Topics in Axion Cosmology

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for Cosmological Physics
at The University of Chicago

Why axions?

For this talk, "axion" means the pseudo-Golstone boson (light pseudoscalar) associated with an anomalous global symmetry.

From the phenomenological perspective we like axions because
... they can solve the strong CP problem
... they provide a dark matter candidate
... they provide an inflaton candidate
… they can generate the baryon asymmetry
... it slices. it dices.

From the theory perspective we like axions because
… they arise generically in string theory
… they arise generically in field theory

Baryogenesis from Axion Inflation via Decaying Magnetic Helicity

… via Chiral Gravitational Waves

Cosmology of the Clockwork Axion

Baryogenesis from Axion Inflation via Decaying Magnetic Helicity

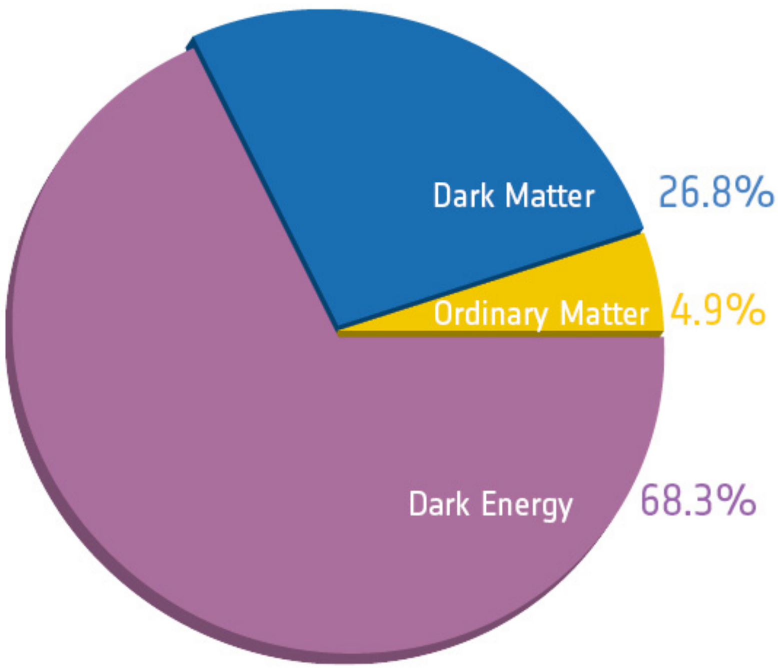
based on work with Kohei Kamada (1606.08891 & 1610.03074)

… via Chiral Gravitational Waves

Cosmology of the Clockwork Axion

The "Ordinary Matter" Problem

Cosmologists say that "We don't understand 95% of the universe."



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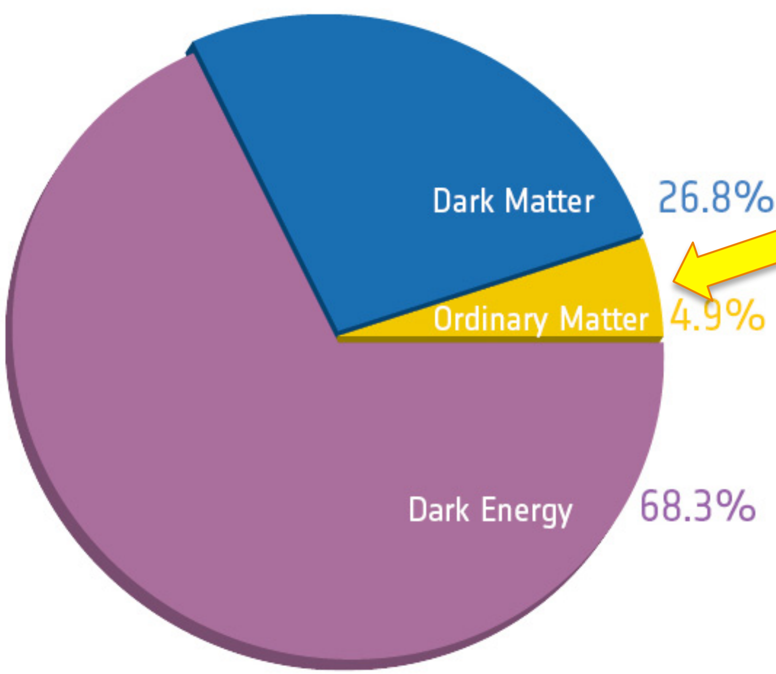
But this is not true … in fact, we don't understand 100%!

The "ordinary matter" problem = why is there more matter than anti-matter? → baryogenesis is the endeavor to solve this problem



Matter / Antimatter Asymmetry

$$\Omega_b \sim 5\% \quad \Rightarrow \quad n_B/s \sim 10^{-10}$$

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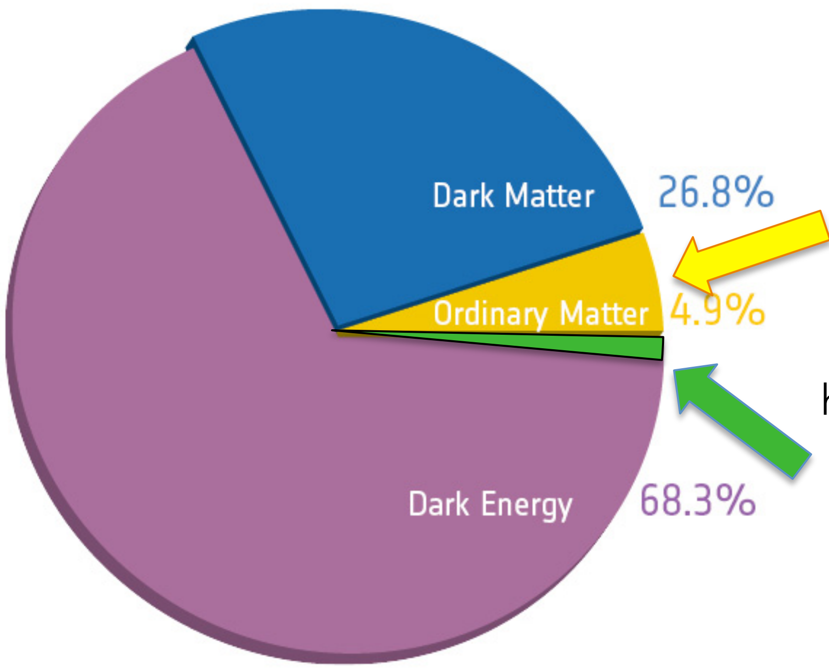


Matter / Antimatter Asymmetry

$$\Omega_b \sim 5\% \quad \Rightarrow \quad n_B/s \sim 10^{-10}$$

hiding in the "cracks" (error bars):

Primordial Magnetic Field

Executive Summary

In this section, I'm going to …

… assume that a helical magnetic field was created in the early universe prior to the EW epoch. (e.g., arises naturally in axion inflation)

… show that the decaying helicity of this field gives rise to a baryon asymmetry through the Standard Model B+L anomaly (builds on earlier work by Joyce, Shaposhnikov, Giovannini, Bamba, Geng, Ho, …)

… calculate the evolution of the magnetic field and baryon asymmetry from magnetogenesis until today, while paying particular attention to the EW crossover (this is my work with Kohei Kamada; see also Fujita & Kamada, 2016)

… conclude that the predicted relic baryon asymmetry suffers from a large theoretical uncertainty, because we don't understand well how magnetic fields behave at the EW crossover (even though this is just SM physics!)

Standard Model anomalies & primordial magnetic fields

Standard Model B & L Violation

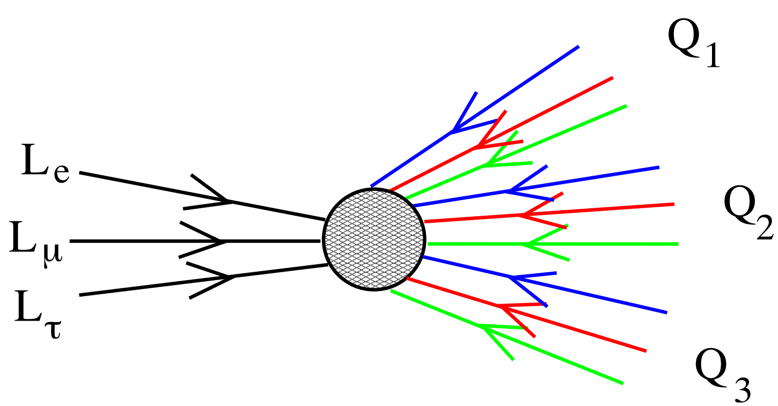
baryon & lepton number | SU(2)$_L$ gauge field | U(1)$_Y$ gauge field

$$\partial_\mu j_B^\mu = \partial_\mu j_L^\mu = N_{\rm gen}\left(\frac{g^2}{16\pi^2}{\rm Tr}\left[W_{\mu\nu}\tilde{W}^{\mu\nu}\right] - \frac{1}{2}\frac{g'^2}{16\pi^2}Y_{\mu\nu}\tilde{Y}^{\mu\nu}\right)$$

't Hooft (1976)

Thermal fluctuations of the weak isospin field (EW sphaleron), provide support for the SU(2)$_L$ term.

Kuzmin, Rubakov, Shaposhnikov (1985)



B-Number from Magnetic Helicity

Joyce & Shaposhnikov (1997); Giovannini & Shaposhnikov (1997) see also Rubakov & Tavkhelidze (1985)

baryon & lepton number — SU(2)$_L$ gauge field — U(1)$_Y$ gauge field

$$\underbrace{\partial_\mu j^\mu_B = \partial_\mu j^\mu_L}_{\text{baryon \& lepton number}} = N_{\rm gen}\left(\underbrace{\frac{g^2}{16\pi^2}{\rm Tr}\left[W_{\mu\nu}\tilde{W}^{\mu\nu}\right]}_{SU(2)_L \text{ gauge field}} - \underbrace{\frac{1}{2}\frac{g'^2}{16\pi^2}Y_{\mu\nu}\tilde{Y}^{\mu\nu}}_{U(1)_Y \text{ gauge field}}\right)$$

A hypermagnetic field provides support for the U(1)$_Y$ term.

$$\langle Y_{\mu\nu}\tilde{Y}^{\mu\nu}\rangle = -4\boldsymbol{E}_Y\cdot\boldsymbol{B}_Y = 2\left[\frac{\partial}{\partial t}(\boldsymbol{A}\cdot\boldsymbol{B}) + \boldsymbol{\nabla}\cdot(\phi\boldsymbol{B} + \boldsymbol{E}\times\boldsymbol{A})\right]$$

To induce a change in B-number, the helicity must change

$$\Delta Q_B = -N_{\rm gen}\frac{g'^2}{16\pi^2}\Delta\mathcal{H}_Y \qquad \text{where} \qquad \mathcal{H}_Y = \int d^3\boldsymbol{x}\,\boldsymbol{A}_Y\cdot\boldsymbol{B}_Y$$

In a plasma, the helicity decays because of ohmic losses

$$\boldsymbol{E}_Y = \boldsymbol{j}_Y/\sigma_Y \approx \boldsymbol{\nabla}\times\boldsymbol{B}_Y/\sigma_Y$$

$$\langle Y_{\mu\nu}\tilde{Y}^{\mu\nu}\rangle = -4\langle\boldsymbol{B}_Y\cdot\boldsymbol{\nabla}\times\boldsymbol{B}_Y\rangle/\sigma_Y$$

Decaying hypermagnetic helicity sources B-number!

E.g., field generation via axion inflation

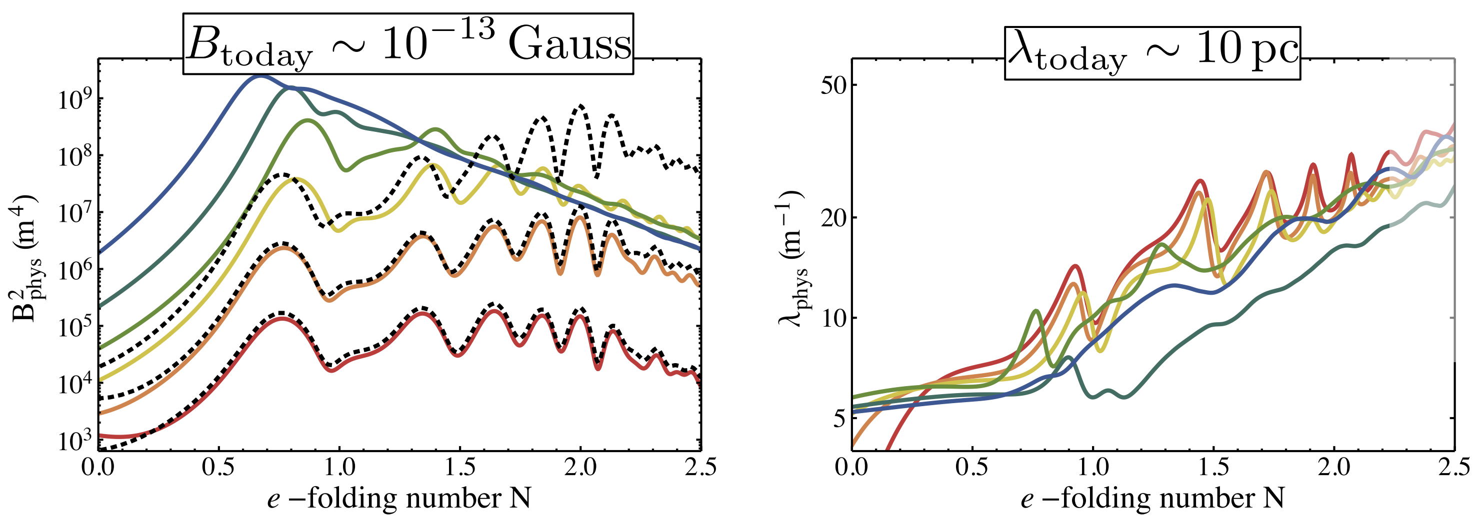
For example, a helical magnetic field may be generated during inflation from a pseudo-scalar inflaton (or spectator field).

Garretson, Field, & Carroll (1992); Anber & Sorbo (2006)
Durrer, Hollenstein, Jain (2010)
Barnaby, Moxon, Namba, Peloso, Shiu, & Zhou (2012)
Caprini & Sorbo (2014)
Fujita, Namba, Tada, Takeda, Tashiro (2015)
Anber & Sabancilar (2015)

axion coupled to EM ... rolling sources helicity ... opens tachyonic instability

$$\xi \equiv \frac{d\varphi/dt}{fH}$$

$$-\mathcal{L}_{\rm int} = \frac{\varphi}{4f} F_{\mu\nu}\widetilde{F}^{\mu\nu} = \frac{d\varphi/dt}{2f}\mathbf{A}\cdot\mathbf{B} + \cdots \qquad \left(\frac{\partial^2}{\partial\eta^2} + k^2 \pm k\frac{\xi}{\eta}\right) A_\pm(\eta, k) = 0$$



Lattice simulation of B-field growth during preheating after axion inflation
Adshead, Giblin, Scully, Sfakianakis (2016)

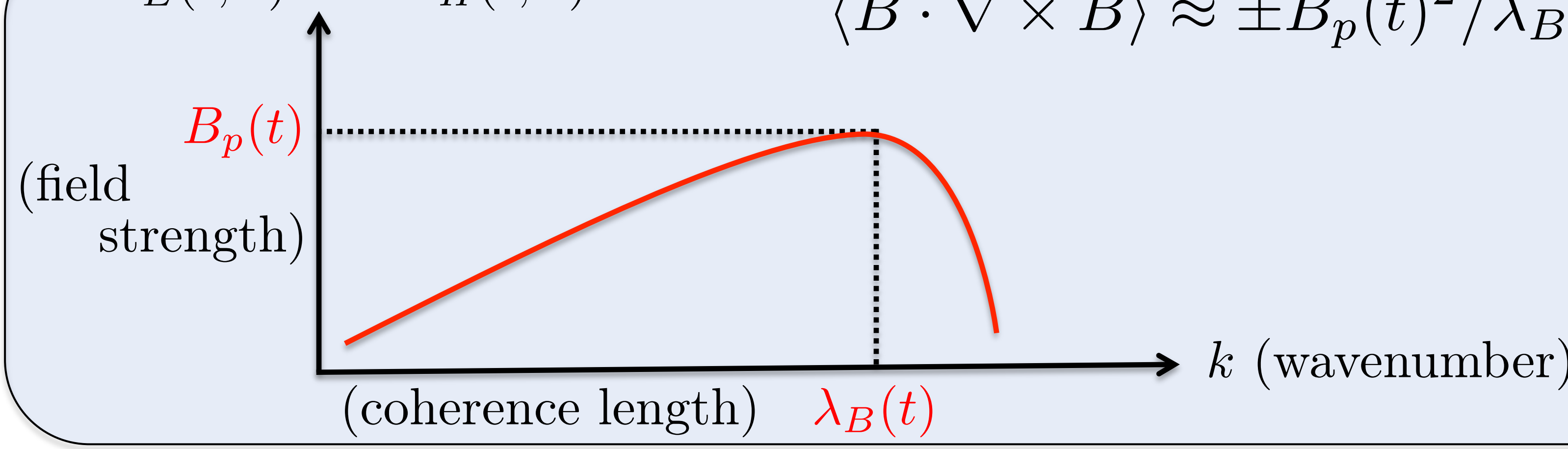
How do we formulate the problem?

