# Introduction

We welcome ideas and criticisms of the manual. If you see an error or unclear writing, let your TA know. We will use your feedback to improve the manual for future students.

The Staff of Physics 109

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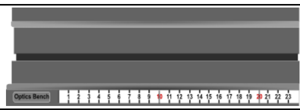
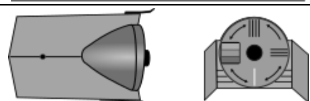
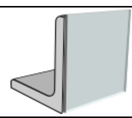
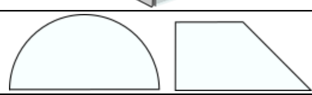
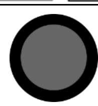
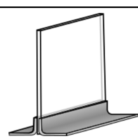
# Lab 1: *Reflection and Refraction*

This half of the semester, we will explore light. Before we get to photography or color mixing, you need to know how light interacts with surfaces or mediums in its path. When light hits a surface, it can be absorbed, transmitted, reflected, or a combination of these. Today's lab focuses on reflected and transmitted light rays. You will examine how light rays change direction at the interface between two mediums.

**Reflection** is when light bounces off of a surface and continues in another direction. **Refraction** is when light changes direction when traveling from one medium into another.

## Equipment

| | |
|---|---|
|  | **Optics Bench**: Helps you line up all of the other equipment in a straight line. |
|   | **Light Source**: Select the single-slit option by rotating the round selector on the end so the single-slit is at the bottom. You will use this again with different settings for the lens lab. |
|   | **Ray Table:** A rotating protractor. Line up your mirror or prism along the COMPONENT line. This ensures that the NORMAL line shows you the normal (perpendicular direction) to the surface. Note: a variation has a single-layer ray table that uses a magnet to sit on a stand. |
|  | **Flat Mirror**: Two of these will be used in various reflection experiments. |
|  | **Acrylic Blocks:** Clear plastic blocks in semi-circular and pane/prism shapes are used to investigate refraction |
|  | **Polarizing Light filter:** As light travels, the electrical field wave can be in any direction perpendicular to the ray. Polarizing filters remove light with particular orientations. |
|  | **Semi-transparent Mirror**: A special type of mirror that lets you see both a reflection in the mirror and something behind the mirror. Used to measure image distance. |

Other Equipment: A wall mirror on the lab room door, dry erase markers, a semi-transparent mirror

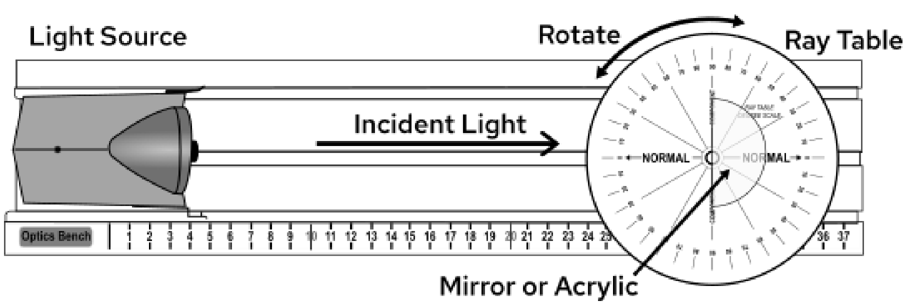



Figure 3. Equipment setup for most of the experiments. The track will help align the incident light ray with the mirror or prism on the ray table.

# Law of Reflection

Light can be reflected off of almost any surface. When light bounces off of a textured surface (like an apple or a wall), the surface variation sends light in all directions and is called **diffuse** reflection.

For today's lab you will focus on **specular** reflection, where light is reflected by a very smooth, shiny surface (like a mirror). In these cases, you can predict the direction the reflected light will travel based on the direction it came from.

Figure 1 shows how you measure the direction of a light ray. Consider an object reflected in a mirror (your face, for instance). An **incident ray** of light comes from one point of the object (the tip of your nose or a single eyelash) to the mirror. Then the **reflected ray** travels away from the mirror.

In order to define the relationship between the directions of the incident and reflected rays, you draw a **normal**. A normal is any line perpendicular to the mirror surface. The useful one for your purpose is drawn at the point where the incident ray hits the mirror.

The angle between the **incident ray** and the **normal** is called the **angle of incidence**. Label it **i**.

The angle between the **reflected ray** and the **normal** is the **angle of reflection**. Label it **r**.

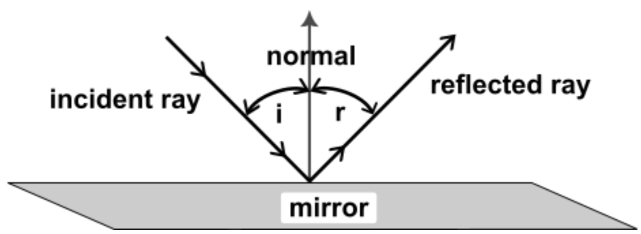



Figure 1. Law of Reflection

# Reflection Experiments

## 1. Angle of Reflection

Place the Ray Table on the Optics Bench. The Ray Table has angle markings to measure the angle of incidence and the angle of reflection for a ray of light. Your Light Source should be about half of a meter from the Ray Table and have light showing through one narrow slit (if no light is showing, plug in the light source).

- To get accurate measurements, you want to be sure our Light Source and Ray Table are aligned.
  - Start with the Ray Table oriented so that the diameter line marked "NORMAL" is along the length of the optics bench.
  - Move the light source so that the light beam aligns with "NORMAL" (see Fig. 3).
  - Place the mirror along the diameter marked "COMPONENT," so that the mirror is perpendicular to the incident light beam – this is called **normal incidence.** If everything is aligned, you will only see one light ray. Where is the reflected ray?
- Rotate the Ray Table so the incident ray is at 20° from the normal and you can see a reflected ray. Measure r to the nearest 1°.
- Using at least five additional angles i (from 20° to 70°), measure angle r. Vary i in equal steps of 10° or less. In your notebook create a table (like on the next page) to show how **r** changes when **i** is varied.

| Angle of Incidence ($i$) | Angle of Reflection ($r$) |
|---|---|
| 20° | ? |
| ⋮ | ⋮ |
| 70° | ? |

- Write 1-2 sentences with your conclusion about the relationship between **i** and **r** in your notebook.

**Parts 2-5 of the Reflection experiments can be done in any order. Stations will be set up around the room for parts 3, 4, and 5. Your group may use them whenever they are available.**

## 2. Corner Reflectors

Astronauts have placed mirrors on the moon to reflect a laser beam from earth. If the angle of the mirror is slightly off, the reflected light beam will miss the observer on earth.

- Set up your optical bench with the light source and ray table at opposite ends.
  - Start with normal incidence: the normal line aligned with the light ray and the mirror on the component line perpendicular. As before, you should only have one ray.
  - Now turn the ray table 3-5° and hold a piece of paper near the light source to find the reflected ray. Use a ruler to measure the distance between the light source is the returning reflected ray.

A "corner reflector" solves this problem by ensuring the beam returning to earth is parallel to the beam sent to the moon.

- Use two flat mirrors to create a corner reflector (to make measurements easier, place at least one along a marked diameter of the ray table). Light should bounce from one mirror to the other and then back toward the source.
  - Change the orientation of one of the mirrors until you get a reflected beam that is parallel to the incident beam. What is the angle between mirrors?
  - Rotate the Ray Table to change the incident angle, is the reflected beam still parallel? What angle between mirrors lets you get a parallel reflected beam for any incident angle?

## 3. Location of Mirror Image

- Use one dry erase marker as your object and look at its image in the semi-transparent mirror. Have your lab partner move a second marker (position tracker) behind the mirror until it seems to be in the same place as the image of the first marker. Measure the distance from the mirror to both the object and the image. Record these measurements in your lab notebook.
- Repeat this process with the object at different distances from the mirror. How does the image position vary with the object position?

## 4. How Large Does a Mirror Need to Be?

When getting ready for an event, you want to be able to see yourself from head to toe. What is the minimum height the mirror need to be compared to your height to do so?

- Make a guess at the answer in your notebook.
- Now do an experiment to find out!
  - Standing about 1 meter from a tall mirror, have your lab partner mark the point where you see the top of your head and the point where you see the bottoms of your feet.
  - Have your partner measure the distance between the two marks, and compare this distance to your height.

- In your notebook sketch yourself and the mirror with light rays traveling from your body to the mirror and then to your eye.
- Does distance from the mirror matter? Take a few steps backwards – and repeat the marks. **Hint: What role does distance to the mirror play in your sketch?**
- How does the law of reflection determine the minimum mirror size?

### 5. Polarization by Reflection

Light is an electromagnetic wave and most of the time the direction of electric oscillation is random, even for rays traveling the same direction. Light can be **polarized**, meaning the electrical oscillations are primarily in one direction.

The difference is similar to a crowd leaving an event vs. a marching band. In both cases, the people walk in the same direction, which is analogous to the direction the ray is traveling. The difference is in the direction which people are looking (analogous to the direction of electrical oscillations). Members of the crowd are looking in every direction while the band members are "polarized," they all face the same direction.

Light reflected from a glass plate or a shiny surface (the lake, a car hood, a linoleum floor) is polarized.

- In the hallway, close one eye and look through a polarizing filter with the other. Focus on the reflections of the ceiling lights in the floor tiles. As you look through the filter, slowly rotate it clockwise or counterclockwise for a full turn. How does the brightness of the reflections change?
- The polarization of the reflected light depends on the angle of reflection. Can you find an orientation of the polarizer that makes at least one of the reflections disappear completely?
- How would this apply to polarized sunglasses? What light would you want to block while driving?

## Law of Refraction (Snell's Law)

A transparent medium (substance) like glass or water also changes the direction of light rays. These materials both reflect and transmit light. In today's lab, you will use acrylic (a clear plastic) to investigate the transmitted, or **refracted**, portion of light.

When a light ray hits the surface of a transparent medium, the incident ray splits into two components (Fig. 2). The **reflected ray** follows the law of reflection, just like a mirror. Unlike a mirror, there is also a **refracted ray** that kinks as it passes through the acrylic. The angle of the refracted ray is determined by the law of refraction, also known as Snell's Law.

Just like reflection, you measure an **angle of incidence** between the incident light ray and the normal to the acrylic surface. For Snell's Law, label the angle of incidence $\theta_1$. If $\theta_1 \neq 0°$, the refracted ray kinks when it passes through the surface (Fig. 2, $\theta_2$). The **angle of refraction**, $\theta_2$, is the angle between the refracted ray and the normal to the surface inside the new medium.

The angle of incidence ($\theta_1$) and the angle of refraction ($\theta_2$) are related by Snell's Law, which uses each materials' **index of refraction** ($\mathrm{n}$) and the sine of each angle.

$$\text{Snell's Law: } n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Index of refraction is a property of a given medium and is related to how fast light travels through that medium. The index of refraction for light in a vacuum is $n = 1$, and for light traveling in air is $n \approx 1$. Other materials have $n > 1$ because light slows down as it moves through higher density materials.

When light goes from a lower density medium (air) to a higher density medium (acrylic) the angle of incidence is larger than the angle of refraction, and the ray kinks **toward the normal**.

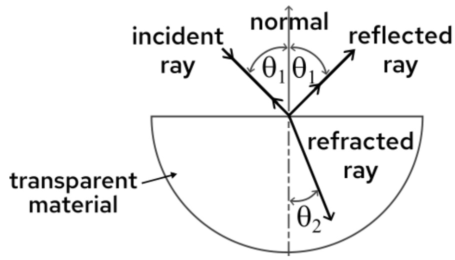



Figure 2. Snell's Law: When light contacts a new surface, it may be reflected away from the surface and refracted through the new medium.

Snell's law lets us predict the direction of the refracted light ray inside a second medium for any angle of incidence $\theta_1$, as long as we know the index of refraction for each medium.

# Refraction Experiments

## 6. Index of Refraction

- Place the half circle of transparent acrylic on the Ray Table with the flat edge along the "component" line (Fig. 3). The light beam should enter the acrylic in the center of the flat edge.
- Make sure you can see the refracted ray of light clearly through the top of the acrylic. If you cannot, ask your TA.

  **Note: If everything is lined up correctly, light should only change direction at the flat surface.**
- Use angles of incidence $\theta_1$ from 20° to 70°, in steps of 10° and measure the angle of refraction $\theta_2$ at each step.
- Record your measurements in a table like the one below. Use a calculator to calculate sin $\theta_1$ and sin $\theta_2$, and finally calculate the index of refraction using sin $\theta_1$/sin $\theta_2$. Recall $n_1 \approx 1$ for air.

| $\theta_1$ | $\theta_2$ | sin $\theta_1$ | sin $\theta_2$ | $n = \frac{\sin\theta_1}{\sin\theta_2}$ |
|---|---|---|---|---|
| 20°<br>⋮<br>70° | | | | |

- What is the index of refraction for the acrylic block?

## 7. Reversibility

Snell's law applies to either direction of the light path – light entering the acrylic block from air or light exiting the acrylic block into air.

- Reverse the light path by turning the Ray Table half-way around (i.e. 180°) so that light enters the curved side of the semi-circle and exits the center of the flat side.
  - Choose one of the $\theta_1$ & $\theta_2$ pairs from the table you made in Experiment 6.
  - Use the $\theta_2$ from your chosen pair as your new angle of incidence inside the acrylic.
  - Measure the new angle of refraction $\theta_2$ in air.
  - What do you notice about the angles in air in both cases?

- We have focused on the flat surface, why didn't the light kink at the curved surface? **Hint: where is the normal to the curved surface?**

If all went well, you found that the path of light is exactly the same for light traveling in either direction. If you look at the semicircle from above, the path looks the same no matter which side the source is on.

## 8. The Critical Angle and Total Internal Reflection

When traveling along a **radius** of our semi-circle, the light from the source is perpendicular to the curved surface and does not kink. This makes it easier to measure the change in angles at the flat surface.

- Using the setup from part 7, look at the light ray as it hits the inside flat face. There is a reflected ray that stays in acrylic and a refracted ray that exits to air. Sketch what you see in your notebook.
- Starting from $\theta_1 \approx 20°$, turn the Ray Table and observe the intensity of the reflected ray as you increase $\theta_1$. At a certain angle, the refracted ray disappears and the reflected ray gets brighter. This $\theta_1$ angle is called the **critical angle**.

The term "critical angle" refers to the angle at which the refracted ray **just** disappears – make $\theta_1$ a little smaller and the refracted ray comes back.

- Measure the critical angle with your setup. Note that at the critical angle the refracted ray made an angle $\theta_2 = 90°$ from the normal.
- Use the index of refraction you measured in part 6 to calculate the critical angle from Snell's law. Set $\theta_2 = 90°$ and solve for $\theta_1$.
- Compare your calculated critical angle to the observed critical angle.

An **internal reflection** is a light ray reflected by the **inner** surface of the acrylic that stays inside the acrylic.

- Watch the internally reflected ray again as you turn the ray table from an angle of incidence below the critical angle to one above it. How do the direction and brightness of the reflected ray change as you pass the critical angle?

  When you go beyond the critical angle, Snell's law gives us an angle $> 90°$ for the refracted ray and it cannot exit the acrylic. This condition is referred to as **total internal reflection**. It happens for all angles $\theta_1$ larger than the critical angle.

  The complete expression for the critical angle is "critical angle of total internal reflection." For any angle of incidence larger than this angle all the light is reflected, and none is refracted.

## 9. Dispersion

What is called **white light** is really made up of all colors of the rainbow. You can observe this by using **dispersion**, a property of some mediums where each color of light has a slightly different index of refraction, meaning refraction causes each color to kink by a different angle, separating the colors.

- Using white light, rotate the Ray Table so that the angle of incidence $\theta_1$ is just below the critical angle. Hold a piece of paper in the path of the refracted beam to see the colors of the rainbow. Note: the colors might blur together, but you should see red on one side and blue on the other.
- Look at the sequence of colors: is blue light kinked more or less than red light? Does this mean that the index of refraction is larger or smaller for blue light?

## 10. Light Passing through a Window Pane or Prism

The semicircular acrylic block let us ignore refraction at one edge by ensuring normal incidence on the curved surface. If we do not have this special condition, we need to consider refraction at both surfaces.

- Use the block of clear acrylic (Fig. 4) to study what happens to light rays traveling through a transparent object with parallel or angled sides. Make sure you can see the ray through the top.
- In your notebook, make a large drawing of a piece of acrylic with parallel faces, and a light ray entering the acrylic at a large angle of incidence.
  - Draw the ray inside the acrylic and the ray coming out the other side.
  - Use the Snell's law to calculate the angle of incidence and angle of refraction at both surfaces and write them in the drawing.

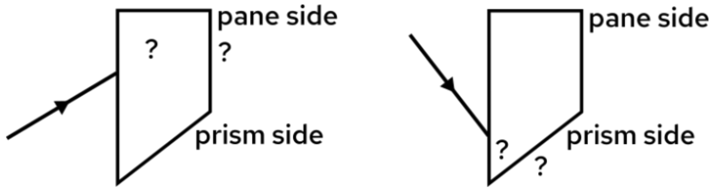
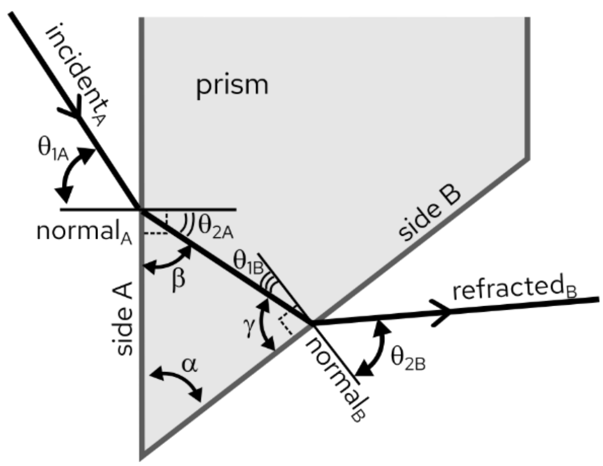



Figure 4. Trapezoidal acrylic block. Parallel sides act as a window pane. The triangular end acts as a prism.

- Do the same for a prism (triangle of acrylic). **Hint:** Measure the angle of the second surface ($\alpha$).




  - At the first surface, solve for $\theta_{2A}$: $n_{1A} sin\ \theta_{1A} = n_{2A} sin\ \theta_{2A}$
  - Then, we can find angle $\beta$ using $\theta_{2A} + \beta = 90°$
  - The ray inside the prism forms a triangle with the two sides of the prism, so $\alpha + \beta + \gamma\ =\ 180°$, and we can solve for the angle between side B and the internal ray.
  - Now find $\theta_{1B}$ using $\theta_{1B} + \gamma = 90°$
  - Finally, find $\theta_{2B}$: $n_{1B} sin\ \theta_{1B} = n_{2B} sin\ \theta_{2B}$

# Lab 2: Lenses

Last week in lab, you studied refraction, how light rays kink as they move from one material into another. Today we look at lenses, which have two curved, non-parallel surfaces. While Snell's Law calculations are valid for lenses, the geometry gets complicated thus, we would need many calculations to get accurate results. In this lab, we will explore lens properties using shortcut methods that simplify calculations.

This lab focuses on **converging lenses**, which are thicker in the middle. A converging lens takes rays parallel to its optical axis (horizontal line in Fig. 1) and kinks them to converge at the **focal point** on the other side of the lens. Converging lenses form a **real image** if an **object** is far enough from the lens. Light rays starting from one point on the object (in Fig. 1a, top of the head) are kinked by the lens to converge at another point.

In Fig. 1a, the ray that hits point A near the top of the lens is kinked down, the ray hitting point B of the lens is kinked up, and the ray through the center of the lens is not kinked. Note that the rays hitting the lens closer to the optical axis (B) are kinked less than those farther away (A). The ray passing through the center of the lens does not kink.

Each point on the lens has a specific kinking angle (the angle between the ray entering the lens and the ray leaving the lens) that is the **same** for every ray, no matter the direction it comes from (Fig. 1b). The kinking angle at point A is the same for rays coming from the top of the head or the bottom of the skirt.

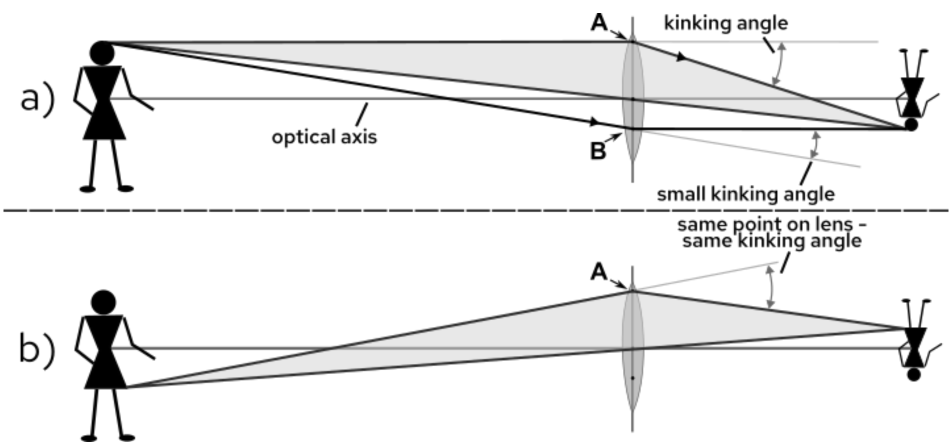



Figure 1. Light rays from two points on the object converge on the opposite side of the lens.

Interestingly, we can cover any part of the lens and still get a **complete** image. Rays from all parts of the person pass through every part of the lens. Therefore, we could cover one half of the lens and the other half will still form a perfect image. The difference is that the image is **dimmer** if we cover half of the lens because we block half of the rays and reduce the intensity of the image.

In practice, all lenses are much smaller than the one in Fig. 1. We sketch human-sized lenses to make it easier to see that they are converging lenses, or to draw the "three easy rays" to locate where the image is formed. We can get a complete (dimmer) image with a much smaller lens. Look at the camera lenses on your phones. They are tiny, but you can still take a head-to-toe photo of a person, or an entire landscape! For any size lens with a certain focal length, the image is formed in exactly the same place because **all** rays from an object at a certain distance cross each other at the same, precise image distance, independent of the lens size.

Since each part of the lens kinks light rays from any direction by the same angle, we can relate **object distance** (**o**, the distance between the object and the plane of the lens) to **image distance** (**i**, the distance between the image and the lens) without any calculation.

In Fig. 2, we show the kinking angle at point A of the lens for a ray coming from the bottom of the candle. Imagine that the candle moves away from the lens. Since the kinking angle at point A is always the same, which direction does the image of the candle move? Is the new image closer to or farther from the lens?

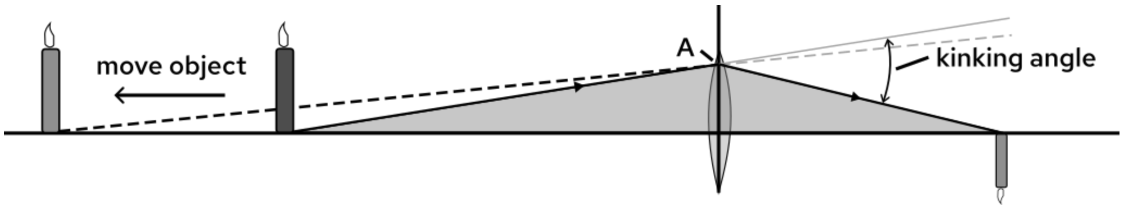



Figure 2. Changing the object distance will change the image distance for a specific lens.

Lenses never create perfect images. If a lens produces a perfect image of the tip of your nose when you are 2 meters from the lens, then **all** rays from the tip of your nose converge to one point on the screen. Rays that come from a little farther away (say your ears) will **not** converge perfectly on the screen.

Lens **aberrations** cause images to be distorted or out of focus in some way because the rays coming from one point of the object do not converge to a single point of the image. For instance, a small dot on the object ( • ) may appear as a short line ( | ) on the screen. We will study **chromatic aberrations** in today's experiment, which are caused by different indices of refraction for different colors of light.

The **focal length** $f$ of a converging lens is the distance from the center of the lens to the point where incident rays parallel to its optical axis converge. Focal length is a property of the lens, determined by the materials and geometry of the lens itself.

Consider the three lenses in Fig. 3: the one on the left has infinite radii of curvature, the one in the middle has large radii, and the one on the left has smaller radii. How do you think the rays will be kinked by each lens?

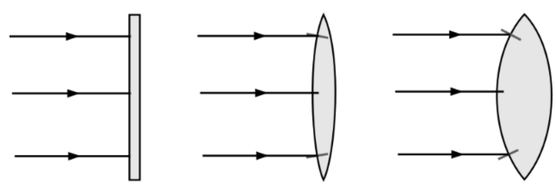


Figure 3. Lenses with different radii of curvature

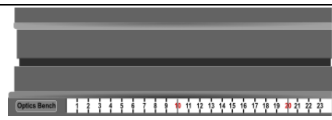
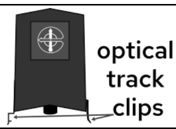
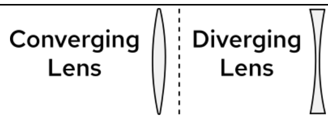
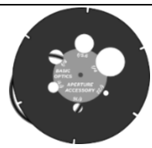
Focal length is usually measured in millimeters (mm). It is positive for converging lenses and negative for diverging lenses. A typical phone camera has f = 26 mm.

The **thin lens formula** predicts where we will find the image given the object distance and the focal length of the lens. Note: You cannot change the focal length of a lens, all you can change is the object distance, and therefore the image distance. The thin lens formula is:

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i}$$

where **f** is the focal length, **o** is the object distance from the lens, and **i** is the image distance from the lens.

## Equipment

| | |
|---|---|
|  | **Optics Track**: Holds and accurately aligns all of the other pieces. A ruler along the edge lets us measure the object distance and the image distance. |
|  | **Light Source**: The light box we used last week can be oriented to project an image with crosshair rulers that make it easier to measure magnification. |
|  | **Stand up Lenses**: These lenses stand in the optics track and have specific focal lengths. They are labelled with various letters and numbers. We will use the ones labelled #1 and S. |
|  | **Ray Optics Kit Lenses:** Simplified converging and diverging lenses that will help us trace our "three easy rays". |
|  | **Apertures**: A disc that attaches to the lens with apertures (holes) of different diameters to cover part of the lens. Note that the diameter of each aperture is given by the focal length divided by a number called the f number and abbreviated f/. The largest and smallest apertures on the disc have f numbers 4 and 22, so they are labeled "f/4" and "f/22," as they are on most cameras |
| Other equipment: a white screen, a red and a blue filter, rulers | |

## Experiments

### 1. Focal Length

- Measure the focal length of lens #1 by producing an image of an object **at infinity**. The incident rays from an object at infinity are parallel. The lens causes these rays to converge and form an image one focal length f from the lens.
    - True infinity is unrealistic, but an object at least 5 m (~16 feet) from the lens will work for this lab. A good option is a window on the other side of the room. At night, you can use a light bulb on the other side of the room.
    - Attach the lens and the screen to the optics track. Organize your optics track so that the lens is between your chosen object and the screen. Adjust the screen location until you find a sharp image of the object.
    - Using the ruler on the optics track, measure the focal length (the lens to the screen).
    - In your notebook, draw a ray diagram with a converging lens, parallel rays from the object to the lens, and converging rays after the lens. Mark f and its value in your drawing.
- If we wanted a lens with a shorter focal length, how would its shape be different? Remember that a shorter focal length lens kinks the rays more – is the lens more curved or less curved?
- Switch to lens S, and measure the focal length the same way you did above. Record the focal length f for lens S in your notebook.
- How do the curvatures of the two lenses compare? Are they what you expect?

