Physics 201/202 & 207/208 Lab Manual

Part I: Mechanics, Heat, Sound/Waves
Part II: Electricity & Magnetism, Light, Modern Physics

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December 22, 2000

NOTE: M=Mechanics, H=Heat, S=Sound/Waves, E=Electricity & Magnetism, L-Light, MP=Modern Physics, C=Computerized Experiment

Contents

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Forward

Spring, 2001

We are gradually revising the manual to improve the clarity and interest of the activities. In doing this, we have discovered the tremendous amount of work that M.J. Winokur and M. Thompson did in order to bring the manual into its current electronic form. I speak for the students and Physics Department when I say: Thankyou!

M. Briggs
J. Sylvester
B. Grogan

In reference to the 1997 edition

Much has changed since the implimentation of the first edition and a major overhaul was very much in need. In particular, the rapid introduction of the computer into the educational arena has drastically and irreversibly changed the way in which information is acquired, analyzed and deseminated. To reflect these changes in the introductory laboratory we have endeavdored to create a educational tool which utilizes this technology to its fullest extent; hopefully while enhancing the learning process and the understanding of physics principles. Thus, when fully deployed, this new edition will be available not only in hard copy but also as a fully integrated web document so that the manual itself has become an interactive tool in the laboratory environment.

The introductory student will find a wealth of learning resources to be available while the laborary is being performed. Too many times students have questions during these lab sessions which could be answered if the student could access additional information. In the past the teaching assistant has been the only source of this additional information and, if already occupied, too often these questions went unanswered. There are also instances in which a student may inadvertently stumble and produce a lab write-up containing a major error. Too often this mistake is only seen at a latter time when the instructor is correcting the lab reports. Students do not always have the opportunity to go back and make it right and so the misunderstanding is never satistisfactorily resolved. By making an interactive lab manual we hope to provide students with immediate feedback so that if they do stray from the straight and narrow, a timely course correction is provided.

M.J. Winokur
M. Thompson

From the original edition

The experiments in this manual evolved from many years of use at the University of Wisconsin. Past manuals have included "cookbooks" with directions so complete and detailed that you can perform an experiment without knowing what you are doing or why, and manuals in which theory is so complete that no reference to text or lecture was necessary.

This manual avoids the"cookbook" approach and assumes that correlation between lecture and lab is sufficiently close that explanations (and theory) can be brief: in many cases merely a list of suggestions and precautions. Generally you will need at least an elementary understanding of the material in order to perform the experiment expeditiously and well. We hope that by the time you have completed an experiment, your understanding will have deepened in a manner not achievable by reading books or by working "paper problems". If the lab should get ahead of the lecture, please read the pertinent material, as recommended by the instructor, before doing the experiment.

The manual does not describe equipment in detail. We find it more efficient to have the apparatus out on a table and take a few minutes at the start to name the pieces and give suggestions for use. Also in this way changes in equipment, (sometimes necessary), need not cause confusion.

Many faculty members have contributed to this manual. Professors Barschall, Blanchard, Camerini, Erwin, Haeberli, Miller, Olsson, Visiting Professor Wickliffe and former Professor Moran have been especially helpful. However, any deficiencies or errors are our responsibility. We welcome suggestions for improvements.

Our lab support staff, Joe Sylvester and Harley Nelson, have made important contributions not only in maintaining the equipment in good working order, but also in improving the mechanical and aesthetic design of the apparatus.

Likewise our electronic support staff (Mike Murrary, Jerry Glowacki, and Art Webb) not only maintain the electronic equipment, but also have contributed excellent original circuits and component design for many of the experiments.

R. Rollefson
H. T. Richards

Introduction to the lab activities for Physics 201/202 and 207/208

General Instructions and Helpful Hints

Goals

Physics 201/202 and 207/208 are introductory calculus-based physics courses which introduce the undergraduate student to a broad spectrum of fundamental physical laws spanning from mechanics to heat and thermodynamics to electricity and magnetism to waves and light. To help develop a meaningful understanding of these physics principles the beginning student is presented with a variety of resources: textbooks, lectures and demonstrations, problem solving, discussion sessions and the laboratory.

Of these, the laboratory component furnishes a unique opportunity for demonstrating physical principles in both a qualitative and quantitative hands-on fashion. An inseparable aspect of this laboratory experience should be the realization that physics is, first or foremost, an experimental science in which the limitations of the instrumentation and the technique of the experimenter can heavily impact the scientific process. Hence this laboratory experience is intended to provide the student with a diverse set of experiences including: a realistic feeling for the origin and limitations of physical concepts; an awareness of experimental errors, of ways to minimize them and how to estimate the reliability of the result in an experiment; an appreciation of the need for keeping clear and accurate records of experimental investigations.

Throughout this laboratory experience there is one crucial step for achieving these stated goals in an enduring way: Simply put, a clearly written laboratory notebook in which each of the aforementioned components is documented and recorded. This lab notebook, at a minimum, should contain the following:

1. *Heading of the Experiment:* Copy from the manual the number and name of the experiment.
   Include both the current date and the name(s) of your partner(s).

2. *Original data:* Original data must always be recorded directly into your notebook as they are gathered. "Original data" are the actual readings you have taken. For example if you know that each in a series of distance measurements is in error by a constant offset of 0.006 mm, then you should record the actual readings (containing this error) and then, afterwards, either correct each data point or the average. In this way it will always be clear that you have made appropriate corrections. Also, when you take 5 or 6 successive readings of a measurement, record each reading, not just the average. From the scatter of the readings, you can estimate the precision of the measurement. Both partners should record data, so that errors of recording show up. (Complete trackability, say if you were producing a part for the space shuttle, would require that you record serial numbers of equipment. You could then find the same equipment to check results later.) Arrange data in tabular form when appropriate, and properly label each item or table.

3. *Housekeeping deletions:* You may think that a notebook combining all work would soon become quite a mess and have a proliferation of erroneous and superseded material. Indeed it might, but you can improve matters greatly with a little housekeeping work every hour or so. Just draw a box around any erroneous or unnecessary

material and hatch three or four parallel diagonal lines across this box. (This way you can come back and rescue the deleted calculations later if you should discover that the first idea was right after all. It occasionally happens.) Append a note to the margin of box explaining to yourself what was wrong.

We expect you to keep up your notes as you go along. Don't take your notebook home to "write it up" – you probably have more important things to do than making a beautiful notebook. (Instructors may permit occasional exceptions if they are satisfied that you have a good enough reason.)

4. *Remarks and sketches:* Avoid, when possible, "pictorial" sketches of apparatus. On the other hand, a simple diagrammatic sketch is useful and is sometimes the simplest and clearest way to define the various quantities indicated in a table of data; *a phrase or sentence introducing each table or calculation is essential for making sense out of the notebook record.* When a useful result occurs at any stage, describe it with at least a word or phrase.

5. *Graphs:* There are three appropriate methods:

   A. Affix furnished graph paper in your notebook with transparent tape.
   B. Affix a computer generated graph paper in your notebook with transparent tape.
   C. Mark out and plot a simple graph directly in your notebook.

   Show points as dots, circles, or crosses, i.e., $\cdot$, $\circ$, or $\times$. Instead of connecting points by straight lines, draw a smooth curve which may actually miss most of the points but which shows the functional relationship between the plotted quantities. Fasten directly into the notebook any original data in graphic form (such as the spark tapes of Experiment M4).

6. *Units, coordinate labels:* Physical quantities *always* require a number and a dimensional unit to have meaning. Likewise, graphs have *abscissas* and *ordinates* which always need labeling.

7. *Final data, results and conclusions:* At the end of an experiment some written comments and a neat summary of data and results will make your notebook more meaningful to both you and your instructor. Note that perfect results are not essential when making a quantitative measurement. "Good" results occur when your value agrees, within appropriate limits of error, with the expected result. "Bad" results occur if the measured value falls outside the range given by uncertainty. This latter result may be perfectly acceptable if a satisfactory explanations (i.e., a legitimate error) for the failure can be forwarded. In fact, most people seem to learn more from their failures rather than their successes.

Expt. M1 Systematic and Random Errors, Significant Figures,
Density of a Solid

NAME: Jane.Q. Student Date: 2/29/00

Partner: John Q. Student

Purpose: To develop a basic understanding of systematic and random errors in a physical measurement by obtaining the density of metal cylinder.

Equiment: Venier caliper, micrometer, precision gauge block, precision balance

Theory: ρ= mass/($\pi *$ r**2 h)

$$\Delta\rho = \sqrt{(\Delta\ m\ /m)^{**}2.+(\Delta\, h/h)^{**}2.+(\ 2\ \Delta\, r/r)^{**}2.}\quad \rho$$

DATA:

1. Calibration of micrometer

Reading with jaws fully closed:

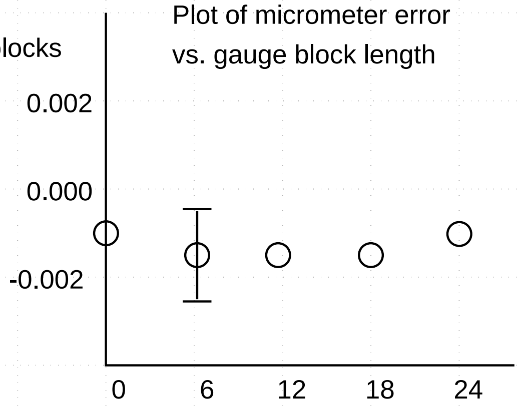
1. 0.000013 mm $\pm$ 0.000001 mm
2. 0.000014 mm
3. 0.000012 mm
4. 0.000014 mm Micrometer exhibits a systematic zero offset
5. 0.000015 mm

Ave, $\pm$ Standard Deviation

Measure four calibraton gauge blocks

Plot of micrometer error vs. gauge block length

Micrometer error (mm)



Gauge block length (mm)

Measure of cylinder diameter:

Measure of cylinder height:

Measure of cylinder mass

CALCULATIONS:

Density= ??? Uncertainty from propagation of error.

RESULTS and CONCLUSIONS:

PARTNERS

Limitations of space and equipment usually require that one works with a partner. In addition, discussing your work with someone as you go along is often stimulating and of educational value.

Independent calculations; checks: If possible both partners should perform completely independent calculations. Mistakes in calculation are inevitable, and the more complete the independence of the two calculations, the better is the check against these mistakes. Poor results on experiments sometimes arise from computational errors.