Background B-Field Evolution

Simplified model for the background B-field:

$$P_E(t,k) = \pm P_H(t,k)$$

$$\langle B \cdot \nabla \times B \rangle \approx \pm B_p(t)^2/\lambda_B(t)$$



MHD evolution of B-field leads to inverse cascade scaling behavior.

$$B_p(t) = (a/a_0)^{-2} (\tau/\tau_{\rm rec})^{-1/3} B_0$$

$$\lambda_B(t) = (a/a_0)(\tau/\tau_{\rm rec})^{2/3} \lambda_0$$

Frisch, Pouquet, Leorat, Mazure, 75,76
Banerjee & Jedamzik, 2004
Campenelli, 2007
Kahniashvilli et. al. 2013

Baryon Asymmetry Evolution

Roughly speaking, you integrate the anomaly equation to obtain the kinetic equation for B-number:

$$\partial_\mu j_B^\mu = N_{\text{gen}} \left(\frac{g^2}{16\pi^2} \text{Tr}\big[W_{\mu\nu} \tilde{W}^{\mu\nu} \big] - \frac{1}{2} \frac{g'^2}{16\pi^2} Y_{\mu\nu} \tilde{Y}^{\mu\nu} \right)$$

$$\frac{d}{dt} n_B = -\Gamma_{\text{sphaleron}} \, n_B + \mathcal{S}_{\text{hypermagnetic}}$$

This glosses over Yukawa interactions which communicate B-number violation between the left- and right-chiral fermions.

SM Boltzmann eqns. w/ anomaly terms

$$\frac{d\eta_{u_L^i}}{dx} = -\mathcal{S}^i_{\rm UDW} - \sum_{j=1}^{N_{\rm g}}\left(\mathcal{S}^{ij}_{\rm Uhu} + \mathcal{S}^{ij}_{\rm Uu} + \mathcal{S}^{ij}_{\rm Uhd}\right) - \mathcal{S}_{\rm s,sph} - \frac{N_{\rm c}}{2}\mathcal{S}_{\rm w,sph} + \left(N_{\rm c} y_{Q_L}^2 \mathcal{S}^{\rm bkg}_{\rm y} + \frac{N_{\rm c}}{2}\mathcal{S}^{\rm bkg}_{\rm w} + N_{\rm c}\frac{y_{Q_L}}{2}\mathcal{S}^{\rm bkg}_{\rm yw}\right)$$

$$\frac{d\eta_{d_L^i}}{dx} = \mathcal{S}^i_{\rm UDW} - \sum_{j=1}^{N_{\rm g}}\left(\mathcal{S}^{ij}_{\rm Dhd} + \mathcal{S}^{ij}_{\rm Dd} + \mathcal{S}^{ij}_{\rm Dhu}\right) - \mathcal{S}_{\rm s,sph} - \frac{N_{\rm c}}{2}\mathcal{S}_{\rm w,sph} + \left(N_{\rm c} y_{Q_L}^2 \mathcal{S}^{\rm bkg}_{\rm y} + \frac{N_{\rm c}}{2}\mathcal{S}^{\rm bkg}_{\rm w} - N_{\rm c}\frac{y_{Q_L}}{2}\mathcal{S}^{\rm bkg}_{\rm yw}\right)$$

$$\frac{d\eta_{\nu_L^i}}{dx} = -\mathcal{S}^i_{\nu\rm EW} - \sum_{j=1}^{N_{\rm g}}\mathcal{S}^{ij}_{\nu\rm he} - \frac{1}{2}\mathcal{S}_{\rm w,sph} + \left(y_{L_L}^2 \mathcal{S}^{\rm bkg}_{\rm y} + \frac{1}{2}\mathcal{S}^{\rm bkg}_{\rm w} + \frac{y_{L_L}}{2}\mathcal{S}^{\rm bkg}_{\rm yw}\right)$$

$$\frac{d\eta_{e_L^i}}{dx} = \mathcal{S}^i_{\nu\rm EW} - \sum_{j=1}^{N_{\rm g}}\left(\mathcal{S}^{ij}_{\rm Ehe} + \mathcal{S}^{ij}_{\rm Ee}\right) - \frac{1}{2}\mathcal{S}_{\rm w,sph} + \left(y_{L_L}^2 \mathcal{S}^{\rm bkg}_{\rm y} + \frac{1}{2}\mathcal{S}^{\rm bkg}_{\rm w} - \frac{y_{L_L}}{2}\mathcal{S}^{\rm bkg}_{\rm yw}\right)$$

$$\frac{d\eta_{u_R^i}}{dx} = \sum_{j=1}^{N_{\rm g}}\left(\mathcal{S}^{ji}_{\rm Uhu} + \mathcal{S}^{ji}_{\rm Uu} + \mathcal{S}^{ji}_{\rm Dhu}\right) + \mathcal{S}_{\rm s,sph} - N_{\rm c} y_{u_R}^2 \mathcal{S}^{\rm bkg}_{\rm y}$$

$$\frac{d\eta_{d_R^i}}{dx} = \sum_{j=1}^{N_{\rm g}}\left(\mathcal{S}^{ji}_{\rm Dhd} + \mathcal{S}^{ji}_{\rm Dd} + \mathcal{S}^{ji}_{\rm Uhd}\right) + \mathcal{S}_{\rm s,sph} - N_{\rm c} y_{d_R}^2 \mathcal{S}^{\rm bkg}_{\rm y}$$

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$$\frac{d\eta_{\phi^+}}{dx} = -\left(\mathcal{S}_{\rm hhw} + \mathcal{S}_{\rm hw}\right) + \sum_{i,j=1}^{N_{\rm g}}\left(-\mathcal{S}^{ij}_{\rm Dhu} + \mathcal{S}^{ij}_{\rm Uhd} + \mathcal{S}^{ij}_{\nu\rm he}\right)$$

$$\frac{d\eta_{\phi^0}}{dx} = \mathcal{S}_{\rm hhw} - \mathcal{S}_{\rm h} + \sum_{i,j=1}^{N_{\rm g}}\left(-\mathcal{S}^{ij}_{\rm Uhu} + \mathcal{S}^{ij}_{\rm Dhd} + \mathcal{S}^{ij}_{\rm Ehe}\right)$$

$$\frac{d\eta_{W^+}}{dx} = \left(\mathcal{S}_{\rm hhw} + \mathcal{S}_{\rm hw}\right) + \sum_{i=1}^{N_{\rm g}}\left(\mathcal{S}^i_{\rm UDW} + \mathcal{S}^i_{\nu\rm EW}\right) .$$

$$\mathcal{S}^{ij}_{\rm Dhu} \equiv \frac{\gamma^{ij}_{\rm Dhu}}{2}\left(\frac{\eta_{d_L^i}}{k_{d_L^i}} + \frac{\eta_{\phi^+}}{k_{\phi^+}} - \frac{\eta_{u_R^j}}{k_{u_R^j}}\right) , \quad \mathcal{S}^{ij}_{\rm Uhu} \equiv \frac{\gamma^{ij}_{\rm Uhu}}{2}\left(\frac{\eta_{u_L^i}}{k_{u_L^i}} + \frac{\eta_{\phi^0}}{k_{\phi^0}} - \frac{\eta_{u_R^j}}{k_{u_R^j}}\right) ,$$

$$\mathcal{S}^{ij}_{\rm Uhd} \equiv \frac{\gamma^{ij}_{\rm Uhd}}{2}\left(\frac{\eta_{u_L^i}}{k_{u_L^i}} - \frac{\eta_{\phi^+}}{k_{\phi^+}} - \frac{\eta_{d_R^j}}{k_{d_R^j}}\right) , \quad \mathcal{S}^{ij}_{\rm Dhd} \equiv \frac{\gamma^{ij}_{\rm Dhd}}{2}\left(\frac{\eta_{d_L^i}}{k_{d_L^i}} - \frac{\eta_{\phi^0}}{k_{\phi^0}} - \frac{\eta_{d_R^j}}{k_{d_R^j}}\right) ,$$

$$\mathcal{S}^{ij}_{\nu\rm he} \equiv \frac{\gamma^{ij}_{\nu\rm he}}{2}\left(\frac{\eta_{\nu_L^i}}{k_{\nu_L^i}} - \frac{\eta_{\phi^+}}{k_{\phi^+}} - \frac{\eta_{e_R^j}}{k_{e_R^j}}\right) , \quad \mathcal{S}^{ij}_{\rm Ehe} \equiv \frac{\gamma^{ij}_{\rm Ehe}}{2}\left(\frac{\eta_{e_L^i}}{k_{e_L^i}} - \frac{\eta_{\phi^0}}{k_{\phi^0}} - \frac{\eta_{e_R^j}}{k_{e_R^j}}\right) ,$$

$$\mathcal{S}^i_{\rm UDW} \equiv \gamma^i_{\rm UDW}\left(\frac{\eta_{u_L^i}}{k_{u_L^i}} - \frac{\eta_{d_L^i}}{k_{d_L^i}} - \frac{\eta_{W^+}}{k_{W^+}}\right)$$

$$\mathcal{S}^i_{\nu\rm EW} \equiv \gamma^i_{\nu\rm EW}\left(\frac{\eta_{\nu_L^i}}{k_{\nu_L^i}} - \frac{\eta_{e_L^i}}{k_{e_L^i}} - \frac{\eta_{W^+}}{k_{W^+}}\right)$$

$$\mathcal{S}_{\rm hhw} \equiv \gamma_{\rm hhw}\left(\frac{\eta_{\phi^+}}{k_{\phi^+}} - \frac{\eta_{\phi^0}}{k_{\phi^0}} - \frac{\eta_{W^+}}{k_{W^+}}\right)$$

$$\mathcal{S}_{\rm s,sph} \equiv \gamma_{\rm s,sph}\sum_{i=1}^{N_{\rm g}}\left(\frac{\eta_{u_L^i}}{k_{u_L^i}} + \frac{\eta_{d_L^i}}{k_{d_L^i}} - \frac{\eta_{u_R^i}}{k_{u_R^i}} - \frac{\eta_{d_R^i}}{k_{d_R^i}}\right) ,$$

$$\mathcal{S}_{\rm w,sph} \equiv \gamma_{\rm w,sph}\sum_{i=1}^{N_{\rm g}}\left(\frac{N_{\rm c}}{2}\frac{\eta_{u_L^i}}{k_{u_L^i}} + \frac{N_{\rm c}}{2}\frac{\eta_{d_L^i}}{k_{d_L^i}} + \frac{1}{2}\frac{\eta_{\nu_L^i}}{k_{\nu_L^i}} + \frac{1}{2}\frac{\eta_{e_L^i}}{k_{e_L^i}}\right)$$

$\eta = n/s$

$x = T/H \sim M_{\rm pl}/T$

k = # degree of freedom

$$\mathcal{S}^{ij}_{\rm Uu} \equiv \gamma^{ij}_{\rm Uu}\left(\frac{\eta_{u_L^i}}{k_{u_L^i}} - \frac{\eta_{u_R^j}}{k_{u_R^j}}\right) ,$$

$$\mathcal{S}^{ij}_{\rm Dd} \equiv \gamma^{ij}_{\rm Dd}\left(\frac{\eta_{d_L^i}}{k_{d_L^i}} - \frac{\eta_{d_R^j}}{k_{d_R^j}}\right) ,$$

$$\mathcal{S}^{ij}_{\rm Ee} \equiv \gamma^{ij}_{\rm Ee}\left(\frac{\eta_{e_L^i}}{k_{e_L^i}} - \frac{\eta_{e_R^j}}{k_{e_R^j}}\right) ,$$

$$\mathcal{S}_{\rm hw} \equiv \gamma_{\rm hw}\left(\frac{\eta_{\phi^+}}{k_{\phi^+}} - \frac{\eta_{W^+}}{k_{W^+}}\right)$$

$$\mathcal{S}_{\rm h} \equiv \gamma_{\rm h}\frac{\eta_{\phi^0}}{k_{\phi^0}} .$$

$$\mathcal{S}^{\rm bkg}_{\rm y} = \frac{1}{sT}\frac{\alpha_{\rm y}}{4\pi}\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\overline{\langle Y_{\mu\nu}\rangle\langle Y_{\rho\sigma}\rangle}$$

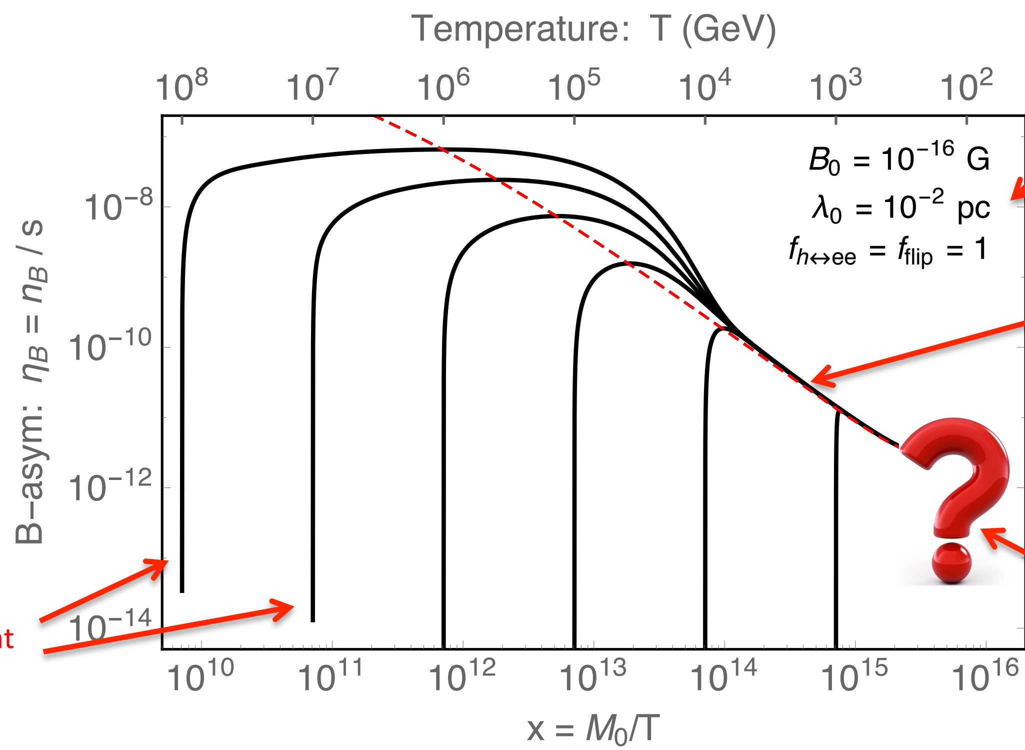
$$\mathcal{S}^{\rm bkg}_{\rm w} = \frac{1}{sT}\frac{1}{2}\frac{\alpha_{\rm w}}{4\pi}\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\overline{\langle W^a_{\mu\nu}\rangle\langle W^a_{\rho\sigma}\rangle}$$

$$\mathcal{S}^{\rm bkg}_{\rm yw} = \frac{1}{sT}\frac{gg'/4\pi}{4\pi}\epsilon^{\mu\nu\rho\sigma}\overline{\langle Y_{\mu\nu}\rangle\langle W^3_{\rho\sigma}\rangle} .$$

Related work: Giovannini & Shaposhnikov; Fujita & Kamada; AL, Sabancilar, & Vachaspati; Semikoz, Dvornikov, Smirnov, Sokoloff, Valle

Numerical Results

Evolution before EW crossover



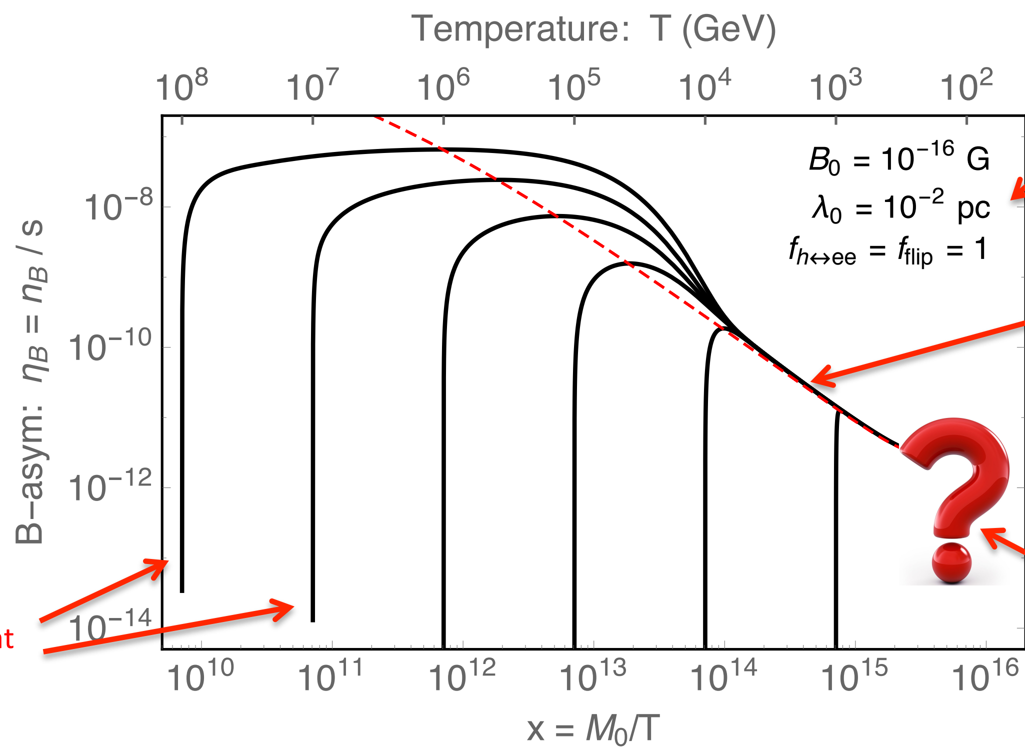
field strength & coherence length today

source term balances against washout by sphaleron

turn on B-field at different times

we'll discuss evolution through the EW crossover next

Evolution before EW crossover



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turn on B-field at different times

equilibrium baryon asymmetry: nB / s

source from decaying magnetic helicity

washout due to sphaleron + Yukawa interactions + chiral magnetic effect

$$\eta_B^{(\mathrm{eq})} \approx \frac{\# \frac{\alpha_{\mathrm{y}}}{sT}\left(\boldsymbol{B}_Y \cdot \boldsymbol{\nabla} \times \boldsymbol{B}_Y\right)/\sigma_Y}{\#|y_e|^2 m_h^2(T)/T^2 + \# \frac{\alpha_{\mathrm{y}}^2}{T^3}|\boldsymbol{B}_Y|^2/\sigma_Y} \simeq \left(4 \times 10^{-12}\right) \frac{B_{14}^2}{\lambda_1} \frac{\left(T/T_{\mathrm{w}}\right)^{4/3}}{0.08\, m_h^2(T)/T^2 + B_{14}^2 \left(T/T_{\mathrm{w}}\right)^{2/3}}$$

order 1 numbers

$$\left(B_{14} \equiv B_0/(10^{-14}\,\mathrm{G})\,,\;\; \lambda_1 \equiv \lambda_0/(1\,\mathrm{pc})\,,\;\; T_{\mathrm{w}} \equiv 162\,\mathrm{GeV} \right)$$