## 2. Magnification

We use the magnification equation to relate the height of the image, $h_i$, to the height of the object, $h_o$. Using similar triangles, we can also relate magnification to image and object distances (Fig. 4).

$$M = \frac{h_i}{h_o} = \frac{i}{o}$$

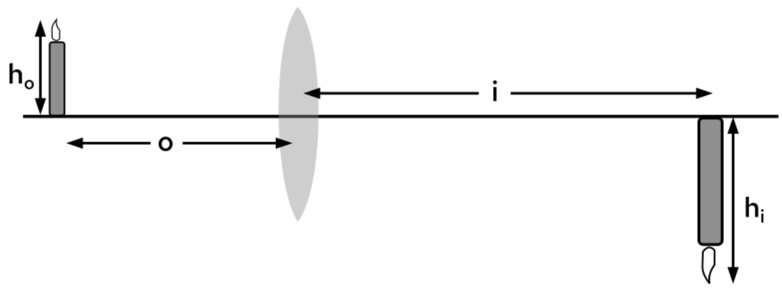



Figure 4. A magnified image using a converging lens.

With one exception (when o = i), the size of the image is different from the size of the object. Here we investigate the definition of magnification and how the magnification depends on the object and image distances.

- Set up the light source at one end of the optics track, with the crosshair light source at 0 cm on the track ruler and facing the rest of the optics track. The crosshair will be our object.
- Set up the screen near the other end of the track (near 100 cm) to display the image.
- Start with lens #1 near the object and move the lens position until you produce a sharp image of the crosshair on the screen. Measure $\mathbf{i}$ and $\mathbf{o}$ for this configuration.
- Without moving your object or screen, move the lens to find a **second** lens position that gives a sharp image. Again measure $\mathbf{i}$ and $\mathbf{o}$.
- What do you notice about your two measurements?
- Choose the lens position that gives the larger image. Use a small ruler to measure the size of the image and the size of the object. Calculate the magnification from your measurements. How many times larger than the object is the image?
- Compare the value of $i/o$ to the magnification M.
- Explain the result geometrically with similar triangles. **Hint:** Sketch the ray through the center of the lens in Fig. 4.

## 3. Thin Lens Formula

The thin lens formula is given in Setting Expectations. We just collected experimental data for these distances using lens #1, so we can compare the experimental values to the equation.

- Use the object and image distances you measured above and calculate the focal length of lens #1 from the lens formula.
- Compare the result to your measured value of f from part 1. **Hint:** values should be similar.

**Application:** Consider the following scenario. A local movie theater wants to change the size of their screens without changing the distance from the projection booth to the screen. In other words, they want magnification to change without changing image distance. Let's explore how this can be done.

First, we need a lens with a different focal length. Test this out on the optics track.

- Reset the light source, lens #1 and screen to give you a clear image of the cross hairs on the screen. (Your system may already be in the configuration.)

- Without changing the distance between the lens and the screen, replace lens #1 with lens S.
- Change the **object distance** by moving the light source to get a clear image on the screen.
- With lens S, is the image larger or smaller? Why?

  **Hint:** Use the fact that the focal length of lens S is smaller. The lens formula tells you how the object distance changes for a smaller focal length if image distance stays the same.

  **Alternatively**: Use the fact that lens S kinks the light rays more than lens #1, thus requiring a different object distance.

Now consider a scenario where the image distance is shortened. We looked at this in Fig. 2. Try it!

- Move the lens closer to the screen and then adjust the object distance to get a clear image. Which way did you need to move the object to accommodate the shorter image distance?

One more to try: What happens if we choose an object distance **o** that is less than the focal length f? Use the lens formula to make a prediction, then try it!

- Set up the light source and lens so that $\mathbf{o} < \mathbf{f}$. Where is the image? Is it real or virtual?

## 4. Depth of Field

In photography, **depth of field** refers to the range of object distances that are in focus. If near and far objects are **both** in focus (selfie with your face and a mountain), we say that the picture has a large depth of field. If only objects at a certain distance are in focus, but the background is blurry (a professional headshot), the picture has a small depth of field. Let's investigate how we can change the depth of field.

- Reset the optics track so that you have lens #1 and the screen separated by at least 35 cm. You will need space to move the light source along the track. Keep using the crosshair ruler as the object.
  - Adjust the lens position to focus the crosshair and the "mm" letters on the screen.
  - Now move the light source on the track to locate the minimum and maximum object distances where you can barely read the mm letters. The distance between these points is called the "depth of field." Record the result in your notebook.
- Now place a small aperture over the lens, for example the 3rd smallest on the disc. How has the depth of field changed?
- To understand why it changes, draw a ray diagram and add a screen that is a little too close to the lens so that the rays converge behind the screen and the image is blurry.*
  - In the diagram, which rays would be removed by using the aperture? Why does this help the sharpness of the image? There are many more rays than the three we draw.

*Normally when we draw ray diagrams, we look at where the rays converge, that is, where we could put a screen to get a sharp image. Here we intentionally put the screen in the **wrong** place, so that the rays from one point on an object make a blurred disc on the screen rather than a sharp point.

On a camera, we can change the **aperture** to change the diameter of the lens used, which means adjusting the depth of field of the picture. Objects at different distances cannot all be perfectly in focus, but we can sharpen their images by using a smaller aperture.

## 5. Chromatic Aberration

Last week, we considered **dispersion**. Materials have different indices of refraction for different wavelengths of light. That also means that a lens will kink light of different colors by different amounts. Then the image distance for blue light is not exactly the same as it is for red light. Thus, if you want to take a photo of a US flag and focus the camera on the red stripes, the blue field will be slightly out of focus. This is referred to as the **chromatic aberration** of a lens.

- Remove the aperture from the lens. Place a red filter over the light source and move the screen to focus the image.
- Now replace the red filter with a blue filter and move the screen to re-focus the image. Did you have to move the screen closer to the lens or farther away?
- What color was kinked more by the lens, red or blue?

Most cameras today use **achromatic** lenses to avoid this issue. Cameras use converging lenses to create real images on the image sensors. These lenses are made with low-dispersion glass to minimize the chromatic aberration, but that alone does not eliminate the problem. To remove the small dispersion from the main lens, a second lens is glued to the first. This second lens is a weakly diverging lens with large dispersion. Different colors of light are split apart by the first lens and are brought back together by the second lens.

## 6. Compound Lenses

Compound lenses combine two or more lenses of different focal lengths to achieve a different result, such as variable magnification (zoom) and achromatic lenses. To study these, use the converging and diverging lenses from the ray optics kit. For your light source, turn the light box to project three almost parallel rays of light through three slits.

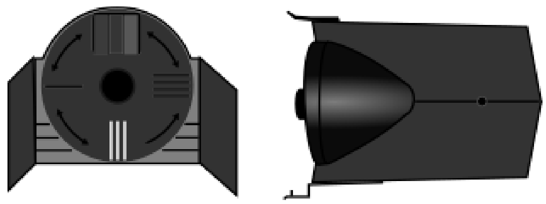


Figure 5. Orient the light source to get three beams of light.

- Remove the light box from the optics track and place it on a blank sheet of paper so that three almost parallel rays of light are visible on the paper.
- Place the converging lens in the path of the rays and trace the rays of light on the blank paper with a ruler. Remember to trace the rays on both sides of the lens and to mark the position of the lens as well.
- Use the ray diagram to estimate focal length of the lens and its dioptric power.
- Repeat this process for the diverging lens. Remember to place the diverging lens so that there is space on the paper to extrapolate the light rays "backwards" to mark the focal point.

Another way to talk about focal length is to use the **dioptric power**, $\mathbf{D}$. The dioptric power of a lens is the inverse of its focal length **in meters**. For reference, typical eyeglass lenses range from -20.0 D to +12.0 D (very nearsighted to very farsighted).

$$\mathrm{D} = \frac{1}{\mathrm{f}\,(meters)}$$

- If we were to glue these lenses together, what would the compound lens do with parallel light rays? Calculate the combined power of the compound lens (add the dioptric powers) and write your prediction in your notebook.
- Now try it: Place the two lenses together so that they form a compound lens, with no distance between the lenses. No glue, please!
    - Record and explain what you observe, using your understanding of what the power of a lens means.

Our lenses are made of the same material. If the shape does not change, but the converging lens had a higher index of refraction than the diverging lens, how would the light rays be affected by the compound lens?

# *Lab 3: Photography*

In this lab you will learn to use and analyze a digital camera to see how it works.

In particular you will learn about **focus**, **exposure time**, **ISO** and **f number** or aperture. On your phone, the camera app automatically chooses the focus, exposure, aperture, and ISO for every picture you take. If you want to take creative photographs, however, you must know how to choose and adjust these settings, either by using an app with more controls or by using a digital or film camera that lets you adjust them.

Each of the options for the three camera settings - exposure time, aperture, and ISO - are determined by the manufacturer of the camera as a specific series of numbers. In the first part of the lab, you are asked to examine the significance of these numbers. In the second part of the lab, you will take photographs of the same scene using different combinations of focus, exposure time, aperture, and ISO. Your goal is to better understand some of the most important measurable optical qualities in photography: **depth of field**, **reciprocity**, and **graininess**. Hopefully, this lab will inspire you to conduct further photographic experiments first in the lab and later on your own.

## *Digital cameras and film cameras*

In all cameras, the lens forms a real image on the **sensor**, a flat device inside the camera behind the lens that detects the light forming the image. Sensors can be either a film or a CCD, in a film or a digital camera, respectively. The most important difference between digital cameras and film cameras, therefore, is the kind of sensor they use. Everything else - lenses, aperture, and shutter - are present in all cameras.

In a film camera, the light sensor is a piece of light-sensitive **film**, which is a thin layer of flexible polymers coated with chemicals that either darken or change color when exposed to light.

In digital cameras, the light sensor is a charge-coupled device, or **CCD**. The CCD is divided up into millions of pixels (picture "pix" + elements "el" = **pixel**). Each pixel on the CCD detects the intensity and color of light in that microscopic location. As they are exposed to light, the CCD pixels build up electrical charge. After the exposure time ends, the electrical charge on each pixel is converted to image brightness.

Each CCD pixel is split into red, green, blue (RGB) with filters. When you view the resulting image on a screen, the color of each pixel is reconstructed from the amount of RGB captured by the CCD. This is called additive color mixing. If you print out the picture in color, the pixels on the paper are colored by mixing different amounts of cyan, magenta, yellow, and black (CYMK) ink. This is called subtractive color mixing. You will experiment with both types of color mixing in the next two labs. There is no perfect method to translate RGB screen images to CMYK printed images. When making prints from digital images, photographers and artists often spend a lot of time adjusting the CMYK ratios to best match the RGB original.

## *Camera Settings*

Generally, you select an object to take a photo of and adjust the lenses to focus the image. Then, you can adjust the exposure time, f number and ISO to get an image that has good exposure.

### ***Lenses***

Most digital cameras have multiple lenses, each with a different purpose. These lenses can move relative to the light sensor (changing the **image distance i**) or relative to one another (changing the **net focal length f**).

**Focusing the camera:** the camera can move all of the lenses relative to the sensor. This changes image distance while keeping focal length constant. This adjustment allows you to select a specific object distance to be in focus, that is, to form a sharp image.

**Zooming in:** the camera can move lenses relative to one another, which changes the effective focal length. If the object distance stays constant, the image will be magnified. To achieve this, the image distance must also increase, which is why professional zoom lenses can be very long.

### *Exposure Time, f number, and ISO*

The **exposure time** of a camera often called the "shutter speed." It is the amount of time the shutter allows light to expose the CCD or film, measured in seconds. The app on the lab computer calls this value the "Time value" or "Tv."

In a digital camera, the aperture is formed by overlapping blades that create an adjustable-diameter opening at the center of the lens. The **f number**, written as f/, is the focal length of the lens divided by the aperture diameter of the lens during the exposure time. Professional photographers refer to the f number setting as the "f-stop." The app on the lab computer refers to the f number as the "Aperture value" or "Av." On another camera you might set a knob to a certain f/, for example f/4.5, which is equivalent to setting your Av value to 4.5.

Setting the **ISO** determines the light sensitivity of the sensor in the camera. Lower ISO numbers (100, 200) mean lower sensitivity and are best for brightly lit environments. Higher numbers mean a more sensitive sensor and allow photographs to be taken in low-light conditions. Digital and film cameras use the same ISO system, but they operate in different ways. In film cameras, ISO is changed by changing the chemicals in the film so you must change the film to change sensitivity. With digital cameras, on the other hand, you can change ISO at will from one picture to the next.

You can adjust the exposure of a photograph by changing exposure time, ISO, and f number. For each of these, one step to the left **doubles** the amount of light or the sensitivity. One step to the right **halves** the amount of light or sensitivity.

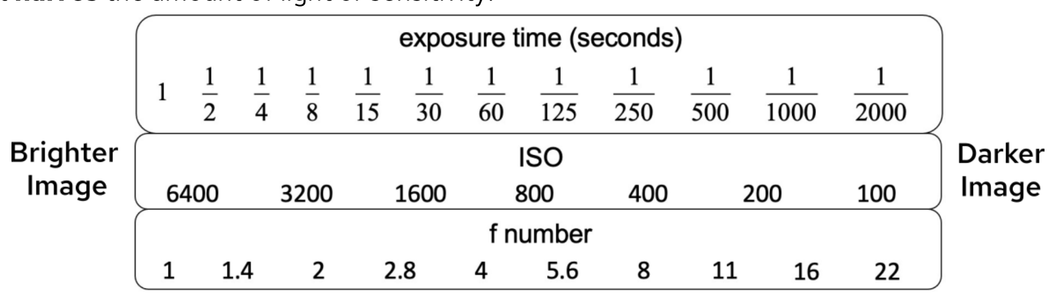
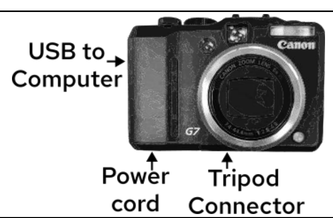
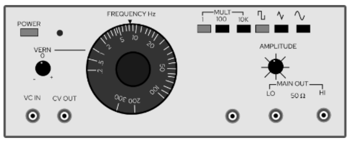
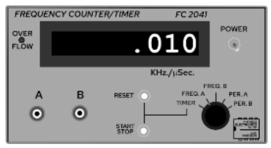
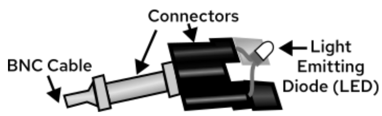



Figure 1. Sample photography setting values and steps.

## *Equipment*

| | |
|---|---|
|   | **Camera**: Canon G7, a very nice digital camera that can be connected to a computer and then controlled from an app. |
| Remote Capture Task App | **App**: The lab computers open this app automatically when the camera is connected and turned on. |

| | |
|---|---|
|  | **Frequency Generator**: This box generates an electrical signal that you will use to make a diode (below) flash on and off. The large dial makes coarse adjustments to the frequency, and the small dial marked VERN makes fine adjustments. Press the 1 multiplier MULT 1 button and the square wave button at the top. |
|  | **Frequency Counter**: This device is used to display the frequency output of the frequency generator. Note that the counter displays frequency in kilohertz (kHz). 1 kHz = 1000 Hz, so you'll need to multiply the reading by 1000 to get the correct reading in Hz. In the image, the display shows 10 Hz. |
|  | **Light Emitting Diode (LED)**: An LED connected to the frequency generator flashes at the frequency displayed by the counter. You will use this to understand exposure times. |

Other equipment: meter stick

## *Experiments*

### 1. Setting up the Camera

- Turn the computer and monitor on, and log in. Create a folder with your section number in the Pictures folder.
    - In the search bar next to the start button, type "File Explorer." Click on the folder icon. Double-click on the Pictures folder in the list (left). On the "Home" tab (top), click New Folder and type in your section number.
- Plug the camera power cord in, close the cover, and turn on the camera by pressing the power button on top of the camera.
- Connect the camera to the computer using the USB cable. When you do so, a "Camera Window—Canon PowerShot G7" window should appear on the computer screen.
- Click on the "Remote Shooting" tab. Click on the "Start Remote Shooting" icon.
- Find the folder you created in the list and select it. Click "OK." All of the photos you take during the lab will be saved in this folder.

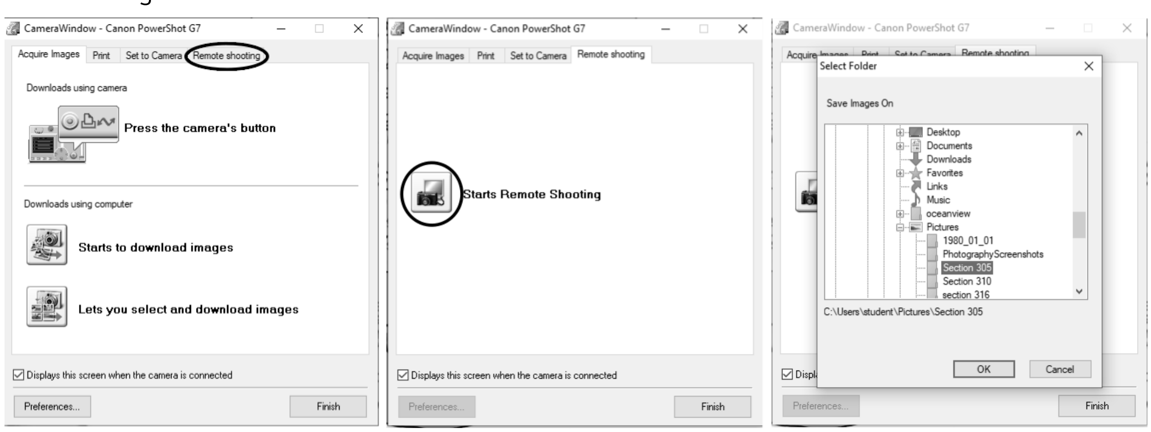



Figure 2. The first window opens automatically. Clicking "Remote Shooting" gets the second window. In the second window click the icon to open the "Select Folder" window.

The Remote Capture Task window should now appear, showing the live preview:

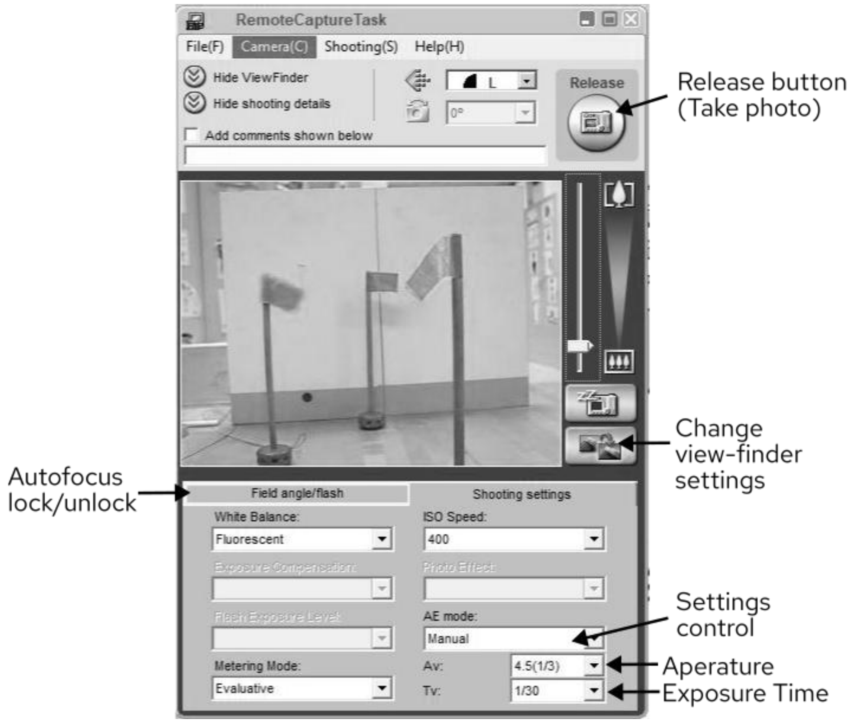



Figure 3: RemoteCapture Task window—live preview

- Under the “Shooting settings” tab, change the following settings to the values shown:

| **Setting Name** | **Value** | **Description** |
|---|---|---|
| ISO | 400 | Sets sensitivity of camera to light – to be discussed in part 6 below |
| AE Mode | Manual | Gives you full control over all camera settings |
| Av | 4.5 (1/3) | Controls the camera aperture—you will experiment with this in part 5 |
| Tv | 1/50 | Controls the exposure time (measured in seconds) |
| White Balance | fluorescent | Not really relevant for this lab, but feel free to play with it! |
| Metering Mode | evaluative | Not really relevant for this lab, but feel free to play with it! |

## 2. Pixels

Recall that a digital image is an array of pixels. 1 megapixel = 1 million pixels, or 1000 pixel x 1000 pixel. The more pixels or megapixels an image has, the smaller and less visible the individual pixels are to the naked eye.

- In “RemoteCapture Task,” press the “Release” button to take your first picture. The “ZoomBrowser EX” window appears automatically. Follow the instructions below to examine the picture.
- Click on the “Scroll Mode” tab near the top of the Zoom Browser EX Window (Fig. 4).

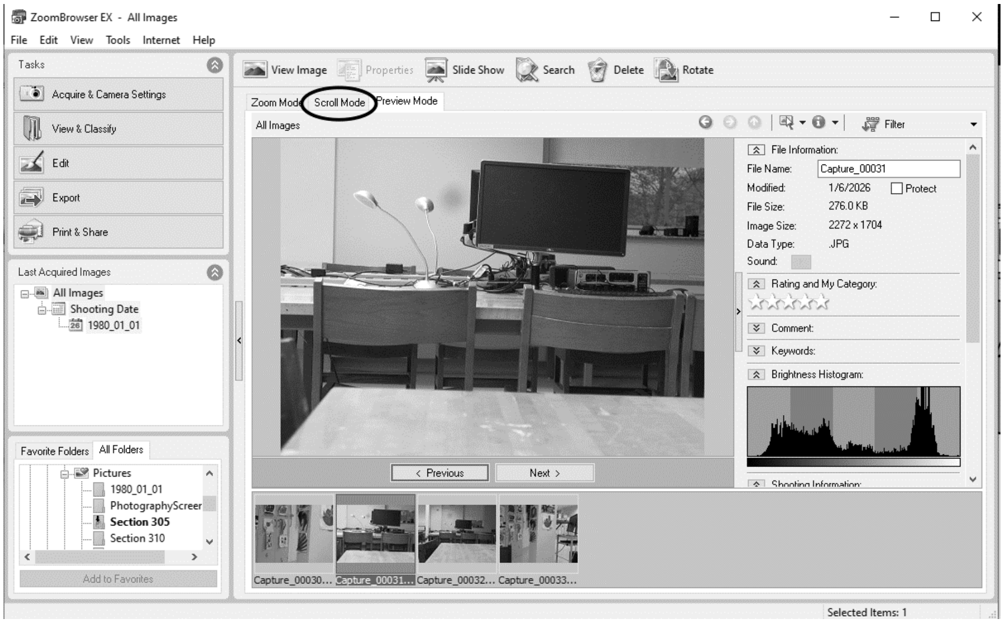


Figure 4: ZoomBrowser EX window

- Double click on the image you want to examine. A new window will appear showing the image.
- Go back to the Zoom Browser EX window and click "Properties," which should now no longer be grayed-out. A new window called "Properties" will appear.

You should now see the two windows shown in Fig. 5.

- Use the slider on the upper left of the window with the large picture in it to zoom in and out.
- Zoom in until you can see the pixels, and then out again until you cannot see the pixels anymore. How many pixels do you think there are in the picture? Make a guess!
- Go to the Properties window and scroll down until you see an entry labeled "Image Size"- this will help calculate the actual number of pixels. Were you close?

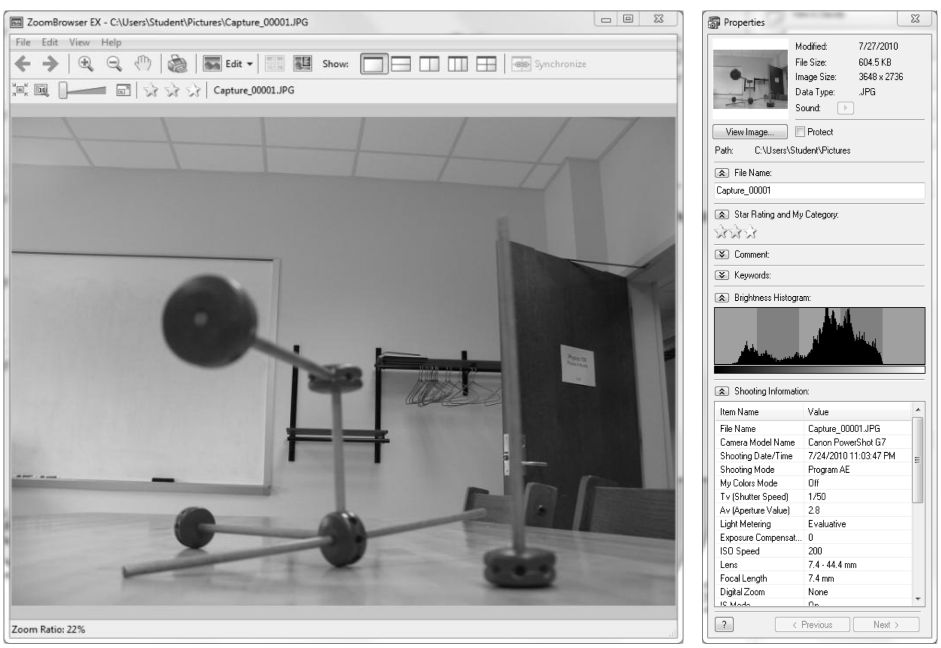


Figure 5: ZoomBrowser EX window, with Properties window

### 3. Focus

As you saw earlier, "zooming in" on an object requires changing the net focal length of the camera.

- The zoom slider is to the right of the preview picture in the RemoteCapture Task window. Listen and watch while you move the zoom slider. You should be able to see and perhaps even hear the camera lenses moving.

The focal length of the G7 can be changed from 7.4 mm to 44.4 mm. Recall that increasing the focal length while keeping the object distance constant increases the magnification of the object.

- Make sure that the zoom is zoomed all the way out before continuing.
- Go to the "Shooting settings" tab in the RemoteCapture window and make sure the "AE mode" is on the "manual" setting. Then go to the tab called "Field Angle/Flash," and set the "AF Operation" to "AF unlock." What this setting does is described in the table below.

| | |
|---|---|
| AF unlock | The camera automatically focuses; specifically, it measures the object distance and adjusts image distance **i** so the lens forms an in-focus image on the CCD. Notice that it moves the lens, not the CCD, to change **i**. |
| AF lock | The lens position is locked, so the image distance is fixed. |

The AF unlock sets the camera to autofocus on the object at the center of the image. But you want to choose to focus on different objects at different distances, one at a time, so you want to focus your camera manually.

- Place a lab notebook 30 cm from the front of the camera lens, and let it autofocus (AF unlock). Press the "change viewfinder settings" button.
- If the picture in the preview window looks like it is in focus, change the focus setting on the computer to "AF Lock." This locks the lenses in place so that objects that are 30 cm from the front of the camera lens are in focus.