CHOICE OF NOTEBOOK

We recommend a large bound or spiral notebook with paper of good enough quality to stand occasional erasures (needed most commonly in improving pencil sketches or graphs). To correct a wrong number always cross it out instead of erasing: thus ~~3.1461~~ 3.1416 since occasionally the correction turns out to be a mistake, and the original number was right. Coarse (1/4 inch) cross-ruled pages are more versatile than blank or line pages. They are useful for tables, crude graphs and sketches while still providing the horizontal lines needed for plain writing. Put everything that you commit to paper right into your notebook. Avoid scribbling notes on loose paper; such scraps often get lost. A good plan is to write initially only on the right-hand pages, leaving the left page for afterthoughts and for the kind of exploratory calculations that you might do on scratch paper.

COMPLETION OF WORK

Plan your work so that you can complete calculations, graphing and miscellaneous discussions before you leave the laboratory. Your instructor will check each completed lab report and will usually write down some comments, suggestions or questions in your notebook.

Your instructor can help deepen your understanding and "feel" for the subject. Feel free to talk over your work with him or her.

Errors and Uncertainties

Reliability estimates of measurements greatly enhance their value. Thus, saying that the average diameter of a cylinder is 10.00 ± 0.02 mm tells much more than the statement that the cylinder is a centimeter in diameter.

To physicists the term "**error**" is interchangeable with "**uncertainty**" and does **not** have the same meaning as "**mistake**". Mistakes, such as "errors" in calculations, should be corrected before estimating the experimental error. In estimating the reliability of a single quantity (such as the diameter of a cylinder) we recognize several different kinds and sources of error:

FIRST, are **actual variations** of the quantity being measured, e.g. the diameter of a cylinder may actually be different in different places. You must then specify where the measurement was made; or if one wants the diameter in order to calculate the volume, first find the average diameter by means of a number of measurements at carefully selected places. Then the scatter of the measurements will give a first estimate of the reliability of the average diameter.

SECOND, the micrometer *caliper used may itself* be in error. The errors thus introduced will of course not lie equally on both sides of the true value so that averaging a large number of readings is no help. To eliminate (or at least reduce) such errors, we calibrate the measuring instrument: in the case of the micrometer caliper by taking the zero error (the reading when the jaws are closed) and the readings on selected precision gauges of dimensions approximately equal to those of the cylinder to be measured. We call such errors **systematic**, and these cause errors on accuracy.

THIRD, Another type of **systematic** error can occur in the measurement of a cylinder: The micrometer will always measure the largest diameter between its jaws; hence if there are small bumps or depressions on the cylinder, the average of a large number of measurements will not give the true average diameter but a quantity somewhat larger. (This error can of course be reduced by making the jaws of the caliper smaller in cross section.)

FINALLY, if one measures something of definite size with a calibrated instrument, errors of measurement still exist which (one hopes) are as often **positive** as **negative** and hence will **average out** in a large number of trials. Such errors are called random, and result in less precision. For example, the reading of the micrometer caliper may vary because one can't close it with the same force every time. Also the observer's estimate of the fraction of the smallest division varies from trial to trial. Hence the average of a number of these measurements should be closer to the true value than any one measurement. Also the **deviations** of the individual measurements from the **average** give an indication of the **reliability** of that average value. The typical value of this deviation is a measure of the precision. This average deviation has to be calculated from the absolute values of the deviations, since otherwise the fact that there are both positive and negative deviations means that they will cancel. If one finds the average of the absolute values of the deviations, this **"average deviation from the mean"** may serve as a measure of reliability. For example, let column 1 represent 10 readings of the diameter of a cylinder taken at one place so that variations in the cylinder do not come into consideration, then column 2 gives the magnitude (absolute) of each reading's deviation from the mean.

Measurements	Deviation from Ave.	
9.943 mm	0.000	
9.942	0.001	
9.944	0.001	
9.941	0.002	
9.943	0.000	
9.943	0.000	
9.945	0.002	Diameter =
9.943	0.000	
9.941	0.002	9.943±0.001 mm
9.942	0.001	
Ave = 9.943 mm	Ave = 0.0009 mm≈0.001 mm	

Expressed algebraically, the average deviation from the mean is $= (\sum |x_i - \bar{x}|)/n)$, where x_i is the ith measurement of n taken, and $\bar{x}$ is the mean or arithmetic average of the readings.

Standard Deviation:

The average deviation shown above is a measure of the spread in a set of measurements. A more easily calculated version of this is the standard deviation σ (or **root mean square deviation**). You calculate σ by evaluating

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2}$$

where $\overline{x}$ is the mean or arithmetical average of the set of n measurements and x_i is the i^{th} measurement.

Because of the square, the standard deviation σ weights large deviations more heavily than the average deviation and thus gives a less optimistic estimate of the reliability. In fact, for subtle reasons involving degrees of freedom, σ is really

$$\sigma = \sqrt{\frac{1}{(n-1)}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

σ tells you the typical deviation from the mean you will find for an individual measurement. *The mean $\bar{x}$ itself should be more reliable.* That is, if you did several sets of n measurements, the typical means from different sets will be closer to each other than the individual measurements within a set. In other words, the uncertainty in the mean should be less than σ. It turns out to reduce like $1/\sqrt{n}$, and is called **the error in the mean** σ_μ:

$$\sigma_\mu = \text{error in mean} = \frac{\sigma}{\sqrt{\mathrm{n}}} = \frac{1}{\sqrt{\mathrm{n}}}\sqrt{\frac{1}{\mathrm{n}-1}\sum_{\mathrm{i}=1}^{\mathrm{n}}(\mathrm{x_i} - \bar{\mathrm{x}})^2}$$

For an explanation of the $(n-1)$ factor and a clear discussion of erros, see P.R. Bevington and D.K Robinson, *Data Reduction and Error Analysis for the Physical Sciences*, McGraw Hill 1992, p. 11.

If the error distribution is "normal" (i.e. the errors, ϵ have a Gaussian distribution, $e^{-\epsilon^2}$, about zero), then on average 68% of a large number of measurements will lie closer than σ to the true value. While few measurement sets have precisely a "normal" distribution, the main differences tend to be in the tails of the distributions. If the set of trial measurements are generally bell shaped in the central regions, the "normal" approximation generally suffices.

Relative error and percentage error:
Let ϵ be the error in a measurement whose value is a. Then $(\frac{\epsilon}{a})$ is the relative error of the measurement, and 100 $(\frac{\epsilon}{a})\%$ is the percentage error. These terms are useful in laboratory work.

SYSTEMATIC ERRORS IN THE LABORATORY STANDARDS OF LENGTH, TIME AND MASS

For the experiments in this manual these systematic errors are usually negligible compared to other uncertainties. An exception sometimes occurs for the larger masses especially the 100 gram, the 500 gram, and 1 kg masses. Some contain drilled holes into which lead shot and a plug have been added to adjust the mass to within tolerance (typically 1.000 ± 0.003 kg). Occasionally a plug works loose and the calibration lead shot is lost. You can check the assigned mass values by weighing them on the triple beam balances. Report any deviations greater than 0.4% to the instructor.

UNCERTAINTY ESTIMATE FOR A RESULT INVOLVING MEASUREMENTS OF SEVERAL INDEPENDENT QUANTITIES

A.) If the desired result is the *sum* or *difference* of two measurements, the *ABSOLUTE uncertainties ADD*:
Let Δx and Δy be the errors in x and y respectively. For the *sum* we have $z = x + \Delta x + y + \Delta y = x + y + \Delta x + \Delta y$ and the relative error is $\frac{\Delta x + \Delta y}{x+y}$. Since the signs of Δx and Δy can be opposite, adding the absolute values gives a pessimistic estimate of the uncertainty. If errors have a normal or Gaussian distribution and are independent, they combine in quadrature, i.e. the square root of the sum of the squares, i.e.,

$$\Delta z = \sqrt{\Delta x^2 + \Delta y^2}$$

For the *difference* of two measurements we obtain a relative error of $\frac{\Delta x + \Delta y}{x-y}$. which becomes very large if x is nearly equal to y. Hence avoid, if possible, designing an experiment where one measures two large quantities and takes their difference to obtain the desired quantity.

B.) If the desired result involves **multiplying (or dividing)** measured quantities, then the **RELATIVE uncertainty** of the result is the *SUM of the RELATIVE errors* in each of the measured quantities.

Proof:

$$\text{Let} \quad z = \frac{x_1\ x_2\ x_3.....}{y_1\ y_2\ y_3.....} \quad \text{and hence}$$

$$\ln z = \ln x_1 + \ln x_2 + \ln x_3 + ... - \ln y_1 - \ln y_2 - \ln y_3 - .....$$

Then find the differential, $d(\ln z)$:

$$d(\ln z) = \frac{dz}{z} = \frac{dx_1}{x_1} + \frac{dx_2}{x_2} + \frac{dx_3}{x_3} + ... - \frac{dy_1}{y_1} + \frac{dy_2}{y_2} + \frac{dy_3}{y_3} - .....$$

Consider finite differentials, Δz, etc. and note that the most pessimistic case corresponds to adding the absolute value of each term since Δx_i and Δy_i can be of either sign. Thus

$$\frac{\Delta z}{z} = \sum_i (\frac{\Delta x}{x}) + \sum_i (\frac{\Delta y}{y})$$

Again, if the measurement errors are independent and have a Gaussian distribution, the relative errors will add in quadrature:

$$\frac{\Delta z}{z} = \sqrt{\sum_i (\frac{\Delta x}{x})^2 + \sum_i (\frac{\Delta y}{y})^2}$$

C.) **Corollary:** If the desired result is a *POWER* of the measured quantity, the *RELATIVE ERROR* in the result is the relative error in the measured quantity *MULTIPLIED by the POWER:* Thus $z = x^n$ and

$$\frac{\Delta z}{z} = n\frac{\Delta x}{x}.$$

The above results also follow in more general form: Let $R = f(x, y, z)$ be the functional relationship between three measurements and the desired result. If one differentiates R, then

$$dR = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$$

gives the uncertainty in R when the uncertainties dx, dy and dz are known.