Let's play with the parameters until we get $\eta_B \sim 10^{-10}$ to match observed BAU

… while keeping $$\left(\frac{\lambda_0}{1\ \mathrm{pc}}\right) \sim \left(\frac{B_0}{10^{-14}\ \mathrm{Gauss}}\right)$$

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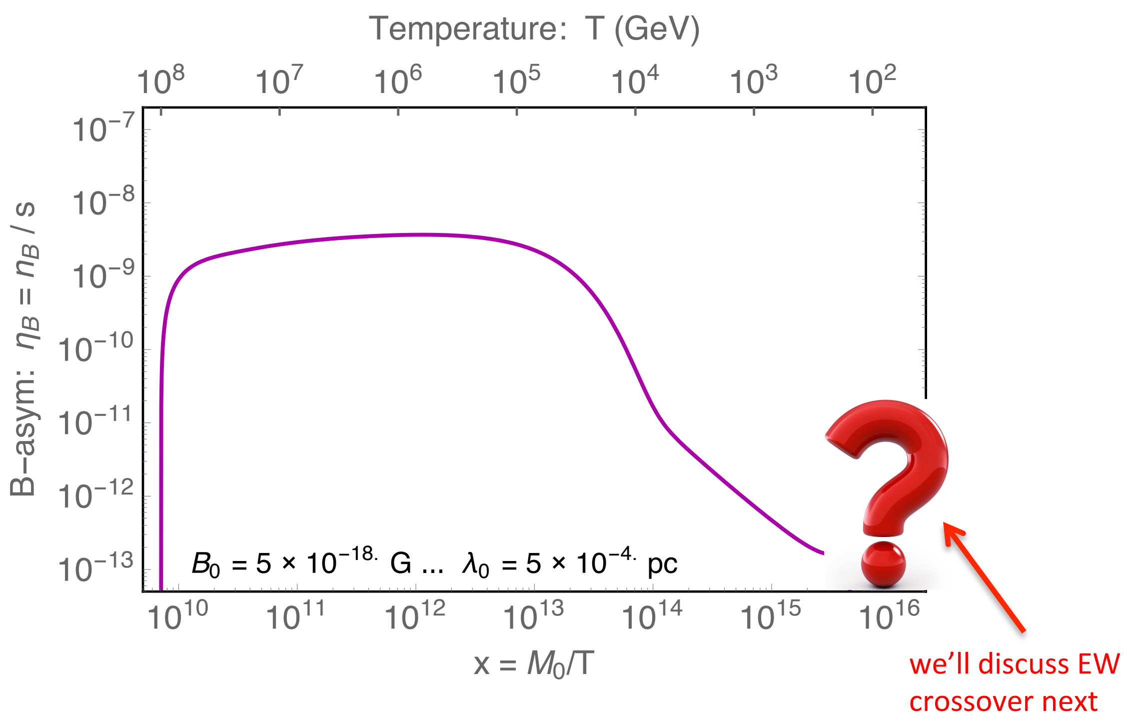

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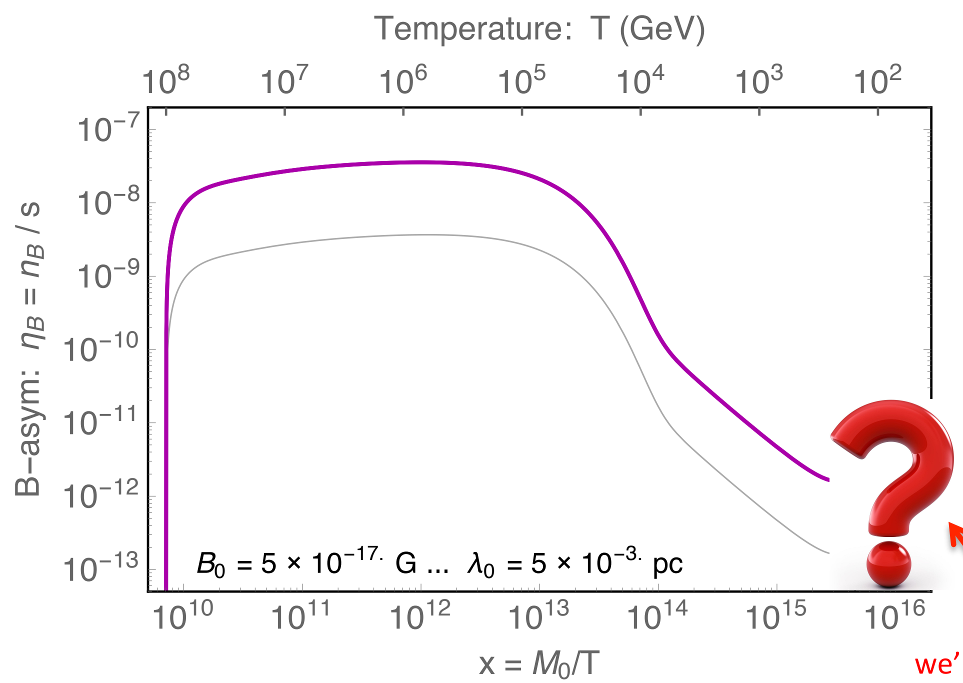

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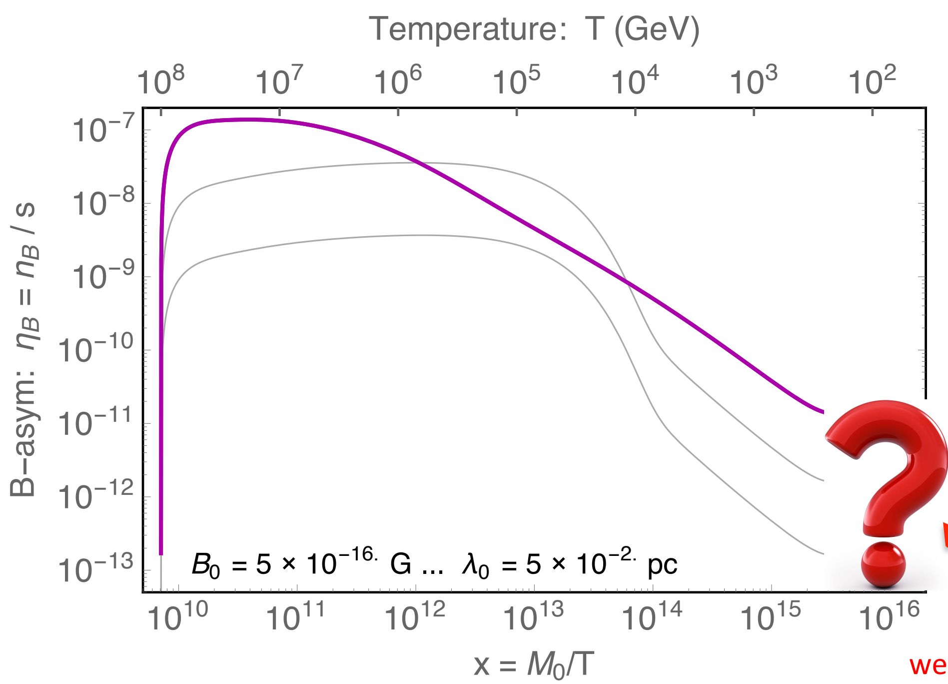

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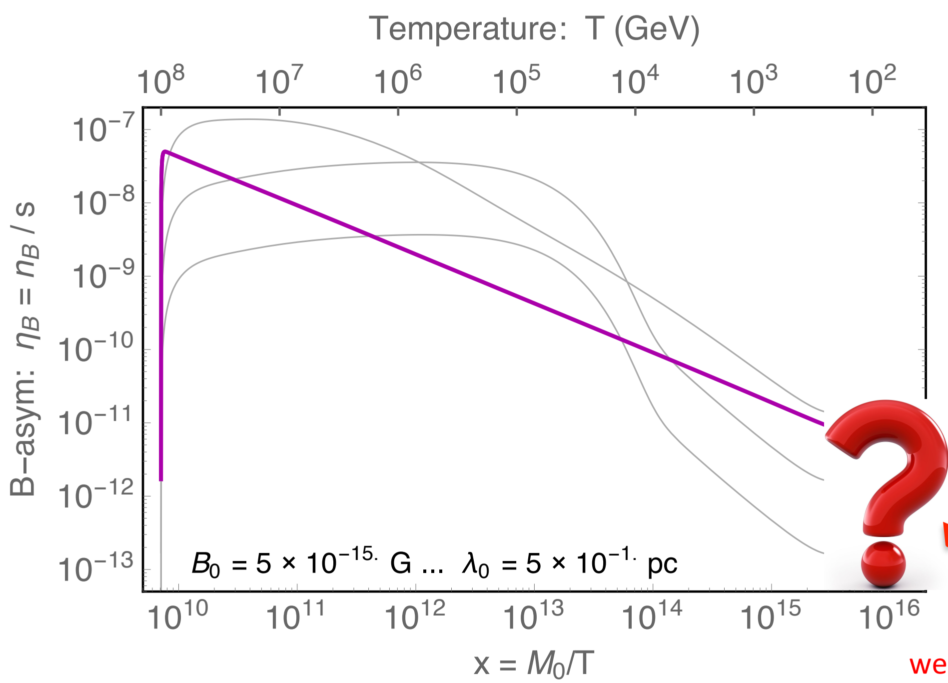

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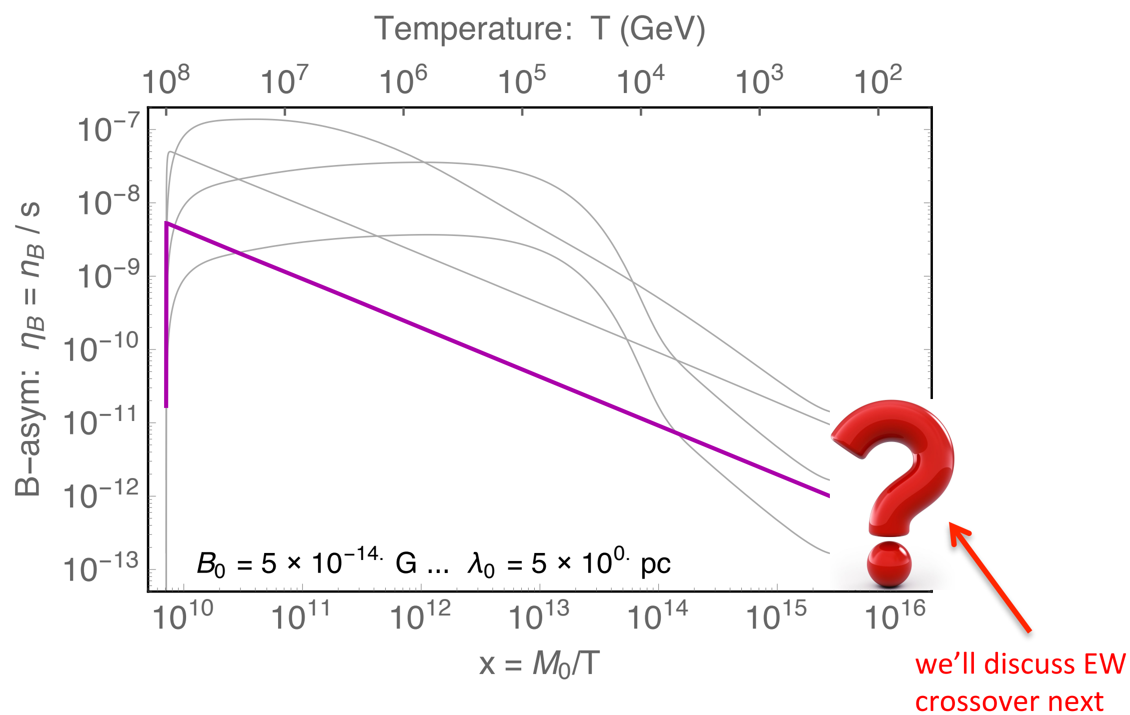

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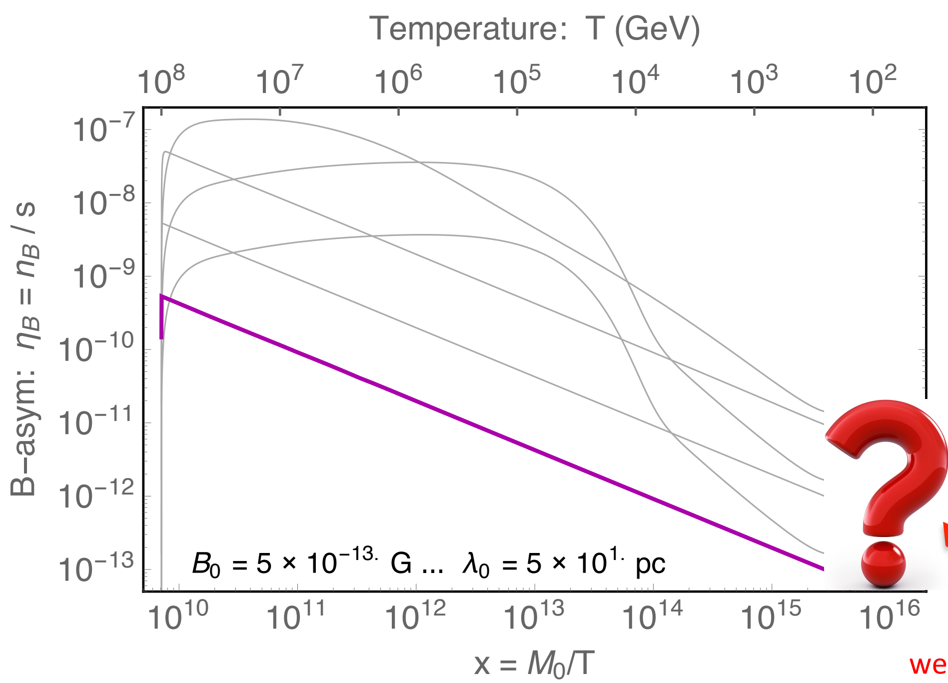

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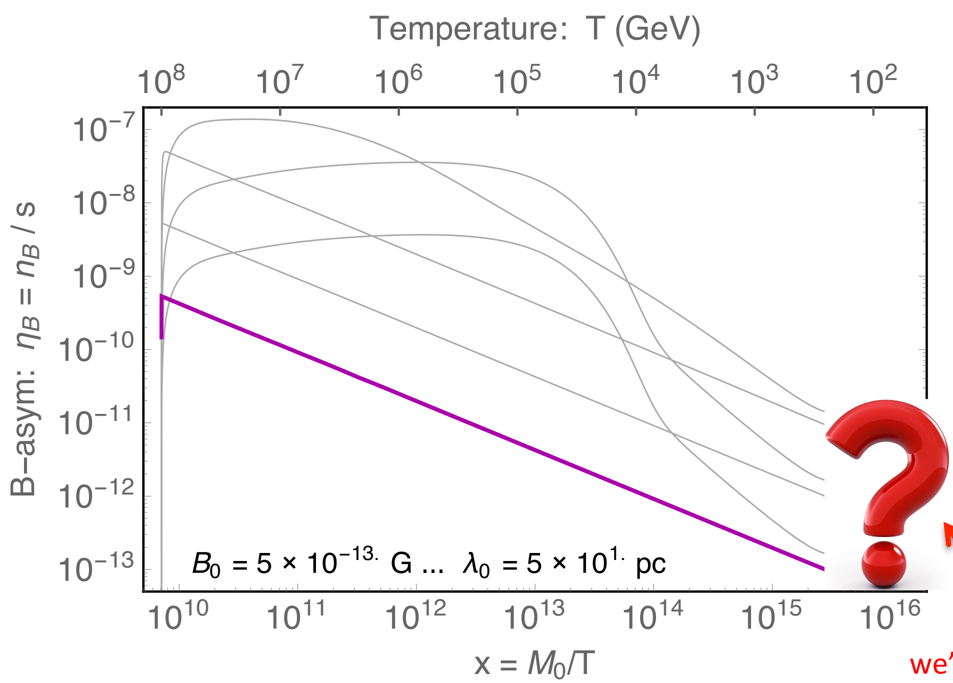

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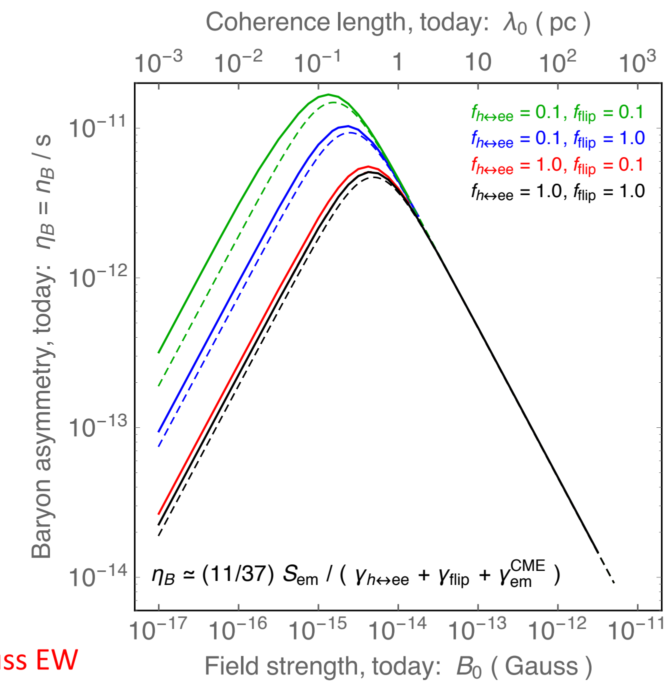

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Washout induced by chiral magnetic effect … prevents η_B from reaching 10^{-10} for large B_0.
This behavior was overlooked in some previous studies. The CME cannot be neglected!

