to t. For $R > 1$, the trigonometric functions are replaced by the corresponding hyperbolic functions, and Ω is replaced by $G = \sqrt{\kappa^2/16 - \lambda^2}$. The corresponding two-qubit entanglement is quantified by the concurrence, given by [33]

$$\mathcal{E}_c(t) = 2|\mathcal{A}(t)\mathcal{B}(t)|/\langle\psi_c(t)|\,\psi_c(t)\rangle. \quad (7)$$

To quantify to what extent the entanglement generation is conditionally speeded up by the non-Hermiticity, we define the conditional rescaled entangling time, $T_c^{\rm rs} = T_c/\tau$, where T_c is the shortest time needed to conditionally produce the two-qubit maximally entangled state for the NH system, and $\tau = \pi/4\lambda$ is the corresponding time for the Hermitian system. For

$R < 1$, the conditional rescaled entangling time is given by

$$T_c^{\rm rs} = 4\frac{\arctan\sqrt{(1-R)/(1+R)}}{\pi\sqrt{1-R^2}}. \tag{8}$$

When $R > 1$, $T_c^{\rm rs}$ is

$$T_c^{\rm rs} = 4\frac{{\rm arctanh}\sqrt{(R-1)/(R+1)}}{\pi\sqrt{R^2-1}}. \tag{9}$$

To quantitatively confirm the non-Hermiticity-enabled probabilistic entanglement speedup, in Fig. 1(b) we present the concurrence, associated with the no-jump trajectory, as a function of λt for different values of R. The results clearly show that $\mathcal{E}_c$ exhibits a nonmonotonous behavior, and can reach the maximum 1 regardless of whether $R < 1$ or $R > 1$. The time needed to reach the maximal entanglement ($\mathcal{E}_c = 1$) is monotonously decreased with the increase of R. When $R > 1$, after reaching the maximum $\mathcal{E}_c$ drops to zero at the time $t = G^{-1}\,{\rm arctanh}(4G/\kappa)$, and then monotonously tends towards $\mathcal{E}_c(t \to \infty) = 2x/(1+x^2)$, where $x = (G - \kappa/4)/\lambda$. For $R \gg 1$, $\mathcal{E}_c(t \to \infty) \simeq 1/(2R)$. Figure 1(c) displays the conditional rescaled entangling time $T_c^{\rm rs}$ vs R. The results clearly show that the entanglement generation is indeed speeded up by the non-Hermiticity in a probabilistic manner. The result can be interpreted as follows. The maximal entanglement is reached when the populations of $|1,0\rangle$ and $|0,1\rangle$, $P_{1,0}$ and $P_{0,1}$, are balanced. For the unitary evolution, this is achieved by coherent excitation transfer. When the excitation is initially populated in the qubit with stronger dissipation, the dissipation helps to conditionally increase the ratio $P_{0,1}/P_{1,0}$ from 0 to 1.

This entanglement dynamics is in stark contrast with that presented in Ref. [31], where the system starts with the initial state $|0,1\rangle$. For such an initial state, the system evolution associated with the no-jump trajectory is given by [31]

$$|\psi_c'(t)\rangle = \mathcal{A}'(t)|0,1\rangle - i\mathcal{B}(t)|1,0\rangle, \tag{10}$$

where $\mathcal{A}'(t)$ is equal to $e^{-\gamma t/4}[\cos(\Omega t) + \frac{\kappa}{4\Omega}\sin(\Omega t)]$ and $e^{-\gamma t/4}[\cosh(Gt) + \frac{\kappa}{4G}\sinh(Gt)]$ for $R < 1$ and $R > 1$, respectively. For $R < 1$, the conditional rescaled entangling time is

$$T_c'^{\rm rs} = \frac{4\arctan\sqrt{(1+R)/(1-R)}}{\pi\sqrt{1-R^2}}. \tag{11}$$

Contrary to the case with the initial state $|1,0\rangle$, $T_c'^{\rm rs}$ increases with R. This is due to the fact that the dissipation plays a negative role in the transfer of the excitation from the strongly dissipative qubit to the weakly dissipative one, so that it takes a longer time to balance $P_{1,0}$ and $P_{0,1}$. When $R > 1$, $\mathcal{E}_c'$ is monotonously increased, tending to a fixed value, which is smaller than 1 and is decreased when R increases. The results imply that the dissipation can speed up entanglement only when the excitation is initially held by the qubit with a larger dissipative rate.