For example, consider the density of a solid (Exp. M1). The relation is

$$\rho \ = \ \frac{m}{\pi r^2 L}$$

where m = mass, r = radius, L = length, are the three measured quantities and ρ = density. Hence

$$\frac{\partial \rho}{\partial m} = \frac{1}{\pi r^2 L} \qquad \frac{\partial \rho}{\partial r} = \frac{-2m}{\pi r^3 L} \qquad \frac{\partial \rho}{\partial L} = \frac{-m}{\pi r^2 L^2}$$

and so

$$d\rho = \frac{1}{\pi r^2 L}\,dm + \frac{-2m}{\pi r^3 L}\,dr + \frac{-m}{\pi r^2 L^2}\,dL.$$

To get the relative error divide by $\rho = m/\pi r^2 L$. The result, if one drops the negative signs, is

$$\frac{d\rho}{\rho} = \frac{dm}{m} + 2\frac{dr}{r} + \frac{dL}{L}$$

and represents a worst possible combination of errors. For small increments:

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + 2\frac{\Delta r}{r} + \frac{\Delta L}{L}$$

and

$$\Delta\rho = \rho\left[\frac{\Delta m}{m} + 2\frac{\Delta r}{r} + \frac{\Delta L}{L}\right]$$

Again if the errors have normal distribution, then

$$\frac{\Delta\rho}{\rho} = \sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(2\frac{\Delta r}{r}\right)^2 + \left(\frac{\Delta L}{L}\right)^2}$$

SIGNIFICANT FIGURES

Suppose you have measured the diameter of a circular disc and wish to compute its area $A = \pi d^2/4 = \pi r^2$. Let the average value of the diameter be 24.326 ± 0.003 mm ; dividing d by 2 to get r we obtain 12.163 ± 0.0015 mm with a relative error $\frac{\Delta r}{r}$ of $\frac{0.0015}{12} = 0.00012$. Squaring r (using a calculator) we have $r^2 = 147.938569$, with a relative error $2\Delta r/r = 0.00024$, or an absolute error in r^2 of $0.00024 \times 147.93\cdots = 0.036 \approx 0.04$.

Thus we can write $r^2 = 147.94 \pm 0.04$, any additional places in r^2 being unreliable. Hence for this example the first five figures are called *significant.*

Now in computing the area $A = \pi r^2$ how many digits of π must be used? A pocket calculator with $\pi = 3.141592654$ gives

$$A = \pi r^2 = \pi \times (147.94 \pm 0.04) = 464.77 \pm 0.11 \text{ mm}^2$$

Note that $\frac{\Delta A}{A} = 2\frac{\Delta r}{r} = 0.00024$. Note also that the same answer results from $\pi = 3.1416$, but that $\pi = 3.142$ gives $A = 464.83 \pm 0.11$ mm^2 which differs from the correct value by 0.06 mm^2, an amount comparable to the estimated uncertainty.

A good rule is to use one more digit in constants than is available in your measurements, and to save one more digit in computations than the number of significant figures in the data. When you use a calculator you usually get many more digits than you need. Therefore **at the end**, be sure to **round off** the final answer to display the correct number of **significant figures.**

SAMPLE QUESTIONS

1. How many significant figures are there in the following number?
   (a) 976.45
   (b) 4.000
   (c) 10
2. Round off each of the following numbers to three significant figures.
   (a) 4.455
   (b) 4.6675
   (c) 2.045
3. A function has the relationship $Z(A, B) = A + B^3$ where A and B are found to have uncertainties of $\pm\Delta A$ and $\pm\Delta B$ respectively. Find ΔZ in term of A, B and the respective uncertainties assuming the errors are uncorrelated.
4. What happens to σ, the standard deviation, as you make more and more measurements? what happens to $\overline{\sigma}$, the standard deviation of the mean?
   (a) They both remain same
   (b) They both decrease
   (c) σ increases and $\overline{\sigma}$ decreases
   (d) σ approachs a constant and $\overline{\sigma}$ decreases

Suggestions on Form for Lab Notebooks:

NUMBER AND TITLE (e.g. E1. ELECTROSTATICS)

Date performed:____________ Partner:__________________

Subdivisions: If appropriate, name and number each section as in the manual.

DATA:

Label numbers and give units. In a few words, state what quantities you measured. If appropriate, record the data in tabular form. Label the tables and give units.

CALCULATIONS:

State the equations used and present a sample calculation. (Inclusion of the arithmetic is not necessary.)

CONCLUSIONS:

If any important conclusions follow from the experiments, state them and show by a brief statement how they follow. Compare your results with accepted values if the experiment has involved the measurements of a physical constant.

Errors:

Some of your experiments will be qualitative while others will involve quantitative measurements of physical constants. Where it is appropriate, estimate the uncertainty of each measurement used in a calculation and compute the uncertainty of the result. Does your estimate of uncertainty indicate satisfactory agreement between your result and the accepted one (or between your several values if you have several)? Intelligent discussion is welcomed, but don't make this section a burden on you.

Mechanics:

MC-0 Notes on the Computers

MC-0a Printing

You will want to avoid printing two copies in rapid succession. Wait for your computer to finish "spooling" before sending your next print job, or you risk crashing your computer and thereby loosing all your data.

Mechanics:
MC-1 Errors & Motion

MC-1a Measurement and Error

OBJECTIVES:

The major objectives of this first, short (est. 1 hr) computer lab are to begin to develop an understanding of what it means to make an experimental measurement and provide a methodology for assessing random and systematic errors in this measurement process. In addition this lab will also give you a minimal framework in which to introduce you to the PASCO© (page 210) interface hardware and software.

THEORY:

By now you should have had numerous opportunities to become familiar with time and the concept of a time interval. The increment of one second will be used as an intuitive reference point. In this lab you will test your ability to internalize this one second time interval by making and recording a repetitive flicking motion with your finger. By flicking your finger back and forth you will move it though an infrared beam sensor (i.e. the PASCO photogate) and each full cycle (back and forth, approximately 2 seconds) will be simultaneously recorded, plotted and tabulated by the PASCO interface software.

Your goal in this experiment is to assess the size of systematic and random errors in your data set and learn a simple methodology for distinguishing between the two.
SYSTEMATIC ERRORS: These are errors which affect the *accuracy* of a measurement. Typically they are reproducible so that they always affect the data in the same way. For instance if a clock runs slowly you will make a time measurement which is less than the actual reading.
RANDOM ERRORS: These are errors which affect the *precision* of a measurement. A process itself may have a random component (as in radioactive decay) or the measurement technique may introduce noise that causes the readings to fluctuate. If many measurements are made, a statistical analysis will reduce the uncertainty from random errors by averaging.

APPARATUS:

⇒ Computer with monitor, keyboard and mouse.

⇒ A PASCO photogate and stand: This device emits a narrow infrared beam in the gap and occluding the beam prevents it from reaching a photodetector. When the beam is interrupted the red LED should become lit. (Plugged into DIGITAL CHANNEL #3.)

⇒ A PASCO Signal Interface (CI-700) monitors the photodetector output *vs* time and can be configured to tabulate, plot and analyze this data.

PROCEDURE:

To configure the experiment you should refer to Fig. 1 below. Adjust the photogate so that one member can easily and repetitively flick his/her finger through the gap. The phone-jack cable from the photogate should be plugged into the DIGITAL CHANNEL #3 socket. Ignore the other sensors which may already be plugged into other sockets. It is *important* that the PASCO interface is turned on before the computer. If not the computer will not recognize it and, therefore, it must be rebooted to properly communicate with the PASCO module.

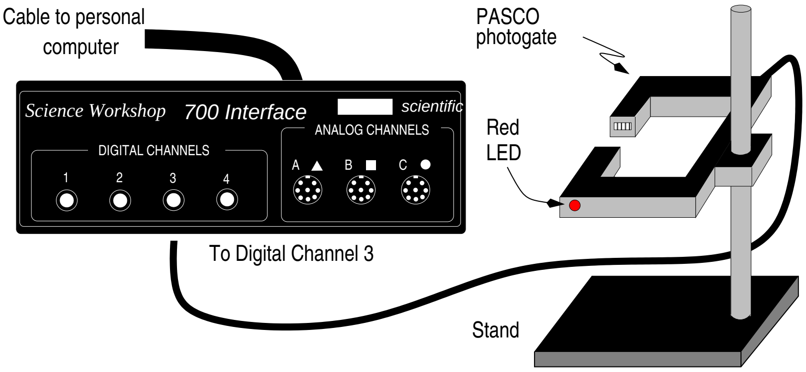


Figure 1: A schematic of the M0C components.

To initiate the PASCO (page 210) interface software you will need to click the computer mouse on the telescope icon in the "toolkit" area below. Fig. 2 below gives a good idea of how the display should appear. Note that, while you are able to reconfigure the display parameters, the default values that are specified on start-up will allow you to do most of this experiment without necessitating any major changes.

You will note that a "dummy" first data set already exists on start-up showing a typical data run. In the table you can view all 35 data points and the statistical analysis, including mean and standard deviation. In addition there should be a plot of this data and a histogram.

SUGGESTED PROCEDURE:

1. Start the preliminaries by CLICKing on the MONitor icon and practice "flicking" a finger back and forth so that a two second interval appears in the window. CLICK on the STOP icon when done. (The upper left window is configured like a tape player). The same person need not perform both operations. The purpose of the "monitor" function is to permit adjustments and trials without storing the results in memory.
2. Start the data run by CLICKing on the RECord icon and stop it by CLICKing on the STOP icon. Each run gets its own data set in the "Data" display window. (If there are any data sets in existence you will *not* be able to reconfigure the interface parameters or sensor inputs.)
3. With the REC option on, cycle a finger back and forth fifty times and STOP the data acquisition. DO NOT watch the time display while you do this, since you want to find out how accurately and precisely you can reproduce a time interval of 2 s using only your mind.
4. What is the mean time per cycle? What is the standard deviation? The mean $\bar{t}$ and standard deviation σ are given by:

$$\bar{t} = \sum_{i=1}^{N} t_i/N \quad \text{and} \quad \sigma = \sqrt{\sum_{i=1}^{N} (t_i - \bar{t})^2/[N-1]} \ .$$