What happens at the EW crossover?

Evolution through EW Crossover

$$\frac{d}{dt} n_B = -\Gamma_{\text{sphaleron}}\, n_B + \mathcal{S}_{\text{hypermagnetic}}$$

At this time…

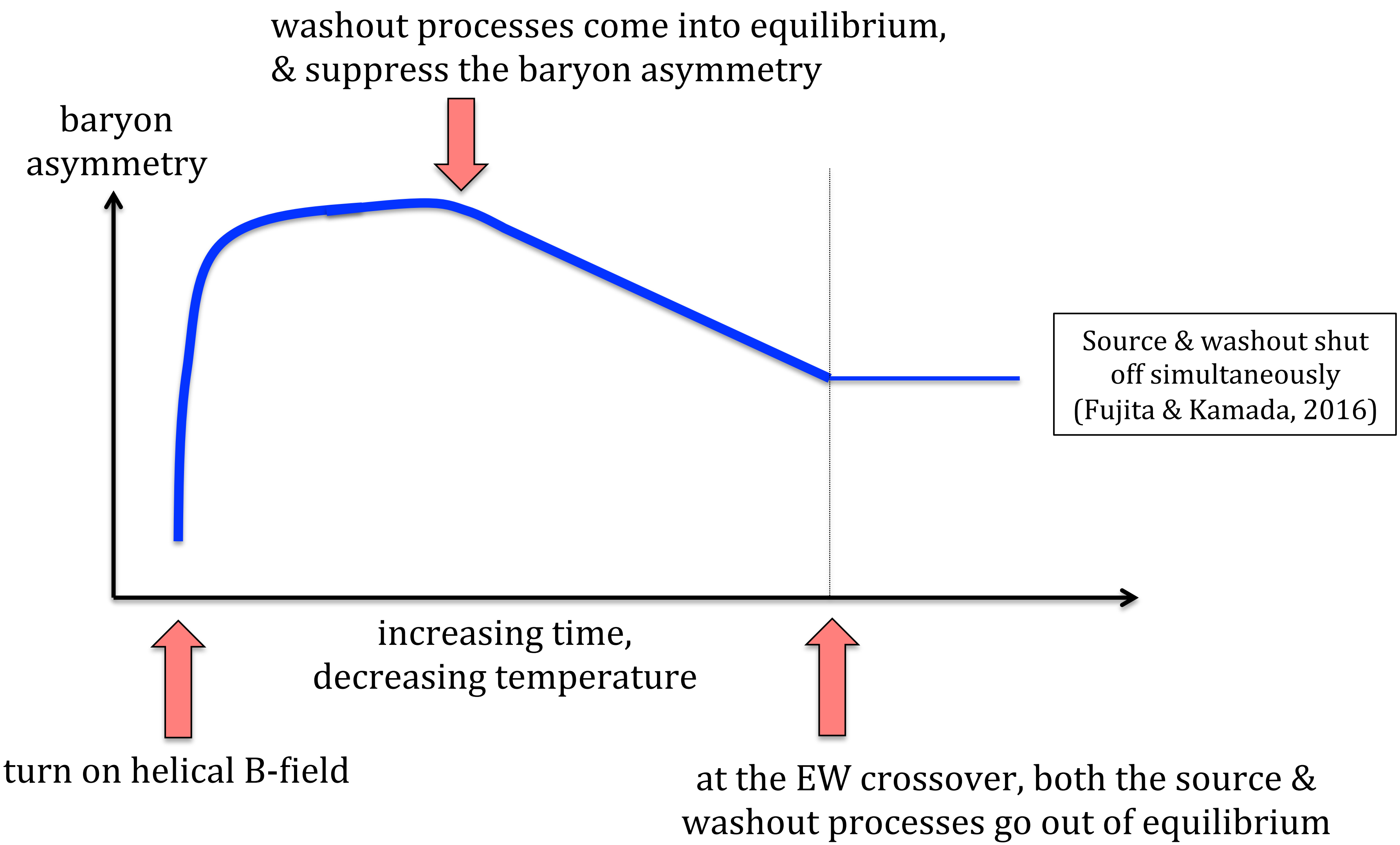
… the *source shuts off*, because the $U(1)_Y$ field is converted into a $U(1)_{em}$ field, which does not source B-number.

$$\partial j_B \sim W\tilde{W} - Y\tilde{Y} \neq F\tilde{F}$$

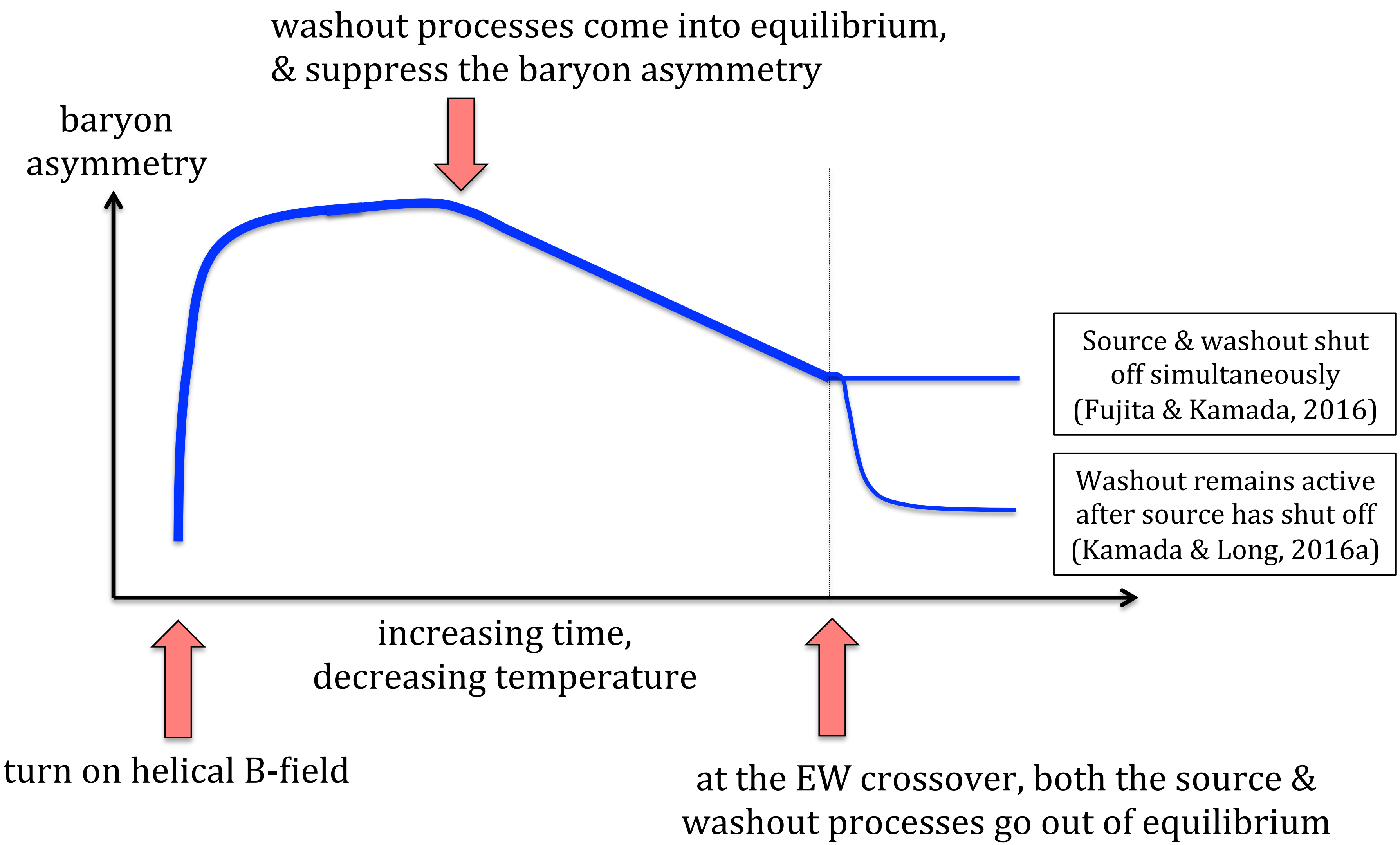
… the *washout shuts off*, because the W-boson mass grows, suppressing EW sphaleron transitions.

$$\Gamma_{\text{sph.}} \propto \exp\left[-\#\, M_W(T)/\alpha_W\right]$$

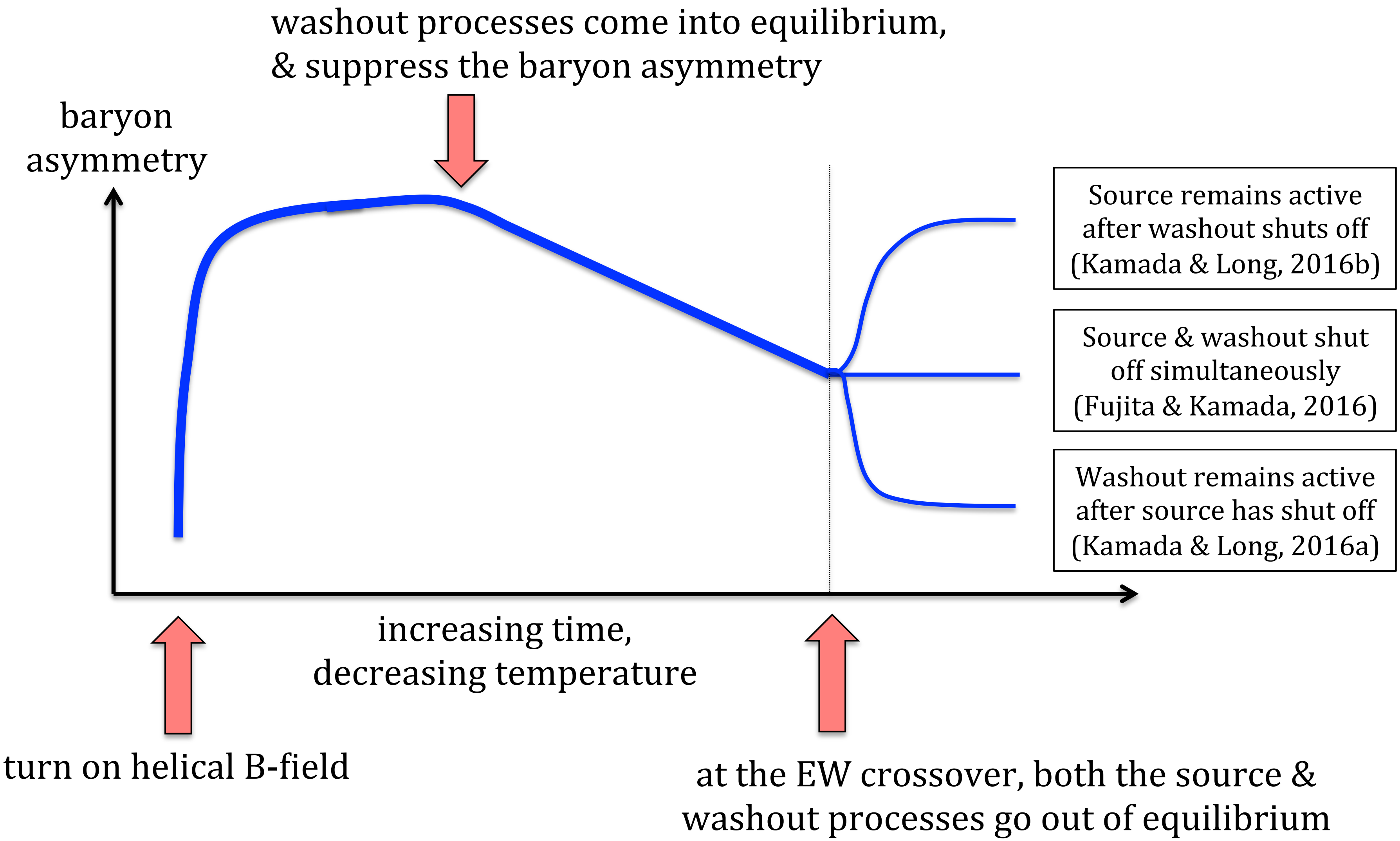
Crossover Evolution Scenarios



Crossover Evolution Scenarios



Crossover Evolution Scenarios

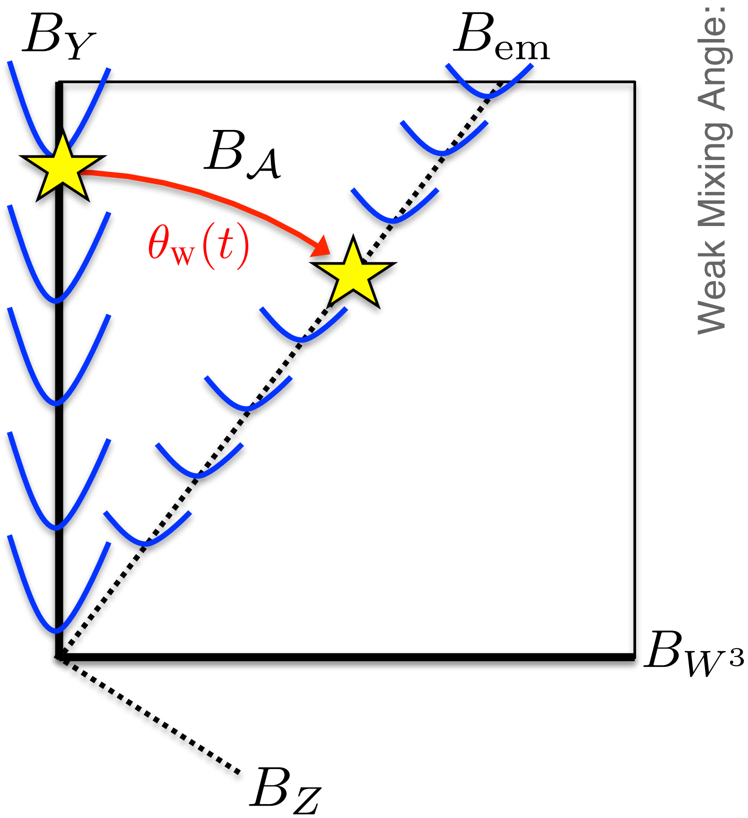


Model the $U(1)_Y$ to $U(1)_{em}$ conversion

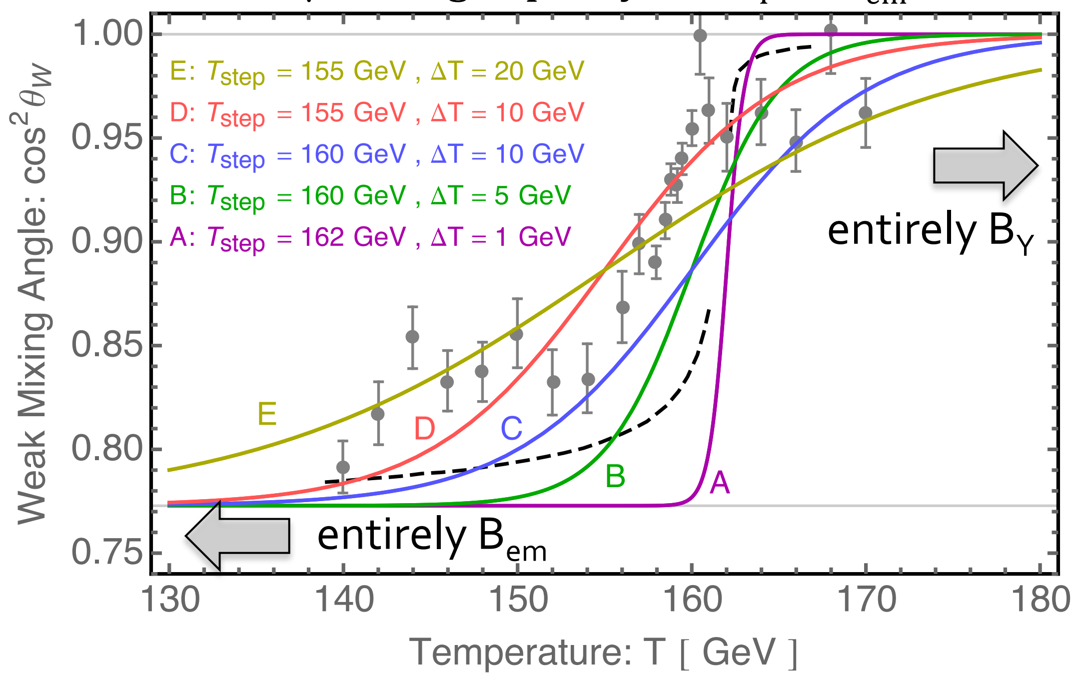
$$\langle W^1_\mu(x)\rangle = \langle W^2_\mu(x)\rangle = 0$$