It is worthwhile to investigate to what extent the success probability of producing the maximally entangled state is reduced with the increase of R, which is defined as the probability that no quantum jump occurs before the system reaches maximal entanglement. This probability is given by

$$\mathcal{P} = \langle\psi_c(T_c)|\psi_c(T_c)\rangle = |\mathcal{A}(T_c)|^2 + |\mathcal{B}(T_c)|^2, \tag{12}$$

where $\mathcal{A}(T_c)$ and $\mathcal{B}(T_c)$ are determined by Eq. (6) with $t = T_c$. When $R < 1$, this probability is

$$\mathcal{P} = \frac{2\exp\left(-\pi R\gamma T_c^{\rm rs}/2\kappa\right)}{1-R^2}\sin^2\left(\frac{\pi}{4}T_c^{\rm rs}\sqrt{1-R^2}\right). \tag{13}$$

For $R > 1$, $\mathcal{P}$ is

$$\mathcal{P} = \frac{2\exp\left(-\pi R\gamma T_c^{\rm rs}/2\kappa\right)}{R^2-1}\sinh^2\left(\frac{\pi}{4}T_c^{\rm rs}\sqrt{R^2-1}\right). \tag{14}$$

Here, $T_c^{\rm rs}$ are given by Eqs. (8) and (9) for $R < 1$ and $R > 1$, respectively. Figure 1(d) presents this probability as a function of R, where we have set $\kappa_2 = 0.1\kappa_1$, i.e., $\gamma = 11\kappa/9$. The result implies that, for a given interqubit coupling strength, the time needed to achieve the maximal entanglement is shortened at the price of the decrease of the success probability. We note that it was recently shown that the entanglement speedup can be realized with a relatively high probability by using a truly $\mathcal{PT}$-symmetric system [34], whose Hamiltonian includes both the loss of the excited state population and the gain of the ground state population of the qubits. However, for any real open quantum system described by the Lindblad master equation [35], the NH Hamiltonian, which corresponds to the no-jump quantum state trajectory, does not contain gain of the ground state population.

Due to the probabilistic nature of the protocol, the average rescaled time needed for the generation of a maximally entangled qubit pair is $T_{\rm av}^{\rm rs} = T_c^{\rm rs}/\mathcal{P}$. Figure 1(e) shows $T_{\rm av}^{\rm rs}$ as a function of R. As expected, $T_{\rm av}^{\rm rs}$ increases with R, as the success probability $\mathcal{P}$ decreases faster than $T_c^{\rm rs}$. Figure 1(f) displays the average entanglement, $\mathcal{E}_{\rm av} = 2|\mathcal{A}(t)\mathcal{B}(t)|$, defined as the concurrence averaging over all the trajectories for the mixed state, as a function of λt for different values of R. The results show that the maximum of $\mathcal{E}_{\rm av}$ decreases with R. Despite these unfavorable features, the probabilistic entanglement acceleration still helps to suppress the effects of dephasing noises. More concretely, for the success events the entangling time is shortened, so that the entanglement procedure is less affected by dephasing noises, which represent the main error source in superconducting circuits [36], where the dephasing times of qubits are typically much shorter than their dissipative times.

III. CONDITIONAL THREE-QUBIT ENTANGLEMENT SPEEDUP

We note that the scheme can be directly generalized to realize dissipation-based three-qubit entanglement acceleration. The system involves three qubits arranged in a linear array, as shown in Fig. 2(a). Q_1 is coupled to Q_2 and Q_3 with the same coupling strength λ. Q_2 and Q_3 have the same dissipative rate $\kappa_2 = \kappa_3$, which is much smaller than that of Q_1, denoted as κ_1. For the no-jump evolution trajectory, the system dynamics is governed by the NH Hamiltonian

$$\mathcal{H}_{\rm NH} = \lambda[\sigma_1^+(\sigma_2^- + \sigma_3^-) + {\rm H.c.}] - \frac{i}{2}\sum_{j=1}^{3}\kappa_j|1\rangle_j\langle 1|. \tag{15}$$