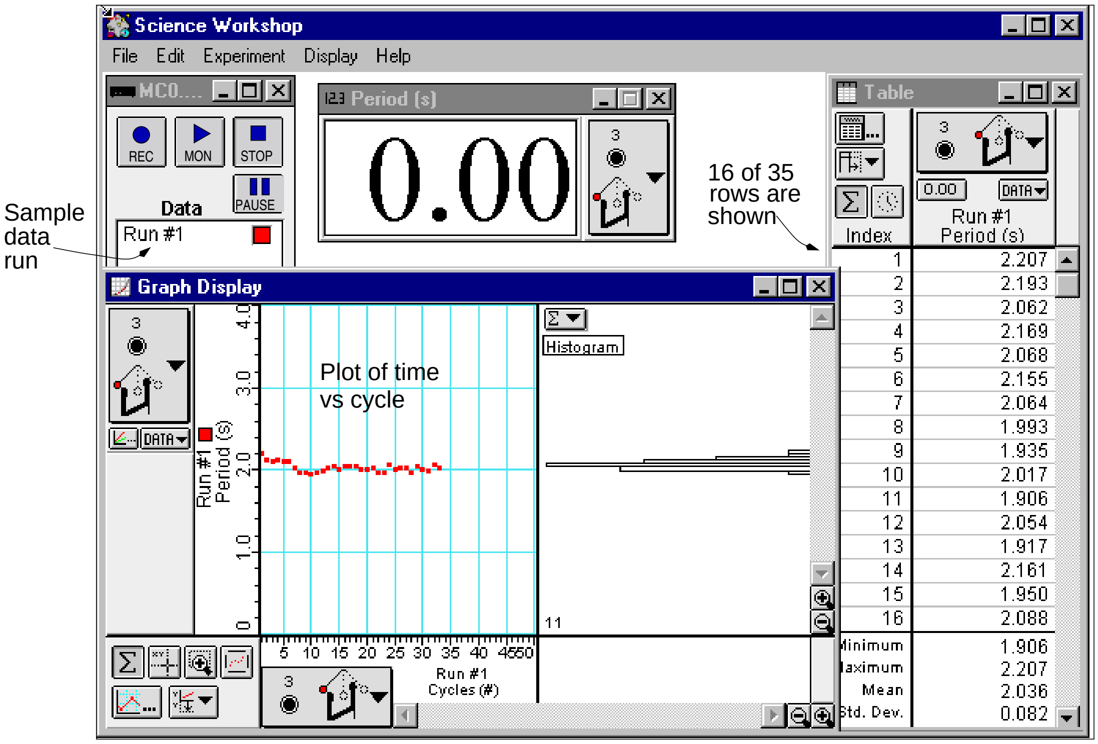


Figure 2: The PASCO SCIENCE WORKSHOP display window

Questions to consider:

I. Is your mean suggestive of a systematic error?

II. Does your data qualitatively give the appearance of a normal distribution (i.e. a Gaussian bell curve.)

5. For analyzing and quantifying random errors, you need to asses how a data set is distributed about the mean. The standard deviation σ is one common calculation that does this. In the case of a normal distribution approximately 68% of the data points fall within $\pm 1\sigma$ of the mean (90% within $\pm 2\sigma$). Is your data consistent with this attribute?

6. Assessing the possibility of systematic behavior is somewhat more subtle. In general σ is a measure of how much a *single* measurement fluctuates from the mean. In this run you have made fifty presumed identical measurements. A better estimate of how well you have really determined the mean is to calculate the *standard deviation of the mean* $\overline{\sigma} = \sigma/\sqrt{N}$. After recording $\overline{\sigma}$ in you lab book, can you now observe any evidence that there is a systemic error in your data? Answer this same question with respect to the first "sample" data set.

7. OPTIONAL: Systematic errors can sometimes drift over time. In the best-case scenario they drift up and down so that they hopefully average out to zero. (Clearly it would be better if they could be eliminated entirely.) With respect to $\overline{t}$ and $\overline{\sigma}$ for the first 25 and second 25 cycles do you observe any systematic trends? Use the "region of interest" feature of the statistical analysis software by using the mouse and highlighting (in black) a data subset (through a CLICK and drag motion) over the rows of interest.

MC-1b Errors and the Density of a Solid

OBJECTIVES:

To learn about systematic and random errors; to understand significant figures; to estimate the reliability of one's measurements; and to calculate the reliability of the final result.

NOTE: This experiment illustrates the earlier sections on Errors and Significant Figures. The actual density of the metal is incidental. However, the accuracy of your estimate of reliability will show whether you have mastered the material in the earlier sections.

APPARATUS:

Metal cylinders of varying sizes, micrometer and vernier calipers, precision gauge blocks, precision balance.

Precautions:

Avoid dropping or deforming in any other way the metal cylinders. Avoid damage to the precision screw of the micrometer by turning only the friction head to open or close the caliper jaws. Be sure to disengage the caliper lock before using. (The caliper lock lets you preserve a reading.) Improper weighing procedures may damage the precision balance. Consult your instructor if in doubt. In handling the gauge blocks avoid touching the polished surfaces since body acids are corrosive.

INTRODUCTION:

First read the material on Errors and Significant Figures in the Manual Introduction). Since density is the mass per unit volume, you must measure the mass (on a balance) and compute the volume ($h\pi r^2 = h\pi d^2/4$) from measurements of the cylinder's dimensions where d is the diameter and h is the height. Any one of three length measuring devices may be used. These include a micrometer, a vernier caliper and/or a simple metric ruler. The micrometer will permit the highest precision measurments but using one can be cumbersome, especially when reading the vernier scale. All methods will demonstrate the aforementioned objectives. Your instructor will give you guidance in choosing an appropriate measurement device.

SUGGESTIONS FOR THE MICROMETER:

Record the serial number of your micrometer. Then familiarize yourself with the operation of the caliper and the reading of the scales. If you have trouble with the principle and operation of the micrometer caliper, ask your instructor for help or refer to Appendix A.. Note that use of the "friction" head in closing the jaws insures the same pressure on the measured object each time. Always estimate tenths of the smallest division. Some micrometers have verniers to assist the estimation.

SUGGESTIONS FOR THE VERNIER CALIPER:

If you are not familiar with the principle of the vernier (refer to Appendix A.), invented 1631 by Pierre Vernier, ask the instructor to explain it or try the java vernier at http://webphysics.davidson.edu/. Experiment with one of the large verniers in the lab until you are sure you understand it. Note that verniers need not be decimal: for many inch scales the vernier estimates 1/8's of the 1/16 inch division, i.e. 1/128's of an inch. However vernier calipers divide the inch into 50 divisions and the vernier estimates 1/25 of the 1/50 inch divisions, i.e. 1/1000 inch or 1 mil.

Precautions on use of the calipers:

1. Unclamp **both** top thumbscrews to permit moving caliper jaws.
2. Open caliper to within a few mm of the dimension being measured.
3. Close right thumbscrew to lock position of lower horizontal knurled cylinder which executes fine motions of caliper jaw. **Never over tighten!**

CALIBRATION OF THE MICROMETER (or VERNIER, ETC.):

1. OPTIONAL: Wipe the micrometer caliper jaws with cleaning paper. Then determine the zero error by closing the jaws. Make and record five or six readings. The variation of these repeated readings gives you an estimate of the reproducibility of the measurements. (For those using the micrometers they have been given a small zero error. Thus a zero error correction is necessary.) In general any measurement device can have a zero error.
2. Measure one or four (OPTIONAL) calibration gauge blocks (6, 12, 18, and 24 mm): Set the gauge blocks on end, well-in from the edge of the table, and thus freeing both hands to handle the caliper. Record the actual (uncorrected) readings and take several readings (3 to 5) of each gauge block used. If measuring only a single block choose one with a thickness intermediate to the diameter and height of the metal cylinder.
3. If using a single precision block obtain a correction factor you will use to minimize systematic measurement errors. Otherwise, plot a correction curve for your micrometer, i.e. plot errors as ordinates and nominal blocks sizes (0, 6, 12, 18, and 24 mm) as abscissa. Normally the correction will not vary from block to block by more than 0.003 mm (for the micrometer). If it is larger, consult your instructor.

DENSITY DETERMINATION:

1. Make five measurements (should be in millimeters) of the height and five of the diameter. Since our object is to determine the volume of the cylinder, distribute your measurements so as to get an appropriate average length and average diameter. Avoid any small projections which would result in a misleading measurement. If not possible to avoid, estimate their importance to the result. Record actual readings and indicate how you distributed them.
2. Calculate the average length and average diameter reading, and then use your correction value (curve) to correct these average readings. If you were to use the uncorrected values, how much relative error would this introduce?
3. Calculate the *standard deviation* for your length and diameter averages.
4. Weigh the cylinder a few times on the electronic balance; estimate to 0.1 mg.
5. From the average dimensions and the mass, calculate the volume and density. Make a quantitative (refer to Section) estimate of the uncertainty in the density. You should use, as your starting point:

$$\text{Density}(\rho \pm \Delta\rho) = \rho(h \pm \Delta h, d \pm \Delta d, m \pm \Delta m) = \frac{m}{\pi (d/2)^2 \ h}$$

6. Compare the density with the tabulated value. Tabulated values are averages over samples whose densities vary slightly depending upon how the material was cast and worked; also on impurity concentrations.

7. In your notebook or lab form summarize the data and results. Also record your result on the blackboard. Is the distribution of blackboard values reasonable, i.e. "normal" distribution (refer to Section , pg. 9)?

   OPTIONAL: To test how accurately you can estimate a fraction of a division, estimate the fractions on the vernier caliper before reading the vernier. Record both your estimate and the vernier reading.

Related facts and URL links:

Question: Why are there are *exactly* 25.4 mm in 1 inch?

Answer(not verified): The Treaty of the Meter (Convention du Metre) in the late 19th century established the first centralized international system of metrology. This defined the meter.

In 1959, the countries of the world that were using Imperial units defined them uniformly based on the metric units. The inch was simply defined that way and agreed to by all. Before 1959, different countries related inches to meters in other ways. Among them was the United States. The Metric Act of 1866 defined the meter in terms of inches (i.e., before the Treaty of the Meter), and that relationship had continued to been used even after the Treaty fixed the length of the meter.

Changing the definition in the U.S. in 1959 caused very little problem, except for the U.S. Geological Survey. When you deal with things 10^5 meters big (like the sizes of the states), even 1 part in 10^5 changes affect the specifications of boundary lines in significant ways. So, to this day the U.S. has two different systems of inches, feet, and yards (in the ratio of 36:3:1, for both). There's the usual inch, foot, and yard; and there's the survey foot (and inch and yard), which is based on the pre-1959 definition. (The ambiguity goes away, of course, when metric specification is used. Newer USGS maps are metric.)

Links to metrology site(s):
National Institute of Science & Technology at http://www.nist.gov/

MC-1c Motion, Velocity and Acceleration

OBJECTIVES:

The major objectives of this exploratory computer lab are two-fold. Since you will be using computer based data-acquisition throughout this course, we expect you to become familiar with the PASCO© (page 210) interface hardware and software. Our second objective is for you to develop an intuition for Newtonian mechanics by experimenting with 1-D motion. There is no extensive write-up in this lab, but only a series of recommended experiments and the requirement to write down your observations in your lab book/form.