  **Note: You should not hear any more whirring noises from the camera because it is no longer moving the lenses' distance around. If you do, that means the camera has gone back into "AF unlock" mode.**
- Move your notebook closer and farther from the camera. How much closer or farther can you get it before the image is blurred?
- Unlock the autofocus (AF unlock) and focus on an object a little farther from the camera (press "change viewfinder settings") then re-lock it (AF lock). You may need to zoom in to ensure your object fills the field of view.

  **Note: If you use the zoom, the camera will take itself out of AF lock mode because it has to move lenses around in order to zoom.**

## *Controlling the amount of light*

There are two ways to control the amount of light let into the camera when you take a photo: the **exposure time Tv** and the **f number** or **aperture Av**. Exposure time is the amount of time the sensor collects light to capture a photograph. Making the mechanical aperture diameter bigger lets in more light and making it smaller lets in less light. In the next two sections, you will examine exposure time and the aperture.

### 4. Exposure time

You can test camera exposure time using a light-emitting diode (LED), a frequency generator, and a frequency counter (Fig. 5). With this set up, the LED will blink on and off as many times per second as the frequency generator tells it to. The frequency counter is there to accurately measure and display the frequency.

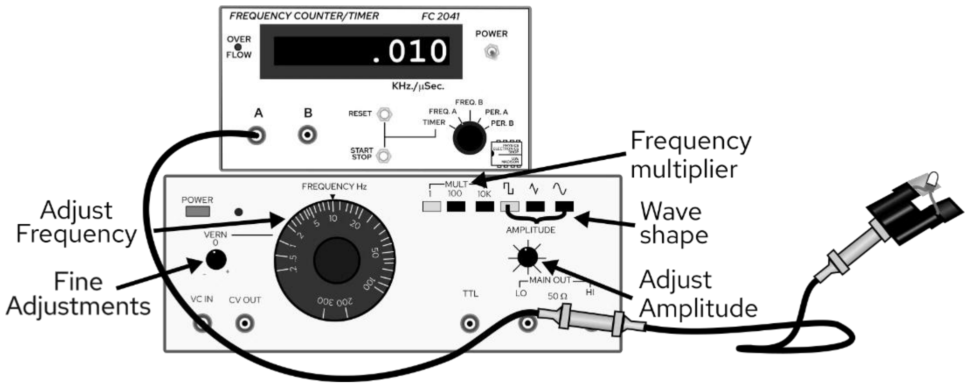



Figure 6: Blinking LED Set-up. On the frequency generator, select MULT: 1 and the square wave shape.

- Set the frequency generator to a 10 Hz (counter will display 0.010 kHz) square wave. This makes the diode blink 10 times per second, which you should be able to see. Try increasing the frequency while watching the LED. What is the highest frequency for which you are still able to tell it is blinking?

Once you have become comfortable with your blinking diode, try taking pictures of it.

- Set the exposure time Tv for the camera to 1/10 of a second, and ISO to 100. Set the frequency generator to 60 Hz (60 flashes per second). How many times do you expect the diode to blink in 1/10 of a second?
- Swing the LED in a circle while a teammate takes a picture (about one circle per second). The resulting photo should show a series of bright dots, separate images of the LED each time the LED blinked during the exposure time, in different positions because you were rotating it.
- Count the number of bright dots and compare this number to the number of flashes you expect to happen during the exposure time.

For example, if you set the frequency generator to 60 Hz and the camera exposure time is set to 1/10 of a second, you should expect to see 1/10 of the 60 flashes that occur every second, so ~6 bright dots in the picture.

- Choose at least **five** additional exposure times, from very long to very short, to take similar photos. For each exposure time, do the following.
  - Choose a blink frequency and set the frequency generator to that frequency, so that the photos have a reasonable number of dots. Record the number of dots you expect to see.
  - Take the photo. Do you see the expected number of dots? That is, do the camera and frequency generator agree on the time?

    **Note**: You may find that sometimes the number of dots in the photo is off by one compared to the expected number. Can you explain why this might be? Think about the unknown timing of the blinking with respect to the exposure time.

You often need to adjust exposure time to avoid motion blurring. As you have found, you can capture a blurry picture if you use too long an exposure time to take a picture of a moving object. You can also take a blurry picture of a static object if you move the camera while the picture is being taken.

- Hold the camera in your hands (not by the tripod) and take a picture without stabilizing your arms on the table. What is the longest exposure time you can use for a non-motion-blurred picture?

## 5. Aperture or f number

- Set the f number Av to 4.5 and take a picture of something on the tabletop.
- Inspect the picture:
    - In the ZoomBrowser EX window, click "scroll mode," and double click on your most recent picture.
    - Then click "properties" (Fig. 5) and locate "focal length" and "Av (Aperture Value)" in the middle of the list.
    - Calculate the diameter of the aperture in mm using $f/ = f_{net}/D$ where f/ is the f number, $f_{net}$ is the net focal length of the combined lenses, and D is the diameter of the aperture, that is the diameter of the lens effectively used to capture the picture.
- Compare your calculation to the camera's current aperture.
    - Hold the camera under a lamp and look inside the lens.
    - The aperture is right behind the lens, at the center of it; you cannot see anything behind it because the shutter is closed and in the way. Use a ruler to estimate the size of the aperture.
    - It is difficult to get an exact measurement, but does the aperture diameter appear to be similar to the diameter you calculated?
- Watch the aperture change as the camera adjusts to different lighting.
    - Under "Shooting Settings" set the AE mode to "Program AE" and the focus to "AF unlock."
    - Move your desk lamp in front of the lens while you look directly at the aperture through the center of the lens. You should be able to see the aperture getting bigger and smaller while the camera tries to decide what is the best size for the aperture.
    - Can you see a difference in aperture size if you point the camera at something bright, as compared to something dark?

**Remember that the amount of light that comes in through the aperture is proportional to the area of the aperture. The area of the aperture is proportional to the radius squared.**

## 6. ISO

The ISO setting controls the sensitivity of the image sensor to light. The higher the ISO number, the higher the sensitivity; that is, dimmer environments can be captured with a higher ISO. The number itself does not mean much (it is an attempt to replicate ISO numbers from film), but it does provide you with a relative scale. For example, if all other settings are identical, a picture taken with ISO 400 will be twice as bright as the same picture taken with ISO 200.

In some ways, ISO is like the volume control on a radio. If the incoming signal is very weak, you can turn the volume control up so that you can hear it. However, if you turn the volume up, you are likely to also hear a lot of unwanted noise or static. This is analogous to using a high ISO number in a low-light setting. You can produce a reasonably bright image, but you will also get more visual noise, which is called **graininess**.

- Set the AE mode under "Shooting Settings" to "Manual."
- Set the ISO to 400, and set the exposure time (Tv) to 1/200 of a second. Take a picture; it should be pretty dark. Label this picture 1.

- Now change the ISO to 1600, and take another picture of precisely the same object with the same conditions of light. Note that it is much brighter. Label this picture 2.
- Now change the ISO back to 400. What exposure time gives you the same exposure (brightness) as picture 2? Make this change and take another picture. This final picture is picture 3.
- Compare picture 2 and picture 3. Describe their similarities and differences in terms of brightness and in terms of graininess.

**Going to high ISO lets you take pictures in the dark, at the price of higher noise.**

## 7. Exposure vs. Aperture—Reciprocity

You have already tested the exposure time settings for the camera. Now you are going to test whether the aperture settings are consistent with the exposure time settings: you are going to test the camera **reciprocity**.

A camera has good reciprocity if a picture taken with f number Av and exposure time Tv looks as bright as a picture taken with f number n×Av and exposure time $T/n^2$. Here n is any number, recall aperture diameter is focal length f/f number Av. For example, if you double the aperture diameter, you have to cut the exposure time by a quarter to maintain the same brightness.

- Set the ISO to 400, the exposure time Tv to 1/40, and the f number Av to 4.5. Take a picture (picture A).
- Change Av to 4.0. Does this make the aperture bigger, or smaller? Take another picture (B).
- Compare the two pictures. Which is brighter?
- Now change Av to 8.0, change the exposure time Tv to 1/10, and take one more picture (C).
- Compare the last two pictures, B and C. Do they look equally bright? Are they exactly the same?

If the camera is well-designed, pictures B and C should be equally bright. The picture got darker by making the aperture smaller, but it also got brighter by the same amount by increasing the exposure time.

- Why did you have to quadruple the exposure time (1/40 s to 1/10 s) in order to cancel out halving the aperture diameter (f number Av from f/4 to f/8)?

The two effects should cancel out exactly for these two pictures. If they do, you have established that the camera has good reciprocity. If they don't, and one picture is brighter than the other, then you have established reciprocity failure, which means that either the aperture or the exposure time is not accurate for this camera.

## 8. Exposure vs. Aperture—depth of field

If you carefully compare the last two pictures you took, you may notice that the two pictures are not exactly the same, even if they are equally or similarly bright: They have different **depths of field**. The depth of field of a picture is the range of object distances over which their images in the picture are in focus. A smaller aperture yields a larger depth of field.

- Go to the RemoteCapture Task window and click the Field/Angle Flash tab. Unlock the autofocus ("AF unlock") and set Macro to "On." ("Macro" is the setting you use to allow the camera to focus on objects that are very close to the camera lens.)
- Position a lab notebook so that it is 20 cm in front of the camera lens and click "change viewfinder settings." Then set "AF Lock." Now any objects placed 20 cm away from the lens are in focus.


Figure 7: Tinkertoy flags at different distances from the camera

- Set up five tinker toy flags so that each is at a different distance from the lens: 7 cm, 20 cm, 33 cm, 46 cm, and 1 m. Make sure you can see all five flags in the camera image.
- Take a picture with a large depth of field: set the f number Av to 8.0 and the exposure time Tv to 1/10 second.
- Take a picture with a small depth of field: set the f number Av to 2.8 and the exposure time Tv to 1/80 second.
- Can you see the difference in depth of field between the two pictures? Which flags are in focus for each?
- Many video conference platforms (Zoom, Teams, etc.) let you "blur the background" of your video. Are they achieving this by changing the camera's depth of field or by some other method? How can you tell?

## 9. Have fun!

For whatever time remains before the end of the lab, you are welcome to try taking creative pictures, whether of your fellow classmates, inanimate objects, or of the natural world.

Hopefully, you will use what you learned in today's lab when you take photographs in the future. Enjoy!

# Lab 4: Additive Color Mixing

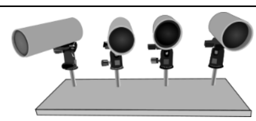
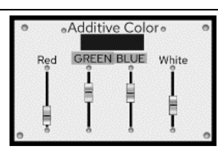
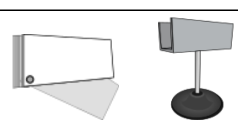
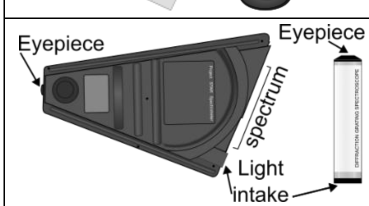
Spectral colors extend from red to orange, yellow, green, cyan, blue, and violet. Other hues are not seen in the visible light spectrum and are therefore called **non-spectral hues**, or **mixed-only hues**. For example, mixed-only hues are magenta or purple, which can only be obtained by adding lights of two spectral colors: red plus either blue or violet.

In your eyes, the light sensor is the retina, where three different kinds of cone cells detect light with long, medium, or short wavelength ranges. Because each type is sensitive to a broad range of wavelengths, it can detect light of many different wavelengths with very different intensities. These cells make it possible to detect and interpret all the different physical colors, spectral or mixed, and generate the sensation you perceive as color.

In today's experiments, you will use four LED projectors, which project colored light - red, green, blue - and white light. You will find that the **three additive primaries**, **RGB** can be combined to mix all of the colors represented in the color triangle. When the light circles from all three projectors overlap to make one circle on the white screen on your lab table, you perceive it as one color. The color you see depends on the RGB mixture in each experiment. Though much larger, this circle behaves like every pixel on your phone or computer screen. On screens, microscopic pixels emit RGB light, which is then mixed in your eye, so you perceive the mixed color, not the separate RGB colors. The additive color mixing happens in your retina, not on your screen.

None of the RGB projectors emits any yellow light. Despite the lack of spectral yellow (yellow wavelengths), you will produce yellow by overlapping R + G lights! This suggests that your eye does not separate light into the different wavelengths, like a prism does. Rather, you perceive a given hue when your long, medium, and short wavelength cones are activated in a particular combination and send that message from your retina to your brain. Many wavelength combinations could activate the same combination of cones and therefore be perceived as the same hue. In the example above, how do you see yellow if there is no spectral yellow light? R and G light stimulates your cones in the same combination as spectral yellow.

## Equipment

| | |
|---|---|
|  | **Light Emitting Diode (LED) Projectors:** Four projectors with circular apertures produce circles of R, G, B, or W light. Knobs on the stands allow you to adjust the positions of the projected light on the screen. |
|  | **Projector Dimmer:** Sliders control the intensity of each LED by controlling the electrical current to it. Your eyes perceive light intensity as brightness. A screen shows the brightness level from 0 – 100. The ON/OFF switch is on the back. |
|  | **Book of Filters and Stand:** The stand holds the book of filters with one color filter at a time in front of the white LED. You will try to match this color using a combination of RGB lights. |
|  | **Spectroscope or Spectrometer:** These tools separate light into different wavelengths like a prism. By looking through the eyepiece and pointing the device at a light source, you can see either individual lines or a continuous spectrum of colors depending on the wavelengths that make up the light. |

Other equipment: white screen, colored papers, tape

## Initial Observations

You have four projectors: R, G, B, and W. Based on what you have learned in class, what wavelength(s) do you think are present in each circle projected by each of the RGB projectors? Do you expect individual lines (single wavelengths present) or a broader band spectrum from each of the projectors?

- In your notebook, create a table like the one below. First write down what you expect to see, then do the experiment and write down what you actually see.
- Use the spectroscope or the spectrometer to observe lines or broad bands.
  - One at a time, set the intensity of each projector to 40 (with the rest at 0).
  - Point the spectroscope at each circle and look through the eyepiece (narrow end or cone-shaped end). You may need to get very close to the screen (30cm) with the spectroscope.

| **projector** | **expectations:** | | **observations:** |
|---|---|---|---|
| | **line(s) or broad band?** | **wavelength(s)** | |
| **red** | | | |
| **green** | | | |
| **blue** | | | |
| **white** | | | |

- When you are done, move all the sliders down to 0.
- Do you see blue or green wavelengths with the red projector on? R or B when G is on?
- Compare your expectations to your observations. Were there any surprises? Comment in your notebook.

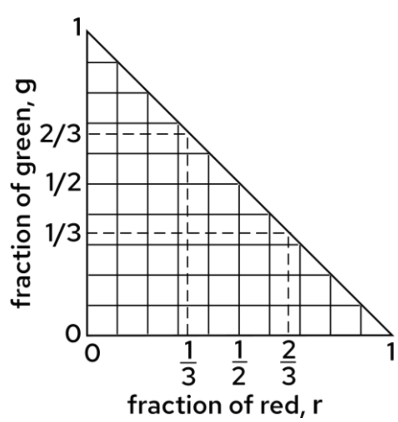
### Color Mixing

In lecture, you have used the color triangle to mix different colors from R, G, and B. You can identify the location of any color on the triangle from the fraction of red and the fraction of green in the mixture.

Recall that:

$$\text{fraction of red, r} = \frac{R}{R+G+B} \qquad \text{fraction of green, g} = \frac{G}{R+G+B}$$

where R,G,B are the intensities (from 0 to 100) of each RGB projector, and r, g, b are the fractions of each color (from 0 to 1). The fraction of blue is not explicitly marked. However, since the sum of the fractions is always $r + g + b = 1$, the fraction of blue can be calculated given the fractions of red and green.



Throughout the experiments in this lab, you can use the spectroscope or the spectrometer to compare the perceived colors to physical colors of light that are actually present in the light circles.

- Sketch a large (full page) color triangle in your notebook. Be sure to include the axes labels and grids shown. Mark the locations of red (R), green (G), blue (B), cyan (C), magenta (M), yellow (Y), and white (W).

# Experiments

## 1. White

- Turn on the R, G, and B projectors and overlap them. Use your predictions to create a white circle without the white projector.
    - Once you have a mixed (R+G+B) white circle, use the spectroscope to look at it on the screen. Get close to the screen and make sure that the overhead lights are off.
    - Record what you see – do you see a continuous spectrum or separate lines or separate broad bands?
    - Calculate the fractions of R and G (**r** and **g**) in your white circle using the projector intensities R, G, B.
- Turn on the white projector to make a second white dot on the screen. Use the spectroscope to compare the RGB white dot and the white projector dot. Record what you see.
- Turn off all the projectors.
- On your color triangle, mark the location of your mixed white as mxW.

## 2. Hue, Saturation, and Brightness

You can describe any RGB mixed color by specifying three parameters: hue (r, g, b or position on the color triangle), saturation (amount of white, or distance from W), and brightness (intensity). You will start with a few experiments to visualize the meaning of the three parameters.

### Hue

- Move the sliders for the R and G projectors to 40% and adjust the projectors so that the two circles overlap. What hue do you see?
- Use the spectroscope to look at the mixed color on the screen. Which bands do you see?
- If your observations surprise you, consider the following: Turn off the R projector, and again use the spectroscope. What do you see? Now turn back on the R projector, and look through the spectroscope. What do you see?
- Calculate the fractions ***r*** and ***g*** for this hue using the projector intensities. Mark the location for this hue on your color triangle as mxY.

### Saturation

- Turn off the R projector so that you have a G circle.
- Turn on the W projector to 40 and move it so that the W circle partly overlaps the G circle. What do you see?
- Calculate the fractions **r**, **g**, and **b** in the G + W color. Treat W as 40 R + 40 G + 40 B.
    - Where is this color on the color triangle?
    - If you draw a line from G to W, all the colors along the line have the same **hue** but different **saturations**.

**The circle is still the same green hue – but it is paler, it has lower saturation, it has a higher amount of white in it.**

### Brightness

- Turn off the W projector and move the sliders for the R and the G projectors to 40. Adjust the projectors to make non-overlapping circles side by side.

  The two spots should have the same perceived brightness (the same intensity). As you saw earlier, the R circle should have **r** = 1, **g** = 0, **b** = 0; and the G circle should have **r** = 0, **g** = 1, **b** = 0.

- Now adjust both the R and G sliders to 20. Compare the brightness of the two circles – are they still equal? How would you describe the colors you see?
- Compute the new fractions **r** and **g** when the sliders are at 20. Where are these dimmer R and G colors located on the color triangle?

  **The color triangle can show you hue and saturation, but does not show brightness. A very bright color is located in the same place as a very dark color.**

## 3. Color Triangle or Chromaticity Diagram

You can describe hues from the color triangle in terms of additive RGB and subtractive CYM primary colors. If a hue is between primaries on the color triangle, use the additive primary as an adjective and the subtractive primary as the noun. For instance, call orange (2 R + 1 G) a reddish yellow or RY since Y = 1 R + 1 G and you have extra R.

### Blue Added to Green

- Add B and G in equal proportions.
- Calculate the fractions r and g. Do you have a hue marked at this point on your color triangle?
  - If so, does the labeled hue match what you see on the screen?
  - If not, mark the point on the color triangle. Label the point with the name of the hue.
- Use two parts blue and one part green, and label this location on your color triangle using the above convention as BC.
- Use two parts green and one part blue, and label this location on your color triangle as GC.
- Note: all mixtures of B and G are represented by points along the line from B to G. Check the relationships below on your color triangle.
  - For equal intensities of B and G, the point lies halfway between B and G.
  - If there is two times more Green than Blue (GC), the point is 1/3 of the way from G and 2/3 of the way from B. The distances are in direct proportion to the mixing ratio.

### Red Added to Green; Red Added to Blue

- You have already added R and G in equal amounts. Reset the projectors to produce a mixed yellow.
- Adjust the projector intensities so that you have two parts R and one part G. What would you call this hue? Add it to your color triangle.
- Mix two parts G with one part R. Name the hue produced and add it to your color triangle.
- Repeat the previous three steps but with R and B. What colors are produced by the following?
  - equal intensities (1 R + 1 B)
  - 2 R + 1 B
  - 1 R + 2 B

### 4. Complementary Hues

Two hues are called complementary when added to one another they produce white.

- Add R and G in equal amounts to get a yellow hue.
- Which hue is complementary to yellow? **Hint**: What must one add to R+G to produce W?
- Project a circle of the complementary hue and then move it so that the two complementary hues are seen side-by-side with some overlap. If done correctly, the overlapping area will appear white.
- Look at the color triangle in your notebook – where are the two complementary hues located with respect to white? **Hint**: draw a line between the two hues.
- What hue do you expect to be complementary to R? And to G? Try It! Then show the complementary hues on the color triangle in your notebook.

## Color Matching

### 5. Matching Filter-colored Light

So far, you have been combining R, G, and B lights the way a computer display or cell phone screen does to produce differently colored pixels. In theatres, stage lights often use colored filters to produce different colors of light from white light. These filters subtract certain wavelengths from white light (you expand on this in the next lab).

Today, you want to match the color of filter-colored light by adding R, G, B in different proportions.

- Choose one of the colored filters (pink, azure, orange, purple, etc.) and make it stick out of the book of filters. Set the book of filters on the stand and place your chosen colored filter in front of the W projector to create a circle of colored light on the screen. You may need to adjust the intensity of the W projector.
- Adjust the position of the R, G, and B projectors so they create one circle, next to (but not overlapping) the colored spot from the white light + filter.
- Adjust the amounts of R, G, B until the two colors match. Write down the three RGB amounts.
- Describe in words the color you have matched (ex. “sky blue” describes a bright, pale blue).
- Repeat this process for two additional color filters.

### 6. Matching Pigments

- Remove the color filter and attach a sheet of colored paper to your screen.
- Illuminate the colored paper with the W projector.
- Repeat the color-matching procedure that you used for the filter in Experiment 5 and record the color name and mix of RGB amounts you used to match it.

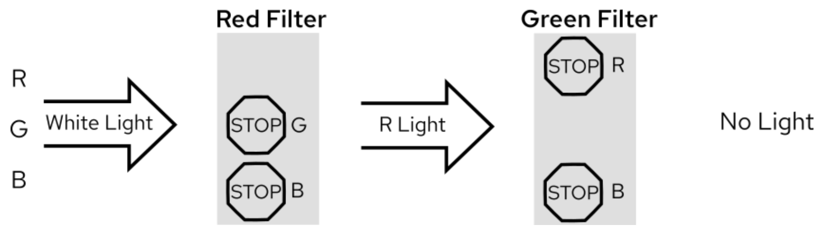
# *Lab 5: Subtractive Color Mixing*

Visible light varies in wavelength from about 700 nm to 400 nm, which includes all spectral colors from red to violet. It is useful to simplify this broad range with lots of colors into three ranges: red or R (700-600 nm); green or G (600-500 nm); and blue or B (500-400 nm).

In the last lab you used **additive** color mixing to combine light sources of different colors. By adding another light, the projected dot got brighter, and combining equal amounts of R, G, and B gave you a white dot.

If, instead of light, you mix R, G, B pigments, you get a muddy, dark color. This is because pigments (and filters) use **subtractive color mixing**. Specifically, R paint is red because it **absorbs** B and G and scatters R. Similarly, G paint is green because it **absorbs** B and R, and B paint is blue because it **absorbs** R and G. Similarly, white light with a red filter transmits R light and absorbs G and B light.

Consider overlapping an R filter and G filter. White light is R+G+B, but the R filter subtracts G and B, the green filter subtracts R and B, leaving:




White light & Red Filter & Green Filter = No Light

(R + G + B) - G - B - R - B = 0 (no negative light!)

- **Try looking through a red filter layered with a green filter, what do you see?**

When mixing pigments or overlapping filters, we use **subtractive color mixing primary colors: cyan (C), magenta (M), and yellow (Y)**. These are a better choice for primary colors with filters and pigments because CYM only subtracts one of the RGB colors of light from white light. This gives us more flexibility to create a wide range of colors.

In addition, when using pigments, black (K) and white (W) are often used in addition to CMY to more efficiently produce a broad range of brightness and saturation. With filters, K is mixed in by using a gray filter, or neutral density filters, which absorb equally all wavelengths of light. Adding white to filtered light usually means using a filter that is semi-transparent at the wavelengths it absorbs, so only a portion of the light is removed at those wavelengths.

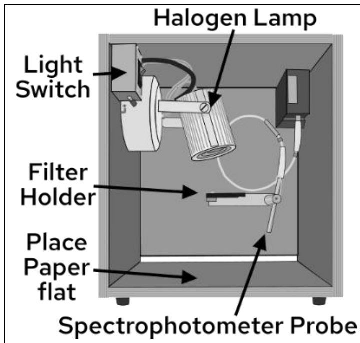
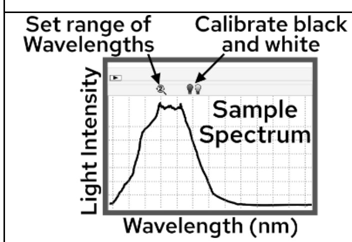
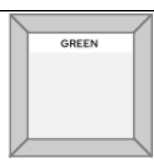
To identify the color a mixture will have, you need to consider what colors are absorbed or **subtracted** by the pigments in the paint from the illuminating white light (with all wavelengths).

Understanding color mixing is essential for artists. Many artists mix pigments intuitively from experience or by trial and error on their palette until the color is just right. Your goal today is to understand the physical process of mixing pigments. Besides painting, subtractive color mixing applies to color printing (inkjet printers, lithography, silkscreen, or photo printing) and color filters (stage lights). Filters use transparent pigments in a transparent gel material, so the filters absorb certain colors and **transmit** (and partly also **scatter**) the remaining colors in white light.

## Equipment

If you have ever gotten paint matched at a hardware store, you have seen a version of today's equipment in action. These instruments identify the wavelengths of light scattered by a paint sample. Then they determine the mix of pigments needed to match the color of the sample.

For this lab you will use a **spectrophotometer** to measure which light wavelengths are scattered by the paint and collected by the probe. This spectrophotometer is much more precise (and much more expensive) than the hand-held spectrometers used in the additive color mixing lab.

| | |
|---|---|
|  | **Spectrophotometer Setup**: 1. The halogen lamp (white light) lights up the sample on paper. 2. The sample and the paper scatter light and the spectrophotometer probe collects it. 3. The probe's fiber optic captures light from a small area and sends it to the spectrophotometer. 4. The spectrophotometer measures the amount of light at each wavelength.<br>The slot in the back of the box lets you slide paper into position underneath the probe so you can put the sample color mixture under the probe for analysis.<br>The filter holder supports a filter (or filters) between the lamp and white paper to analyze the color of transmitted light. |
|  | **Spectrum Analysis App**: Displays the intensity of light reflected at each wavelength on the lab computer.<br>You can calibrate the intensity range using a white sample ( ) and a black sample ( ). May need to recalibrate between trials.<br>The app can also zoom in ( ) on a specific range of wavelengths. Note the range goes from short wavelengths to long wavelengths, the opposite of our usual RGB order. |
|  | **Gel sheet filters**: Identified by plastic or carboard frames. These filters are relatively inexpensive, but imperfect. They have gradual rather than abrupt cutoffs between transmitted and absorbed wavelengths.<br>**Avoid touching the gel sheet with your fingers.** |
|  | **Dichroic filters**: Identified by metal tape frames. These more expensive glass filters have sharper wavelengths cutoffs; most wavelengths are fully transmitted (~70%) or fully blocked (<1%).<br>**Avoid touching the glass with your fingers.** |
|  | **Acrylic paints in CYMKW**: You use these to mix pigments. The five tubes of paint can be used to mix nearly any color you want. |

Other equipment: white paper, paint brushes, water container

## Experiments

### 1. Setting up the spectrophotometer:

- Open the "Lab Software/109" folder on the lab computer. Start the "OceanView" app.
    - If the app does not recognize the spectrophotometer, disconnect the USB cable from the spectrophotometer unit, then reconnect it. If it fails again, ask your TA for help.
- Take the blue cap off the end of the spectrophotometer probe.