$$\langle W^3_\mu(x)\rangle = \sin\theta_{\rm w}(t)\,\mathcal{A}_\mu(x)$$

$$\langle Y_\mu(x)\rangle = \cos\theta_{\rm w}(t)\,\mathcal{A}_\mu(x)$$

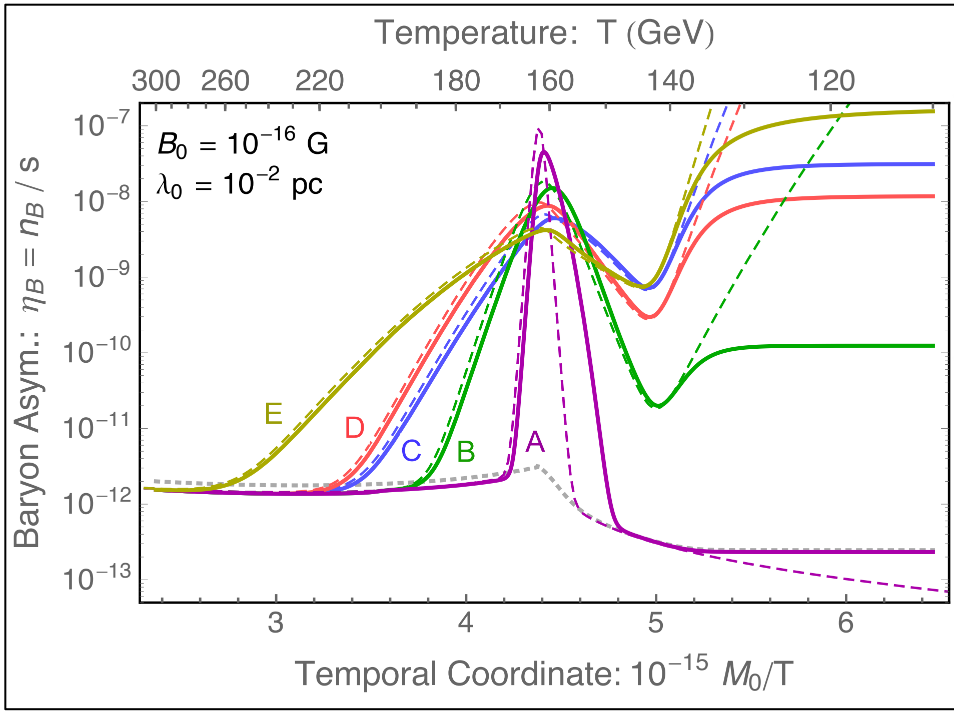


Z-γ mixing is proxy for B_Y to B_{em}



dots & error bars = lattice simulation from D'Onofrio & Rummukainen (2015).
black dashed = analytic approx. from Kajantie, Laine, Rummukainen, & Shaposhnikov (1996)
colored curves = we use tanh functions to model the crossover

BAU Evolution through EW Crossover



realistic regime … source active after washout shuts off

pessimistic regime … washout active after source shuts off

$$\eta_B^{\mathrm{eq}} \approx \frac{11}{37} \frac{g'^2 \left(\cos^2\theta_w \, \mathcal{S}_{\mathrm{BdB}} + \frac{d\theta_w}{d\ln x} \sin 2\theta_w \, \mathcal{S}_{\mathrm{AB}}\right)}{\frac{1}{2}\left(\gamma^{11}_{\mathrm{Ehe}} + \gamma^{11}_{\nu\mathrm{he}}\right) + \gamma^{11}_{\mathrm{Ee}} + g'^4 \cos^4\theta_w \gamma^{\mathrm{CME}}}$$

$A \cdot B$

$B \cdot \nabla \times B$

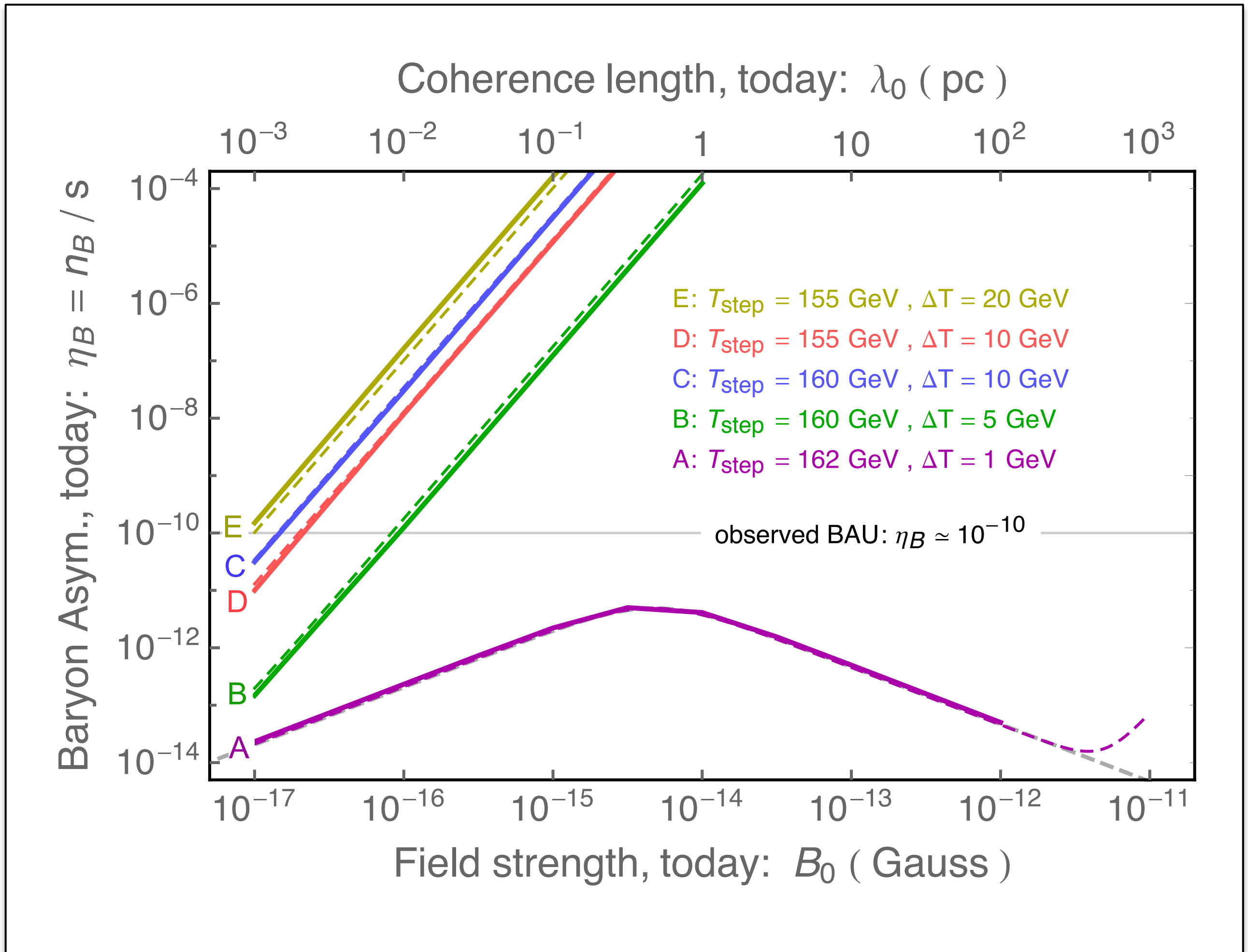
Result: Predicted Baryon Asymmetry

The conversion of $U(1)_Y$ B-field into $U(1)_{em}$ B-field at the EW crossover is not well-understood.

However, the relic baryon asymmetry depends sensitively on these details.

Consequently, the predicted baryon asymmetry is very uncertain.

Need to understand the crossover better!



Where to go from here?

Refinements:

Study conversion of magnetic fields at EW crossover

➔ main message in this section

Calculate helicity decay directly with MHD simulations

➔ "not difficult" to implement

Implications & Applications:

Study baryogenesis from axion inflation (etc.) self-consistently

➔ Various studies: Anber & Sabancilar (2015); Cado & Sabancilar (2016); Jimenez, Kamada, Schmitz, & Xu (2017)

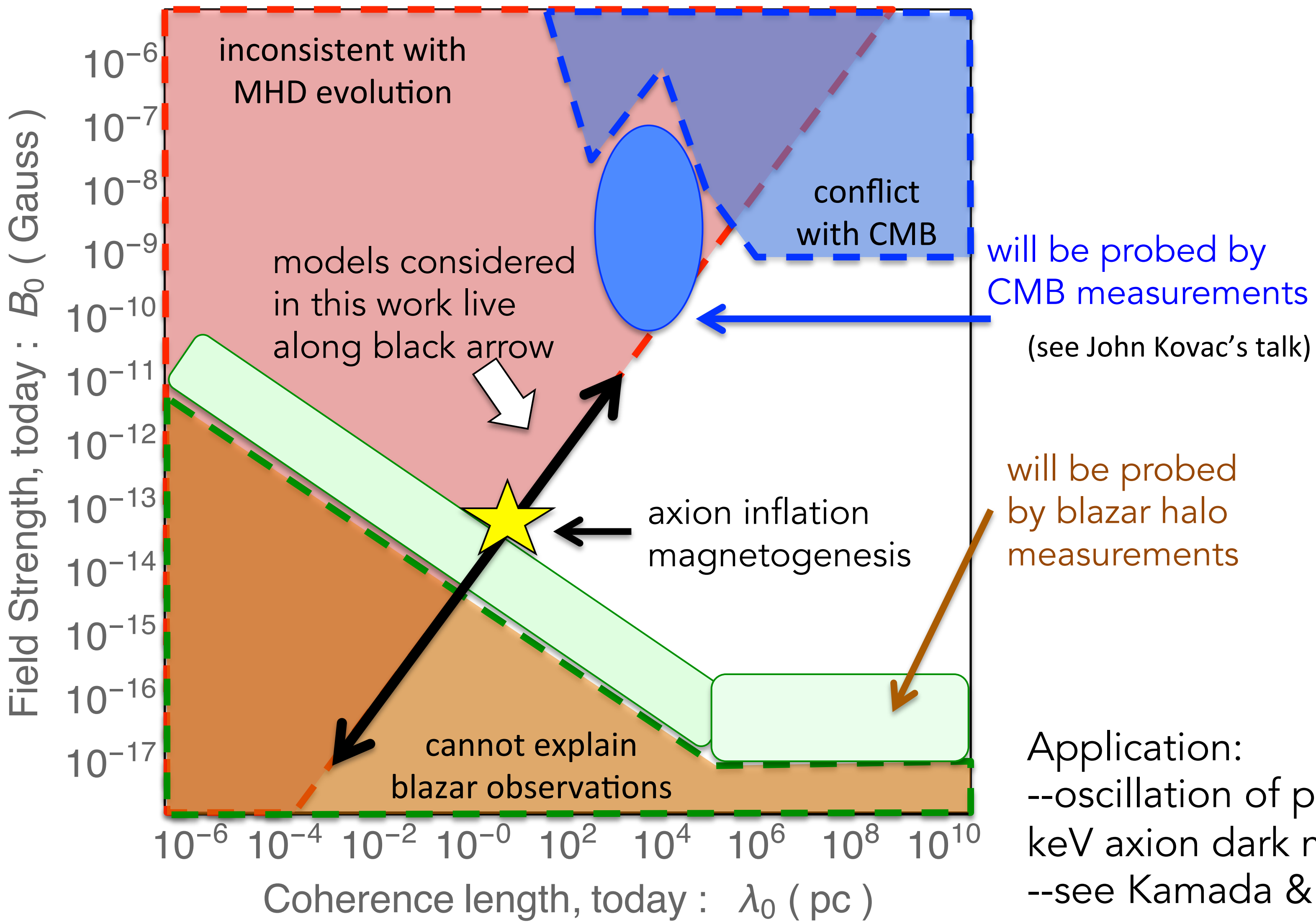
Observation side – develop new probes of relic (helical) magnetic fields

➔ E.g., using cascade halos around TeV blazars

Study "dark" magnetic field (hidden sector U1) and /or dark matter production

➔ E.g., Cado & Sabancilar (2016)

Implications & Applications



Application:
--oscillation of photons into keV axion dark matter
--see Kamada & Nakai (2017)

(figure adapted from
Durrer & Neronov, 2013)

Magnetic Field Scaling Law

Comoving quantities: $\tilde{B}(\tau) = a(t)^2 B_p(t)$ $\quad \tilde{\lambda}(\tau) = a(t)^{-1}\lambda_B(t)$

Adiabatic evolution after recombination: $\tilde{B}_{\rm rec} = B_0$ $\quad \tilde{\lambda}_{\rm rec} = \lambda_0$

Coherence length tracks eddy scale:

$$\tilde{\lambda}(\tau) = C\, v_A(\tau)\tau$$

$$v_A(\tau) = c/\sqrt{1+(\rho+P)/(2P_m)} \propto \tilde{B}(\tau)$$

$$P_m(\tau) = \tilde{B}(\tau)^2/2$$

Helicity is quasi-conserved: $\tilde{\lambda}\tilde{B}^2 = \tilde{\lambda}_{\rm rec}\tilde{B}^2_{\rm rec}$ $\quad$ H ~ λ B^2 for maximally helical field

Solution:

$$B_p = (a/a_0)^{-2}(\tau/\tau_{\rm rec})^{-1/3} B_0$$

$$\lambda_B = (a/a_0)(\tau/\tau_{\rm rec})^{2/3}\lambda_0$$

"inverse cascade"

Baryogenesis without (B-L)?

Recall that (B-L) = 0 at all times! But, Kuzmin, Rubakov, & Shaposhnikov ('85) taught us that B → 0 and L → 0 in equilibrium. **How is washout avoided?**

In the **symmetric phase** (T > 160 GeV), the EW sphaleron tries to drive (B+L) to zero, but the $U(1)_Y$ field sources (B+L) and prevents B,L → 0.

$$\partial j_B \sim W\tilde{W} - Y\tilde{Y}$$

In the **broken phase** (T < 160 GeV), the EW sphaleron remains in equilibrium until T~140 GeV. Since the $U(1)_{em}$ field doesn't source B-number (because, vector-like interactions), why doesn't B-number washout? … The $U(1)_{em}$ field sources chiral charge (like in QED) and prevents B-washout in the R-chiral fermions.

toy model

$$\frac{d\eta_L}{dx} = -\gamma_{\rm sph}\eta_L + \gamma_{\rm flip}\big(\eta_R - \eta_L\big) - \mathcal{S}_{\rm em}$$
$$\frac{d\eta_R}{dx} = -\gamma_{\rm flip}\big(\eta_R - \eta_L\big) + \mathcal{S}_{\rm em}$$

⇒

$$\eta_{L,\rm eq} = 0$$
$$\eta_{R,\rm eq} = \frac{\mathcal{S}_{\rm em}}{\gamma_{\rm flip}}$$

Baryogenesis from Axion Inflation via Decaying Magnetic Helicity

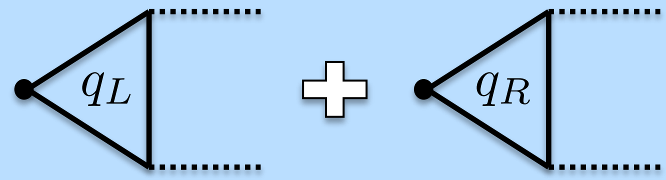
… via Chiral Gravitational Waves

work in progress with Peter Adshead & Evangelos Sfakianakis

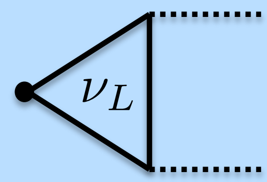
Cosmology of the Clockwork Axion

Gravitational Anomaly

$$\partial_\mu j^\mu_B = 0 =$$



$$\partial_\mu j^\mu_L = \frac{N_{\rm gen}}{24}\frac{1}{16\pi^2}\, R\tilde{R} =$$



Delbourgo & Salam (1972); Eguchi & Freund (1976); Alvarez-Gaume & Witten (1984)

Gravitational interactions are vector-like, but SM matter content is chiral: SM includes L-handed ν's but not R-handed ν's.

What sources the anomaly?

The source term is

$$R\widetilde{R} \equiv \frac{1}{2}\epsilon^{\alpha\beta\gamma\delta} R_{\alpha\beta\rho\sigma} R_{\gamma\delta}{}^{\rho\sigma}$$

In terms of linearized metric perturbations,

$$\int \mathrm{d}^3x\, R\widetilde{R} = \frac{16}{a(\tau)^4}\frac{2}{M_{\mathrm{pl}}^2}\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3}\Big(k^3\partial_\tau h_R(\tau,\boldsymbol{k})^* h_R(\tau,\boldsymbol{k}) - k\partial_\tau^2 h_R(\tau,\boldsymbol{k})^*\partial_\tau h_R(\tau,\boldsymbol{k}) - \big(R \longrightarrow L\big)\Big)$$

The source term arises in a background of polarized gravitational waves with a changing chirality. (Recall that F.Fdual comes from magnetic field with changing helicity.)

Source is absent in vacuum in GR.

Can we use the SM gravitational anomaly to do baryogenesis?