THEORY:

The motion of an object is described by indicating its distances x_1 and x_2 from a fixed reference point at two different times t_1 and t_2. From the change in position between these two times one calculates the average *velocity* (remember that direction is implied) for the time interval:

$$\text{average velocity} \equiv \overline{v} = \frac{x_2 - x_1}{t_2 - t_1} \quad = \quad \frac{\Delta x}{\Delta t} \; m/s$$

The *acceleration* of an object is found by finding its velocity v_1 and v_2 at two different times t_1 and t_2. From the change in velocity between two different times one calculates the average acceleration for the time interval:

$$\text{average accelation} \equiv \overline{a} = \frac{v_2 - v_1}{t_2 - t_1} \quad = \quad \frac{\Delta v}{\Delta t} \; m/s^2$$

FUNDAMENTAL CONCEPTS:

1. The equation that describes the motion of an object that moves with *constant velocity* is: $x = A + B \cdot t$.
   If you make a plot of x versus t, you find that it describes a straight line. The letter A indicates the position of the object at time $t = 0$. The letter B is the slope of the line, and is equal to the velocity of the object. So we can rewrite this $x = x_0 + v \cdot t$.

2. The equation that describes the motion of an object that moves with *constant acceleration* is: $x = x_0 + v_0 \cdot t + \frac{1}{2}at^2$ and $v = v_0 + at$.
   So we can rewrite this as: $x = A + Bt + Ct^2$ and $v = B + 2Ct$. The letter A indicates the position of the object at time $t = 0$. The letter B is the the velocity of the object at time $t = 0$, and is the slope of the graph at this time. The letter C is equal to half the acceleration.

PRECAUTIONS:

In order for the position sensor to work properly it must be pointed in such a way that it "sees" the vane, and doesn't identify the front of the cart; that means that it must be pointed slightly upwards. The sonic ranger tends to "see" the closest reflecting surface. In addition the minimum range is approximately 40 *cm*.
Make sure you do not drop the carts or allow them to roll off the table, because it damages the bearings and they begin to suffer too much friction. Try to arrange to keep the cart on the track all the time.

APPARATUS

- ⇒ Computer with monitor, keyboard and mouse.
- ⇒ A PASCO position sensor; this device emits a series of short pulses of sound, and receives the echo of the sound reflected by a nearby object. The length of the time interval between the emission and the reception of the sound pulse depends on the distance to the reflecting object.
  This method of locating an object is the same as the one used by bats to find flying insects or by navy ships to locate submarines.
- ⇒ A PASCO Signal Interface converts the time interval between the emission and reception of the sound pulse to digital form , i.e., numbers that can be then plotted on the monitor.
- ⇒ PASCO dynamic track with magnetic bumpers; cart with reflecting vane; meter stick; one or two steel blocks.

PROCEDURE:

Your instructor will demonstrate how to configure the experiment. To initiate the PASCO(page 210) interface software you will need to click the computer mouse on the telescope icon in the "toolkit" area below. The Fig. 1 below shows how the display should appear. Note that, while you are able to reconfigure the display parameters, the default values that are specified on start-up will allow you to do this experiment without necessitating any changes. All three measured quantities, position, velocity, and accelation, are displayed simultaneously. Since velocity is determined from the position data and acceleration from the velocity the "scatter" in the data will become progressively more pronounced.

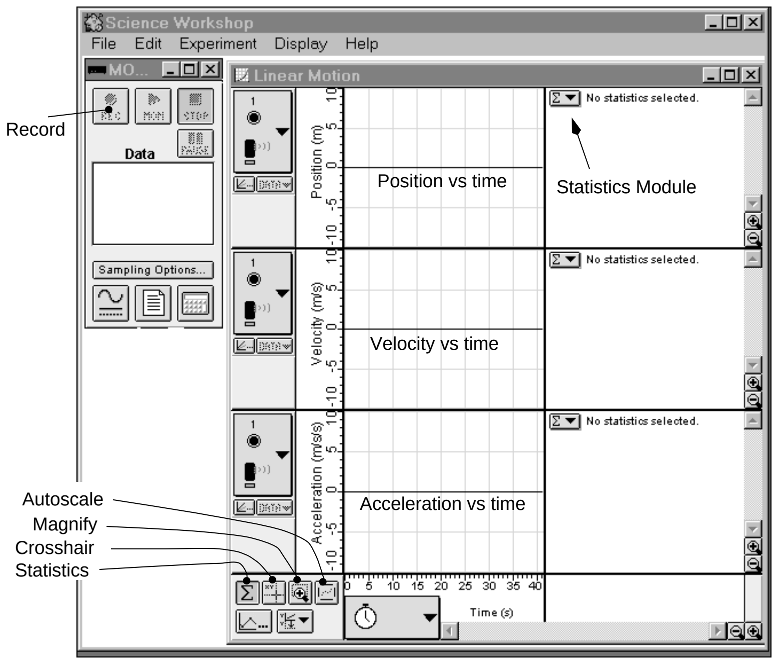


Figure 1: The PASCO scientic workshop display format

Experiment I, Basic Operation and Sonic ranger calibration:

1. To start the data acquisition CLICK on the REC icon. To stop it CLICK on the STOP icon (the upper left window is configured like a tape player). Each run gets its own data set in the "Data" display window. If there are any data sets in existence you will *not* be able to reconfigure the interface parameters or sensor inputs, unless you clear (delete) all data.

2. With the REC option on move the cart *to* and *fro* and watch the position, velocity and acceleration displays.

3. STOP the data acquisition.

4. CLICK once (or twice) on the AUTOSCALE icon. CLICK on the MAGNIFY icon and move to one of graph regions and CLICK again. CLICK on the CROSSHAIR icon and move about on the various graphs. Change the plot display region be manual adjusting the x-min, x-max, etc. values. To do this CLICK in the various scale (time, position,etc.) areas. CLICK on the STATISTICS icon once and then again. *Make* sure that all three members of the team have an opportunity to test these components. It is imperative that the basic operations on the software interface are understood by everyone!

5. DELETE the data set by a CLICK on the RUN #1 item in the "Data" window and then striking the "Delete" (<DEL>) key.

6. Start the data acquisition and observe the closest distance to the sonic ranger at which it still functions. This value is supposed to be close to 40 cm. If it is much larger, re-aim the position sensor.

7. Configure the distances so that when the cart nearly touches the near magnetic bumper the sonic ranger still records accurately.

8. Measure the position at two distances approximately 1 m apart and compare the printed centimeter scale with the position sensor readout. By how much do the readings differ?

Experiment II, Inclined Plane and Motion:

1. Raise the side of the track furthest from the position sensor using one of the supplied blocks. Hang the 500 g weight off the other side to weigh the front end down and prevent unwanted slippage of the track (See the demonstration set-up to see how the weight should be positioned.)

2. Find an appropriate release point that allows the cart to roll down the track *without* striking the magnetic bumper.

3. CLICK the REC button and release the cart letting it bounce three or four times and the CLICK the STOP button.

4. Qualitatively describe the shape of the three curves: position, velocity and accelation and discuss how they evolve with time.

5. Obtain a hard copy of this data by simultaneously pressing the **Alt-p** keys.(Only one copy is made even if you ask for more.) Repeat this step to get additional copies.
6. Label/identify the various key features in the various curves by writing directly on the hard copies. Paste the graphs in your note bood.

QUESTIONS: (to be discussed as a group)

1. Does the velocity increase or decrease linearly with time when it is sliding up or down the track?
2. When the velocity is close to zero can you observe any discrepancies in the data? Can you think of a reason for any deviations from linearity?
3. Why does the maximum of the position readout fall with each subsequent bounce?
4. Is the collision with the magnetic bumper or the residual friction in the bearing the most likely source of loss?
5. Can you think of a method using your data to determine which of these two proposed mechanisms is the most likely culprit?

Experiment III, Reproducing Expt. II manually:

1. Remove the block so that the cart is no longer raised.
2. Start a new data set (by clicking the REC button) and try to move the cart back and forth at constant velocity using your hand from the side. Alternatively have one team member aim the position sensor at another member holding a book waist-high and walk towards the sensor or away from the sensor.
3. Using the cross hair estimate your velocity in either m/s or cm/s.
4. Repeat step 3. but now try to obtain a region of constant accelation.

Experiment IV, Accelation at g (9.8 m/s^2):

1. Have one member of the group stand *carefully* on a chair and hold the position sensor facing downward above another member's head while he or she is holding a notebook on their head.
2. Start a new data set and have the student holding the notebook jump up and down a few times.
3. Stop the recording and determine whether free-fall yields a constant accelation close to the accepted value.
4. You may wish to try the statistical analysis module to measure your acceleration. Ask you lab instructor for a demonstration.
5. Fini.

M-2 Equilibrium of Forces

OBJECTIVE: To experimentally verify vector addition of forces.

APPARATUS:

Circular horizontal force table with 360° scale, pulleys, weight hangers, slotted masses, protractor, rulers.

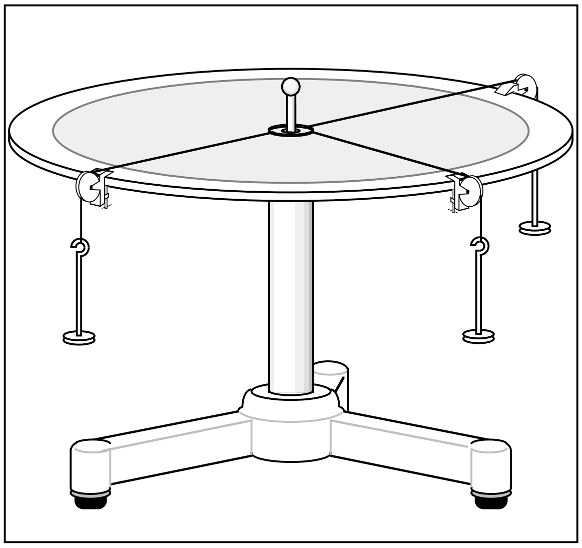


Figure 1: The horizontal force table

PROCEDURE:

Choose two concurrent forces, each between 100 and 500 gram weight and not in the same direction. Add these two vectors to find the resultant force: use both a graphical construction and a numerical computation.

Check your result by setting up the two original forces on the force table, and finding experimentally the single force which will hold them in equilibrium. Random errors are due mainly to friction. You can estimate them by repeating the measurement several times. Are your measured values consistent with the computation and graphical construction?

Repeat the experiment but use three different original forces.

QUESTION: If the weight hangers used all have identical masses, can their weight be neglected?