- Click the “manually set numeric ranges” button. Set the **x-axis** minimum to 400 and maximum to 700. This adjusts the **range** to display only the **visible** part of the spectrum (400 – 700 nm).

  The y-axis is labeled “Light Intensity (counts).”

- Place a white sheet of paper on the bottom of the box and point the probe at it. Turn on your lamp to see the lamp spectrum displayed. Make sure the probe is in the center of the light circle to get the best results.
- You need to adjust some of the app default settings to get good readings. In the app, click "Window", then click "AcquisitionGroup".

You want to get a maximum light intensity of about 3000 counts during the “integration time.” Try to keep the maximum intensity below 4000 counts to avoid overexposure.

- Settings that usually work well:
  - **Integration Time**: **5 ms**

    The spectrophotometer continuously scans the light intensity in 2000 wavelength channels for 5 msec. This is analogous to the exposure time of a camera.
  - **Scans to Average: 10**

    To reduce effects of light changing over time, the app averages 10 scans of the spectrum.
  - **Boxcar: 10**

    To reduce light variation over an area, the app averages the value of a pixel with ten pixels on either side.
- Feel free to play with these settings to understand how they affect the displayed spectrum.

Because light in the laboratory changes throughout the day, you need to calibrate the app to “white light” and to “no light” or black.

- First, click the lit light bulb icon: .

  A new window will open showing the percentage of light detected, compared to the current calibration’s white reference.

- Now, turn off the lamp (or cover the probe opening) and then select the darkened light bulb icon at the top of the screen: . This defines “no light,” or 0% at all wavelengths.
- Next, place a white piece of paper underneath the probe, turn on the lamp, and select the lit light bulb icon: . This defines “white light”, or 100% at all wavelengths. You will need to reset the wavelength range to 400 – 700 nm.

Ideally, the graph should now show 100% “Transmission” because you have defined this paper to scatter 100% R, G, and B. However, two things can affect this: 1) The light intensity of the bulb changes a bit with time. You may have to redefine “white” reflected from paper by selecting the lit light bulb icon throughout the lab. 2) Different white surfaces scatter different amounts of light, so use the same white paper throughout the lab.

Before experimenting with the pigments and filters, familiarize yourself with the spectrophotometer app.

- Try a different white (perhaps a page from your notebook). What intensities does the app show? To define the new surface as “white,” click the lit light bulb icon and the plot on the screen will again show 100% for all wavelengths

- Use the probe to look at two different sheets of colored paper. How much of each wavelength does each scatter compared to the white paper? In your notebook, name the color and sketch the spectrum for scattered R, G, and B light ranges.
- If you redefined "white" as a notebook page, reset it to the white paper you will use during the rest of the lab. Now you are all set – you can switch between the tabs any time you like to see either the Spectrum "Intensity (counts)" or the "Transmission (%)".

## Experiments Mixing Filters:

**Please note that filters can be damaged by the heat of the lamp. Please limit the time that filters are exposed to the lamp light. Record the spectrum for a short time and pause the graph, then remove the filter. Thank you!**

**Place the filter(s) in the holder, turn on the lamp and click Start to acquire the desired spectrum. Then click Pause to freeze the graph. Now turn off the lamp and remove the filters. This keeps the lamp and the filters safe and cool while you analyze the spectrum. When you are ready to record another spectrum, place the next filter(s), turn on the lamp and click Restart . !!**

When you overlap filters, the color of light is determined by the light that is transmitted through both filters.

### 2. Subtractive Effects of Layered Filters

**Plastic-frame filters:** you have gel sheet filters in a range of hues, well beyond the usual additive or subtractive primary colors (reddish-magenta, reddish-yellow, etc.).

- Locate three identical reddish-magenta filters with plastic frames. Place a sheet of white paper in the spectrophotometer box, and turn on the lamp.
    - Start with observation: Place 1, then 2, then 3 filters on top of each other on the filter holder. **Describe** how you see the transmitted light changing as you add each filter. What happens to the hue and to the brightness?
    - Now use the spectrophotometer to examine the spectra with one, two, and three filters.
    - In your notebook, sketch the spectrum for 1 and for 3 filters. Why are the spectra different? As light reaches each filter will it transmit more light in the red or the blue? What happens with 2 or 3 filters?

**Dichroic filters:** You have nearly ideal optical filters for the additive (R, G, B) and subtractive (C, M, Y) primary colors. These filters transmit almost all light in specific wavelength regions, while almost completely absorbing light in other regions.

- **One filter at a time**: Select three of the six dichroic filters.
    - For each filter, measure and sketch the transmitted spectrum.
    - Make a table like the one below with the color of each filter, and identify which parts of the spectrum (R, G, B) are absorbed and which are transmitted.
    - For each color transmitted, estimate the percentage of light transmitted (% Transmission for the each RGB range).

| | **dichroic filter color** | **absorbs R, G, or B** | **transmits R, G, or B** |
|---|---|---|---|
| Filter 1 | | | |
| Filter 2 | | | |
| Filter 3 | | | |

- Does the yellow filter transmits yellow light? What does the spectrophotometer tell you?
- **Two filters overlapped**: Choose two dichroic filters from the subtractive primaries CMY.
    - Predict the color of light transmitted when you combine these two filters. **Hint**: Use a table like the one above for these two filters, and use it to figure out the result of combining these two filters!
    - Now do the experiment with the two filters, and use the spectrophotometer to analyze the result. What wavelengths remain? What color is transmitted?
- Artists see that combining blue and yellow paints makes green. Is this true for filters?
    - Sketch the transmission spectrum for a B filter and the transmission spectrum of a Y filter. What do you expect to be transmitted if you combine B&Y?
    - Try the filter combination with the spectrophotometer, and draw the resulting transmission spectrum.
- **Challenge:** Using the subtractive primary filters, either alone or in pairs, how many different hues can you produce? Record the color names and the filter(s) used to create them.

## Experiments Mixing Paints

As you work with paint mixing, ask yourself: Do the paint mixtures behave as overlapping filters? For instance: does mixing Y and C paint yield the same hue as overlapping a Y filter and a C filter?

**Exercise in acrylic paint mixing**: your TA will give you sheets of paper and paint tubes to mix paint in different proportions (see Experiments 3 and 4 below).

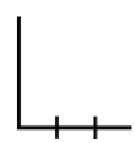
For **each** of the following paint-mixing questions, mix the paints, write down the mixture in numbers and letters, e.g. 2 Y & 1 M = ?, and plot the spectrum as shown below. As you mix each pigment to the mixture, notice the effect it has on the color and on the spectrum. You might consider 1 part to be the size of a pea and 0.2 part to be like a grain of rice.

- Paint the resulting color next to the question mark in your equation.
- Use the spectrophotometer to acquire a spectrum of the resulting color, and sketch the spectrum in your notebook.
- Name the resulting color.

3. Mixing pigments in acrylic paints: unequal parts of Y and M.

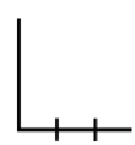
a. Mix 2 parts of Y and 1 part of M.

2Y & 1 M= ?



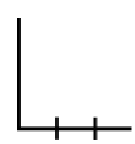
b. Divide the color paint you obtained in step a. into two halves, then mix one half with K.

2Y & 1M & 0.2K = ?



c. Mix the other half with K and W.

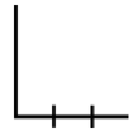
2Y & 1M & 0.2K + 1W = ?



4. Mixing unequal parts of M and C.

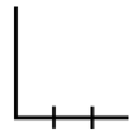
a. Mix 2 parts of M and 1 part of C.

2M & 1C= ?



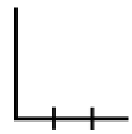
b. Divide the color you obtained in step a. into two halves, then mix one half with K.

2M & 1C & 0.2K = ?



c. Mix the other half with K and W.

2M & 1C & 0.2K & 1W = ?



5. Have fun!

For whatever remains of lab time, you are welcome to use the paints to experiment with other color mixtures or paint a picture! Examples from previous semesters are around the room.

**Please rinse your paint brushes before you leave.**

Real-World Application of Subtractive Color Mixing:

What you learned about subtractive color mixing is relevant to color painting and printing. Your experiment with filters showed you how the subtractive primaries (CMY) are combined to produce other hues, such as red, green and blue. In color printers, the three colors in your ink cartridges are always CMYK. Black ink is present because the mixture of C&M&Y does not produce a very good black, plus it uses three times the amount of ink. Thus, it produces better prints and is more economical to use an additional cartridge for K.

# Lab 6: Oscillators and Resonance

All musical instruments have an **oscillating** component: a vibrating string, a column of air, or the head of a drum. Most instruments have an oscillator and a resonator that also vibrates. For instance, when a guitar's string vibrates, the body of the guitar resonates (starts vibrating at the same frequency) to amplify that sound.

An **oscillator** has **periodic motion**—motion that repeats itself at regular time periods. This class focuses on oscillators that move about an **equilibrium position** (Fig. 1). At equilibrium, all forces on the oscillator balance. Away from equilibrium, forces on the oscillator are unbalanced and the net force (a **restoring force**) pulls it back towards equilibrium.

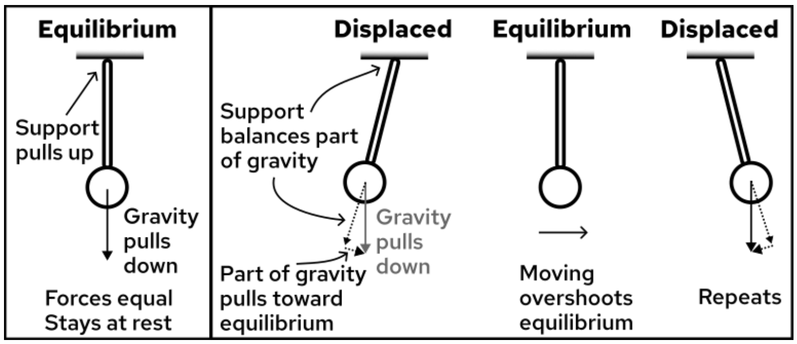



Figure 1. Equilibrium and periodic motion of a pendulum

Today's lab uses two **harmonic** oscillators. These are special because the restoring force is **proportional** to the displacement from equilibrium. The farther the system is from equilibrium, the stronger the restoring force is. The result is sinusoidal oscillations at the same frequency for any amplitude.

Today's oscillators will move **at a single frequency**. In contrast, musical instruments oscillate at multiple frequencies at once. In a future lab, you will explore how the combination of frequencies affects the sound quality of instruments.

An ideal system would oscillate forever at the same frequency and amplitude. Some of the data in the second part of today's experiment will look like the sinusoidal trace in Fig. 2. This figure shows readings from a sensor as a pendulum swings back and forth 5 times. With 5 cycles in 0.1 s, the frequency is 50 Hz.

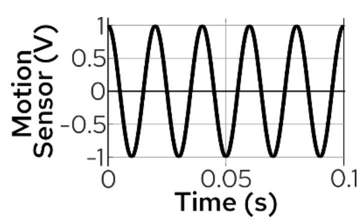



Figure 2. Sample recording of pendulum motion

Real oscillators have friction. This leads to two other qualities with musical value: **damping** and **resonance.**

**Damping** decreases oscillation amplitude over time. For instance, once a piano key is pressed, the sound gets quieter over time. The time for the amplitude to decrease by half is called the **damping time** $\tau$. With lots of damping, the oscillations decrease rapidly, and the damping time is short. When there is little damping, the damping time is long.

Damping is combatted by using a **driving force**, a periodic force that causes oscillation at a specific frequency. If the oscillator is driven at its **resonant frequency** (its natural frequency), the periodic **driving force** reinforces the system's natural oscillations and the amplitude increases. This is called **resonance**.

You can also drive a system to oscillate at other frequencies, but the amplitude of the system will be smaller, even if the amplitude of the driving force is the same. By driving the system at a variety of different frequencies (Fig. 3, top), you can measure the amplitude at each and build a resonance curve (Fig. 3, bottom).

The width of the resonance curve at half of its maximum (Δf) is related to the damping time. The product of the two values equals 0.4, a value that does not change.

$$\Delta f \times \tau = 0.4 \qquad (3)$$

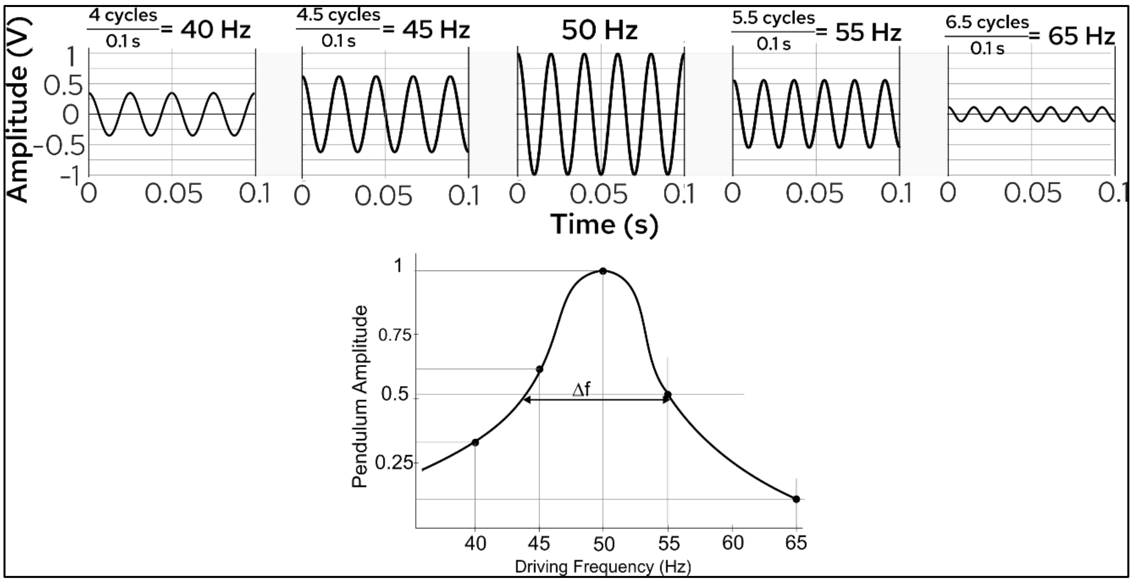
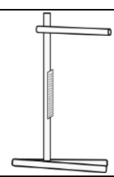
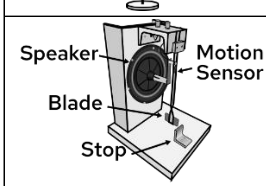
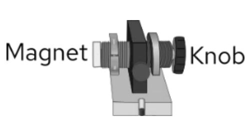
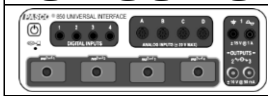



Figure 3. The system is driven at multiple frequencies near the natural frequency and the amplitude is measured at each (top). Then those amplitude values are plotted vs. the driving frequencies to produce a resonance curve (bottom).

## Equipment

| | |
|---|---|
|  | **Spring stand:** Holds a spring with a hanging mass so that you can measure the elongation of the spring and the oscillations of the spring. The scale on the vertical support is used to measure spring elongation. Verify that connections between the two bars are tight. |
|  | **Spring with hanging mass:** A spring with loops at the top and bottom. It hangs from the horizontal support on the spring stand. A 5 g mass weight hanger can be hung from the bottom of the spring. |
|   | **Blade Pendulum Apparatus:** A vertical pendulum blade with a sensor at the top to measure its motion. It can be manually set in motion or driven using the built-in speaker to explore resonance. A strong magnet can be placed nearby to increase damping. |
|   | **Strong Magnet:** Added to the platform of the Blade Pendulum Apparatus to damp the motion of the blade. Notches fit over posts on the platform and the distance between magnet and blade is adjusted by turning the knob. |

| | |
|---|---|
|  | **Pendulum Control Box:** Reads motion data from the pendulum and drives oscillation at a set frequency and amplitude |
|  | **Pasco Interface:** Connects our control box to the computer app. Make sure that the ON/OFF button is lit. |

Other equipment: stopwatch

## Simple Mass-Spring Oscillators

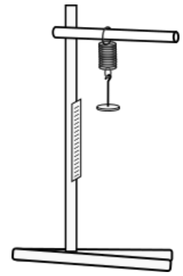


Figure 4. Mass-spring setup

Mass-spring systems are the simplest harmonic oscillators and are described by **Hooke's Law**. It says that the force **F** needed to stretch (or compress) a spring by some amount equals the **elongation x** multiplied by the **spring constant k**.

$$F = kx \qquad (1)$$

**Elongation** is the difference in a spring's length when a force is applied compared to its length with no force applied. The **spring constant** tells us the stiffness of a spring, and is a property of the spring. It acts as a **proportionality constant** between applied force and elongation.

For a given **spring**, a small force creates a small elongation, and a large force creates a large elongation. For a given **force**, a spring with a small spring constant (low stiffness) will stretch more than a spring with a large spring constant.

An oscillator has a **natural frequency f** that it will move at if it starts moving and is allowed to move on its own. For a mass-spring system, the natural frequency is determined by the mass **m** in kg and the spring constant **k** in N/m.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad (2)$$

Hooke's Law and natural frequency can both be used to find the spring constant of an unknown spring. Equation 2 can be rearranged to solve for k:

$$k = 4\pi^2 f^2 m \qquad (3)$$

### 1. Spring Constant

We will use both elongation and oscillation to estimate the spring constant of a small spring.

- Open the file **SpringConstant.xlsx** on the lab computer desktop. It does some calculations for you.
- Hang the spring on the stand and adjust the scale so that the 0 cm mark aligns with the bottom of the spring loop. Keep the scale in this position to measure the elongation of the spring.
  - Add the mass hanger (mass = 5 g) to the bottom of the spring and measure how far the spring has stretched (likely 0.5 mm to 5 mm).
  - Convert this value to meters and note it in the spreadsheet.
- Add slotted masses to the weight hanger so that the total hanging mass matches the first value in the spreadsheet. Record the total mass (in kg), and the resulting elongation (in meters) in the spreadsheet. Measure elongation at the bottom of the spring loop when the mass is at rest.
- Pull the mass down 1 cm, and release it, allowing it to oscillate. Make sure the spring and mass have space to oscillate freely. Time 30 oscillations. Record the time in the

spreadsheet. **Note**: if the mass stops before you get to 30 oscillations, you may need to start with a higher mass.

- The spreadsheet computes the period (T=$\frac{\text{measured time}}{\text{30 oscillations}}$) and the frequency (f = 1/T).
- The spreadsheet also calculates k from the oscillations (using f and m and equation 3).

- Repeat the above steps for the additional mass values found in the table.

Note: 1 kg = 1000 g, 1 m = 100 cm, weight = mass kg x 9.8 m/s$^2$.

| Total Mass (grams) | Total Mass (kilograms) | Elongation (meters) | Weight (Newtons) | Time for 30 oscillations (seconds) | Period $T$ (seconds) | Frequency $f$ (Hz) | Spring constant $k$ (N/m) |
|---|---|---|---|---|---|---|---|
| 20 | | | | | | | |
| 25 | | | | | | | |
| 30 | | | | | | | |
| 35 | | | | | | | |
| 40 | | | | | | | |

- Average the spring constants from the last column, and report the value in your notebook. You can use Excel's =average() function, where you might put "H2:H6" into the parentheses to average the values in column H from rows 2 through 6.

The spreadsheet also makes a graph based on Hooke's Law, using the weight as the force (F) that causes the elongation (x). Recall: $k = {}^{F}/_{x}$

- Do your measurements fit on a straight line? If not, call over your TA.
- The slope of the line is the spring constant k. Find and write k (with the correct units!) in your lab notebook.

- Print the spreadsheet and graph, and tape it into your lab notebook.
- Compare the value of k that you measured using the two methods. Do they agree to within 20%? To within 10%? **Note: k from f is likely an underestimate (spring has mass too).**
- How does the oscillation frequency change as you add mass? What implications could this have for musical instruments? Think about how this relates to string thickness on a guitar.

## Natural Frequency

For this part of the lab, you will use a set up (Fig. 5) that makes it easier to see the effects of damping and resonance. The metal blade is stiff and oscillates at a higher frequency. A sensor (strain gauge) at the top of the blade converts its motion to an electrical signal that is sent to the computer (like Fig. 2).

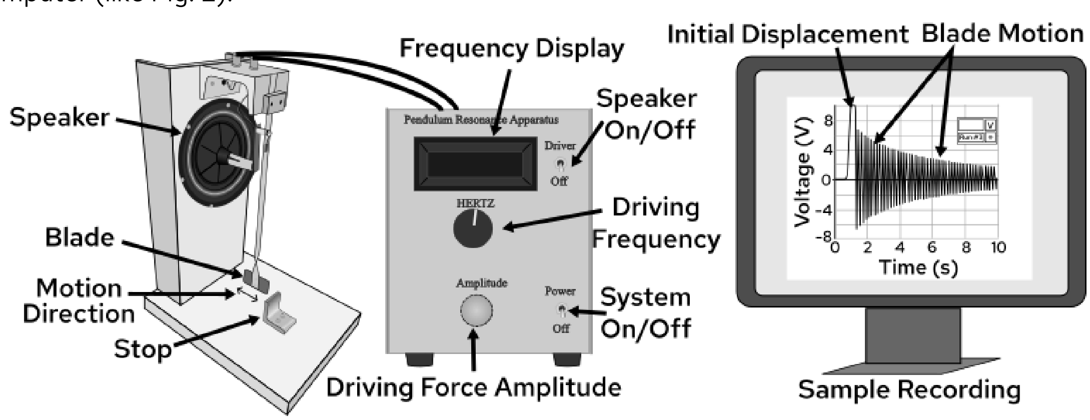



Figure 5. The blade's motion is converted to an electrical signal and displayed on the computer. The control box can also send a frequency to the speaker to drive oscillations.

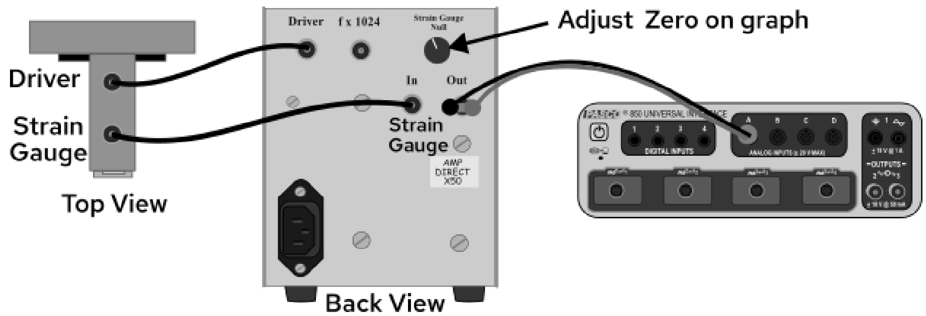



Figure 6. Connections for the blade pendulum set-up. The back of the control box has a knob marked “Strain Gauge Null” that can be used to center graphs of motion at zero.

## 2. Natural Frequency of Metal Blade

- On the control box, turn the “power” switch ON. The “driver” switch should be OFF.
- On the computer, open the “Lab Software/109” folder on the desktop. Double-click on “blade-experiment.cap” to start the recording app, Pasco Capstone.
- Once the app starts up, you should see sample data from a damped oscillation (Fig. 7).

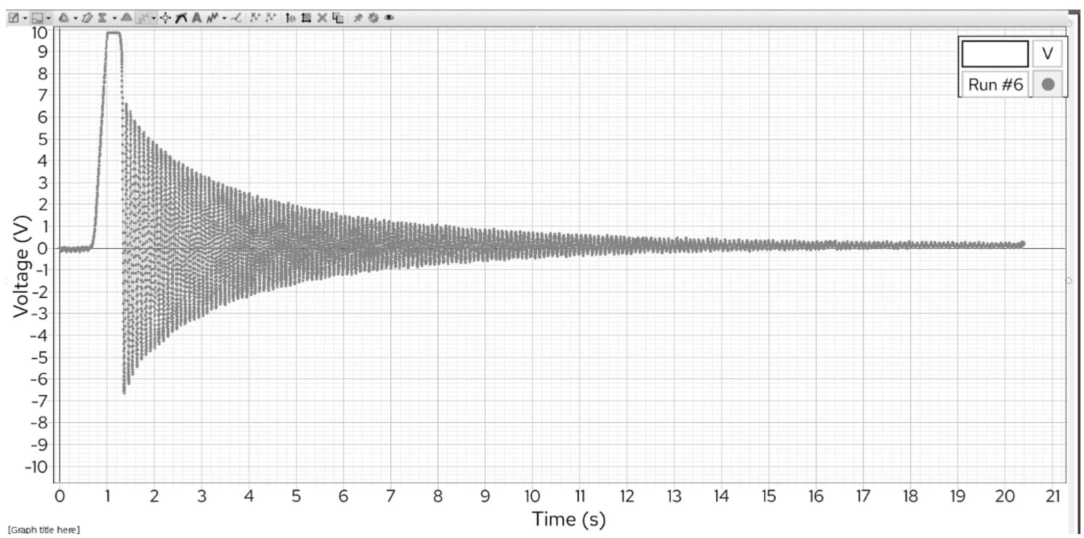


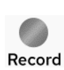

Figure 7. The app graphs the blade’s motion as a function of time, with motion converted to voltage (volts) and time in seconds.

Before you start recording data, familiarize yourself with the graph window and some of the most useful buttons on the Toolbar.


The “Coordinates” button displays the voltage (displacement) and time at any point;


The “Highlight” button enables us to select an interesting portion of the graph;


The “Scale to fit” button rescales the axes so that the data fills the graph window. This can be done for the entire dataset or for a highlighted section.



The “Record” button begins data plotting and recording. Click “Record” before starting the pendulum.

The “Stop” button replaces the “Record” button while data is being plotted. This button stops data recording.

When you are ready to record data,

- Remove the magnet assembly from the stand, and place it on the far end of the table.
- Click “Record”. Pull the blade forward to the stop and release it. Watch the oscillations on the screen.
- Click the “Stop” button to end recording. To see the entire pattern on the screen, click the “Scale to fit” icon.
    - **Note: If the oscillations are not centered at 0 V**, adjust the “Strain Gauge Null” (Fig. 6) on the back of the control box while recording to re-center the data. You will need to collect a new recording once this is done.
- Maximize the graph window, then click on the “Highlight” icon  on the Toolbar.
    - A shaded rectangle should appear. Reposition and resize it to cover **12** oscillations. **Note**: one oscillation includes a positive and a negative half.
    - Click inside the rectangle. Its shading should become darker. Now press the “Scale to Fit” icon. The area you selected should now occupy the entire graph window.
- Use the app to measure the differences in time and voltage:
    - Click on the “Coordinates” icon , choose “Add Coordinates/Delta Tool”,
    - Right-click on the square that appears, select “Show Delta Tool”, and reposition the corners of the displayed rectangle to cover **10** oscillations.
    - The time difference as well as the voltage difference show up in the middle of the guide lines created by the dragging action (Fig. 9).
    - Use the Show Delta Tool to measure the time taken for 10 oscillations, then find the period T and the **natural frequency** $f_N = 1/T$ of the oscillating blade. Record this in your notebook.