Ibanez & Quevedo (1992) – first to consider the question
Alexander, Peskin, & Sheikh-Jabbari (2006) –concrete implementation
Fischler & Paban (2007) – address UV sensitivity
Maleknejad & Sheikh-Jabbari (2011, 2013) – further refinements
Maleknejad (2016) – further refinements
Caldwell & Devulder (2017) – does work in chromo-natural inflation
Papageorgiou & Peloso (2017) – doesn't work in natural inflation

Gravitational Leptogenesis

Alexander, Peskin, & Sheikh-Jabbari (2006)

(1) Let inflation be driven by a slowly-rolling axion.

(2) Introduce a non-minimal gravitational coupling for the axion.

$$\mathcal{L}_{\mathrm{int}} = \frac{1}{16\pi^2}\frac{\phi}{f}R\tilde{R}$$

(3) Rolling axion leads to growth of chiral gravitational waves.

$$\left.\begin{aligned}\Box h_L &= -2i\Theta\dot{h}'_L/a \\ \Box h_R &= +2i\Theta\dot{h}'_R/a\end{aligned}\right\} \quad \text{with} \quad \Theta = \frac{8H}{M_{\mathrm{pl}}^2}\frac{1}{16\pi^2}\frac{\dot{\phi}}{f}$$

(4) Which sources lepton-number

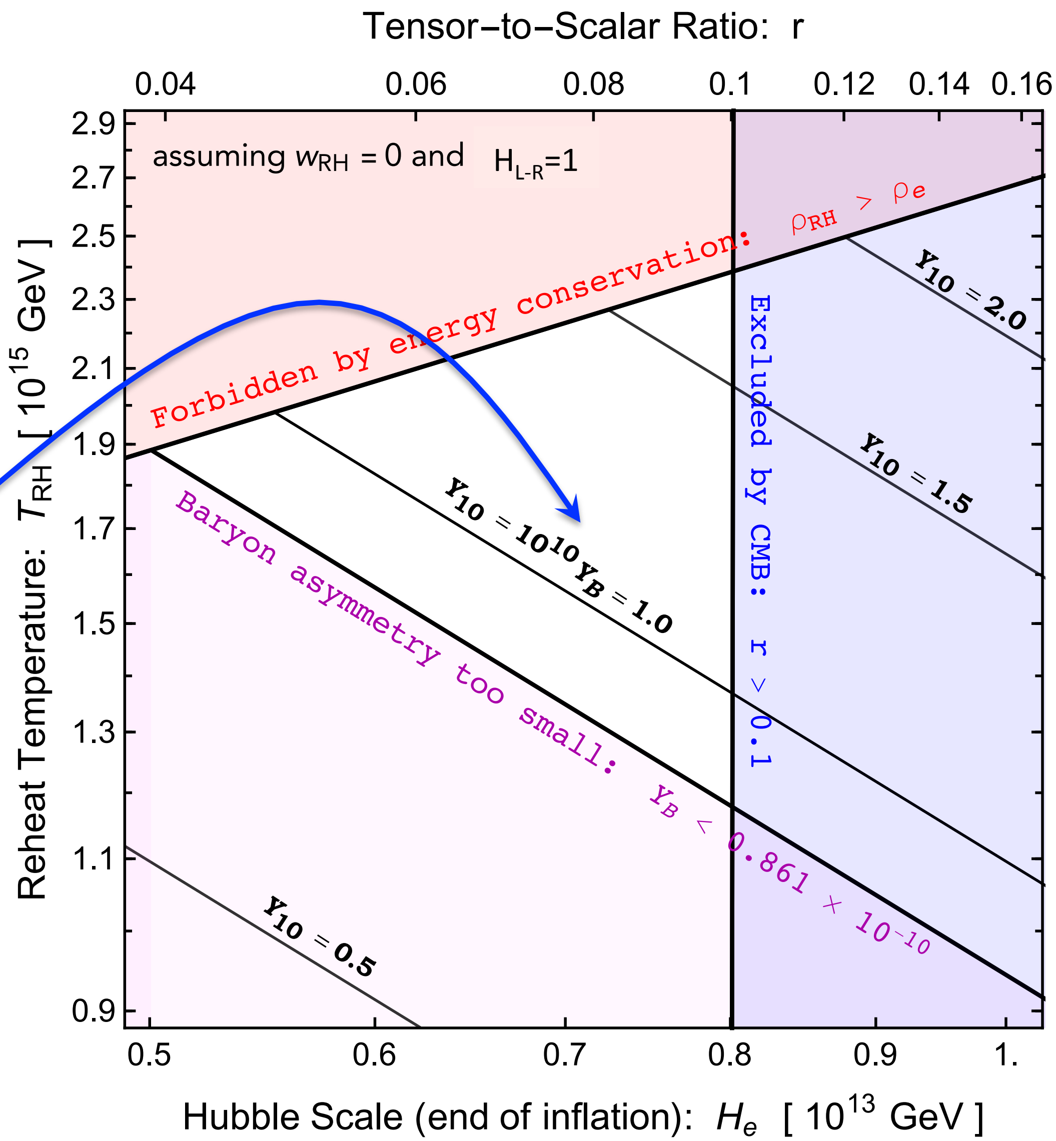
$$Y_L \sim (1\times 10^{-10})\left(\frac{H_e}{10^{13}\ \mathrm{GeV}}\right)\left(\frac{T_{\mathrm{RH}}}{10^{15}\ \mathrm{GeV}}\right)\mathcal{H}^{\mathrm{GW}}_{L-R}$$

$$\mathcal{H}^{\mathrm{GW}}_{L-R} \equiv H_e^{-3}M_{\mathrm{Pl}}^{-2}\int \mathrm{d}\ln k\left[k^3(\Delta_R^2-\Delta_L^2) - k(\Delta_R'^2-\Delta_L'^2)\right]$$

$$\sim (\Delta\rho_{\mathrm{GW}}/H_e^2M_{\mathrm{pl}}^2)\times(d_H/\lambda_{\mathrm{GW}})$$

Gravitational Leptogenesis

Difficult!



Predictive!
Testable!
Signature = stochastic gravitational wave background is chiral

Neutrino Masses

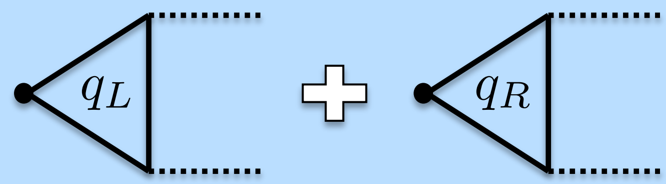
The Standard Model does not accommodate light ν masses.

Solving the ν mass problem kills gravitational leptogenesis.

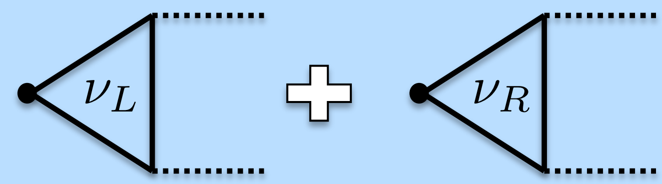
Neutrino Masses -- Dirac

➔ No gravitational anomaly = no source for lepton-number

$$\partial_\mu j^\mu_B = 0 =$$



$$\partial_\mu j^\mu_L = 0 =$$



Neutrino Masses -- Majorana

➔ New heavy particles violate L-number & washout asymmetry.

E.g., for the type-I seesaw

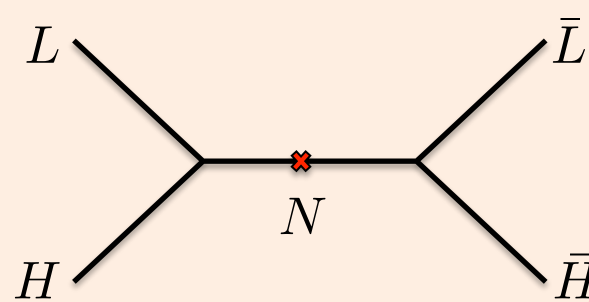
$$\mathcal{L} = \lambda_N L H N + m_N N N + \text{h.c.}$$

$$m_\nu \sim \frac{\lambda_N^2 v^2}{m_N} \sim (0.1\,\text{eV}) \left(\frac{\lambda_N}{1}\right)^2 \left(\frac{10^{14}\ \text{GeV}}{m_N}\right)$$

If $m_N \sim 10^{14}$ GeV and $T_{RH} \sim 10^{15}$ GeV, then heavy Majorana neutrinos (N's) are present in the plasma after reheating.

Their interactions violate L-number ===>

They will washout the lepton asymmetry!



Including washout

Washout rate

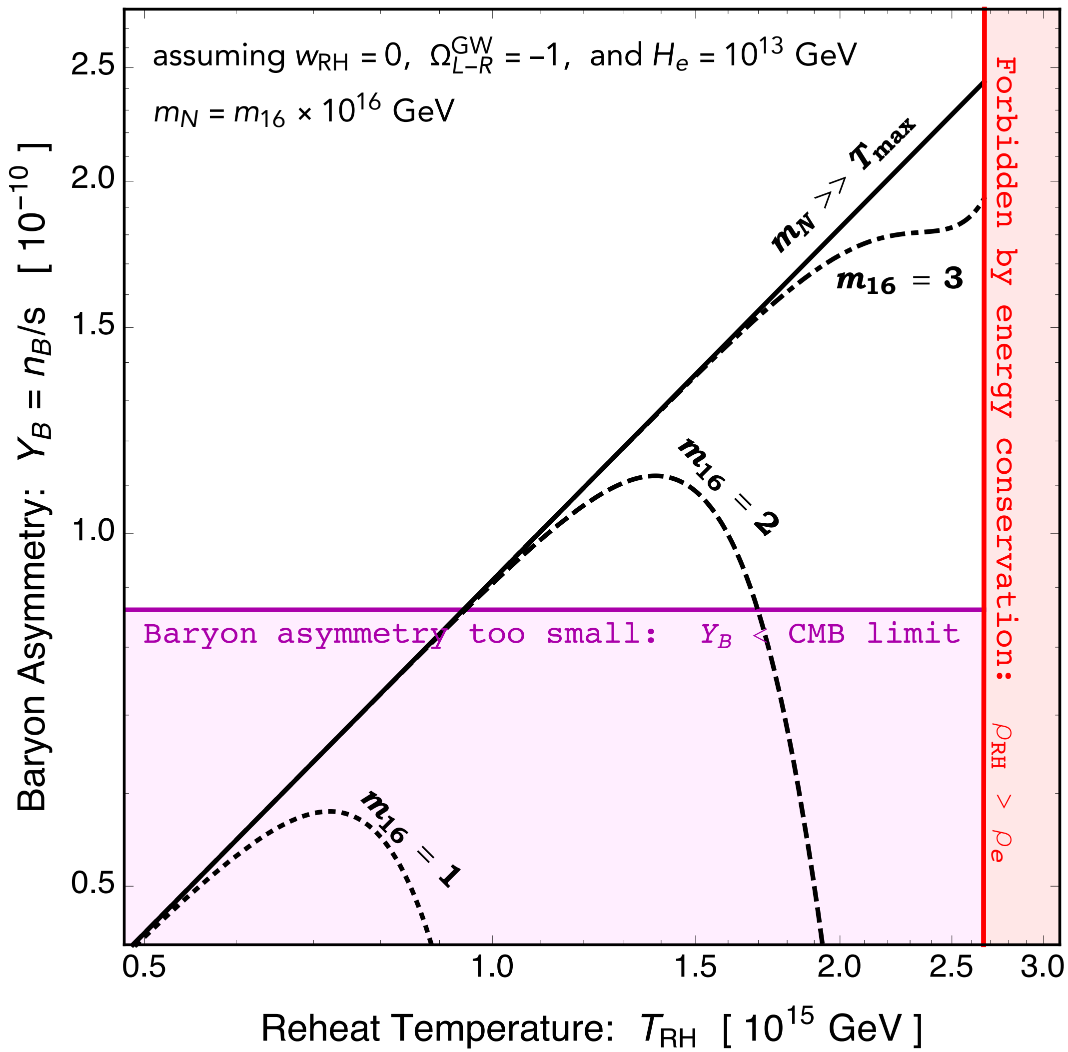
$$\Gamma_{\mathrm{w.o.}} \approx \frac{\lambda_N^2}{24\pi\zeta(3)}\,\frac{m_N^3}{T^2}\,K_1\big(m_N/T\big)$$

After reheating…

$$\frac{d}{dt}n_L + 3Hn_L \approx -\Gamma_{\mathrm{w.o.}}n_L$$

Results ========>

Washout avoidance requires: $m_N >\sim 10\, T_{RH}$



How to save gravitational LG?

Delayed reheating into the SM sector.

To keep the canonical seesaw scale, $m_N \sim 10^{14}$ GeV, the SM must not reheat above $T_{SM} \sim 10^{13}$ GeV.

$H_e \sim 10^{13}\ \mathrm{GeV}$

$\rho_{\rm inf} \sim 100 \times \left(10^{15}\ \mathrm{GeV}\right)^4$

$T_{\rm RH} \sim 10^{15}\ \mathrm{GeV}$

$T_{\rm SM} \lesssim 10^{13}\ \mathrm{GeV}$

Baryogenesis from Axion Inflation via Decaying Magnetic Helicity

… via Chiral Gravitational Waves

Cosmology of the Clockwork Axion

work in progress with Lian-Tao Wang & Andrea Tesi

The Clockwork Axion

Kaplan & Rattazzi (2015)

Consider a family of N+1 complex scalar fields $\phi_n(x)$.

$$\mathscr{L} = \sum_{n=0}^{N}\left[\left|\partial_\mu \phi_n\right|^2 - \lambda\left(|\phi_n|^2 - f_{\rm PQ}^2\right)^2\right] + \epsilon \sum_{n=0}^{N-1}\left[\phi_n^* \phi_{n+1}^3 + \text{h.c.}\right]$$

Respects

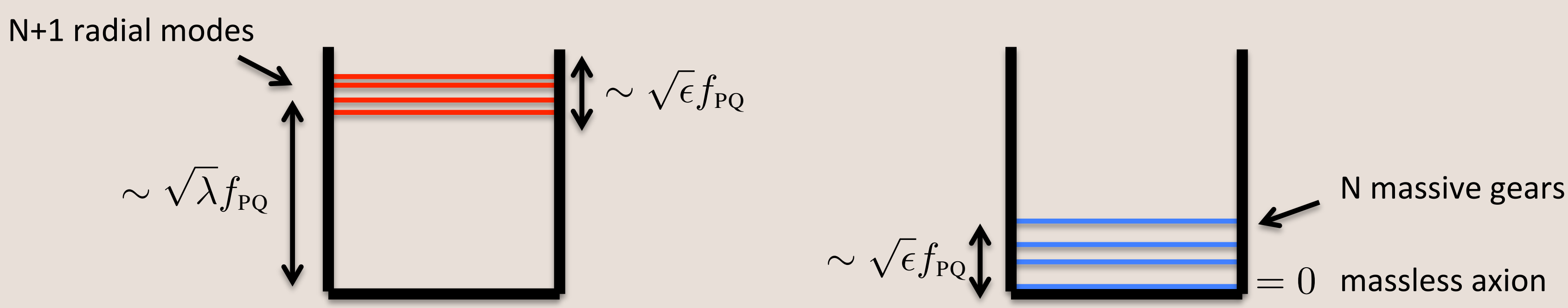
$$\mathrm{U}(1)^{N+1} \quad : \quad \phi_n \to \exp\left[i\theta_n\right]\phi_n$$

Explicitly broken to $(\, q \equiv 3 \,)$

$$\mathrm{U}(1)_{\rm PQ} \quad : \quad \phi_n \to \exp\left[i q^{-n}\theta\right]\phi_n$$

Spectrum.

$$\phi_n = \frac{1}{\sqrt{2}}\left(v_{\rm PQ} + \boldsymbol{\rho}_n\right)\exp\left[i\boldsymbol{\pi}_n / v_{\rm PQ}\right]$$



Typical parameters

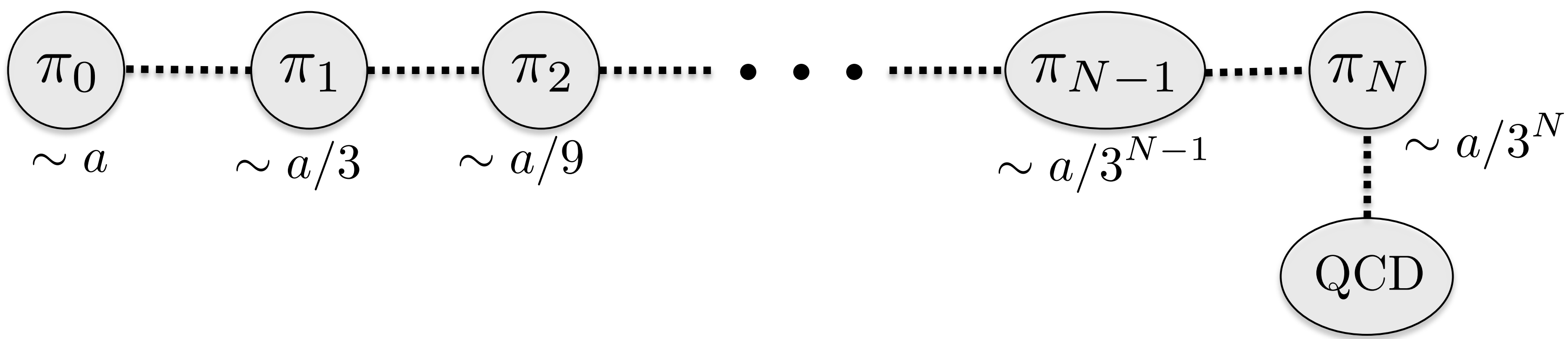
$N \sim 15$
$f_{PQ} \sim 10$ TeV
$m_\rho \sim \lambda^{1/2} f_{PQ} \sim 1$ TeV
$m_G \sim \varepsilon^{1/2} f_{PQ} \sim 100$ GeV
$m_a = 0$ [before QCD effects]

Big problems if axion couples to QCD with strength $1 / f_{PQ}$.
Needs to be much weaker!