M-3 Static Forces and Moments

OBJECTIVE: To check experimentally the conditions for equilibrium of a rigid body.

APPARATUS:

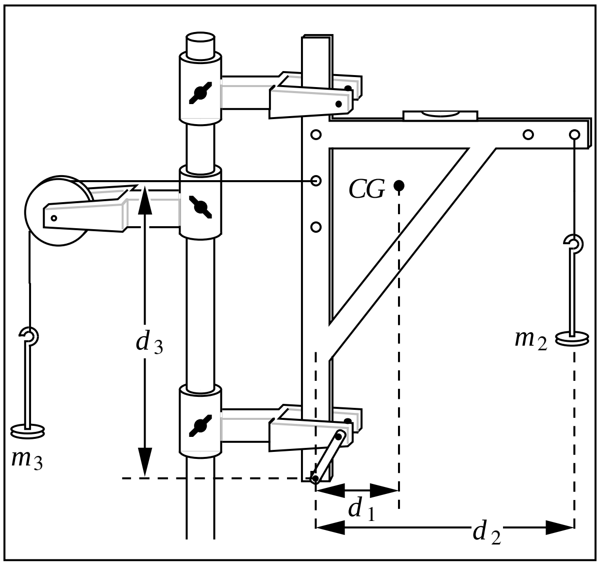
Model of rigid derrick, slotted masses and weight hangers, knife edge (mounted in wall bracket), vernier caliper, single pan balance.

INTRODUCTION:

Equilibrium requires that the net linear acceleration and the net angular acceleration be zero. Hence $\sum \vec{F}_{\text{ext}} = 0$ and $\sum \vec{\tau}_{\text{ext}} = 0$. We treat the rigid derrick as a two dimensional structure so the vector equations become: $\sum \vec{F}_x = 0$, $\sum \vec{F}_y = 0$ and $\sum \vec{\tau} = 0$. The choice of the perpendicular axis about which one computes torques is arbitrary so in part 4 below we choose an axis which simplifies calculations.

SUGGESTED PROCEDURES:

1. Place a load of about 2 kg for m_2.
2. Determine experimentally the force m_3 to hold the derrick in equilibrium with the top member level. Since this is an unstable equilibrium, adjust m_3 so that the derrick will fall either way when displaced equally from equilibrium. To find the uncertainty in m_3 increase or decrease m_3 until you know the smallest force m_3 which is definitely too large and the largest which is definitely too small.



3. Weigh the derrick (use single pan balance) and find the horizontal distance between the center of the stirrup and the vertical line through the center of gravity (c.g.) of the derrick. (Use the knife edge mounted in a wall bracket for locating the c.g.)
4. Calculate how accurately the $\sum \vec{\tau}_{\text{ext}} = 0$ condition is satisfied about the point where the lower stirrup supports the derrick. Note: distance from rotation axis must include stirrup axle radius (use vernier caliper).
5. Calculate the force $\vec{F}$ exerted by the stirrup on the derrick.
6. Choose an axis that is *not* on the line of action of any force and calculate how closely is satisfied about that axis. Is the discrepancy reasonable? Make your answer as quantitative (refer to Section) as you can. (Include the uncertainty in $\vec{F}$ and any other measurements).

M-4 Acceleration in Free Fall

OBJECTIVE: To measure "g", the acceleration of gravity.

APPARATUS:

Free fall equipment: cylindrical bobs (identical except in mass) which attach to paper tape for recording spark positions; spark timer giving sparks every 1/60 s; cushion; non-streamline bobs to study air resistance (by adding a front plate).

INTRODUCTION:

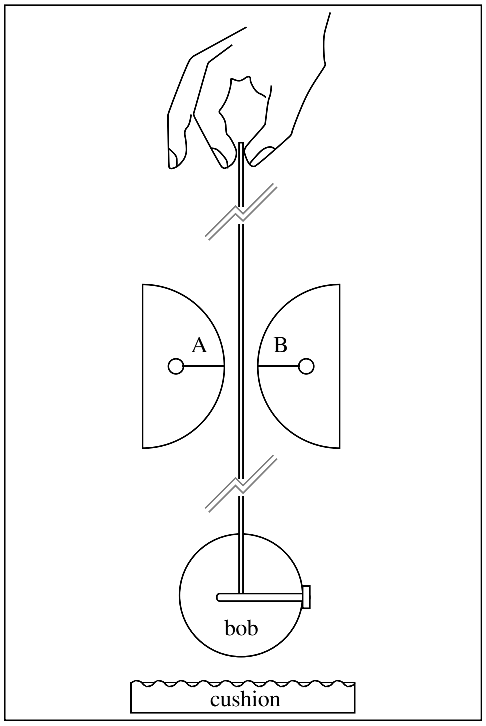
As shown in figure below the spark timer causes sparks to jump from sharp point A (flush with the convex surface of the plastic insulator.) through the falling vertical paper tape to the opposite sharp point B flush with the other insulator surface. As the bob (plus paper tape) undergoes free fall, sparks from A to B mark the paper tape's position every 1/60 *s*. These data give the bob's acceleration in free fall, g, if air resistance is negligible. We'll think about air resistance later.

EXPERIMENTAL SUGGESTIONS:

1. Position spark timer chassis near table edge and with the convex surfaces (A and B) extended beyond table edge. Put cushion on floor directly under A and B.

2. Select the heaviest bob (no front plate). Insert one end of about a meter of paper tape between the two halves of the cylindrical bob and fasten together with thumb screw.

3. Insert paper tape between A and B. Hold the tape end high enough (vertically) above A and B that the bob just touches below A and B (and is centered). Start the spark and immediately release tape plus bob. Discard any part of tape which fell through the spark gap after bob hit the cushion.



4. Fasten sparked tape to table top with masking tape. Place a meter stick on its side (on top of tape) so that ends of the mm graduations touch the dot track on the tape; this avoids parallax error (Appendix 4.)

5. **Ignore** the first spark dot; then mark and measure the position of **every other** of the first 24 dots, thus using 1/30 s as the time interval instead of 1/60 s.
Estimate the dot positions to 0.1 mm, and assume this is your uncertainty, $\delta_r = 0.1\,$**mm. Don't move meter stick between readings!** Tabulate as in sample below.

6. Check the measurement set by remeasuring the tape **after moving the meter stick so that the recorded dot positions will be different**. The differences should be the same. How close are they?

COMPUTER GENERATED TABULAR FORM FOR DATA:

Spark interval	Real time	Position of every 2nd spark (dot)	Average velocity	Average acceleration
i	$t(i)$	$\overline{r}(i) \pm \Delta r$	$\overline{v}(i) \pm \Delta v$	$\overline{a}(i) \pm \Delta a$
Units	sec	mm or cm	...	...
Uncertainty	$\delta_t =$	$\delta_r =$	$\delta_v =$	$\delta_a =$
0	$t(0)$	a	—	—
1	$t(1)$	b	$b-a$	$(c-b)-(b-a)$
2	$t(2)$	c	$c-b$	$(d-c)-(c-b)$
3	$t(3)$	d	$d-c$	$(e-d)-(d-c)$
4	$t(4)$	e	$e-d$	$(f-e)-(e-d)$
5	$t(5)$	f	$f-e$	$(g-f)-(f-e)$
6	$t(6)$	g	$g-f$	$(h-g)-(g-f)$
7	$t(7)$	h	$h-g$	$(i-h)-(h-g)$
8	$t(8)$	i	$i-h$	$(j-i)-(i-h)$
9	$t(9)$	j	$j-i$	$(k-j)-(j-i)$
10	$t(10)$	k	$k-j$	$(l-k)-(k-j)$
11	$t(11)$	l	$l-k$	$(m-l)-(l-k)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

SUGGESTIONS ON HANDLING DATA:

1. Let $t = 0$ be where your readings start. Tabulate the actual readings, r, on the meter stick at the end of each time interval (as in column 3 above); calculate the average velocity, $\overline{v}$ in each interval; then calculate the average acceleration, $\overline{a}$ during each interval. If you do the table by hand it may be easier to compute everything in per time interval units and scale your answer when everything is done.

2. Find the **average** of the **average** $\overline{a}$ in each interval and convert (if necessary). This average of the average $\overline{a}$, $\langle\overline{a}\rangle$" is much more accurate than fluctuations in the $\overline{a}$ column might indicate. Explain why. (Hint: Are the values independent? See last part, Suggestion 4) below.

3. With 1/30 s as the time interval, plot "r" vs time, $\overline{v}$ vs time, and $\overline{a}$ vs time. Note that for $\overline{v}$ the value is the average of v between to adjacent rows, i and $i+1$, and therefore equals the **instantaneous** v at the *middle* of the interval.

4. Use the slope of the velocity curve to find acceleration. Your can do this by hand but graphical analysis tools should be available. This approach is a better way of using all the data than the above numerical one since averaging the $\overline{a}$ column involves summing the $\overline{a}$ column and in such a sum all readings except $(b-a)$ and $(m-l)$ drop out!

ERROR ANALYSIS:

If readings a, b, c, etc., are good to $\approx \frac{1}{2}$ mm because of irregularity in the spark path, the *worst case* would be for $(b - a)$ to be in error by 1 mm. Since these two measurements are to first order, uncorrelated, the errors should add in quadrature [see section on Errors (refer to Section), pg. 9]. Using every $v(i)$ for your slope determination is problematic because each $v(i)$ and $v(i+1)$ is correlated. This means that if $v(1) \propto b - a$ then $v(2) \propto c - b$, so that a fluctuation in b will always push the adjacent values of v apart. Thus a linear-fit of slope suffers from the same shortcoming as the average of the average acceleration calculation noted in Suggestion 4! Calculate the slope using only every other $v(i)$ because $v(1)$, $v(3)$, $v(5)$ etc. use independent data. Does your new value for g vary much? Estimate the uncertainty in the slope. Spark timing errors are negligible. Air resistance is a systematic error, small for the streamline bobs at low velocities, but see the optional experiment below.

EQUIVALENCE OF GRAVITATIONAL AND INERTIAL MASS:

Galileo showed (crudely) that the acceleration of falling bodies was independent of the mass. Use the light plastic bob (identical in size and shape to the massive bob) to repeat the free fall experiment and thus check quantitatively this equivalence (when air resistance effects are small enough). If you are planning to do the optional experiment which follows, you should *skip* this part.

LOCAL VALUE OF g: The UW Geophysics Department determined g accurately for room 4300 Sterling Hall. A plaque on the northwest window sill gives $g = 9.803636 + 0.000001$ m/s^2.