Time for 10 oscillations = ________ seconds

$T$ = ________ seconds

$f_N$ = ________ Hz

## Damped Oscillations

You will use the magnet to increase damping in this system. The magnet induces eddy currents in the metal of the moving blade. These currents interact with the magnetic field to create a force that opposes the blade’s motion (like magnetic brakes on trains). The closer the magnet is to the moving blade, the larger the damping.

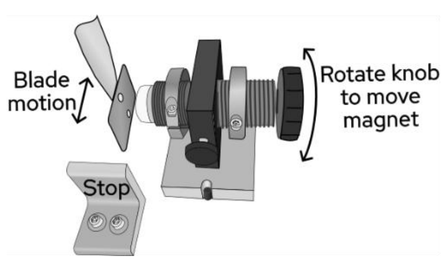



Figure 8. To increase damping, place the magnet stand on the pegs. The white capped magnet should be close to the copper blade.

3. Observation of damping

- With the magnet far away, pull the copper blade forward to the stop, and let it go. Watch as the oscillations decrease until the blade is no longer visibly moving.
- Put the magnet on the stand and mount it near the blade (Fig. 8).
- Again pull the blade forward to the stop and let it go. Is there a noticeable difference in the way the oscillations decrease?

Now you will use the sensor to make a more quantitative observation of damping time.

4. Measurement of damping

- Take the magnet off the stand and move it far from the pendulum.
  - Press Record, pull the blade forward to the stop, and release it. Record data until the oscillations are no longer visible (this may take a minute or so).
  - **Note**: If the voltage when the blade is at rest is not zero, use the Strain Gauge Null knob on the back of the control unit to re-center the data. Record new data.
  - Use the Show Delta Tool to measure the original amplitude (1.830 V in Fig. 9).

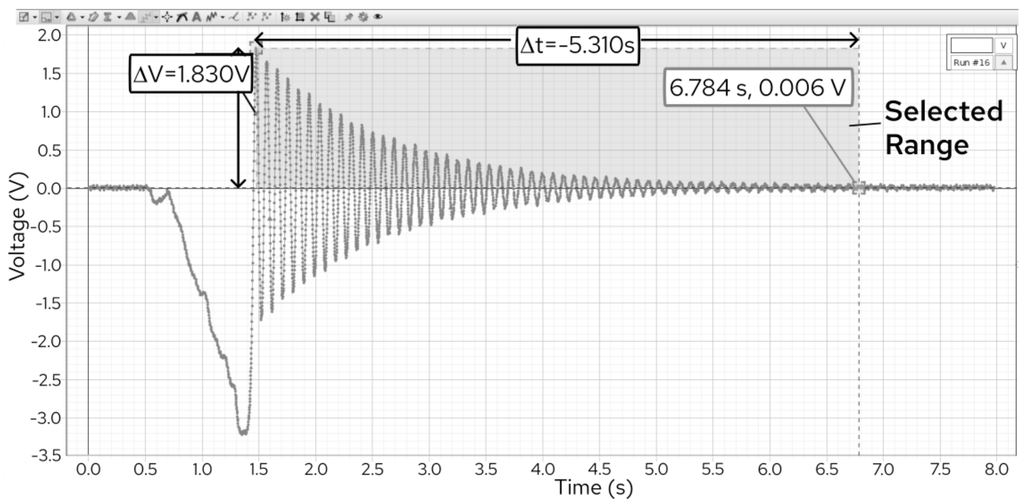



Figure 9. Sample recording with magnet near the blade.

- Calculate half the original amplitude, and use this to determine the damping time $\tau$. In Fig. 9, the initial amplitude is 1.830 V, so you would find an amplitude of ±0.915V to measure $\tau$.
- Approximately how many oscillations occur during one damping time?
- Put the magnet back on the stand. Repeat the steps above to measure the new damping time.

A nice feature of the app –the DATA button –allows you to choose multiple data sets and compare multiple graphs superimposed on the screen. Try it out for the two magnet positions.

- How much does the position of the magnet affect the damping time and the amount of damping?
- Did the change in the amount of damping affect the period of oscillation?

## Resonance

Some musical instruments can "hold" a tone that does not fade because a **driving force** sustains the oscillation. For example, a violinist's bow provides the driving force to keep the string vibrating. On a trumpet, the player's airstream is the driving force. For many instruments, the driving force is built up of many tiny, periodic "pushes" that are timed to correspond to the period of the oscillating system.

This equipment has a loudspeaker connected to the blade near the upper end. The control box sends a low-frequency signal to the speaker to push on the blade at the selected frequency.

## 5. Resonance Curve

Before taking any measurements, take a look at the oscillating blade while varying the driving frequency.

- Move the magnet far from the apparatus to minimize damping.
- On the control box, set the "amplitude" knob to zero (fully counter-clockwise) so that the driving frequency is "off."
- Adjust the frequency knob (labeled "Hertz") so that the driving frequency is equal to the natural frequency $f_N$ that you measured earlier in the lab. (The value should be about 10.6 Hz.)
- Turn the "driver" switch ON. The blade should remain motionless because the amplitude of the driver is set to zero.
- We want to maximize the blade's amplitude without letting it run into the stop. To set the amplitude:
  - Slowly increase the amplitude. The blade amplitude will lag behind the knob adjustments, so give it a few seconds each time you make a change. If the blade hits the stop, decrease the amplitude.
  - Notice that the amplitude varies with the driving frequency. As the driving frequency moves away from the natural frequency $f_N$ (in either direction), the amplitude decreases. This is an example of resonance.
  - Very slowly adjust the frequency of the driver down to 8 Hz and then up to 13 Hz and back to $f_N$. If the blade runs into the stop, decrease the amplitude slightly. Repeat the process until you find an amplitude setting that allows you to scan the entire frequency range without hitting the stop.

    **Once you have found a good amplitude setting, keep this setting for the rest of the experiment.**

Now, you will record the amplitude at different frequencies to build a resonance curve. On the Desktop open the spreadsheet **Resonance.xlsx**, which has rows and columns as shown below. Note that $f_N$ is the natural frequency that you measured earlier.

- Begin by filling in the Frequency column based on your measured natural frequency.

| | Frequency (Hz) | Amplitude (V) |
|---|---|---|
| $f_N$ + 1.0 Hz | | |
| $f_N$ + 0.5 Hz | | |
| $f_N$ + 0.2 Hz | | |
| $f_N$ + 0.15 Hz | | |
| $f_N$ + 0.10 Hz | | |
| $f_N$ + 0.05 Hz | | |
| $f_N$ + 0.02 Hz | | |
| $f_N$ | | |
| $f_N$ – 0.02 Hz | | |
| $f_N$ – 0.05 Hz | | |

| | | |
|---|---|---|
| $f_N$ – 0.10 Hz | | |
| $f_N$ – 0.15 Hz | | |
| $f_N$ – 0.2 Hz | | |
| $f_N$ – 0.5 Hz | | |
| $f_N$ – 1.0 Hz | | |

- Set the driver to the highest frequency in the table.
- When the amplitude has stopped varying[1], use the Show Delta tool to measure the amplitude. See Fig. 10 for an example: the amplitude is 0.415 V (=0.829 V divided by 2). Record the value in your notebook.
- Adjust the frequency to the next highest step, wait for the oscillation to settle, and measure the amplitude. Repeat for the rest of the frequencies in the table.

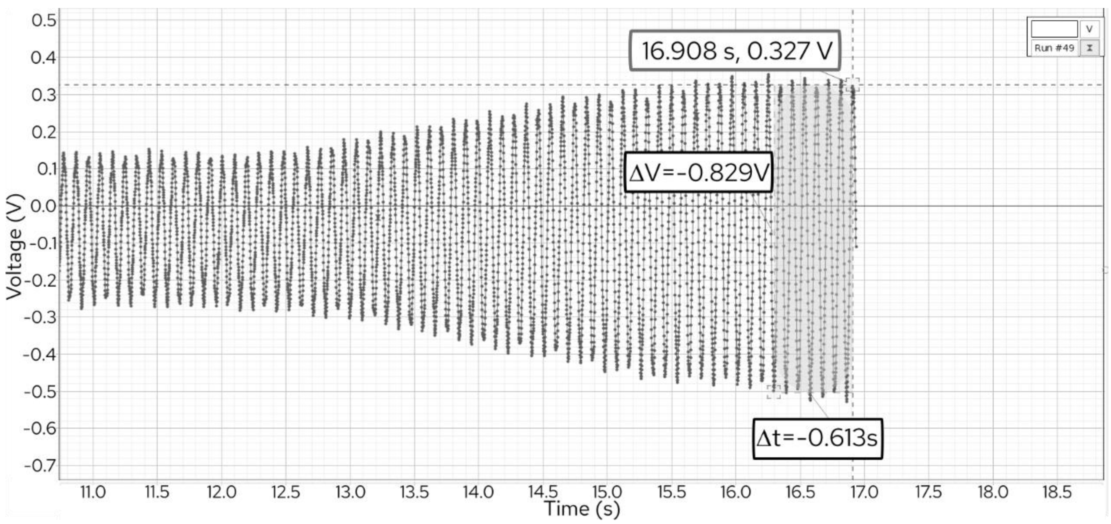
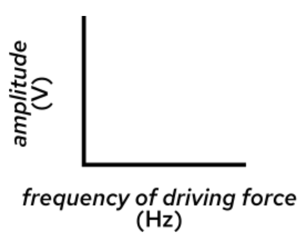
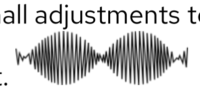



Figure 10. Sample Resonance Recording. Note that the amplitude grows for several seconds before reaching a constant value.

- The spreadsheet makes a graph of the results showing frequency on the horizontal axis and amplitude of oscillation on the vertical axis. Print out this graph.
- Connect adjacent points on the graph with lines.
- Use your graph to calculate an estimated:
  - Resonance frequency, $f_N$. This is the frequency of the largest amplitude.
  - Resonance width $\Delta f$. Recall that the width is the difference in frequency between the half-maximum points of the resonance curve




[1] Note that some frequencies will produce a beat frequency so that amplitude gets larger then smaller. Small adjustments to the frequency dial may help, but if not, use the maximum amplitude of one beat. 

Theory predicts the resonance width $\Delta f$ and damping time $\tau$ are related by $\Delta f \times \tau = 0.4$.

- For your data, calculate $\Delta f \times \tau =$ ______.

**Note**: Your value of $\Delta f \times \tau$ may be a little different than predicted by theory. Your set-up let you measure $\tau$ fairly accurately, but your measurement of $\Delta f$ is likely a little larger or smaller than the correct value.

If you look closely at the graph, you may see that the curve fits better if the maximum amplitude is at a slightly different frequency from the natural frequency $f_N$. There may not be a data point right at the maximum value of the curve. In your data, that means that the closest values to half-maximum are a little farther from the middle. If there were more time available, you could acquire more data points to form a more precise resonance curve, leading to a more accurate measurement of $\Delta f$.

## 6. Buildup Time of Oscillation

The sound of wind instruments or bowed strings does not start immediately; it takes a fraction of a second for the oscillations to build up. If time remains, observe the buildup time of our pendulum as follows:

- Turn off the driver, set the driver frequency to $f_N$, and clear the screen. Touch the pendulum to dampen any remaining oscillations.
- Click the "Record" button and then turn the driver back on.
- The buildup time is the time it takes the amplitude to reach half its final value. Measure the buildup time with the magnet far from the apparatus. Is the buildup time similar to the damping time?
- Now put the magnet back near the blade, and again observe the buildup time. Which is faster—the buildup time with or without added damping?

# *Lab 7: Strings*

As early as 3000 B.C.E., string instruments such as the lyre and the harp developed in cultures around the world. Today, the guitar, the violin, the piano, and many other instruments continue to use the vibrations of strings to make music.

In this lab, you will have the opportunity to see and to understand how strings move when plucked, bowed, or struck. You will observe the standing wave modes that compose the complex motion of strings.

## Standing waves

Suppose you have a string under tension that is fixed at its ends. It will vibrate when plucked (like a guitar) or bowed (like a cello). The shape of the string changes in a periodic way over time. Under the right conditions, the oscillations of the string form simple sinusoidal shapes. Further, every individual point on the string moves sinusoidally in time. We call this simple sinusoidal oscillation a normal mode. Since there is no change of the overall shape as a function of time, the oscillation is also known as a "standing wave." A string oscillating in various normal modes is shown in Fig. 1.

The normal modes on a string have a characteristic node and belly pattern. Nodes are specific points with zero amplitude (still points) that sit at the ends of each belly. Bellies are half-wavelength segments of string that move together. The belly's midpoint is called an antinode (maximum amplitude).

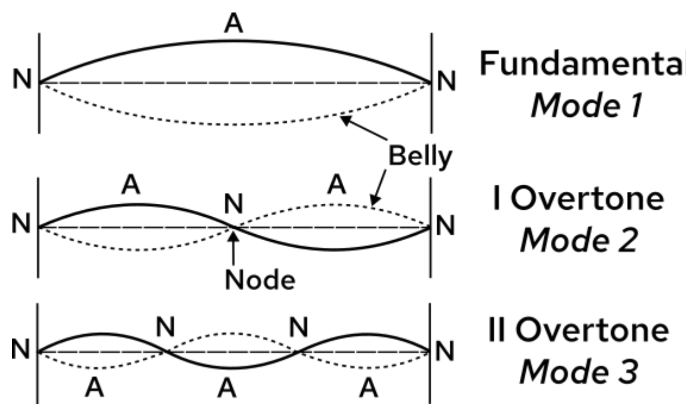



Figure 1. The first three modes of a string. The points marked N are called nodes, and the points marked A are called antinodes. Strings move through the three shapes shown (solid line, dashed line, dotted line) in each cycle.

When a string instrument is played, the string motion is usually much more complicated than the normal modes shown. However, the complicated motion from a pluck or a bow can be represented as a superposition (addition) of many normal modes.

The first mode is called the **fundamental**, while higher modes are called **overtones** of the fundamental. The $n^{th}$ normal mode of a string has a frequency (measured in Hz) of

$$f_n = n\frac{v}{2L}$$

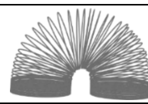
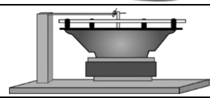
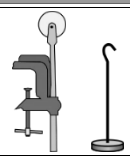
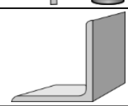
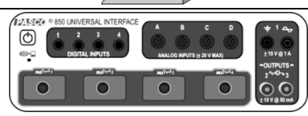
Where $L$ is the length (measured in m) between the two fixed end points of the string; $n$ is a positive integer (1, 2, 3, ...); and $v$ is the wave speed measured in $m/s$.

Normal mode frequencies depend on physical conditions and properties of the string: tension, length, and linear mass density (mass per unit length). In today's lab, you will explore this dependence.

Wave speed on a string depends on the tension force (F, in Newtons) on the string and the string's mass per unit length (ρ, in kg/m).

$$v = \sqrt{\frac{F}{\rho}} = \sqrt{\frac{F}{m/L}}$$

## Equipment

| | |
|---|---|
|  | **Slinky:** Explore the first few modes of a string under tension. Be careful not to tangle the slinky. |
|  | **Modified Loud Speaker:** Drives the oscillation of one end of the string. |
|  | **String pulley and Weight hanger:** Roller supports the opposite end of the string. The 50g weight hanger holds slotted masses to adjust the tension on the string. |
|  | **Bridge:** Can be used to support the roller end of the string and shorten the overall length. |
|  | **Pasco Interface:** Connects software to speaker so we can select the frequency and amplitude of string oscillation. |

Other equipment: String, meter stick, slotted masses

## Slinky Experiments

A slinky essentially acts like a string. You will use one to observe standing waves because its motion is slow enough to be followed by the eye.

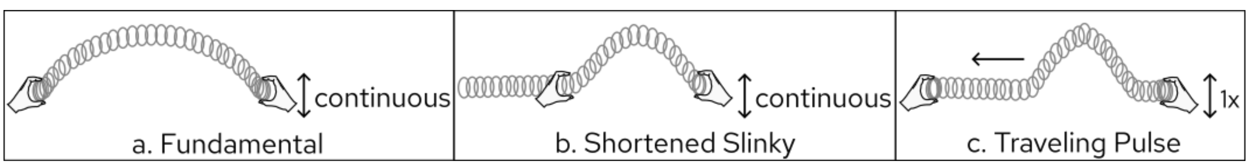



Figure 2. Slinky Motion

### 1. Normal Mode Frequencies

- Stretch the slinky across the short side of the table between yourself and your partner. Keep the length and the tension on the slinky consistent (hold each end at the edge of the table or keep a consistent number of floor tiles between you and your partner).
- Make the slinky move on the tabletop in the fundamental mode by gently moving one end back-and-forth along the table edge at a steady frequency. Adjust your speed until you observe the motion shown in Fig. 2a.

  **You may need to gather some of the slinky in your hand to make this work. Remember how much of the slinky you gathered so that you can reproduce the same tension.**
- Use a stop watch to time ten oscillations. Calculate the period **T** and then the frequency **f** of the fundamental mode.

$$T = \frac{time}{cycles} \qquad f = \frac{1}{T} = \frac{cycles}{time}$$

- **Without changing slinky tension**, move the slinky faster to get the second normal mode (with two bellies). Measure the period and frequency as above.
- **Again, without changing the slinky tension**, repeat the process for the third mode and, if possible, the fourth mode.
- Sketch each mode and write the corresponding frequency next to each picture. How do the frequencies of the modes relate to one another?
- How does the fundamental change if tension stays the same, but you use only half of the length (see Fig. 2b)? Have the partner holding the still end use their other hand to grasp the slinky in the middle to create a new fixed end in the middle of the table.
- How does the frequency of the half-length slinky relate to the modes of the full-length slinky?
- Which mode requires more effort to sustain the oscillation: higher modes or the fundamental?

2. Pulse on Slinky

- Return to the original slinky length and tension. Create a single pulse on the slinky (Fig. 2c). How long does it take the pulse to travel from one end to the other and back? Make five measurements and take the average.
- Compare the round-trip travel time of the pulse to the fundamental period T. How do they relate?

## Experiments with String

Set up the strings experiment according to the diagram in Fig. 3. The string should be about a meter long from the speaker to the pulley. You will use weights to put tension on the string. Musical instruments use very high tension on strings to increase the force on the bridge and produce louder, brighter tones. Today, you want low tensions so that the string oscillates with big amplitudes you can easily see. A tension of 3-4 N works well.

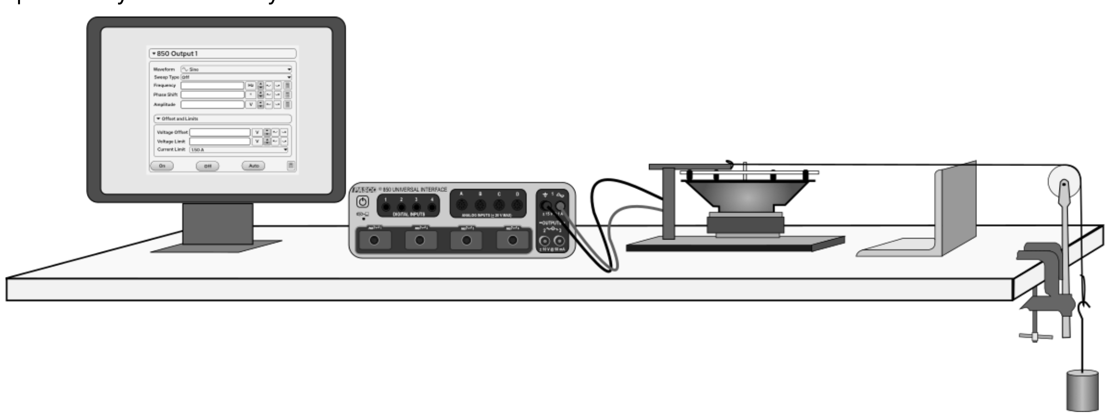


Figure 3. Schematic of equipment set up for string portion of lab.

The loudspeaker provides the sinusoidal driving force. You will adjust the driving frequency to make the string move in results in a **standing wave** patterns or normal modes. When you find a frequency that **resonates**, waves traveling away from the speaker align with waves reflected by the bridge or pulley and the string oscillates with nodes and bellies (Fig. 1). For all other frequencies the waves traveling each direction do not align, causing the string to move irregularly and very little.

### Computer As Driver Control

From the Lab Software/109 folder on the desktop, double-click on the "string-experiment.cap" icon, which opens the string experiment control shown in Fig. 4.

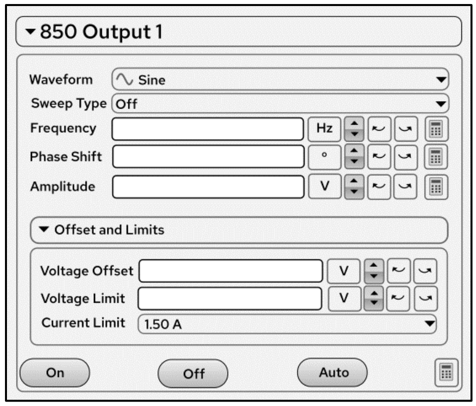



Figure 4. Loud speaker controller.

The control panel allows you to select both the amplitude and the frequency of the signal sent to the loudspeaker. **KEEP AMPLITUDES LESS THAN 2 V. Large amplitudes can break the loudspeaker.** Please check this **before** turning the speaker on. Amplitudes of 1-2 V work well for creating visible waves on the string.

- Set the waveform to Sine and amplitude to <2 V.
- The ON button sets the string into motion. The OFF button stops oscillations.

## 3. Finding the Fundamental Mode

- Add tension to the string by hanging weights from a loop at the end of the string that goes over the pulley. Use a 50 g weight hanger and slotted masses to make a total mass of 300 g to 400 g.
- Calculate the tension on the string. The string tension will equal the weight of the mass you hung from the end. Weight (force) in Newtons equals mass in kilograms multiplied by acceleration due to gravity (9.8 m/s$^2$).

$$\text{Force} = \text{mass} \times \text{gravity}$$

- Start at a frequency around 30 Hz and click ON. Increase (or decrease) the frequency by clicking on the appropriate arrow next to the "frequency control" on the control panel.

  **Note**: By default, clicking on the ▲ or ▼ buttons next to Frequency changes the driving frequency in steps of 1 Hz. The curved arrow buttons change the step size of the up and down arrows. Clicking  will make frequency steps larger. Clicking  will make frequency steps smaller.

- Watch the string carefully as you change the frequency. At some frequency, you will see the string start to resonate and oscillate in the fundamental mode.

  **Note:** Placing plain white paper underneath the string can help to increase its visibility.

- Slowly adjust the frequency to maximize the amplitude of the string. Note: Resonance curves have a range of frequencies with some resonance, you want the frequency that resonates the most.
- Set up a table in your notebook like the one in part 4. Record the fundamental frequency, sketch the shape of the string in the fundamental mode, and count the number of nodes.

## 4. Higher Modes

Based on the measured frequency of the fundamental and what you know about normal modes of strings, predict the frequencies of the first five or six overtones. In your notebook, create a table like the one below. Write in your predicted frequencies.

- To find the other modes, keep watching the string as you increase the frequency of the driver. Try to find the first six or seven modes. For a long string, some students find as many as ten modes. When you think you have reached a higher mode:
  - Record the frequency in your table.
  - Compare the measured and predicted frequencies.
  - Sketch the shape of the string.
  - Count the number of nodes.

How do the frequencies of the modes relate to one another? Are the mode frequencies perfect multiples of the fundamental frequency, as you were told in lecture?

**Note** that the simple formula says all modes of a string are exact multiples of the fundamental frequency. This is not quite correct for a real string because they have some stiffness that the simple formula ignores.

| | Sketch mode | Mode number (n) | Predicted frequency (n x fundamental, Hz) | Measured frequency (Hz) | Number of nodes |
|---|---|---|---|---|---|
| Fundamental, $f_1$ | | | | | |
| 2nd mode/1st overtone | | | | | |
| ⋮ | | | | | |
| 7th mode/6th overtone | | | | | |

## 5. Changing the Length of The String

One common way of changing frequencies in string instruments is by placing a finger on the fingerboard. This changes the length of the string that is free to vibrate and therefore changes its frequency. Frequency depends inversely on the length of the string, so the shorter the length, the higher the frequency.

- Use the bridge provided to make the length of the string half of what it was.
- Find the fundamental and next harmonic just as you did above.
- Compare the two frequencies you find to the normal modes of the longer string.
  - What aspect of the wave on the string has NOT changed by shortening the string? Explain how this is used by musicians to change the tone of a guitar or violin string.

## 6. Changing the Tension on the String

Another way to change the frequency of a guitar or violin string is to change the tension.

- Change the tension on the string by removing half the hanging weight. How does this affect the fundamental frequency? Adjust the speaker frequency to find the new fundamental.
- Calculate the new tension on the string from the formula in the part 3.

The frequencies of the different modes can be calculated using the string formula in the introduction.

- Calculate the speed of the wave on the string for each string tension you have used.
  The **mass per unit length** of the string in this lab is 0.33 g/m = 0.00033 kg/m.

- Use the formula to calculate the expected fundamental frequency f. Compare it to your measurements. You can try this for any combination of string length and string tension.

## 7. The "Plucking Game"

The string set-up does not allow you to investigate this portion experimentally. Ask your TA to use one of the guitars in the lab. Answer the following questions theoretically, based on what you know about the different modes.

- If you pluck the string at the center, what modes will be excited?
- What modes will be excited when you pluck the string:
  - 1/3 the distance between the two ends?
  - 1/5 the distance?
- Can you hear a difference between these on a guitar?

**The different modes of a plucked string get quieter at different rates**: right after plucking, there are many modes present, but the higher modes fade more quickly. After a while, only the fundamental remains. As higher modes fade, the sound of the guitar becomes closer to a pure tone.

## 8. Changing the Mass Per Unit Length of the String

The frequency of the string can be changed by changing the thickness of the string, or, more precisely, the mass per unit length of the string. Strings on musical instruments are made of several materials under different tensions.

In today's lab, you will not be able to directly test the dependence of the frequency on the mass per unit length of the string, but you can make some observations of musical instruments.

- Look at the piano or the guitar. How do the strings that produce low pitches (bass notes) compare to those that produce high pitches (treble notes)?
  - How do their masses per unit length (thickness) compare?
  - Does it look like the strings that produce high pitches made of the same materials as those for low pitches? Why or why not?
  - Beyond adjusting string thickness, are there ways that the manufacturer has increased mass per unit length of the lowest notes?

# *Lab 8: Pipes*

Many musical instruments are made of pipes in which sound waves travel. These instruments have many different features: flared ends, reeds, finger holes, or a coil-like shape. Instruments with these features are still pipes that are subject to the physical rules for sound waves in air. The goal today is to understand how sound waves in pipes behave. You will use simple pipes, but the same principles apply to sound made by more complicated brass and woodwind instruments.

In the previous lab, you looked at normal modes on strings. You saw the standing wave patterns formed by the motion of the string. In strings, different points along the string move with different amplitudes. In pipes, different points along the pipe experience different air **pressure** variations. While the movement of the string is visible to the naked eye, you will use a microphone to measure pressure variations in a pipe.

You will explore two different types of pipes in lab: open and closed pipes.