In the single-excitation subspace $\{|1,0,0\rangle, |0,1,0\rangle, |0,0,1\rangle\}$, the system has three eigenenergies, given by

$$E_0 = 0,\ \ E_\pm = -i\gamma/4 \pm \Lambda, \tag{16}$$

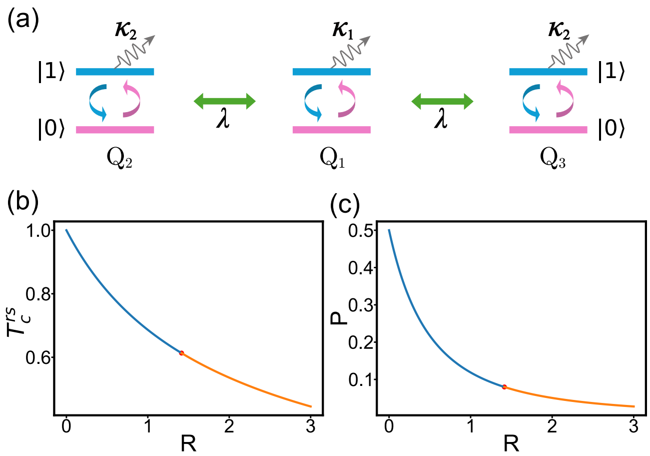

FIG. 2. (a) Sketch of the three-qubit system. Q_1 is symmetrically coupled to two qubits, Q_2 and Q_3, with the strength λ. Q_2 and Q_3 have the same dissipative rate κ_2, which is much smaller than that of Q_1, κ_1. (b) Conditional rescaled entangling time T_c^{rs} vs the R. Here, T_c^{rs} is defined as the ratio between the times needed to reach the W-type maximally entangled state under the NH and Hermitian Hamiltonians with the same interqubit coupling λ. The system is initially in the state $|1, 0, 0\rangle$ and evolves under the three-qubit NH Hamiltonian. (c) Success probability for obtaining the W-type maximally entangled state as a function of R. In the simulation, κ_2 is set to be $0.1\kappa_1$.

where $\Lambda = \sqrt{2\lambda^2 - \kappa^2/16}$. The corresponding eigenstates are

$$|\Phi_0\rangle = \frac{1}{\sqrt{2}}(|0, 1, 0\rangle - |0, 0, 1\rangle),$$
$$|\Phi_j\rangle = \mathcal{N}_\pm(E_\pm|1, 0, 0\rangle + \sqrt{2}\lambda|\phi_b\rangle), \tag{17}$$

where $|\phi_b\rangle = \frac{1}{\sqrt{2}}(|0, 1, 0\rangle + |0, 0, 1\rangle)$. When the system is initially in the state $|\psi(0)\rangle = |1, 0, 0\rangle$, the state evolution associated to the no-jump trajectory is

$$|\Psi(t)\rangle = \mathcal{C}(t)|1, 0, 0\rangle - i\mathcal{D}(t)|\phi_b\rangle. \tag{18}$$

When $R < \sqrt{2}$, the coefficients $\mathcal{C}(t)$ and $\mathcal{D}(t)$ are given by

$$\mathcal{C}(t) = e^{-\gamma t/4}\left[\cos(\Lambda t) - \frac{\kappa}{4\Lambda}\sin(\Lambda t)\right],$$
$$\mathcal{D}(t) = \frac{\sqrt{2}\lambda}{\Lambda}e^{-\gamma t/4}\sin(\Lambda t). \tag{19}$$

For $R > \sqrt{2}$, the trigonometric functions are replaced by the corresponding hyperbolic functions, and Λ is replaced by $\Gamma = \sqrt{\kappa^2/16 - 2\lambda^2}$.