QUESTION: Do your value(s) of “ g ” agree within your assigned errors?

OPTIONAL EXPERIMENTS

A. EFFECT OF AIR RESISTANCE: While air effects are small for streamlined objects at low velocities, they can become large, e.g. on a parachute. To observe and correct for them, first make the bobs **non-streamline** by inserting into the bottom of the bob the banana plug holding a small flat plate. Then repeat the experiment for the **non-streamline bobs (front plate attached), identical except for mass.** There are four possible mass combinations. Since the force of air resistance, $f(v)$, is a function of only velocity if the size, shape and roughness are the same, then $f(v)$ on the two bobs will be almost the same since their velocities are similar. The net force on the falling body is then $F = mg - f(v)$. Hence

$$a = \frac{F}{m} = \frac{mg - f(v)}{m} = g - \frac{f(v)}{m}$$

and

$$\langle a \rangle \approx g - \left(\frac{1}{m}\right) \langle f(v) \rangle \quad .$$

Thus if we measure $\langle a \rangle$ for bobs identical except in mass and plot $\langle a \rangle$ against $1/m$, we should obtain a straight line whose extrapolation to $(1/m) = 0$ should give g. To further test the validity of this hypothesis you should plot the same data as recorded by other lab groups on your plot. If the data permits a simple linear fit you should be able extrapolate to infinite mass (i.e. the y-intercept).

B. MEASUREMENT OF REACTION TIME BY FREE FALL:

(1) With thumb and forefinger grasp a vertical meter stick at the 50 cm mark. Release and grab it again as quickly as possible. From the distance through which the 50 cm mark fell, calculate the time of free fall of the meter stick. This time is your total reaction time involved in releasing and grasping again.

(2) Have your partner hold the vertical meter stick while you place your thumb and forefinger opposite the 50 cm mark but not grasping it. When your partner releases the stick, grab it as soon as possible. Again from the distance through which the 50 cm mark fell, calculate the time of free fall of the meter stick. Compare this time with the other method. Why may these reaction times be different?

OPTIONAL QUESTIONS:

1. Estimate the effect on your g value of the air's buoyant force, F_a for air density, $\rho = 1.2$ kg/m^3 and brass bob density, $\rho_{\text{bob}} = 8700$ kg/m^3.
   Hint: $F_a = \rho_a m_{\text{bob}} g/\rho_{\text{bob}}$ (why?), and $a = g[1 - (\rho_a/\rho_{\text{bob}})]$ (why?).

2. According to universal gravitation, the moon also accelerates the bob with a value a_m of

   $a_m = \frac{GM_m}{r^2} = \frac{6.67\times10^{-11}(7.34\times10^{22})}{(3.84\times10^8)^2} = 0.000033\ m/s^2$

   where G is the constant of universal gravitation, M_m is the mass of the moon, and r is the distance between the moon and the bob. Since this a_m is 33 times the uncertainty quoted for the local g value, why doesn't the plaque also indicate the position of the moon at the time of measurement?

   Hint: Remember the acceleration g in an orbiting earth satellite provides the centripetal acceleration for the circular motion but does not appear as "weight" of an object in the satellite. While to first order the moon and sun effects are negligible, there are detectable tidal effects in the earth ($\approx 10^{-7}\ g$) which one corrects for in the absolute measurements. See Handbuch der Physics, Vol XLVIII, p. 811; also Wollard and Rose, "International Gravity Measurements", UW Geophysical and Polar Research Center, (1963) p. 183. On pages 211 and 236, they also describe the accurate absolute g determination with two quartz pendula in room 70 Science Hall from which the plaque value in Room 4300 Sterling derives.

M-5 Projectile Motion

OBJECTIVE: To find the initial velocity and predict the range of a projectile.
APPARATUS:

Ballistic pendulum with spring gun and plumb bob, projectile, single pan balance, elevation stand.

PART I. BALLISTIC PENDULUM INTRODUCTION:

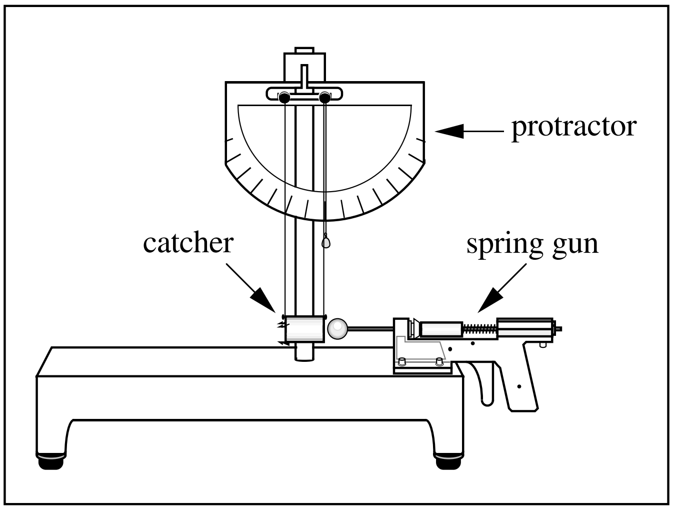


Figure 1: The spring gun.

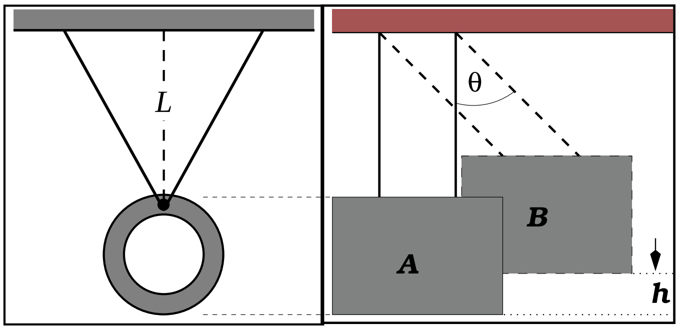


Figure 2: A side view of the catcher

A properly aligned spring gun shoots a ball of mass m into a pendulum catcher of mass M. See Figs. 1 and 2. The pendulum traps the ball; thereafter the two move together in **pure translation**. Since ball and pendulum have no relative motion, the collision is inelastic and thus does not conserve mechanical energy (where does it go?). Of course linear momentum is still conserved, and hence momentum of ball before impact, mu, equals the momentum of ball plus pendulum after impact, $(m + M)V$:

$$mu = (m + M)V \tag{1}$$

where u = ball's velocity before impact and V = initial velocity of combined pendulum plus ball.

To find V, note that although the impact doesn't conserve mechanical energy, the motion **after** impact is almost frictionless and thus conserves mechanical energy. Hence the kinetic energy of the ball plus pendulum at A in Fig. 2, just after impact, equals the potential energy of the two at the top of the swing (at B). Thus

$$\frac{1}{2}(M+m)V^2 = (M+m)gh \ . \qquad \text{Hence,} \qquad V = \sqrt{2gh} \ . \tag{2}$$

ALIGNMENT:

If properly aligned, our bifilar type of suspension for the pendulum (see Figs. 1 and 2) prevents rotation of the bob of mass M. Hence the motion is pure translation. To ensure proper alignment, adjust the three knurled screws on the base so that

A. A plumb bob hangs parallel to the vertical axis of the protractor, and
B. The uncocked gun axis points along the axis of the cylindrical bob. (You may need to adjust the lengths of the supporting strings.)

SUGGESTED PROCEDURE:

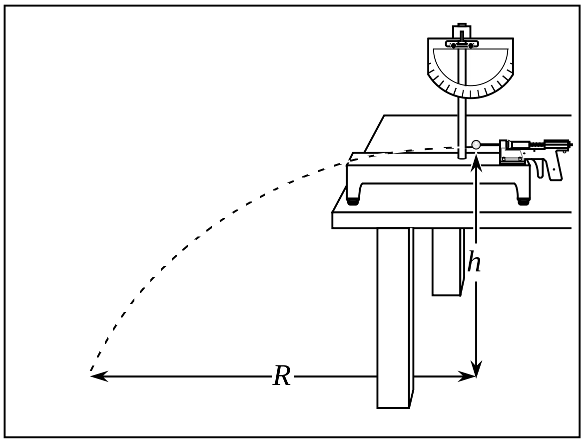
> *PRECAUTIONS:* Some spring guns have three different cocking stops. For consistent results be sure to recock the gun to the same stop.

1. Find m and M by weighing on the single pan balance.
2. Find the height h by measuring angle θ and the length L of the pendulum. *NOTE:* $h = L - L\cos\theta$ and L is not the length of the string, (see Fig. 2).

   Suggestions on finding θ: Find an approximate θ, and then make a masking tape slit on the back of the protractor at this approximate θ so your eye can locate the proper viewing area to find a more accurate θ on subsequent firings. To avoid parallax (see Appendix 4) in reading the protractor choose a line of sight determined by the string in front of the protractor and the string in back of the protractor. Careful observation after a little practice will enable you to get θ to within a degree.
3. Calculate the initial velocity V of the combined pendulum bob and ball.
4. Calculate the initial velocity of the ball, u, as it leaves the gun.
5. Estimate the uncertainty in u. [Hint: Since the largest uncertainty is likely ΔV, then Δh is important. While h is a function of the measured L and θ, the uncertainty in the angle measurement, $\Delta\theta$, will probably dominate. To manually calculate the uncertainty in the resulting trigonometric function, sometimes it is simpler to calculate separately the function for $\theta+\Delta\theta$ and for $\theta-\Delta\theta$. Otherwise the smart form can do this for you. Remember to use absolute, not relative errors when propagating errors through addition (See "Errors and Uncertainties" in the manual.)

PART II. RANGE MEASUREMENTS HORIZONTAL SHOT:

1. After finding u, (the velocity of the ball leaving the gun) predict the impact point on the floor for the ball when shot horizontally from a position on the table.
2. To check your prediction experimentally:



(i) Use the plumb bob to check that the initial velocity is horizontal.
(ii) Measure all distances from where the ball starts free fall (not from the cocked position). All measurements of course refer to the bottom of the ball so $x = 0$ corresponds to the radius of the ball beyond the end of the gun rod. Check that the gun's recoil does not change x.
(iii) Fasten (with masking tape) a piece of computer paper at the calculated point of impact, and just beyond the paper place a box to catch the ball on the first bounce.
(iv) Record results of several shots. (The ball's impact on the paper leaves a visible mark.) Estimate the uncertainty in the observed range.
(v) Is the observed range (including uncertainty) within that predicted.