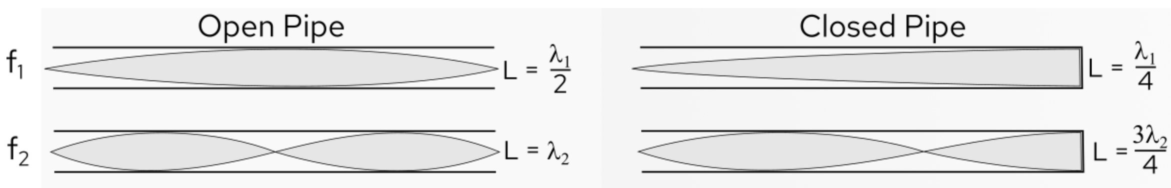



Figure 1. Normal Modes in Open Pipes and Closed Pipes

### Open Pipes

The modes of open pipes follow the same pattern as modes of strings (Fig. 1). **Open** pipes are open to the atmosphere on both ends. Thus, pressure at the ends always equals atmospheric pressure and there is a pressure node at each end.

### Closed Pipes

**Closed** pipes are closed on one end and are open on the other. As before, there is a node at the open end. At the closed end, pressure can build up to form an **antinode** (or the biggest amplitude in a belly). This means normal modes of a closed pipe follow a different pattern than normal modes of an open pipe.

The fundamental mode has a node at the open end and an antinode at the closed end (Fig. 1). The length of the pipe is one quarter of the fundamental wavelength. The first overtone adds a node between the ends of the pipe (and ¾ of a wavelength fits in the pipe), the second overtone adds another node, and so on.

## Equipment

You want to find frequencies that resonate in the pipes. You will use equipment that generates sound waves outside the pipe and equipment that records sound waves inside the pipe.

### Generate Sound Waves

You will use a frequency generator connected to a speaker to generate sound waves at particular frequencies. The sound waves then enter the pipe and travel back and forth along it. At resonant frequencies, constructive interference creates standing waves.

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<tr>
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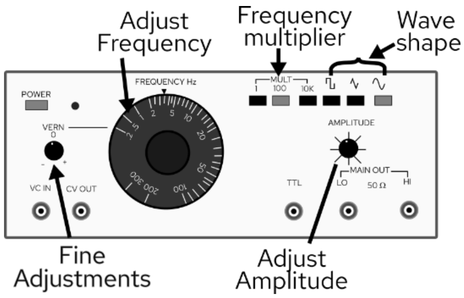

</td>
<td>
Frequency Generator: Generates an electrical signal with a particular shape and frequency that can be sent to the speaker. Knobs adjust the frequency and amplitude. Buttons select a frequency multiplier and wave shape.<br>
Use the 100 multiplier and sine wave shape.<br>
To select 340 Hz, adjust the large frequency knob to between 2 and 5. Check the frequency counter and use the fine adjustment knob (VERN) to make final adjustments.
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</tr>
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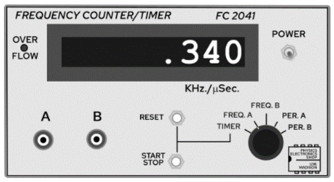
<td>


</td>
<td>
Frequency Counter: Reads the frequency sent to the speaker. The display is in kHz, so a 340 Hz signal is displayed as .340. Make frequency adjustments slowly, as it will take the counter a little time to read the frequency.
</td>
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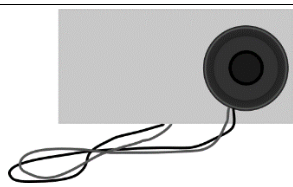
<tr>
<td>

</td>
<td>
Speaker: Converts the electrical signal from the frequency generator to a pure tone. The speaker is located inside the wooden box. Put the end of the pipe near the speaker edge without covering it. This allows the sound wave to enter the pipe.
</td>
</tr>
</table>

On the frequency generator, use a **sine wave**-shaped signal and the **100** multiplier. The 100 multiplier means that, for example, setting the frequency dial at 2 gives a 200 Hz wave. The frequency generator has a fine adjustment (called vernier frequency) labelled “VERN”. Read the frequency being generated from the frequency counter, multiplying by 1000 because the display is in kHz.

## Record Sound Waves

The microphone converts air pressure to an electrical signal. The oscilloscope displays this signal. The microphone only measures the pressure at its tip. When you find a resonant frequency, move the microphone tip to find antinodes (maximum amplitude variations) and nodes (points of little to no variation). The microphone is thin enough to not meaningfully disrupt the movement of the air in the pipe.

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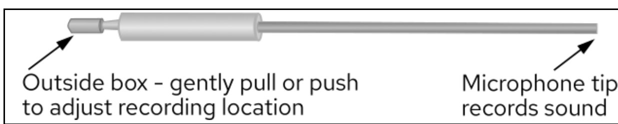
<td>


</td>
<td>
Microphone: Records sound at the tip and converts pressure at that location to an electrical signal
</td>
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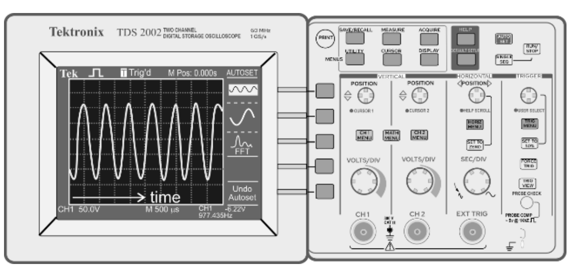
<tr>
<td>


</td>
<td>
Oscilloscope: Displays the electrical signal from the microphone as a function of time.<br><br>
Note that this is showing pressure at one location in the pipe vs. time, not along the pipe’s length. See below.<br><br>
Refer to the end of the lab (Using the Oscilloscope) for more details on the buttons and operation.
</td>
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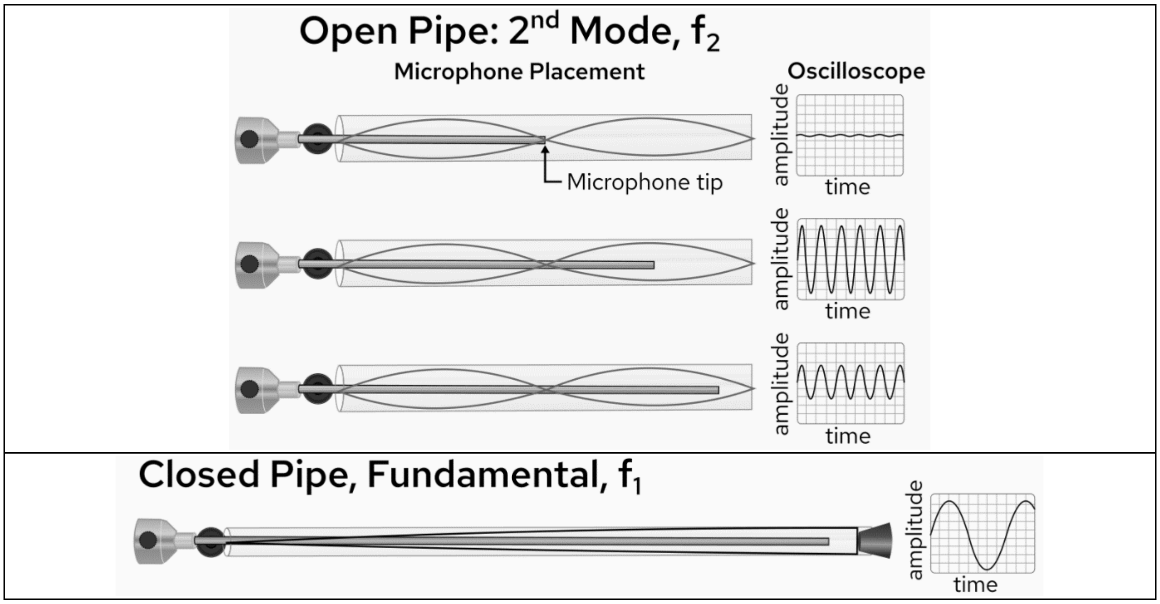
</table>

Sound waves from the speaker have the same amplitude for all frequencies. The amplitude you see on the oscilloscope screen is from air pressure variation inside the pipe. As you change the frequency and get close to a resonant frequency, you will see a larger signal on the oscilloscope because there is a standing pressure wave inside the pipe.




When you find a frequency with a larger response, adjust the frequency in small steps (VERN knob) to find the one with the largest amplitude. This will be a normal mode frequency. If you keep the lid of the box open, you will even be able to hear some of these resonances just by listening to the sound from the pipe.

Figure 2 shows the full experimental set-up. The equipment above the dotted line will sit on the table next to the wooden box. Note that the end of the pipe is next to, but not covering, the speaker.

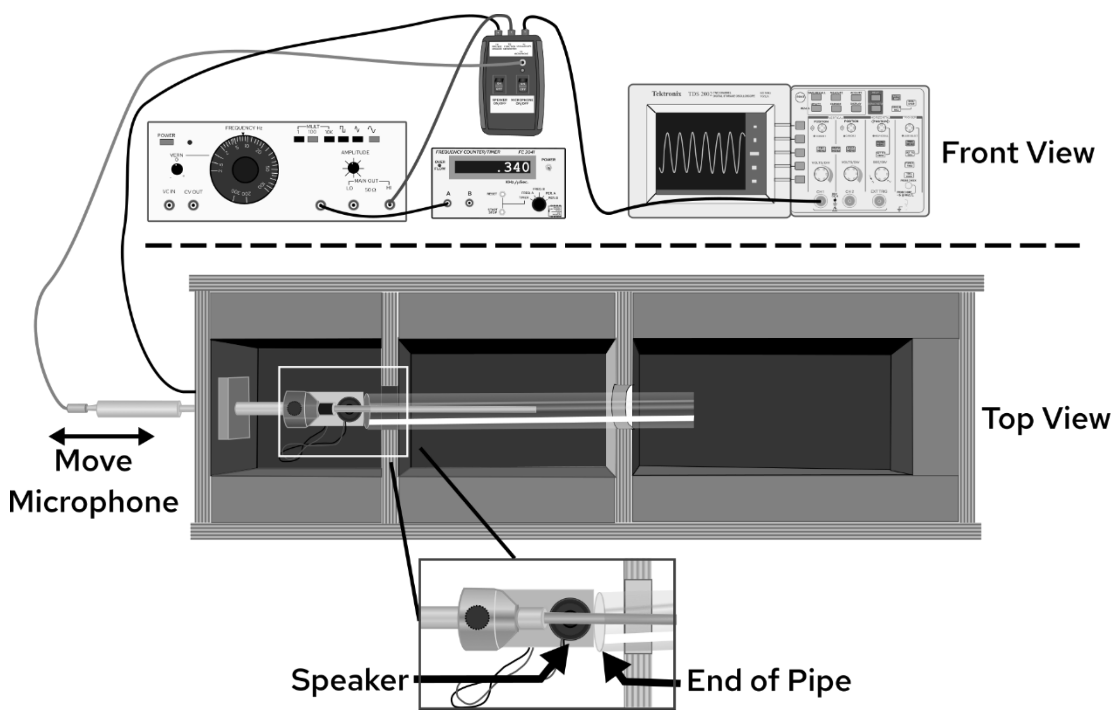
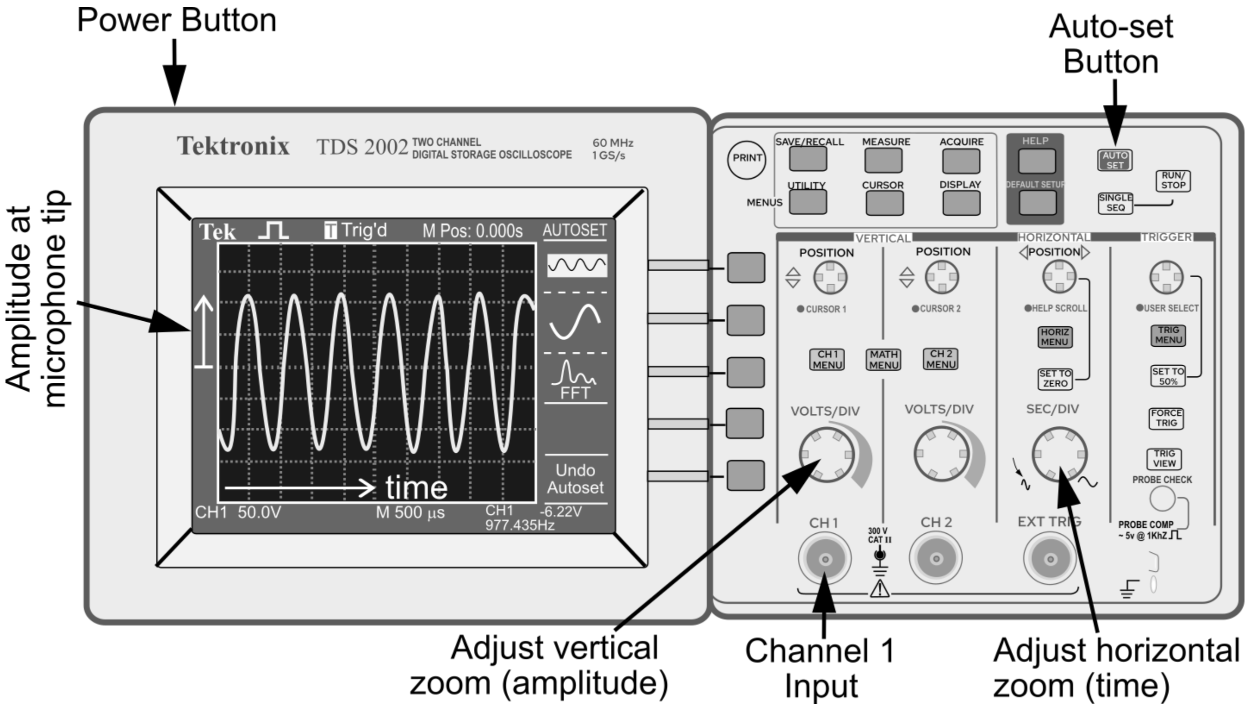



Figure 2. Equipment setup and connections. Note the end of the pipe is next to the speaker.

### Using the Oscilloscope

The oscilloscope is a powerful tool for analyzing electronic signals. For this lab you will need to use a few simple features labeled in the figure below and can safely ignore the myriad other buttons.




Press the Power Button and wait a minute for the oscilloscope to boot. Plug your signal source into the Channel 1 Input using a cable with an end that looks like the picture to the right.

Once the scope is booted, press the Auto Set Button at the top right of the panel. This will ask the scope to detect the features of the signal source and set up the scope automatically. The figure shows a signal displayed using the Auto Set button.

Now you can 'zoom' the horizontal and vertical axes to see whatever feature interests you. The Vertical Zoom is controlled by the Volts/Div knob directly above the Channel 1 Input. The Horizontal Zoom is controlled by the Sec/Div knob. Turning the knobs clockwise will zoom 'in' and counter-clockwise will zoom out.

If you ever get into trouble and lose the signal, do not panic. Just press the Auto Set button again and it will take you back to a good starting spot. If you have trouble beyond that, just ask your TA.

## Experiments

### 1. Modes of the Open Pipe

- Start with the long narrow pipe with no finger hole.
  - Measure the pipe's length with a meter stick.
  - Place the pipe in the as shown in Figure 2.
  - Move the microphone so its tip is near the middle of the pipe (but not exactly at the middle, since some of the modes have nodes there).
- In your notebook, set up a table like the one below. Determined the mode number and calculate expected frequencies for the first five modes (remember to use length in meters).

- The formula to calculate the normal mode frequencies for open pipes is the same as for strings.

$$f_n = n\frac{v}{2L} \quad \text{(open pipe)}$$

  **L** is the length of the pipe in meters, and n is a positive integer. **v** is the speed **of sound** in the pipe. The speed of sound varies slightly with temperature.

$$v_{sound} = [\,331 + 0.6(\text{Temp in °C})]\ \text{m/s}$$

  If no temperature is given, you can assume that it is room temperature, and $v_{sound} = 344\ \text{m/s}$.

- Using these expected frequencies for this pipe as a guide, turn on the speaker and adjust the frequency until you find a frequency that resonates.
- When you find a resonant frequency, you should be able to see a clear sine wave on the oscilloscope, and its amplitude should be larger than at any surrounding frequencies. Based on your predictions, which mode is this?
- Continue to find the resonance frequencies for the lowest five modes, $f_n$ and record the values in the Measured Frequency column. (Do not forget the fundamental mode!)

| | Sketch of mode | Measured frequency (Hz) | Overtone multiple of fundamental | Difference: measured vs. predicted |
|---|---|---|---|---|
| Fundamental | | | | |
| Second mode | | | | |
| Third mode | | | | |
| Etc. | | | | |

- For the first mode and the fifth mode, locate the nodes and antinodes of the standing waves.
  - Call the end of the pipe nearest the speaker 0 cm.
  - For the first mode, move the microphone back and forth in the pipe to find the locations of the largest (antinode) and smallest (node) amplitudes. Are the ends of the pipe nodes or antinodes?
  - Repeat these steps for the fifth mode
- For the higher modes, divide the measured mode frequency by the measured fundamental frequency. Are all of the higher modes exact multiples of the measured fundamental frequency?
- For each mode, calculate and record the difference between the expected and measured frequency in the "difference" column.

## 2. Effect of Length and Diameter of the Pipe

Compare the fundamental frequency you measured in part 1 with the frequency you calculated using the open pipe equation. Do your predicted and measured values exactly agree? They likely will not!

- If not, use your measured fundamental frequency to solve for the length of pipe L that "should" have this fundamental frequency.
  - To do so, rearrange the formula for the fundamental frequency to solve for length, $L = \frac{v}{2f}$.
  - How does this length compare to the actual length of the pipe?

The value of L you obtained above is called the **acoustic length** of the pipe. The acoustic length is a bit larger than the physical length of the pipe because the outermost pressure nodes are always a little outside the ends of the pipe. The larger the diameter of the pipe, the further outside the nodes are.

How does the length of the pipe affect the normal mode frequencies? Let's find out.

- Use a shorter pipe and measure its length.
- Use the procedure from part 1 to find the fundamental frequency and the frequency of the first three overtones (4 total modes) of this pipe. Record these in your notebook.
- Given the formula $f_n = n\frac{v}{2L}$, the frequency and the length should be inversely proportional: $f_n \propto \frac{1}{L}$.
    - Divide the length of the **longer** pipe by the length of the **shorter** pipe.
    - Next, divide the fundamental frequency of the **shorter** pipe by that of the **longer** pipe.
    - What do you notice about the two ratios? Notice the order of division ($\frac{\text{longer L}}{\text{shorter L}}$ vs. $\frac{\text{shorter f}}{\text{longer f}}$) used in each case.

When you studied strings, you found that thick strings have a lower fundamental frequency than thin strings (for the same tension and length). Now find out if the same true for pipes of larger diameter.

- Use one of the larger-diameter pipes and measure the first four resonance frequencies.
- Compare these frequencies to the ones that you measured for the same length of pipe with the smaller diameter.
- Can you make sense of your result based on the discussion of acoustic length above? (Does a longer length increase or decrease the frequency? Which **should** have a longer acoustic length?)

## 3. Closed Pipes

On a pipe organ, the bass pipes are closed at one end. Closing one end not only lowers the fundamental frequency, but also changes the pattern of overtones. The normal mode frequencies for a closed pipe are given by the formula below.

$$f_n = (2n-1)\frac{v}{4L} \quad \text{(closed pipe)}$$

Note that the frequencies are now only **odd** integer multiples of the fundamental frequency.

- Using the long narrow pipe without holes, use the stopper (rubber cork) to close the end away from the speaker. Place the microphone inside the pipe near the closed end.
- Measure the fundamental and the first three overtones. Make a table of the resonant frequencies as you did before.

| | Sketch of mode | Measured frequency (Hz) | Overtone multiple of fundamental | Difference: measured vs. predicted |
|---|---|---|---|---|
| Fundamental | | | | |
| Second mode | | | | |
| Third mode | | | | |
| Fourth mode | | | | |

- What multiples are present?

- How does the fundamental frequency compare to the one measured for the open pipe of the same length? Give a qualitative answer (lower, higher) as well as a quantitative answer (by what multiple are the two frequencies different)?

## 4. Pipes with a Finger hole

Woodwind instruments have finger holes that change the effective length of the pipe and thus its fundamental frequency.

- Select an open pipe with a finger hole. Place it in the box with the finger hole away from the speaker.
- Measure the distance from the speaker end to the finger hole and the overall length of the pipe.
- Measure the first five resonant frequencies. Adjust the frequency carefully.
    - Are the modes still equally spaced (i.e., are each two neighboring modes still a fundamental frequency apart)?
    - Are the resonant frequencies you measured similar to what you found for the long and/or short pipe without a hole?
    - For the pipe without a finger hole, what is the pattern of overtone frequencies? Is this pattern similar for the pipe with a finger hole?
- Using the procedure from part 2, find the acoustic length of the pipe with a finger hole.
- Does the finger hole act like an open end of the pipe? Why or why not?

## 5. Rise and Decay of Pipe Oscillations

A switch on the box lets you abruptly turn on and off the speaker sound. This allows you to study how the volume inside a pipe rises and falls off. Musical tones from wind instruments do not turn on or off suddenly, even if the musician starts and stops blowing abruptly.

- Measure rise time and decay time. To see the rise and fall:
    - On the oscilloscope, change the SEC/DIV setting to make a slow sweep
    - On the control box, turn the speaker on (or off), and quickly press the RUN/STOP button on the oscilloscope to freeze the picture. It may take a few tries and two people for you to hit the right moment.
    - Once you have a picture stored on the scope, you can expand it with the SEC/DIV setting to look at the decay in detail. You can also change the horizontal position by using the small knob above the SEC/DIV knob.
- If you have time: Explore the relationship between rise/decay time and resonance width. First, as you did in the Resonance Experiment, carefully trace out a resonance curve with small step size. Then compare this time to a careful measurement of the resonance width.

# *Lab 9: Musical Scales*

A musical scale is characterized by a sequence of frequencies. In this lab you study the relationships between the frequencies that make up the scales commonly used in western cultures. You will focus on the differences between **just** scale tuning and **tempered** scale tuning. Specifically, you test their musical advantages and disadvantages. As part of the lab, you will also learn terminology used by musicians. Among other things, you will learn about **transposition** and about the difference between **major** and **minor** scales.

Musicians describe **intervals** between two pitches. Intervals are defined by frequency ratio and are named by the number of natural notes (white piano keys) from one pitch to the next. For instance, the interval from C to G is called a **fifth** because G is the fifth white key from C.

If you consider the frequency ratio of one note in a scale to the one before it (D/C, F/E), the ratios are not all the same size. Scales are made up of both **whole tones** (whole steps) and **semitones** (half steps). From the first note to the last, the scales we use in this course have five whole tones and two semitones. The order of these defines a major scale versus a minor scale, which you will see in parts 7 and 8.

- → The ratio between C and D is a whole tone because there is a pitch between them (a black key on a piano) that we call C-sharp (C#) or D-flat (D♭).
- → The ratio between E and F is a semitone because there is no pitch between them. Similarly, the ratio between C and C# is a semitone.

Intervals can be defined by the number of semitones between two pitches. When you count semitones, count the semitone intervals between pitches. For instance, C to E is four semitones (C → C#, C#→D, D→D#, D#→E), which is also called a major third.

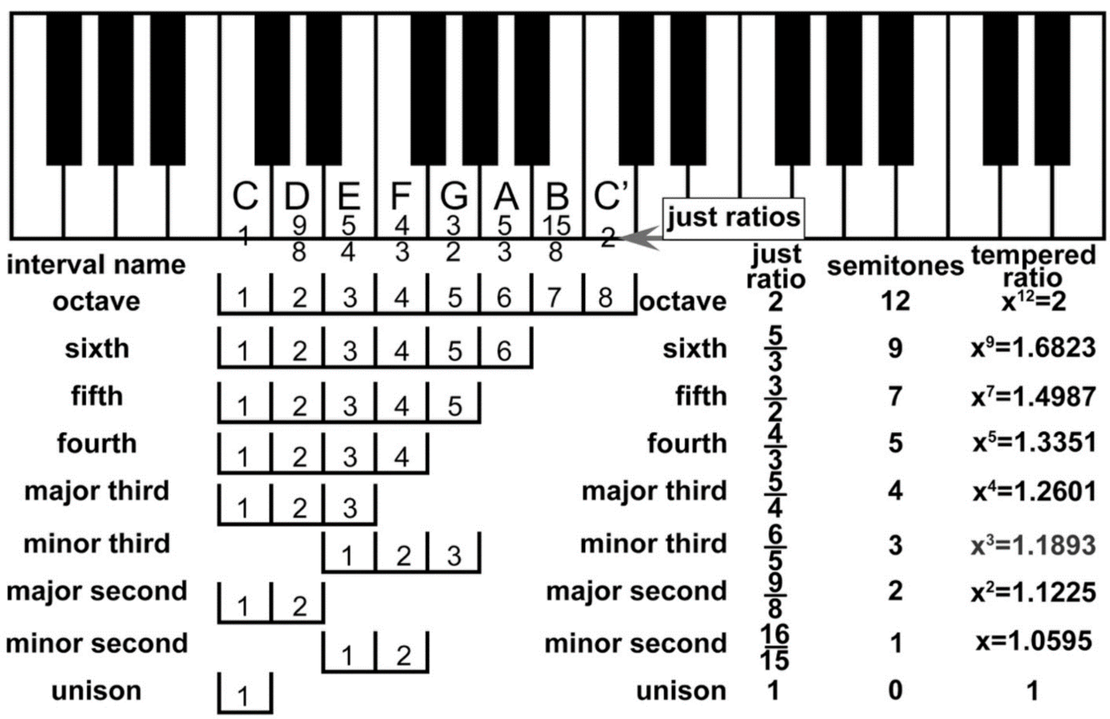
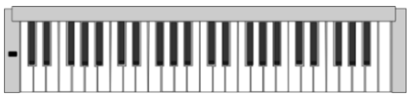
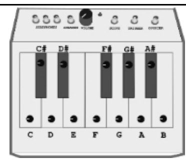
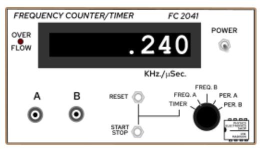



Figure 1. Scales and Intervals reference graphic.

## Equipment

| | |
|---|---|
|  | **Tunable keyboard**: The frequency of each tone is set using the Control Box. Within an octave, each key within is tuned separately, but the same notes in different octaves are related by their normal factor-of-2 relationship. When you tune one C, you tune them all. |
|  | **Control Box**: Each knob sets the frequency for that tone. To tune a key, turn the knob while playing the note on the keyboard and watch the frequency displayed on the frequency counter. |
|  | **Frequency Counter**: Reads the frequency sent to the speaker. The display is in kHz, so a 240 Hz signal is displayed as .240. Make frequency adjustments slowly, it takes time for the counter to read the frequency. |

Other equipment: speaker

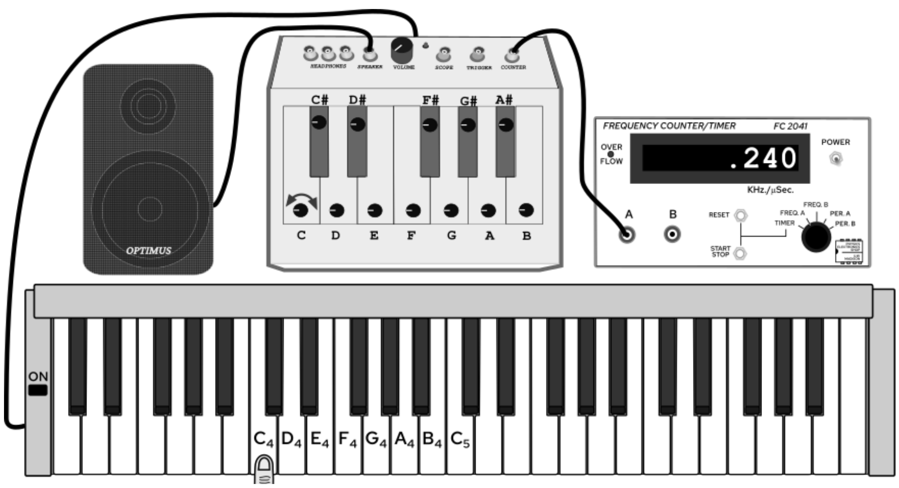



Figure 2. Equipment schematic. To set the frequency for $C_4$, press the key on the keyboard and turn the C knob until the frequency counter reads the desired frequency.