When $|\mathcal{C}(T)| = |\mathcal{D}(T)|/\sqrt{2}$, $|\Psi(T)\rangle$ corresponds to the three-qubit W-type maximally entangled state [37]

$$|\Psi_w\rangle = (e^{i\phi}|1, 0, 0\rangle + |0, 1, 0\rangle + |0, 0, 1\rangle)/\sqrt{3}. \tag{20}$$

For such a W state, any two qubits have a concurrence of 2/3. When $R < \sqrt{2}$, the rescaled three-qubit W-state entangling time is given by

$$T_c^{rs} = \frac{\arctan[\sqrt{2 - R^2}/(1 + R)]}{\alpha\sqrt{2 - R^2}}, \tag{21}$$

where $\alpha = \arccos(1/\sqrt{3})/\sqrt{2}$. Here, T_c^{rs} is defined as the ratio between the W-state generation times for the NH and Hermitian systems, $T_c^{rs} = T_c/\tau_w$, where $\tau_w = \alpha/\lambda$. For $R > \sqrt{2}$, the rescaled W-state entangling time is

$$T_c^{rs} = \frac{\text{arctanh}[\sqrt{R^2 - 2}/(1 + R)]}{\alpha\sqrt{R^2 - 2}}. \tag{22}$$

The rescaled W-state generation time T_c^{rs}, as a function of R, is presented in Fig. 2(b), which confirms that T_c^{rs} is smaller than 1 when $\kappa \neq 0$, verifying the conditional non-Hermiticity-enabled acceleration of the W-state generation. The result demonstrates that the time needed to reach the W-type maximally entangled state monotonously decreases with the increase of κ for the no-jump case. When $R < \sqrt{2}$, the success probability for producing the W-type maximally entangled state is

$$\mathcal{P} = \frac{3\exp\left(-2\alpha R\gamma T_c^{rs}/\kappa\right)}{2(2 - R^2)}\sin^2\left(\alpha T_c^{rs}\sqrt{2 - R^2}\right). \tag{23}$$

For $R > \sqrt{2}$, $\mathcal{P}$ is given by

$$\mathcal{P} = \frac{3\exp\left(-2\alpha R\gamma T_c^{rs}/\kappa\right)}{2(R^2 - 2)}\sinh^2\left(\alpha T_c^{rs}\sqrt{R^2 - 2}\right). \tag{24}$$

Here, T_c^{rs} are given by Eqs. (21) and (22) for $R < \sqrt{2}$ and $R > \sqrt{2}$, respectively. $\mathcal{P}$ as a function of R is displayed in Fig. 2(c), where κ_2 is set to be $0.1\kappa_1$. The result shows $\mathcal{P}$ decreases with the increase of R.

IV. CONCLUSION

In conclusion, we have presented a scheme for exploiting dissipation to speed up entanglement evolution for two qubits with different dissipative rates, interacting with each other via swapping coupling. When the excitation is initially possessed by the qubit with the larger dissipative rate, the system can conditionally evolve to a maximally entangled state within a time shorter than that needed for the Hermitian system with the same interqubit coupling strength. Neither the three-level configuration nor individual driving of the qubits are required. The time needed to achieve the maximal entanglement monotonously decreases with the increase of the dissipation rate at the price of decreasing success probability. We further show that the scheme can be extended to the three-qubit case, where one qubit initially in its excited state is coupled to two identical qubits initially in their ground states. For the no-jump evolution trajectory the time required to produce the W state is reduced by the dissipation of the initially excited qubit, which is assumed to be much stronger than that of the other two qubits.

ACKNOWLEDGMENT

This work was supported by the National Natural Science Foundation of China under Grant No. 12474356 and No. 12375018.