Suppressing the coupling to QCD

Axion overlap decreases exponentially with each site

$$\pi_n = \sum_{i=0}^{N} O_{ni}\, A_i = O_{n0}\, a + \sum_{i=1}^{N} O_{ni}\, G_i \qquad O_{n0} = \frac{C}{q^n} \quad \text{with} \quad q \equiv 3$$



Couple last site to QCD

$$\mathcal{L}_{\rm int} = \phi_N Q Q^c$$

$$\Rightarrow \quad \mathcal{L}_{\rm eff} = \frac{\pi_N}{f_{\rm PQ}} G\tilde{G} = \frac{a}{f_a} G\tilde{G} \quad \text{with} \quad f_a = q^N f_{\rm PQ}$$

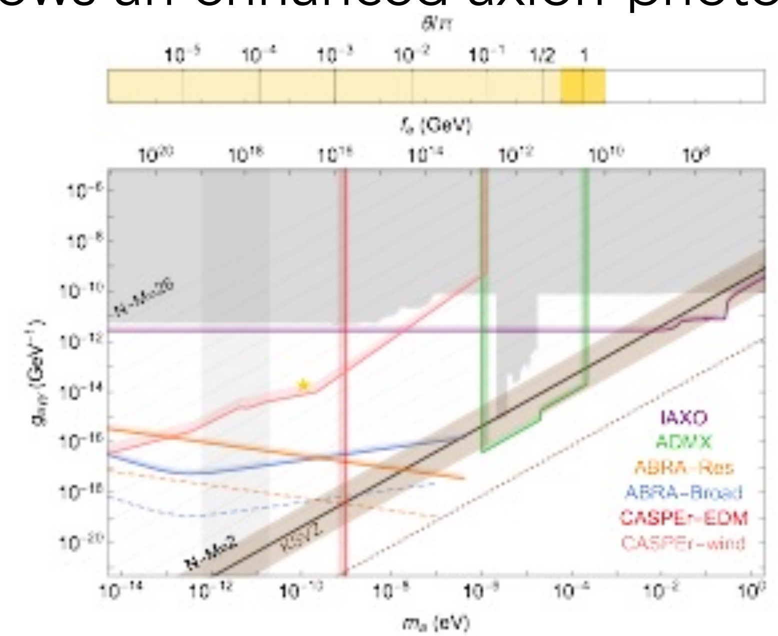
An exponential hierarchy from O(1) parameters!

Phenomenology Implications

Lowers PQ scale to within reach of colliders

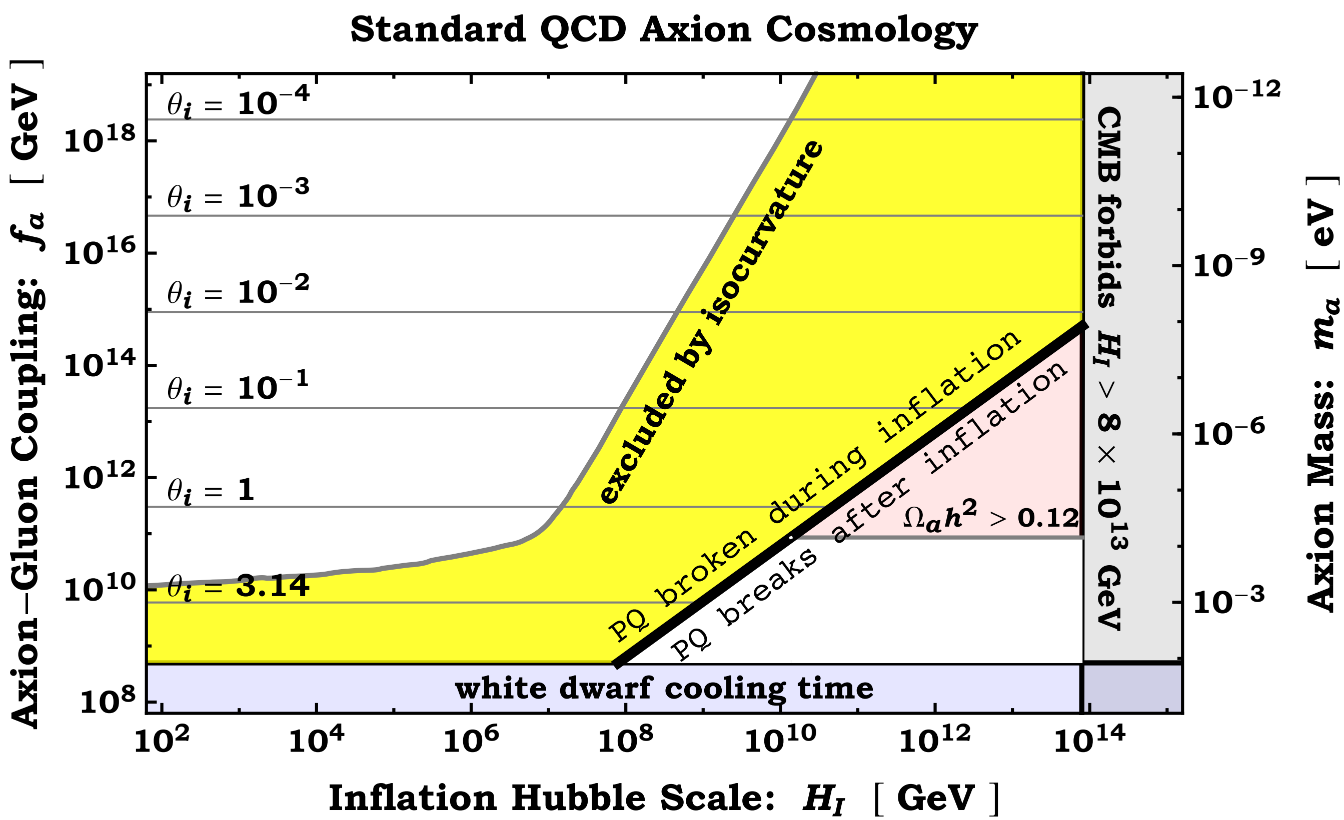
$$\left.\begin{aligned} q &= 3 \\ N &= 15 \\ f_{\rm PQ} &= 10\ \text{TeV} \end{aligned}\right\} \Rightarrow\ f_a = q^N f_{\rm PQ} \sim 10^{11}\ \text{GeV}$$

Allows an enhanced axion-photon coupling



Farina, Pappadopulo, Rompineve, & Tesi (2016)

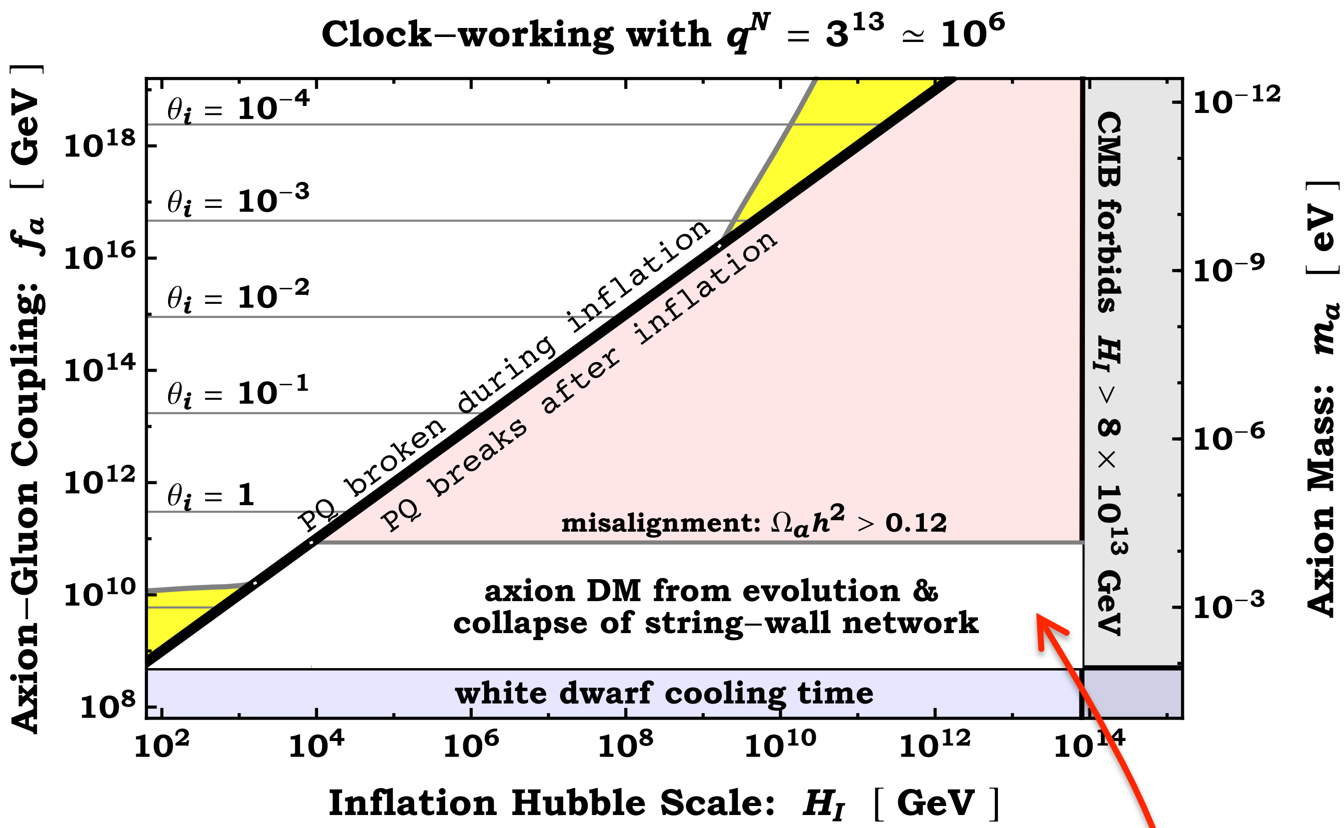
What happens to cosmology?



axion dark matter relic abundance from the misalignment mechanism

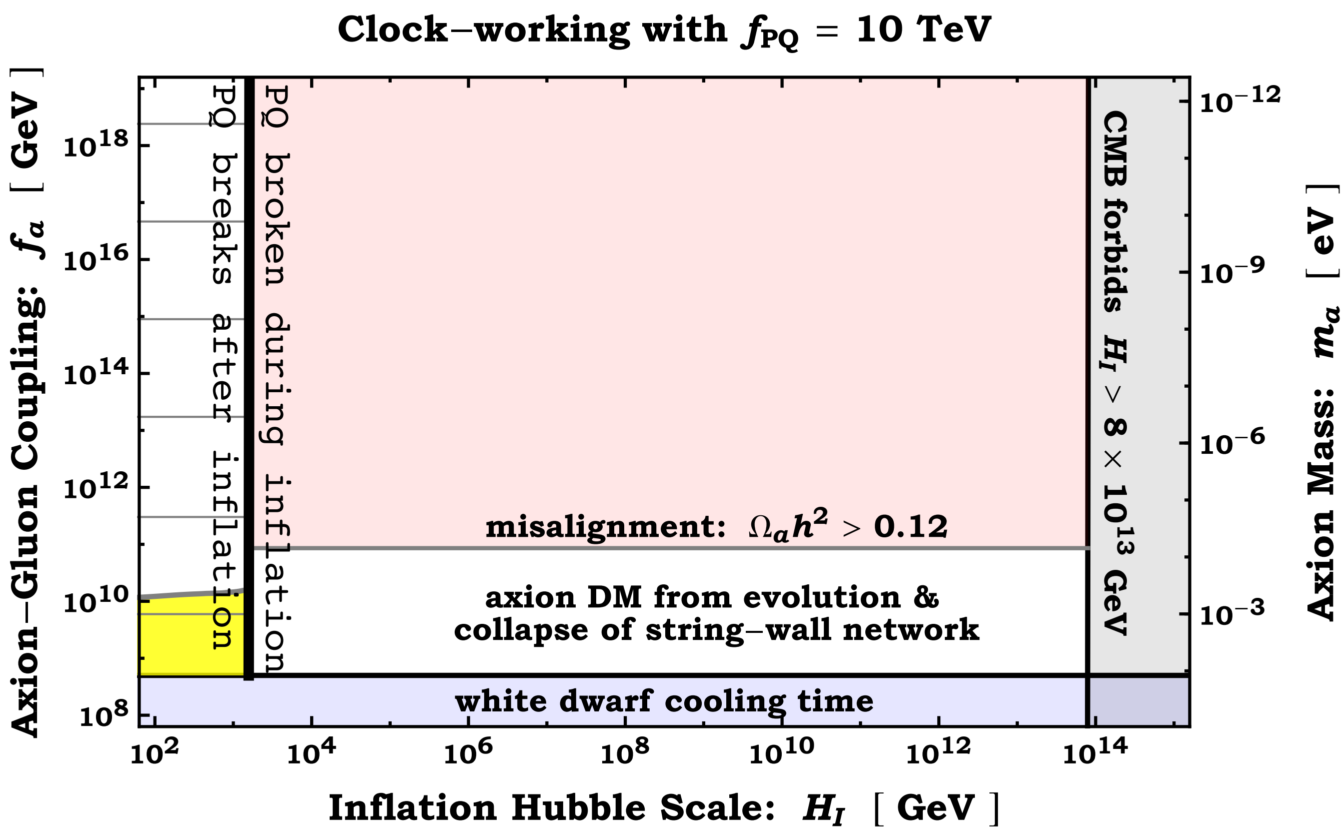
$$\Omega_a h^2 \sim 0.1 \left(\frac{f_a}{10^{11}\ \mathrm{GeV}} \right)^{7/6} \frac{\theta^2}{\pi^2/3}$$

What happens to cosmology?



clock-working widens the
"classical axion window"

What happens to cosmology?

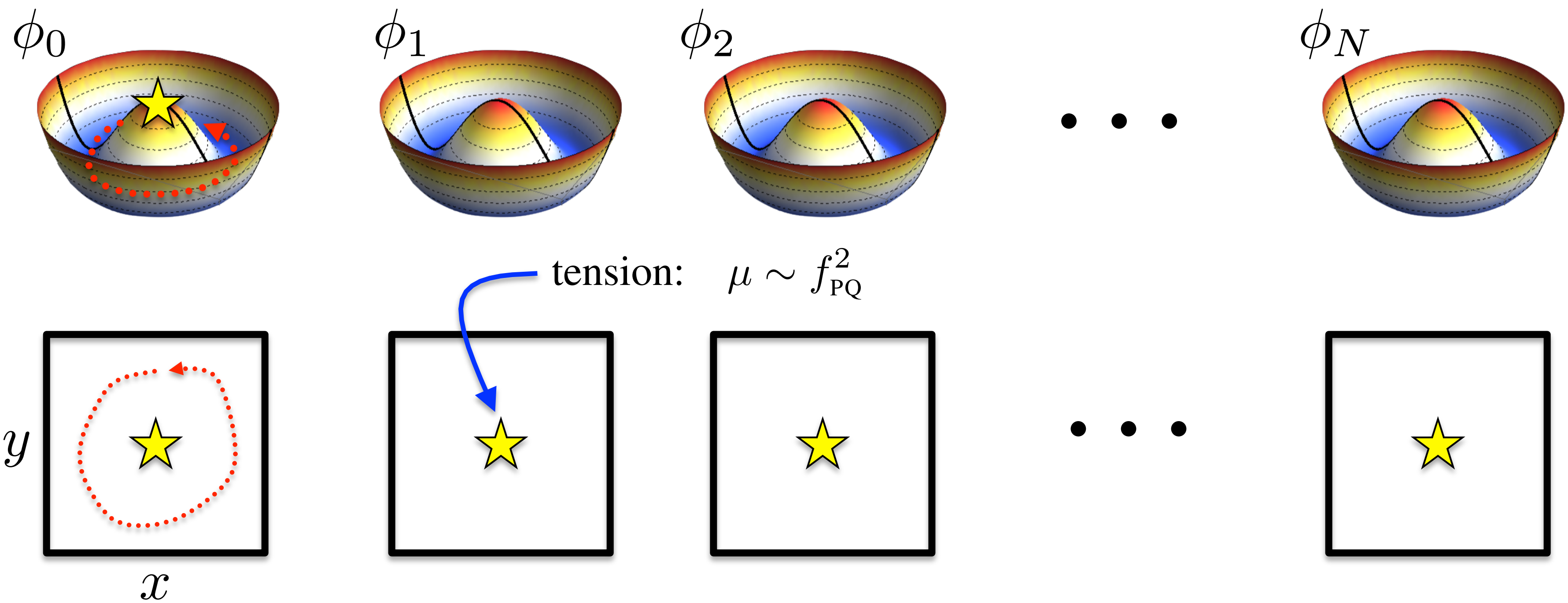


The non-trivial effects of clock-working show up in the defect network …

Of strings & walls

Formation of a string wall-network at T ~ f_{PQ} via Kibble mech.

$$\mathscr{L} = \sum_{n=0}^{N} \left[\left|\partial_\mu \phi_n\right|^2 - \lambda\left(|\phi_n|^2 - f_{\rm PQ}^2\right)^2 \right]$$