## Frequency Difference vs. Frequency Ratios

You might be tempted to think that any consistent spacing of frequencies makes a useful musical scale. Before we get to Just and Tempered tuning, you can tune a keyboard such that the frequency difference from one white key to the next are all the same.

→ If C is tuned to 238 Hz, this makes the C one octave higher (238 Hz × 2) = 476 Hz.

→ Since there are seven intervals between white keys within an octave, we will divide 238 Hz into seven equal intervals (238 Hz ÷ 7 = 34 Hz).

- Tune your keyboard so that the keys have the frequencies below. **Note that you will not independently tune $C_5$. It is set by your tuning of $C_4$.**

| $C_4$ | $D_4$ | $E_4$ | $F_4$ | $G_4$ | $A_4$ | $B_4$ | $C_5$ |
|---|---|---|---|---|---|---|---|
| 238 Hz | 272 Hz | 306 Hz | 340 Hz | 374 Hz | 408 Hz | 442 Hz | 476 Hz |

- Now play a familiar song like “Frère Jacques:” (C-D-E-C, C-D-E-C, E-F-G, E-F-G). Then play chords like C-E-G. How does it sound? Even though this scale is regular, it is useless to a musician. **Note: Frère Jacques is just an example. Feel free to use any familiar song!**

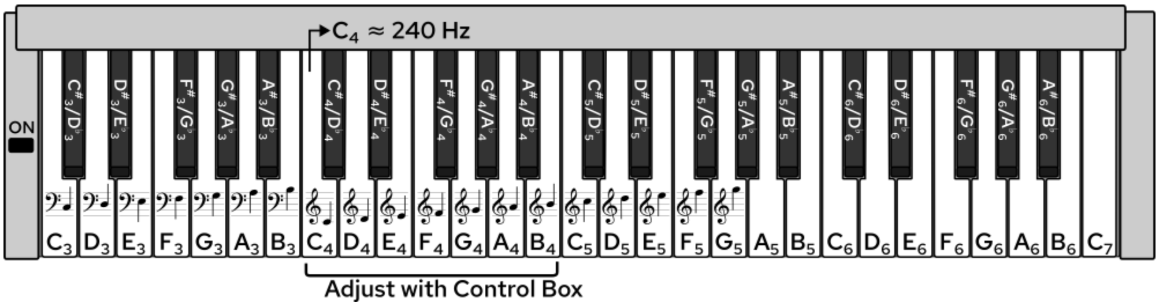



Figure 3. Details of the keyboard. We will tune the 2nd octave from the left to be the 4th octave on a full-size piano. $C_4$ is also called “middle C.”

Rather than using frequency differences to define a scale, useful scales use frequency ratios that stay the same from one octave to the next.

The exact ratios used for intervals, semitones, and whole tones depend on whether you use Just Scale tuning or Tempered Scale tuning. You will now look at each of these tuning approaches and consider their advantages and disadvantages.

## The Just Scale

The **just scale** is made up of three major **triads**. A major triad is a set of three tones with a **major third** between the lower two tones and a **fifth** between the lowest and highest tones. This gives the three tones a frequency ratio of 4:5:6.

### 1. Tuning Triads

- Tune the second C key from the left to 240 Hz. Your triad C-E-G should have a 4:5:6 frequency ratio. What frequency for E will give E:C a ratio of 5:4? What value of G gives G:C the ratio 6:4?
- In your notebook set up a table similar to Table 1 and enter the frequencies of E and G in the white boxes of the 1st Triad. Then tune the E and G keys to these frequencies.
- Play the C-E-G triad together and listen to it. Does it sound harmonious?
- Build another triad starting on G: G-B-D′, again with the ratio 4:5:6. Fill in the frequencies for the 2nd Triad and D in the table; then tune B and D on your keyboard.
- Finally, make a triad that ends on the upper C: F-A-C′ with the ratio 4:5:6 and add the frequencies to the table.
- Listen to the triads one after the other. Are they all good harmonies?
- Copy the frequencies from the three triads into the Just Scale row. You now have a just scale.
- Now play “Frère Jacques” (C-D-E-C, C-D-E-C, E-F-G, E-F-G). How does it sound to you?

Table 1. Fill in the white boxes with your calculated frequencies

| | C | D | E | F | G | A | B | C' | D' |
|---|---|---|---|---|---|---|---|---|---|
| **1st Triad** | **240** | | | | | | | **480** | |
| **2nd Triad** | **240** | | | | | | | **480** | |
| **3rd Triad** | **240** | | | | | | | **480** | |
| **Just Scale** | **240** | | | | | | | **480** | |

## 2. Sensitivity to Tuning

- Pitch Memory Game: Verify that C is tuned 240 Hz and play the C. Then substantially detune C by turning the frequency knob up or down. Without looking at the Frequency Counter, try to retune it to exactly 240 Hz. How close can you get? See which lab partner can get closest!

Let's test how sensitive YOU are to tuning in other scenarios.

### Sensitivity to tuning in chords

- Have one of your lab partners look away from the Control Box and Frequency Counter while you play a chord (notes played simultaneously): C-E-G.
- While playing the chord, slowly de-tune the E until your partner can tell **just by listening** that it is out of tune.
- Measure the frequency of E on the frequency counter. How many Hz did you change before your partner realized the E was out of tune?
- Repeat the previous steps a couple of times, detuning both higher and lower in frequency.
- Rotate roles so that another partner is listening and repeat the process.

### Sensitivity to tuning in songs

- Repeat the experiment above, but instead of a chord, play a song like "Frère Jacques."
- What do you conclude? Are you more sensitive to frequency inaccuracies when you listen to a chord or to a song?

Most (musically untrained) people only notice exact tuning when playing chords. Historically, tuning only became important when people started to use harmonies (polyphonic music).

## 3. The Black Keys of the Keyboard

So far, we have only experimented with three major triads: C-E-G, F-A-C, and G-B-D. Suppose you want to make a major triad starting on D (with the frequency found earlier in lab).

- On the keyboard, play the three triads we used to tune the scale (above), then play D-F-A. Does it sound like a major triad?
- Check your frequency table to see if D-F-A would give the 4:5:6 ratio. It shouldn't! You need another key whose frequency is between F and G. We call this key F# or "F sharp" (it can also be called G♭ or "G flat"). Calculate this frequency.
- Tune the black key between F and G to the frequency you calculated. Now you can play a major triad starting on D!
- Compare the sound of your new triad D-F#-A (a major triad) to the triad D-F-A (a minor triad), which you would be forced to play if you had only white keys.

## 4. The Missing Black Keys

Now you will investigate the function served by the black keys, with no black key between E and F nor between B and C.

- Calculate the ratios of adjacent frequencies of the just scale (D/C, E/D, F/E, etc.).
  **Note: The ratios should be greater than one and in decimal form.**
- Where are the larger intervals (whole tones)? Where are the smaller ones (semitones)?
- What function do the black keys serve?

## 5. Problems With the Just Scale

If you play a song in D instead of C by using F#, you may notice that some of the intervals do not sound quite right (if you're using another song, you may need a C# as well – try triad $A_3$-C#-E to tune it).

- Compare (C-D-E-C, C-D-E-C, F-E-G, F-E-G) to (D-E-F#-D, D-E-F#-D, F#-G-A, F#-G-A). Do they sound similar?
- Listen to the fifth D-A. How does it sound to you?
- Calculate the D-A frequency ratio from your table. Is it a perfect 6/4 ratio, as it should be?

## The Tempered Scale

Using just scale tuning, you have perfect ratios for some triads but imperfect ratios for other triads. You could retune the entire keyboard each time you want to play a song in a different key, but that is not very practical. An alternative is **Tempered Scale** tuning, which divides each octave into 12 equal-ratio semitones. Since you want the frequency to double after 12 semitones (an octave), the frequency ratio for adjacent semitones (C#/C, F/E, etc.) must be the twelfth root of two. $\sqrt[12]{2} = 2^{1/12} = x \approx 1.05946$.

### 6. Define the Tempered scale

- Set up a table like Table 2 in your lab notebook. Use the method outlined in lecture to fill in the frequencies for each note of the tempered scale starting with C=240 Hz.

Table 2. Tempered Scale with C = 240 Hz

| C | C#, D♭ | D | D#, E♭ | E | F | F#, G♭ | G | G#, A♭ | A | A#, B♭ | B | C′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | |

- Tune the keyboard to this tempered scale.
- For your own reference, you may want to fill in the white boxes in Table 3 (in your lab manual) to easily compare the just and tempered tunings.

Table 3. A comparison of frequencies for the Just and Tempered Scales

| | **Just Scale** | | | **Tempered Scale** | | | |
|---|---|---|---|---|---|---|---|
| | Ratio | Standard A = 440 | Lab C = 240 | Ratio | Standard A = 440 | Lab C = 240 | |
| C | 1 | 264 | **240.0** | 1 | 261.6 | **240.0** | C |
| C#, D♭ | | | | 1.059 | 277.2 | | C#, D♭ |
| D | 9/8 = 1.125 | 297 | | 1.122 | 293.7 | | D |
| D#, E♭ | | | | 1.189 | 311.1 | | D#, E♭ |
| E | 5/4 = 1.25 | 330 | | 1.260 | 329.6 | | E |
| F | 4/3 = 1.333... | 352 | | 1.335 | 349.2 | | F |
| F#, G♭ | | | | 1.414 | 370.0 | | F#, G♭ |
| G | 3/2 = 1.50 | 396 | | 1.498 | 392.0 | | G |
| G#, A♭ | | | | 1.587 | 415.3 | | G#, A♭ |
| A | 5/3 = 1.666... | **440** | | 1.682 | **440.0** | | A |
| A#, B♭ | | | | 1.782 | 466.2 | | A#, B♭ |
| B | 15/8 = 1.875 | 495 | | 1.888 | 493.9 | | B |
| C | 2 | 528 | 480.0 | 2 | 523.3 | 480.0 | C |

- Play a song starting on C, then play one starting on D. (See section 5 for Frère Jacques notes.)
- Listen to the triads C-E-G and D-F#-A.

  To musically trained people, the Tempered major third, such as C-E, sounds noticeably worse (more dissonant) than the Just major third.

- o Tune E according to the Just Scale and listen to the major third C-E.
- o Now tune E according to the Tempered Scale and again listen to the major third C-E. Do you notice a difference?

  You can also work with a nearby group to listen to the two tunings more quickly.

## Transposition

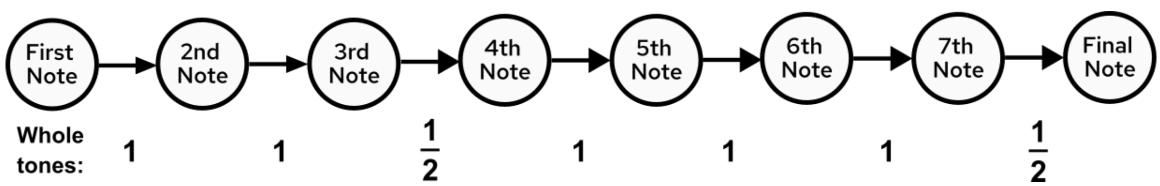
Beyond tunings, western music uses **major scales** and **minor scales**. These scales use eight pitches beginning on a selected tone and ending on the tone of the same name one octave up.

Major and minor scales select specific pitches from the 12 semitones that make up an octave by using different arrangements of whole tones and semitones. Both use five whole tones and two semitones to get from the first pitch to the octave above, but arrange them in different orders.

These scales are named for the first note of the scale and its type (C Major, E Minor), and are used to define the **key** of a musical piece. Songs can be **transposed** from one key to another by preserving the musical intervals from one note to the next.

### 7. Major Scale

If you play the white keys starting on C, you are playing the C major scale. You will recognize the location of the semitones from the location of the missing black keys. The **major** scale is characterized by the following sequence of intervals:




The final note is one octave above the first note.

- To play the same-sounding scale in D, you would have to keep the same order of whole tone and semitone intervals. Write the pitches of the D major sequence in your notebook, then play it.

In musical notation, the sharp symbol, #, indicates that the tone is a semitone above the named pitch. The flat symbol, ♭, indicates that the tone is a semitone below the named pitch.

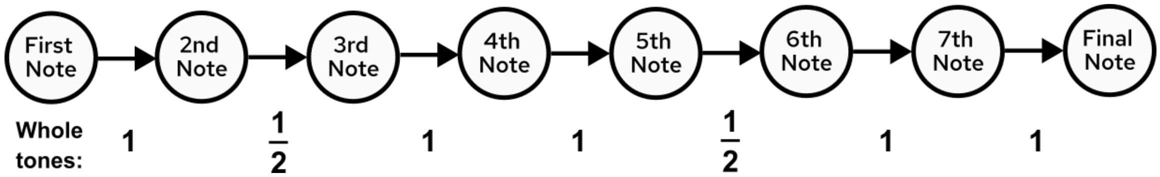
A scale will only use either sharps or flats, but the choice of which to use is based on the notes in the scale. If you need a tone halfway between F and G, you name it F# or G♭ according to the letter that would otherwise be skipped.

- → For instance, G-major has G-A-B-C-D-E-?-G′. In this case, we use F# because we do not want G♭ and G in the same scale.
- → Alternatively, F-major needs a tone halfway between A and B: F-G-A-?-C-D-E-F. In this case, we do not want to have A and A#, so we call the note "B♭" instead.

- Write out the A major scale. Is it better to use sharps or flats?
- Do the same for E major.

## 8. Minor Scale

If you were to play only the white keys starting on A rather than C, you get the sequence of intervals below. Check by looking at the keyboard and watching for the semitone intervals!




This sequence of intervals characterizes the **minor** scale. The minor scale beginning on A is called an A Minor scale.

- Play the above sequence of intervals starting on C. Write out the tones of the C minor scale. Remember: We typically only repeat a letter once in a scale. Is it better to use sharps (#) or flats (♭) for a C minor scale?
- Play "Frère Jacques" in C minor. Does it sound different than it did in C major?

**Note:** there are several types of minor scales in western music. This one is called the Natural Minor scale and is the minor scale used to define the key of a song. We will only use natural minor scales in this course.

**Note:** there are also a number of other kinds of scales we do not cover in this course, but these are the most common in western music.

# *Lab 10: Fourier Analysis and Musical Instruments*

Pure tones have simple sinusoidal waves. The sounds from musical instruments are more complicated because they include higher harmonics. Even for a steady, single note from an instrument, the waveform (or wave shape) can be quite complicated. A waveform repeats itself **f** times a second, where **f** is the **fundamental frequency**. You have learned that **f** is related to the pitch of the tone.

However, pitch is not the only quality that determines the sound of a tone. Different instruments playing tones of the same pitch do not sound the same. Although the fundamental frequency is the same, the wave shape is different. Musicians describe this as a difference in **timbre** or **tone color**.

How does one describe wave shape in physics? Using **Fourier analysis**, you can represent the complex wave shape as a sum of sine waves (or **partials**) of different amplitudes. If the wave shape is periodic (Fig. 1), the frequencies of the partials are multiples of the fundamental frequency. In this case, the partials are called the **harmonics** of the tone being played.

Suppose that the frequency of a musical tone is 200 Hz. 200 Hz is known as the **fundamental** (also called the **first harmonic**). The **first overtone** (or **second harmonic**) would then be at 400 Hz, the **second overtone** (or **third harmonic**) would be at 600 Hz, and so on. Many musical instruments, including voices, have ten or more overtones that contribute to their complex timbres.

A **Fourier analyzer**[2] is a device that measures the amount of various harmonics present in a sound. Most Fourier analyzers display a graph of the amplitude and the frequency of the contributing harmonics.

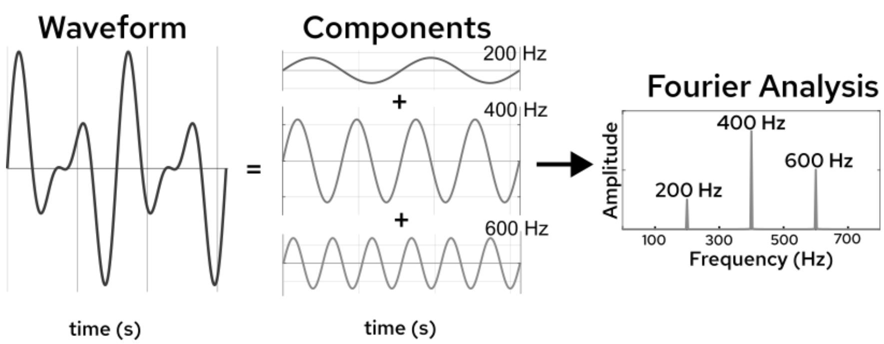



Figure 1. A periodic waveform is is made by adding three component frequencies. These are identified by using Fourier Analysis.

## Fourier Synthesis

To better understand Fourier analysis, you will start with its opposite: Fourier synthesis. While Fourier analysis separates a complex waveform into its partials (sine waves), Fourier synthesis adds up sine waves to construct complex waveforms. You will use a few different tools to explore Fourier synthesis.

### 1. Two Sine Waves of the Same Frequency

When two waves reach the same point in space at the same time, the resulting wave is the addition, or **superposition**, of the individual waves.

[2] Although this lab will concentrate on sound, Fourier analyzers also exist for other types of periodic signals.

Before you look at musical tones with multiple harmonics, consider the simplest case: adding two sine waves of the same frequency.

In a browser, navigate to https://www.desmos.com/calculator/vntvxa3wuu. On this Desmos calculator, you should see two sine functions, $y_1$ and $y_2$, in blue and red, respectively. The calculator also displays the superposition (sum) of the two as a black dotted line ($y_s$). For each of the sine waves, you can independently control the amplitude ($A$), the frequency ($f$), and the phase ($\phi$). Try adjusting the various sliders on the left-hand side to get a feel for how these quantities are related to the wave shape.

It may help to start with a single wave. To hide a particular plot, you can click on the icons near the y formulas on the left. Click on the icon again to bring back the hidden plot.

**When you are done exploring, be sure to set the frequencies of the two waves equal ($f_1 = f_2$).**

- Adjust the phase and amplitude of each sine wave. Can you maximize the amplitude of their sum?
- When two sine waves of the same frequency are added, what is always true about the wave shape that results?
- Keeping the frequencies equal, adjust the other parameters until the two waves cancel one another. What are the conditions required for cancellation?

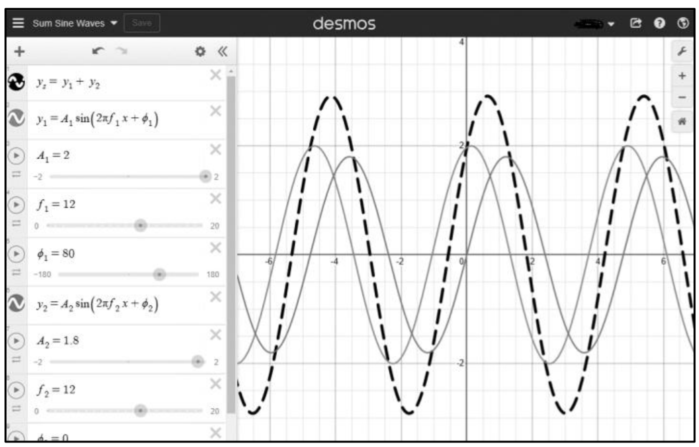


Figure 2. Desmos calculator for summing sine waves

Hopefully you found that the phase difference between the two waves plays an important role in achieving cancellation. When this phase difference occurs, the two waves are said to be (completely) **out of phase**.

What phase condition is required for **maximizing** the resulting wave? When this phase condition occurs, the two waves are called **in phase**.

## 2. Building a Square Wave from Sine Waves

Now let's move on to more complex wave shapes. You will try to build a square wave by adding harmonics. Look at it as a puzzle.

- Open a new browser tab and navigate to:

  https://phet.colorado.edu/sims/html/fourier-making-waves/latest/fourier-making-waves_all.html
  Select the **Discrete** option.

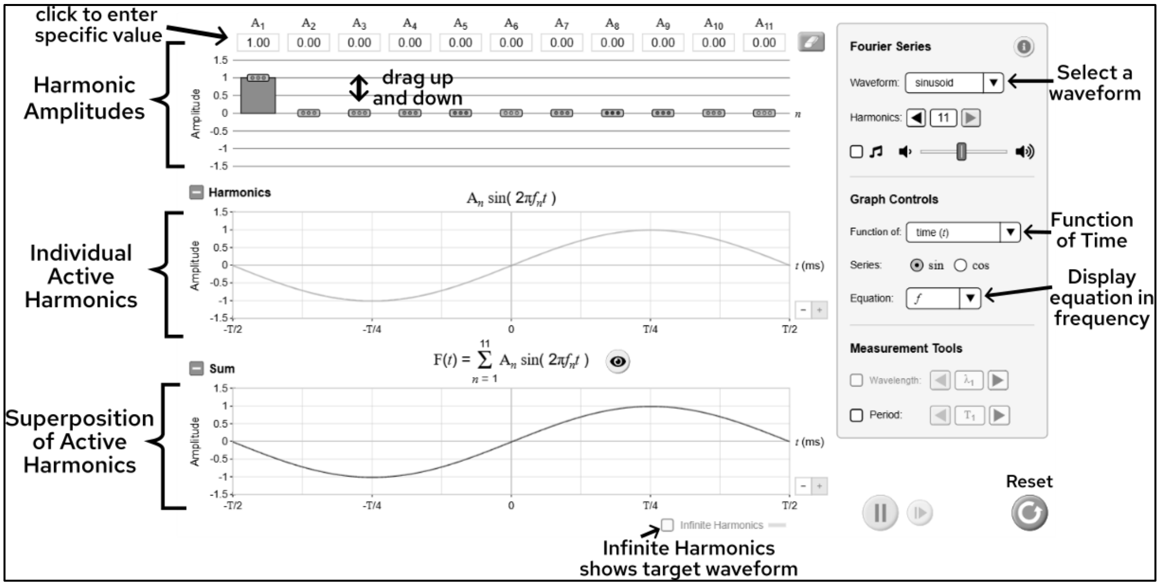



Figure 3. PhET Fourier Simulation

In this simulation (Fig. 3), you can adjust the amplitude of up to 11 harmonics. The individual harmonics are displayed on one graph and their sum is displayed on another. You can click the box next to the musical notes ♫ to hear the current waveform.

The simulator should open with just the fundamental frequency displayed.

- On the menu on the right, change the Graph Controls to display the plot as a function of time (t) and the equation in terms of frequency (f).
- In your notebook, sketch a big square wave (⎍). On top of it, draw a sine wave that resembles the square wave as closely as possible.
- Where is the sine wave are too high to match the square wave? Where is it too low?
- Consider the sine wave that you have already drawn to be the fundamental frequency.
    - To get closer to the square wave shape, you can add another harmonic. Would the second or the third harmonic do a better job? Does one do a better job maintaining the symmetry of the square wave? You may want to try sketching both options in your notebook or using the PhET simulation.
- Now add more components to improve your square wave! Click and drag the amplitudes in the PhET simulation to approximate a rough square wave. You can also enter specific values by clicking on the amplitude value. If you're stuck, the waveform pictures below offer some clues.
- Once you have created a decent square wave, use the amplitude values to try to figure out the pattern of harmonics that creates a square wave. **Hint: Try dividing the amplitude of the fundamental by the amplitudes of the higher harmonics.**

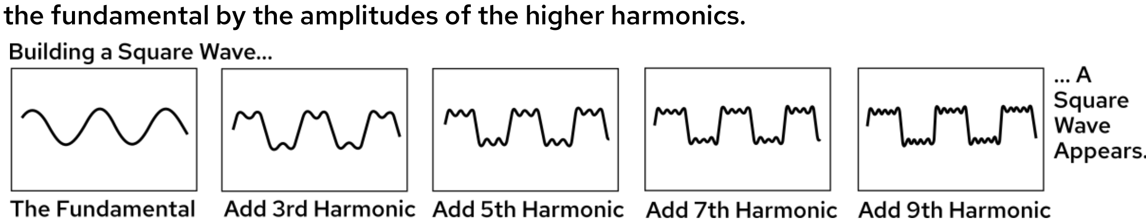



Figure 4. Building a Square wave by adding harmonics. Each frame adds one harmonic to the previous frame.

### 3. Does One Hear Phase?

Checking the sound box ♫ allows you to hear the waves. Listen to the sounds as you change the wave properties.

- Add two or more sine waves (like harmonics 1, 2, and 3), each with an amplitude of 0.75.
- Now let's change the phase of one of these harmonics. While there is not an explicit "Phase" control in this simulator, making one of the amplitudes negative effectively changes its phase by 180°. Change one of the amplitudes to -0.75 while watching and listening.
    - Does the wave shape change?
    - Does the sound that you hear change?

  Since you are changing the bar graph, you might be tempted to say that the Fourier spectrum of the tone changes. However, a Fourier spectrum only shows the **magnitude** (absolute value) of each harmonic's amplitude.

- Given all of this, which would be more useful for specifying the timbre of an instrument: the wave shape or the Fourier spectrum?

## Fourier Analysis

### 4. Fourier Analysis of Sine Waves

You will keep working with the PhET Fourier simulation to explore Fourier analysis by analyzing the simulation's preset functions. A Fourier synthesizer is also able to produce complex waveforms more cleanly and reliably than instruments or your voice. You may want to listen to the difference in waveforms.

- On the menu, select "Sinusoid" as the waveform.
- Look at the Fourier spectrum of the sine wave (the amplitudes at the top). Does it look like you expect? Explain.

### 5. Fourier Spectrum of the Square Wave

- Now, select "square" as the waveform. The Fourier spectrum of this square wave is displayed.

  From the screen, measure and record the amplitude of each frequency. Write the frequency in terms of the fundamental frequency, $f_1$. For example, the second harmonic has a frequency of $2f_1$.

    - Which harmonics contribute?
    - Do you see a pattern in the amplitudes of those harmonics?
    - How do the values compare to the square wave you built in the first part of the lab?
- Examine the other preset options.
    - In the waveforms that you have considered thus far, the fundamental frequency has the largest amplitude, and all of the amplitudes are positive. Is that always the case?
    - Now choose the triangle waveform. How does the waveform change if you reduce the number of harmonics included to 3 or 4 instead of 11?
    - Change the number of harmonics to 1, and then check the box for Infinite Harmonics (available for triangle, square, and sawtooth) at the bottom of the screen. This will show the "ideal" shape of the wave. Slowly increase the number of harmonics while watching the waveform. Describe how the wave changes as you add more harmonics.

## 6. Fourier Analysis of Musical Instruments and the Voice: Introduction

For the rest of the lab, you will use a microphone and a Fourier analysis program.