[1] W. H. Zurek, Decoherence, einselection, and the quantum origins of the classical, Rev. Mod. Phys. **75**, 715 (2003).
[2] M. Brune, E. Hagley, J. Dreyer, X. Maitre, A. Maali, C. Wunderlich, J. M. Raimond, and S. Haroche, Observing the progressive decoherence of the "meter" in a quantum measurement, Phys. Rev. Lett. **77**, 4887 (1996).
[3] S. Deléglise, I. Dotsenko, C. Sayrin, J. Bernu, M. Brune, J.-M. Raimond, and S. Haroche, Reconstruction of non-classical cavity field states with snapshots of their decoherence, Nature (London) **455**, 510 (2008).
[4] C. J. Myatt, B. E. King, Q. A. Turchette, C. A. Sackett, D. Kielpinski, W. M. Itano, C. Monroe, and D. J. Wineland, Decoherence of quantum superpositions through coupling to engineered reservoirs, Nature (London) **403**, 269 (2000).
[5] N. Moiseyev, *Non-Hermitian Quantum Mechanics* (Cambridge University Press, Cambridge, UK, 2011).
[6] S. K. Özdemir, S. Rotter, F. Nori, and L. Yang, Parity-time symmetry and exceptional points in photonics, Nat. Mater. **18**, 783 (2019).
[7] M.-A. Miri and A. Alù, Exceptional points in optics and photonics, Science **363**, eaar7709 (2019).
[8] E. J. Bergholtz, J. C. Budich, and F. K. Kunst, Exceptional topology of non-Hermitian systems, Rev. Mod. Phys. **93**, 015005 (2021).
[9] K. Wang, C. Fang, and G. Ma, Non-Hermitian topology and exceptional-point geometries, Nat. Rev. Phys. **4**, 745 (2022).
[10] K. Wang, L. Xiao, J. C. Budich, W. Yi, and P. Xue, Simulating exceptional non-Hermitian metals with single-photon interferometry, Phys. Rev. Lett. **127**, 026404 (2021).
[11] F. E. Öztürk, T. Lappe, G. Hellmann, J. Schmitt, J. Klaers, F. Vewinger, J. Kroha, and M. Weitz, Observation of a non-Hermitian phase transition in an optical quantum gas, Science **372**, 88 (2021).
[12] L. Ding, K. Shi, Q. Zhang, D. Shen, X. Zhang, and W. Zhang, Experimental determination of $\mathcal{PT}$-symmetric exceptional points in a single trapped ion, Phys. Rev. Lett. **126**, 083604 (2021).
[13] W.-C. Wang, Y.-L. Zhou, H.-L. Zhang, J. Zhang, M.-C. Zhang, Y. Xie, C.-W. Wu, T. Chen, B.-Q. Ou, W. Wu *et al.*, Observation of $\mathcal{PT}$-symmetric quantum coherence in a single-ion system, Phys. Rev. A **103**, L020201 (2021).
[14] P. Peng, W. Cao, C. Shen, W. Qu, J. Wen, L. Jiang, and Y. Xiao, Anti-parity-time symmetry with flying atoms, Nat. Phys. **12**, 1139 (2016).
[15] J. Li, A. K. Harter, J. Liu, L. d. Melo, Y. N. Joglekar, and L. Luo, Observation of parity-time symmetry breaking transitions in a dissipative Floquet system of ultracold atoms, Nat. Commun. **10**, 855 (2019).
[16] Z. Ren, D. Liu, E. Zhao, C. He, K. K. Pak, J. Li, and G.-B. Jo, Chiral control of quantum states in non-Hermitian spin-orbit-coupled fermions, Nat. Phys. **18**, 385 (2022).
[17] Y. Wu, W. Liu, J. Geng, X. Song, X. Ye, C.-K. Duan, X. Rong, and J. Du, Observation of parity-time symmetry breaking in a single-spin system, Science **364**, 878 (2019).
[18] W. Liu, Y. Wu, C.-K. Duan, X. Rong, and J. Du, Dynamically encircling an exceptional point in a real quantum system, Phys. Rev. Lett. **126**, 170506 (2021).
[19] W. Zhang, X. Ouyang, X. Huang, X. Wang, H. Zhang, Y. Yu, X. Chang, Y. Liu, D.-L. Deng, and L.-M. Duan, Observation of non-Hermitian topology with nonunitary dynamics of solid-state spins, Phys. Rev. Lett. **127**, 090501 (2021).