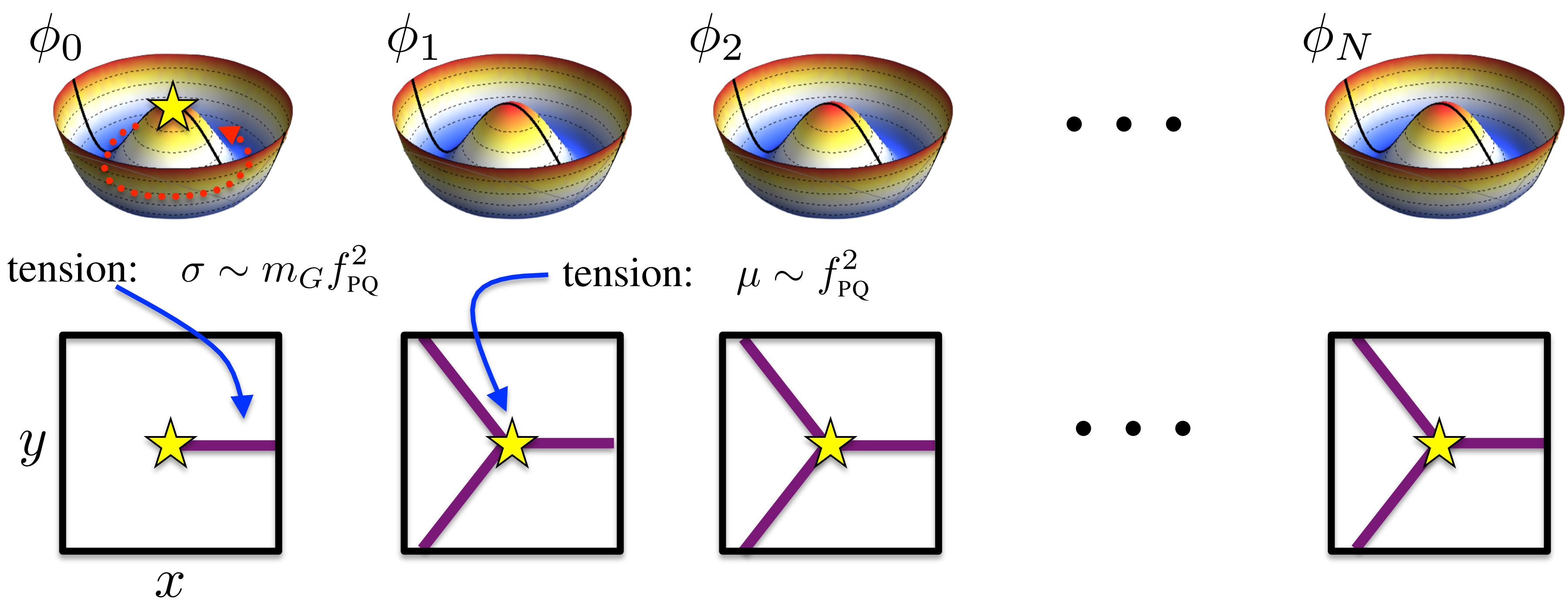


N+1 "flavors" of cosmic strings ...

Of strings & walls

Formation of a string wall-network at T ~ f_{PQ} via Kibble mech.

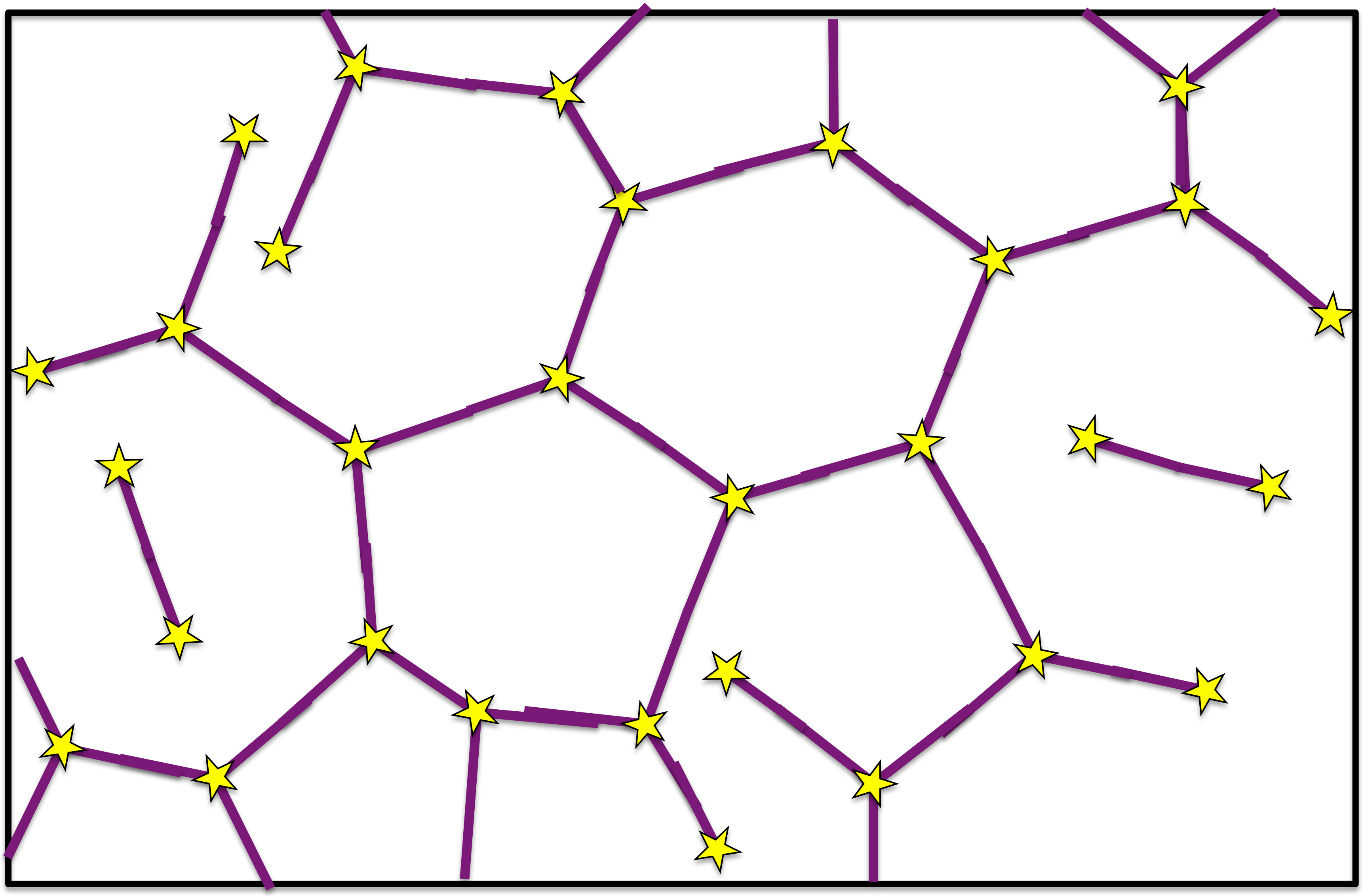
$$\mathscr{L} = \sum_{n=0}^{N}\left[\left|\partial_\mu\phi_n\right|^2 - \lambda\left(|\phi_n|^2 - f_{\rm PQ}^2\right)^2\right] + \epsilon\sum_{n=0}^{N-1}\left[\phi_n^*\phi_{n+1}^3 + \text{h.c.}\right]$$



N+1 "flavors" of cosmic strings … connected by lots of domain walls!

Complicated defect network!

Higaki, Jeong, Kitajima, & Sekiguchi (2016)



Axion Dark Matter from Defects

Naïve estimates …

...imply strong upper limit on m_G & f_{PQ}!

Energy in the strings?

$$\rho_{\text{strings}} \sim \mu H_{\text{QCD}}^2 \sim f_{\text{PQ}}^2 H_{\text{QCD}}^2$$

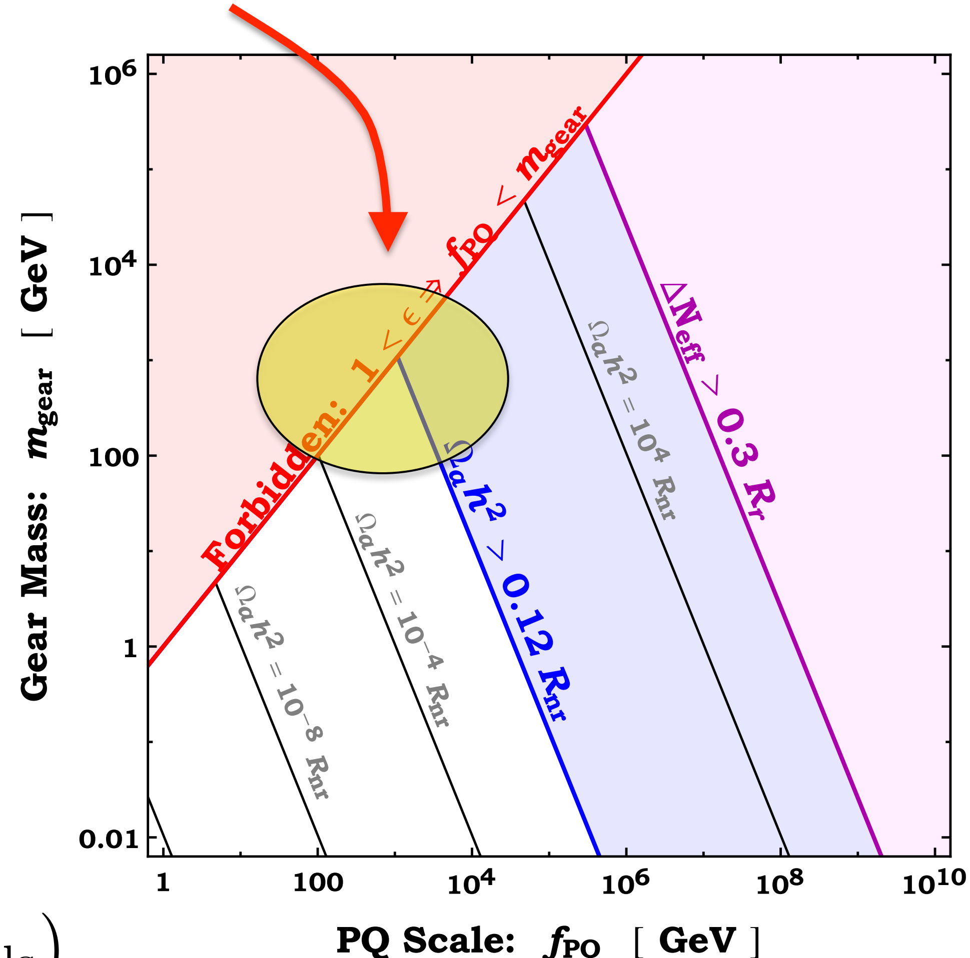
Energy in the walls?

$$\rho_{\text{walls}} \sim \sigma H_{\text{QCD}} \sim m_G f_{\text{PQ}}^2 H_{\text{QCD}}$$

➔ (walls dominate at QCD)

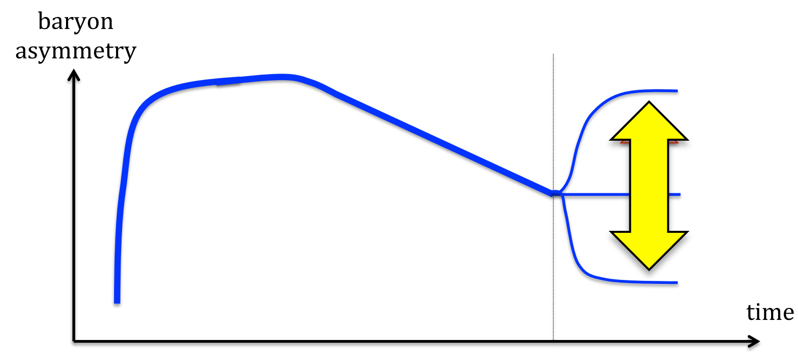
Efficiency of axion emission?

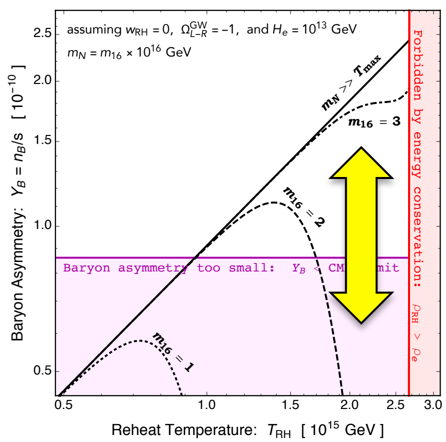
$$\rho_{\text{axions}} = R_{\text{nr}} \times \left(\rho_{\text{strings}} + \rho_{\text{walls}}\right)$$



Hidden theme: be wary of exponentials

Sec 1) The relic baryon asymmetry is *very sensitive* to how hyper-magnetic fields are converted into EM fields at the electroweak crossover. (source competes with exponential washout by sphalerons)





Sec 2) The relic lepton asymmetry is *very sensitive* to the Majorana mass scale & reheat temperature. (exponential washout by L-violating interactions)

Sec 3) How robust are the naïve estimates of axion relic abundance? They could be as wrong as $q^N \sim f_a / f_{PQ} \sim 10^7$. (many domain walls & suppressed couplings)
➔ work in progress

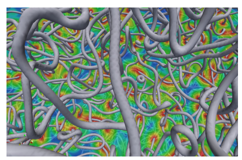
Summary & Outlook

We have considered a few models of high energy physics with axions.

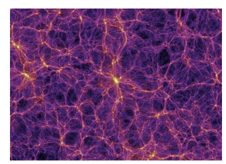
How do these axions modify cosmology? ➔ Rich Physics!

We can probe this physics through cosmological relics!

Gravitational Waves



Topological Defects

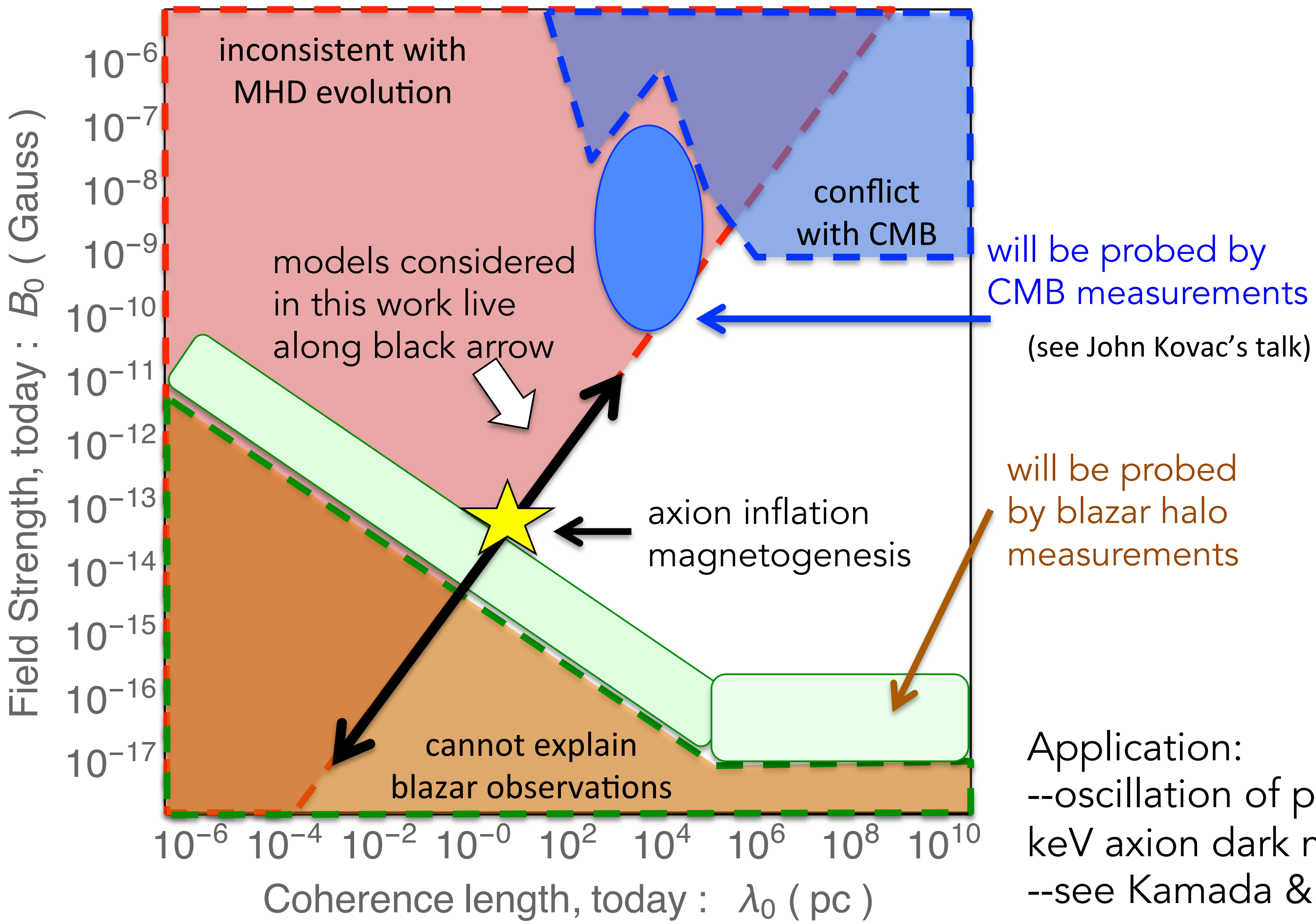


Relic Matter DM & DR

Primordial Magnetic Fields

Matter / Antimatter Asymmetry

Implications & Applications



Application:
--oscillation of photons into keV axion dark matter
--see Kamada & Nakai (2017)

(figure adapted from Durrer & Neronov, 2013)