- Be sure a microphone (black sphere on a tripod) is connected to the computer via USB cord.
- Then, in the Lab Software/109 folder on the desktop of the lab computer, open the file called Simulink_FourierAnalyzer_2021a.slx.
- It may take a few minutes for the program to start up. The program is ready when you have a window called Spectrum Analyzer and another called Time Scope (Figure 4). You may want to rearrange the screen to make these two windows easily visible.

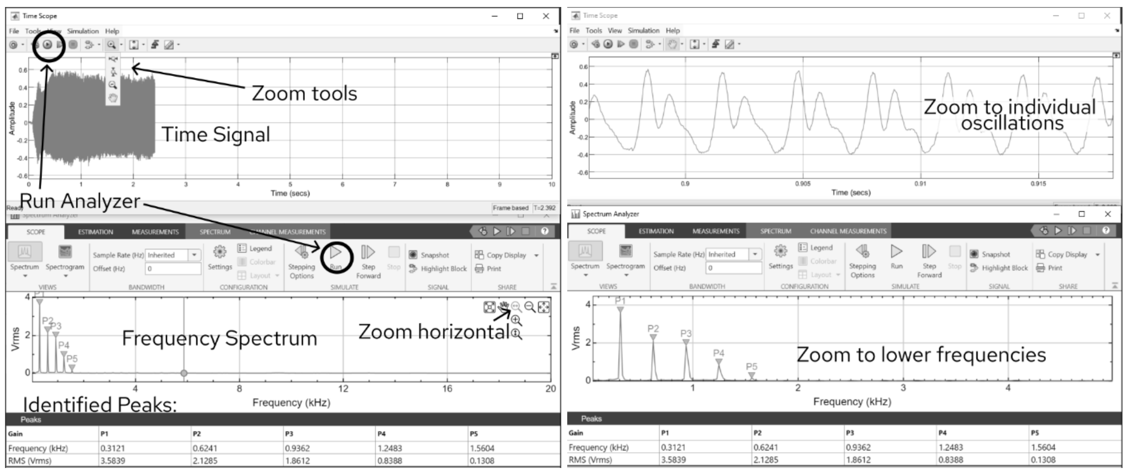



Figure 5. Fourier Spectrum Analyzer windows. Left: The default view with the recording waveform on the top and the spectrum on the bottom. Right: Graphs are zoomed in to see the shape of the oscillations in time and the lower frequency portion of the spectrum.

Get to know the program by analyzing your voice. Click one of the "Run" buttons ( or ) on either window to begin.

- Sing a steady tone into the microphone, for instance a tone like the "aah" in father. Hold a steady tone for a few seconds and while you are still singing, have a teammate press pause ( or ) on either window.
  - Use the zoom tools in each scope window to look at the details of the time signal and of the frequency spectrum.
  - You may need to zoom in more than once on the time scope to see the waveform. Use the dropdown next to the magnifying glass icon () to select the zoom in horizontal () option. You can also select "Pan" from the Tools menu to drag the graph forward or backward in time.
  - The spectrum analyzer automatically identifies the highest peaks in order of amplitude. The values are given in kHz in the box labeled "Peaks."
- Click the "Run" button again and try different tones and vowel sounds to explore how it changes the waveform and the spectrum. Make sure to hold each sound for a few seconds to let the spectrum catch up.
- If you wish, you can save the Fourier spectrum of your voice.
  - To save an image, in the Spectrum Analyzer window, select the Scope menu and use the Copy Display button ().
  - Paste the image into a program like Paint and save it to print or email.
- When you are done exploring, click the "Stop" button ().

**Choose any two (or more!) of the following seven experiments.** Your TA may have additional instructions on which you can choose.

## 7. The Voice

The human voice is one of the most versatile musical instruments because you have so much control over the physical characteristics of the instrument. Among other things, you can dynamically modify:

→ The rate of air flow

→ The shape of the throat

→ The shape of the oral cavity

Each of these change as you sing different word sounds.[3] For example, the word "shake" has three distinct sounds ("sh", "ā," and "k"). As the shapes of your throat and oral cavity change, different frequencies resonate and there are changes to the Fourier spectrum of the sound produced. Different spectra let your ears differentiate the "ā" sound in "shake" from the "oo" sound in "shook."

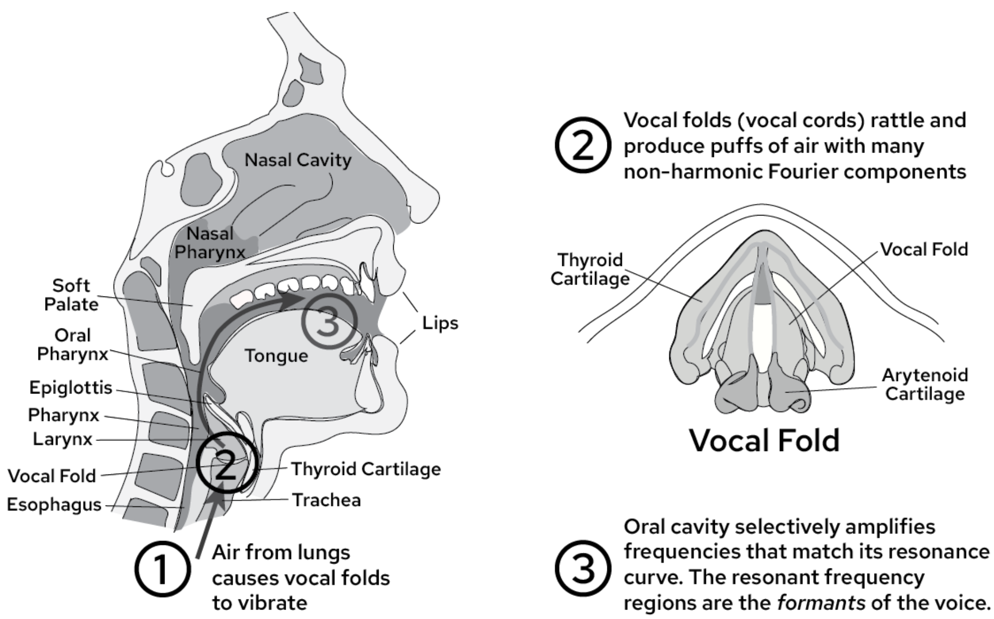



Figure 6. Human Vocal Instrument.

When you sing, you push air out of your lungs through the windpipe, or trachea (see Fig. 6). In the windpipe, the air passes through the narrow slot made by the **vocal folds** (often called vocal cords). The motion of the vocal folds is **not** harmonic (that is, not all frequencies are integer multiples of the fundamental).

As the vocal folds rattle back and forth, they produce puffs of air that have a large number of non-harmonic Fourier components (overtones). These puffs of air are the driving force that make the air in your throat and **oral cavity** begin to move. The oral cavity then selectively amplifies some of these frequencies based on its current shape. With a Fourier analyzer, you can see which frequencies are amplified by changes in the amplitudes of the Fourier components.

For this experiment, you will focus on vowel sounds because they are easier to sustain for enough time to record the spectra.

[3] The smallest part of speech (distinct sound) in language is called a phoneme. These can be smaller than a syllable and are used to construct all spoken words in a language.

### A. Vowel Sounds

You have already looked at the spectrum of one vowel sound: the “aah” in “father.” To produce that vowel, you shape your oral cavity to reinforce and amplify the frequencies that form that sound.

- Run the Spectrum Analyzer to re-record the “aah” sound.
- You will compare this to other spectra, so save a screenshot with the Copy Display button ( ) and paste it into a document or Paint file.
- Record the frequency peaks. Note that the program calls the largest peak $P_1$, which may not be the fundamental. In your notebook, $f_1$ should correspond to the fundamental, $f_2$ to the 2nd mode, etc.
- Beyond the peaks identified by the program, do you see others at higher frequencies?
- Opera singers are often trained to produce additional peaks around 3000 Hz that allow them to be heard through an orchestra.

### B. Same Vowel – Different Pitch

- Have the same singer repeat singing “aah”, but at a higher pitch. The singer should try to keep their mouth in exactly the same position as before. Record a Fourier spectrum.
- In switching from a lower pitch to a higher one, what does the singer notice about their mouth or throat? What changes reinforce the higher frequencies of the new pitch?
- Compare the two spectra: the frequencies should be different, but is there a similar pattern?

### C. Different Vowel – Same Pitch

- Before recording more Fourier spectra, take a little time to sing some other vowels: “ē” as in “bean”, “oo” as in “food”, etc. Pay attention to how you must change the shape of your mouth to form each sound. Note that you must move your lips, tongue, and jaw.
- As you change the shape of your mouth, the frequencies that resonate in the oral cavity change. Thus, you reinforce different frequencies. Some vowels reinforce more harmonics than others.
- Now record Fourier spectra and waveforms for the different vowels. Do your best to sing each vowel at the same pitch. Save screenshots of each spectrum to compare.
- What do you observe in the spectra and waveforms? Which waveforms look the simplest? Do they correspond to simpler spectra? Is the fundamental frequency always the strongest harmonic or do some vowels have stronger overtones?

## 8. Guitar

Guitars typically have six strings and each string is used for a range of frequencies. A guitarist changes the frequency by shortening the vibrating length of the string along the fingerboard. The frets on the guitar help the player to get the proper length of the string, and thus the proper tone.

The frets on a guitar are spaced so that the musician can change the frequency of a given string in semitone steps on the tempered scale. The six strings on a guitar are generally tuned to $E_2$, $A_2$, $D_3$, $G_3$, $B_3$, and $E_4$.

### A. Why Frets?

- Frets help you to play in tune, so why are frets not found on other string instruments like violins. Can you think of any disadvantages?

- On a guitar, one places the finger not **on** the fret, but just beyond the fret, farther from the bridge. Why might that be? **Hint: take a look at what part of the string is free to vibrate in each case.**

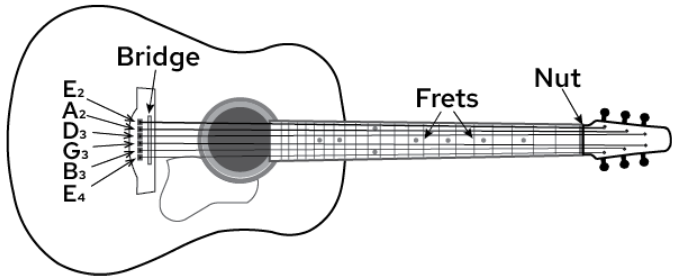



Figure 7. Guitar strings and parts

## B. Placement of Frets

- Measure the vibrating length of an open string on a guitar. The term **open string** refers to a string without fingers on the fingerboard so that the full bridge-to-nut length of the string vibrates. The frequency a string vibrates at is inversely proportional to its length (L).

$$f = \frac{v}{2L}$$

- Suppose you want to produce the tones below, what should the shortened length of the string be to play:
  - One octave (2:1) above the open string? Double the frequency.
  - A fifth (3:2) above the open string? 3/2 times the frequency of the open string.
  - A fourth (4:3) above the open string? 4/3 times the frequency of the open string.
- Are there frets on the fingerboard in the corresponding locations? Do you notice any special marks on the fingerboard at any of these locations?
- Run the Fourier Analyzer and take screen shots of the spectra for an open string and for tones a fourth, a fifth, and an octave higher.
  - Do the fundamental frequencies have the expected ratios?
  - How does the frequency spacing between harmonics change for the higher notes? Why?

## C. Semitone Intervals on Guitar

- A semitone interval, such as G to G#, corresponds to a frequency ratio of about 1.06 – a six percent change in frequency.
  - How large should the first fret spacing be if it is to raise the frequency of the open G-string by one semitone?
  - Compare your estimate with the actual spacing of the first fret (from the nut).
  - Record spectra for the open G-string ($G_3$) and for G#.
- Suppose you now wanted to go up a semitone **from an octave above the open G-string** (which is $G_4$).
  - To play a G#, how wide should this fret spacing be?
  - Measure it on the guitar and compare it to the spacing you calculated.
  - Compare the spectra for these higher G and G# tones.

**This example should explain why the spacing between frets get smaller and smaller as you move along the guitar string to higher pitches. Ask your TA for help if you don't understand it.**

# 9. Timbre of Bowed String – Violin

Bowing a string results in a waveform similar to a sawtooth (see the PhET simulation). Higher even and odd harmonics are present in decreasing intensity. Figure 7 shows a waveform of the motion of a bowed string.

- Observe the Fourier spectrum of a bowed violin string. Be sure to bow between the bridge and the fingerboard, near the bridge. Are even and odd harmonics present? Save a screenshot.
- Now bow the string near the middle (measured bridge to nut) instead of near the bridge.
    - In what way does the Fourier spectrum change?
    - Which harmonics are stronger or weaker? Why might that be?
- A violin body has elegantly shaped holes to the left and right of the strings called F-holes (Fig. 8). One might wonder if they serve a purpose or if they are purely decorative.
    - Compare the Fourier spectrum with the F-holes open to the spectrum with the holes covered (with tape or sticky notes).
    - Play a relatively low tone and a higher tone. Can you hear a difference in tone with the holes closed?
- When you are finished, please **carefully** remove the tape to avoid damaging the violin's varnish.

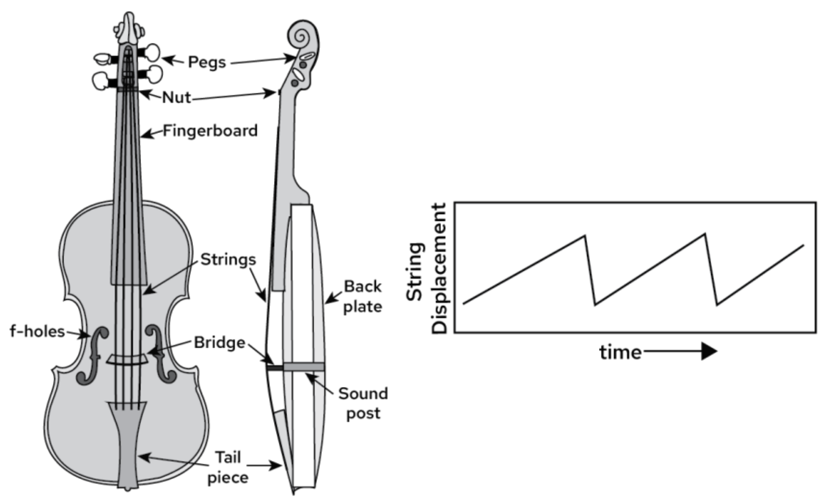



Figure 8. Parts of a violin and saw-tooth motion of bowed string

# 10. Piano Hammers

The striking mechanism of a piano is made up of a complex system of levers (Fig. 9). The mechanism is designed to quickly move the hammer away from the string after striking it and to make sure that the hammer does not bounce back and strike the string a second time.

- Inspect the piano hammer mechanism.
    - Why does one need to remove the hammer from the string (metal bar in lab model) immediately after striking the string?
- The hammer in a piano is set up so that it hits the string at $^1/_7$ of its length from the end.
    - What harmonic does this eliminate? Why would one want to eliminate this harmonic?
    - If the fundamental note is C, determine to what tones the first, second, third, etc. harmonic correspond. Does the seventh harmonic fit into the musical scale?

- If the single piano string demo is available, pluck the piano string in the center.
  - What harmonics are suppressed by doing this?
  - Is there a difference in tone quality between plucking and striking with the hammer?
  - Pluck and strike the string about one-third of the way down the length of the string. What harmonics are suppressed? What harmonics are excited?

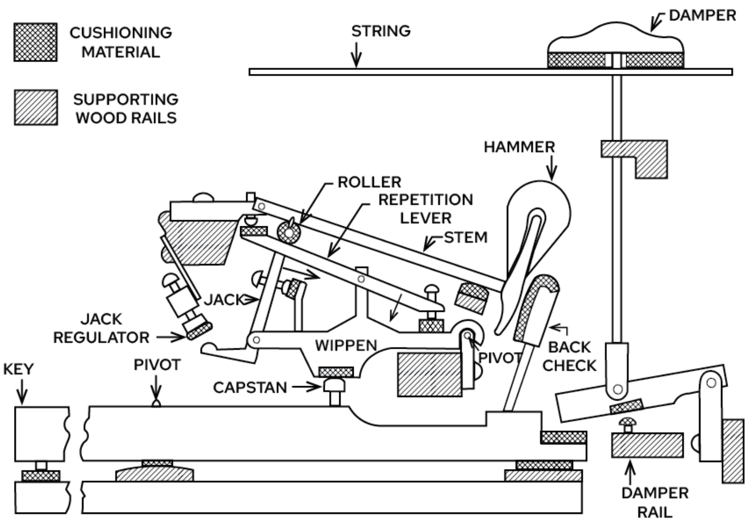


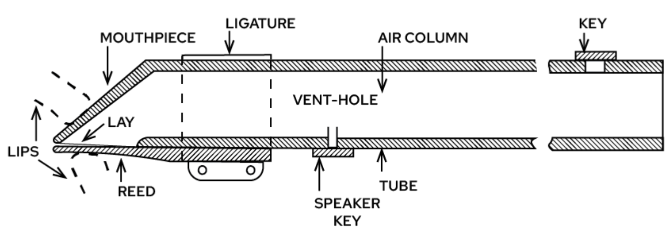

Figure 9. Piano hammer mechanism. When a key is pressed capstan rises and the various linkages cause the hammer to strike the string.

## 11. Wind Instruments I: Reeds, Lip Reeds, and Air Reeds

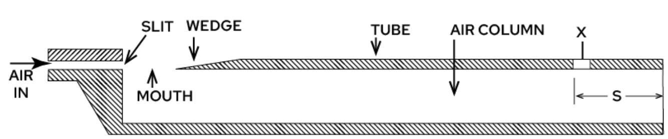
Wind instruments produce sound by setting a column of air in motion. This requires a periodic driving force. For the voice, that driving force is comes from vibration of the vocal folds when air flows through the narrow gap between them. This vibration causes the intensity of the air stream to vary periodically.

The same principle applies to **reed instruments**, like clarinets, oboes, etc. A clarinetist blows air between the solid mouthpiece and a flexible reed (see Fig. 10a). The reed oscillates and periodically blocks part of the air stream. This variation sets the clarinet's air column in motion with pressure waves. Pressure waves are reflected at the far end of the clarinet and cause the reed to oscillate at the same frequency.

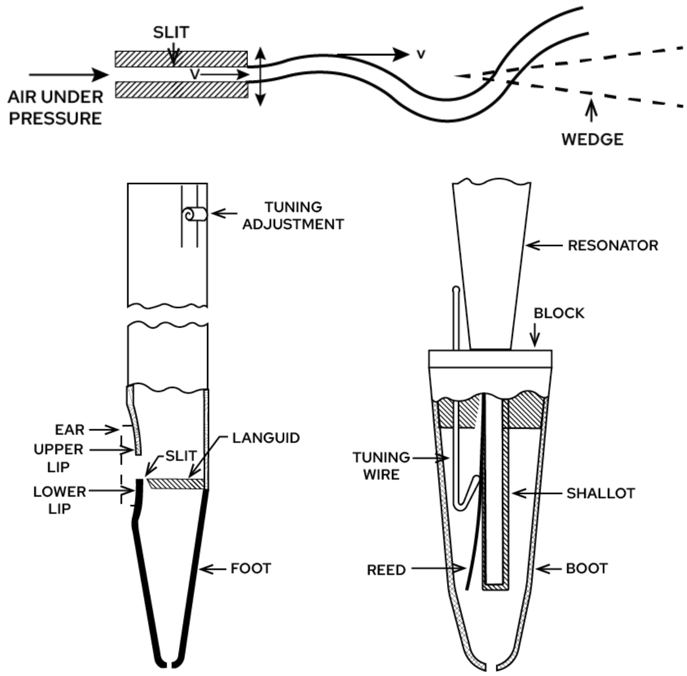
However, the clarinetist can adjust the air flow and the pressure on the reeds with their lips so that the reed frequency is more or less near the resonance frequency of the desired tone.




a) Cross-sectional view of the basic clarinet




b) Cross-sectional view of a recorder (air reed).




c) An oscillating air stream through a flue in an air reed instrument (close-up view of 9b)

d) Organ pipes:
Left: *Diapason pipe*
Right: *Reed pipe*

Figure 10. Wind Instruments using reeds and wedges to oscillate air columns

The reed oscillation and air column oscillation influencing one another is called a **feedback** mechanism. The feedback keeps the driving force steady and at just the right frequency. In **double reed** instruments, like the oboe, two reeds are used instead of one. In **lip reed** instruments like the trumpet, the lips play the role of the flexible reeds.

Another way to excite the air column is by an **air reed**. In instruments like certain organ pipes and the recorder, air is blown through a narrow slit called the **flue** (Fig. 10c). The airstream is directed toward a sharp edge, where it is deflected by turbulence to oscillate back and forth, causing an **edge tone** (Fig. 10b). The flute has no flue, but it operates on similar principles. The narrow slit made by the lips of the flutist replaces the flue.

For all these instruments, the tone quality depends on the Fourier spectrum of the reed oscillations **and** on the resonances of the air column excited by the reed. Just like voices, wind instruments have frequency regions that are reinforced by resonances.

## A. Excitation Mechanism

- Inspect the organ pipes and the wind instruments in the lab. Identify which ones are single reed, double reed, air reed, and lip reed instruments.
- In brass instruments, the lips of the player form a vibrating double reed and act as a tone generator. Remove the mouth piece from a brass instrument and clean it with alcohol. Try to play it by creating a buzzing sound with your lips into the mouthpiece.
  - Try to vary the frequency by changing the **embouchure**, i.e., by changing the pressure on your lips and the speed of air flow between them.

    **Embouchure is a French word, derived from bouche = mouth, which refers to the way one blows into a wind instrument.**
  - Experience the effect of feedback on the lip reed: Attach the mouthpiece to the instrument and blow it again. Can you still vary the frequency at will?
  - Record spectra of at least two different pitches. How do the fundamentals relate?

### B. Length of Pipe

- How long of an open pipe is needed to play a fundamental frequency that corresponds to middle C (262 Hz) of the piano? Use the pipe formula to calculate the length.
- Compare your calculation to the length of the flute. Clean the mouthpiece with alcohol and try to play a tone with all of the keys closed. What fundamental frequency does the Fourier analyzer record?
- Measure the length of the recorder (an open pipe). Calculate the fundamental frequency with all finger holes closed. What tone of the musical scale does it play if all fingerholes are closed? Try it! Clean the mouthpiece and record a Fourier spectrum

## 12. Wind Instruments II (Natural Scale and Brass instruments)

### A. Natural Scale

The bugle has no valves or keys, but the player can produce different modes by adjusting the pressure on the lips and the rate of air flow. Bugle compositions, by necessity, are limited to tones whose frequencies are multiples of the fundamental. The sequence of tones whose frequencies are 2f, 3f, 4f, 5f, etc., are called the **natural scale**. Usually the lowest frequency, 1*f*, cannot be produced. If you assume that the fundamental frequency (mode #1) corresponds to the tone $C_3$, the tones for the higher modes are:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| $C_3$ | $C_4$ | $G_4$ | $C_5$ | $E_5$ | $G_5$ | X | $C_6$ | $D_6$ | $E_6$ |

Note that the 7th harmonic falls between an $A_5$ and an $A\#_5$, and is avoided.

Other instruments that are limited to the natural scale are the Bach trumpet, the natural horn, and the Alphorn. You can also produce a natural scale by rapidly swinging a corrugated pipe.

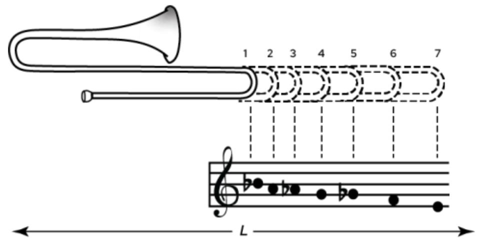
- Try "playing" the corrugated pipe by swinging it at different speeds.
  - Use the Fourier analyzer to determine which tones you can produce.
  - It can be tough to produce the fundamental frequency. However, even if you cannot produce the fundamental, you should be able to tell what it is. How?
- On a piano keyboard, mark the keys that correspond to the natural scale with pieces of tape on the keyboard. Use these notes to play a bugle tune, such as "Taps":

  $G_4$ $G_4$ $C_5$ $G_4$ $C_5$ $E_5$ $G_4$ $C_5$ $E_5$ $G_4$ $C_5$ $E_5$ $G_4$ $C_5$ $E_5$ $C_5$ $E_5$ $G_5$ $E_5$ $C_5$ $G_4$ $G_4$ $G_4$ $C_5$

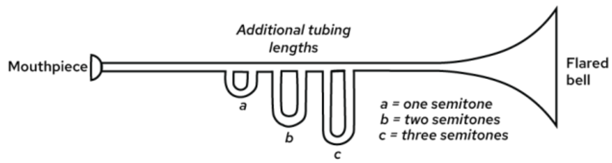
### B. Changing the Length of the Pipe with Valves or Slides

The pitch of the trombone is changed by changing the length of the pipe with a slide. For most other brass instruments, the pitch can be changed in steps of a semitone by opening valves which route the air into additional lengths of piping (See Fig. 11b, c.).

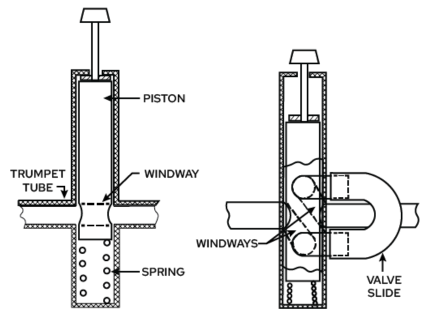
- Use a flexible tape measure to measure the lengths of the additional tubing sections on the trumpet or French horn. Note which valve connects to which length of additional tubing.
- Compare the added length for each of the three valves to the overall length. For each tube section, determine the number of semitones added. Remember that it takes a 6% increase in length to reduce the frequency by a semitone.




a) The trombone slide positions




b) Diagram of additional tubing in brass instruments.
The valves that redirect the air into the added sections are not shown.




c) Structure of a trumpet valve—
Left: Piston up
Right: Piston down

Figure 11. Methods of increasing pipe length for brass instruments

Bugles and natural scale instruments have limited tones; there are gaps in the scale. However, trumpets and horns do not have these gaps; they can play all twelve semitones. How do they fill in the missing tones? Consider the biggest gap in the natural scale, from $G_4$ (mode #3) to $C_4$ (mode #2).

- How many semitones are there from C4 to G4?
- Suppose you play G4 (mode #3) and now want to lower the pitch in semitone steps until you get to $C_4$ (mode #2).
    - → You have available three valves: (a), (b), and (c). When the valve ("piston" in Fig 10c) is depressed, it adds a certain length of pipe (Fig 10b).
    - → Pressing valve (a) lowers the pitch one semitone, valve (b) lowers the pitch two semitones and valve (c) lowers the pitch three semitones.
    - o Set up a table showing all of the semitones between $C_4$ and $G_4$, and make columns for the three available valves.
    - o For each semitone, use a ● to indicate a depressed valve (length added), and a O to indicate the valve is not depressed (no length added).

| Pitch | Valve (a) | Valve (b) | Valve (c) |
|---|---|---|---|
| $G_4$ | O | O | O |
| $F^{\#}_4$ | ● | O | O |
| … | | | |
| $C^{\#}_4$ | ● | ● | ● |
| $C_4$ | O | O | O |

- Record the spectra of few tones from your table. Can you play the notes from $G_4$ down to $C_4$? Note: Valve (a) is the closest to the mouthpiece, Valve (c) is farthest from the mouthpiece.