[20] M. Naghiloo, M. Abbasi, Y. N. Joglekar, and K. W. Murch, Quantum state tomography across the exceptional point in a single dissipative qubit, Nat. Phys. **15**, 1232 (2019).
[21] Z. Wang, Z. Xiang, T. Liu, X. Song, P. Song, X. Guo, L. Su, H. Zhang, Y. Du, and D. Zheng, Observation of the exceptional point in superconducting qubit with dissipation controlled by parametric modulation, Chin. Phys. B **30**, 100309 (2021).
[22] Y. Choi, S. Kang, S. Lim, W. Kim, J. R. Kim, J. H. Lee, and K. An, Quasieigenstate coalescence in an atom-cavity quantum composite, Phys. Rev. Lett. **104**, 153601 (2010).
[23] T. Gao, E. Estrecho, K. Y. Bliokh, T. C. H. Liew, M. D. Fraser, S. Brodbeck, M. Kamp, C. Schneider, S. Höfling, Y. Yamamoto *et al.*, Observation of non-Hermitian degeneracies in a chaotic exciton-polariton billiard, Nature (London) **526**, 554 (2015).
[24] J. Doppler, A. A. Mailybaev, J. Böhm, U. Kuhl, A. Girschik, F. Libisch, T. J. Milburn, P. Rabl, N. Moiseyev, and S. Rotter, Dynamically encircling an exceptional point for asymmetric mode switching, Nature (London) **537**, 76 (2016).
[25] D. Zhang, X.-Q. Luo, Y.-P. Wang, T.-F. Li, and J. Q. You, Observation of the exceptional point in cavity magnon-polaritons, Nat. Commun. **8**, 1368 (2017).
[26] W. Tang, X. Jiang, K. Ding, Y.-X. Xiao, Z.-Q. Zhang, C. T. Chan, and G. Ma, Exceptional nexus with a hybrid topological invariant, Science **370**, 1077 (2020).
[27] K. Wang, A. Dutt, K. Y. Yang, C. C. Wojcik, J. Vukovi, and S. Fan, Generating arbitrary topological windings of a non-Hermitian band, Science **371**, 1240 (2021).
[28] Y. S. S. Patil, J. Höller, P. A. Henry, C. Guria, Y. Zhang, L. Jiang, N. Kralj, N. Read, and J. G. E. Harris, Measuring the knot of degeneracies and the eigenvalue braids near a third-order exceptional point, Nature (London) **607**, 271 (2022).
[29] Q. Zhang, Y. Li, H. Sun, X. Liu, L. Zhao, X. Feng, X. Fan, and C. Qiu, Observation of Acoustic non-Hermitian Bloch braids and associated topological phase transitions, Phys. Rev. Lett. **130**, 017201 (2023).
[30] W. Tang, K. Ding, and G. Ma, Realization and topological properties of third-order exceptional lines embedded in exceptional surfaces, Nat. Commun. **14**, 6660 (2023).
[31] P.-R. Han, F. Wu, X.-J. Huang, H.-Z. Wu, C.-L. Zou, W. Yi, M. Zhang, H. Li, K. Xu, D. Zheng *et al.*, Exceptional entanglement phenomena: Non-Hermiticity meeting nonclassicality, Phys. Rev. Lett. **131**, 260201 (2023).
[32] Z.-Z. Li, W. Chen, M. Abbasi, K. W. Murch, and K. B. Whaley, Speeding up entanglement generation by proximity to higher-order exceptional points, Phys. Rev. Lett. **131**, 100202 (2023).
[33] W. K. Wootters, Entanglement of formation of an arbitrary state of two qubits, Phys. Rev. Lett. **80**, 2245 (1998).
[34] B.-B. Liu, S.-L. Su, Y.-L. Zuo, Q. He, G. Chen, F. Nori, and H. Jing, Using $\mathcal{PT}$-symmetric qubits to break the trade-off between fidelity and the degree of quantum entanglement, arXiv:2407.08525.

[35] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, UK, 2000).

[36] C. Song, K. Xu, W. Liu, C.-p. Yang, S.-B. Zheng, H. Deng, Q. Xie, K. Huang, Q. Guo, L. Zhang *et al.*, 10-qubit entanglement and parallel logic operations with a superconducting circuit, Phys. Rev. Lett. **119**, 180511 (2017).

[37] W. Dür, G. Vidal, and J. I. Cirac, Three qubits can be entangled in two inequivalent ways, Phys. Rev. A **62**, 062314 (2000).