3. Work backwards from the observed range to calculate the initial velocity u. This u is probably more accurate than the value obtained with the ballistic pendulum.

ELEVATED SHOT:

1. Use the stand provided to elevate the gun at an angle above the horizontal. The plumb bob will give the angle of elevation, e.g. 90° - protractor reading. For the elevated gun, be sure to include the additional initial height above the floor of the **uncocked** ball.
2. Before actually trying a shot at an angle, again predict the range but use the value of u which you found from the horizontal shot. (See item 3 above).
3. Make several shots, record the results and compare with predictions.

QUESTION:

From the measured values of u and V in Part I of this experiment, calculate the kinetic energy of the ball before impact, $\frac{1}{2}mu^2$ and the ball and pendulum together after impact, $\frac{1}{2}(m+M)V^2$. What became of the difference?

OPTIONAL:

1. Show that for momentum to be conserved: $\dfrac{KE_{\text{before impact}}}{KE_{\text{after impact}}} = \dfrac{m+M}{m}$
2. Find the spring constant k of the gun from $\frac{1}{2}mu^2 = \frac{1}{2}kx^2$.

M-6 Uniform Circular Motion

OBJECTIVE: To measure the centripetal force, F_c, and compare to

$$F_c = \frac{mv^2}{r} = m\omega^2 r$$

APPARATUS:

Fig. 1 is a schematic of the equipment. The bobs and springs are removable for weighing. Not shown are table clamp and pulley, slotted masses and weight hanger.

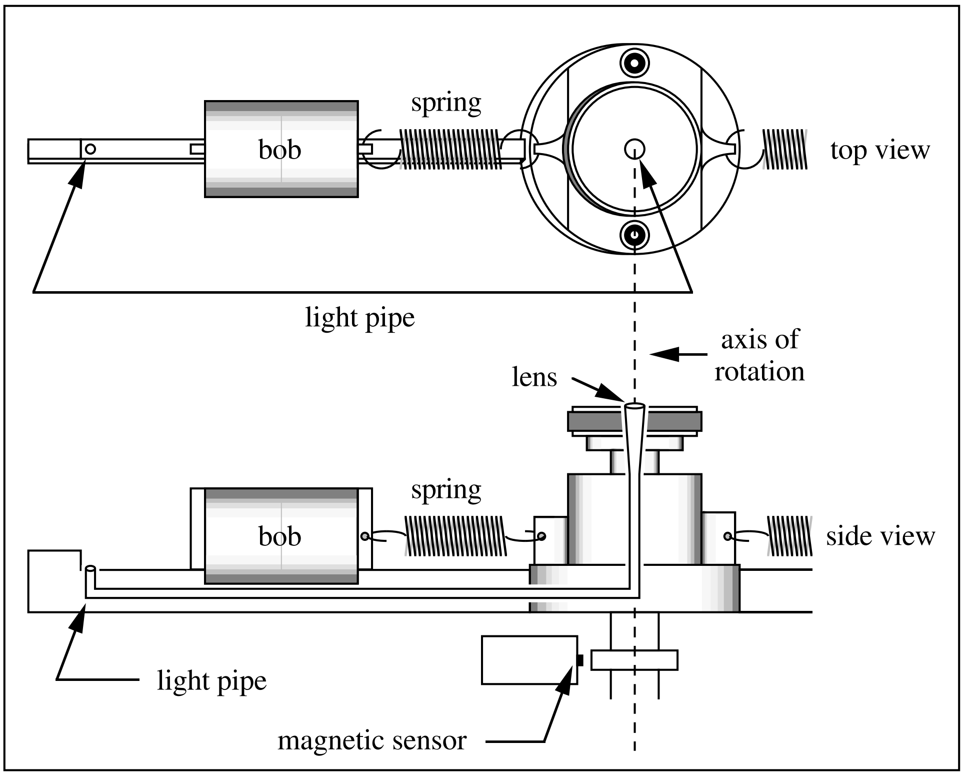


Figure 1: The UCM apparatus.

INTRODUCTION:

A variable speed motor drives the rotating system which has two slotted bobs which slide on a low-friction bar. One adjusts the speed until one bob just covers the optical light pipe and thus reduces the signal seen at the center of the rotating system to zero. A revolutions counter is on the shaft. The counter operates by sensing the rotating magnetic poles and electronically reads out directly the frequency of revolution in rpm. A spring (plus any friction) supplies the centripetal force required to keep the bob traveling in a circle.

If you measure first the frequency of rotation required to make the bob just cover the optical light pipe, and if you then measure the force required to pull the bob out the same distance when the system is not rotating, you can determine F_c and compare your result to

$$F = m\omega^2 r\,,$$

where r is the distance from the axis of rotation to the center of mass of the bob.

SUGGESTIONS:

1. Find the mass of the nickel plated brass bob; also the aluminum bob.
2. **Dynamic measurement of the force**: Attach the brass bob to the spring. Replace the lucite cover, and adjust the motor speed until the light from the light pipe at the center of the rotating system goes to zero. Record the rotation frequency.

   To correct for frictional effects of the bob on the bar, record the frequencies both as the speed is slowly *increased* to the correct value and and as the speed is *decreased* from too high a value. Since the direction of the frictional force reverses for the two cases, the average should eliminate the frictional effect.

   Repeat several times so you can estimate the average and the standard deviation of your values.
3. **Static measurement of the force**: Use the string, pulley and weight holder plus slotted weights to measure the force required to stretch the spring so that the optical light pipe is again just covered. Devise a way to avoid error caused by the friction at the pulley and of the sliding bob on the bar.
4. While the spring is stretched to its proper length (item 3 above), measure the distance r from the axis of rotation to the center of mass of the bob. The center of mass is marked on the bob.

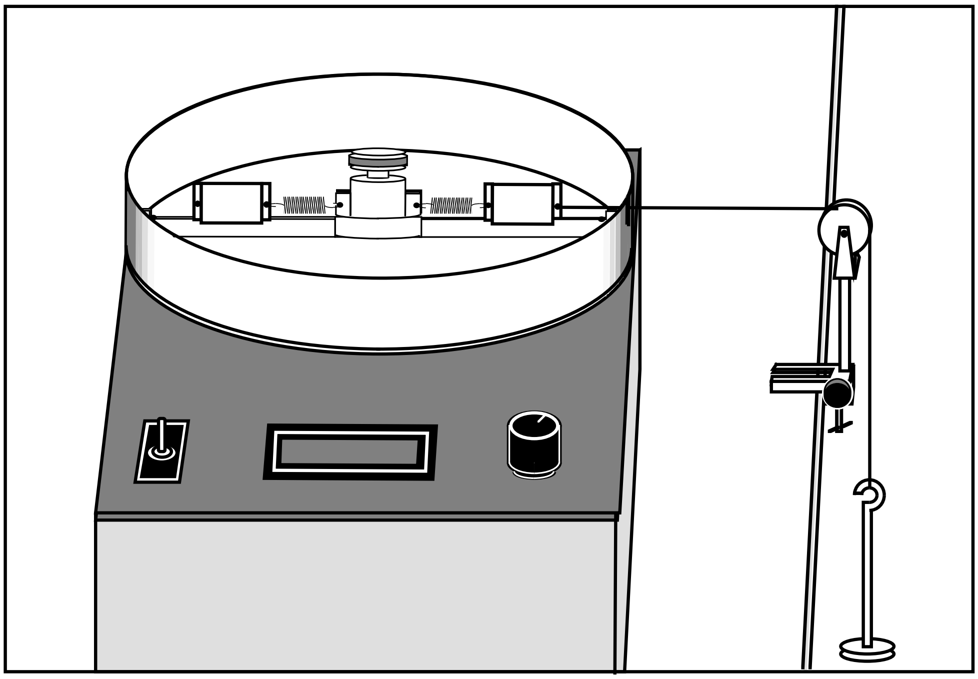


Figure 2: Static measurement of the force using hanging weights

5. Compare the measured centripetal force to the computation $F_c = m\omega^2 r$. In computing the centripetal force, also take into consideration the mass of the spring. One can show (Weinstock, *American Journal of Physics*, 32,p. 370, 1964) that $\approx (1/3)$ of the spring mass should be added to the mass of the bob to obtain the total effective mass.

6. Repeat the above item 1 through item 5 but for the aluminum bob.

QUESTIONS:

1. Estimate the reliability of your measurements. How well do the measured and computed forces agree? Try to account for any discrepancy.

MC-7 Simple Pendulum

OBJECTIVES:

1. To measure how the period of a simple pendulum depends on amplitude.
2. To measure how the pendulum period depends on length if the amplitude is small enough that the variation with amplitude is negligible.
3. To measure the acceleration of gravity.

VIRTUAL PRE-LAB EXPERIMENT:

1. For students wishing to try this experiment on-line, there is a simulation of a pendulum included in the web version of the lab manual...click on the Launch Virtual Pendulum button.
2. Start the pendulum swinging and then let it swing for about 10 periods. Estimate the mean and standard deviation of a single measurement.
3. Perform the required investigations as below except use the virtual pendulum.

APPARATUS:

Basic equipment: Pendulum ball with bifilar support so ball swings in a plane parallel to wall, protractor, infrared photogate and mount, single pan balance.

Computer equipment: Personal computer set to the MC-7 lab manual web-page; PASCO interface module; photogate sensor and extension jack.

NOTE: The period is the time for a complete swing of the pendulum. For the most sensitivity the start and stop of the photogate timer should occur at the bottom of the swing where the velocity is maximum.

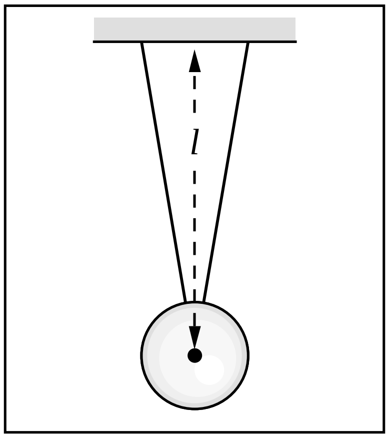


Fig. 1: Side view of pendulum ball and bifilar support. NOTE: L and l are equivalent.

SUGGESTED EXPERIMENTAL TECHNIQUE:

1. Adjust the infrared gate height so that the bob interrupts the beam at the bottom of the swing. (Make sure the PASCO interface is on and that the phone jack is plugged into the first slot.)
2. To initiate the PASCO interface software click the computer mouse on the telescope icon in the "toolkit" area. There will be just a single table for recording the measured period.
3. Start the pendulum swinging and then start the data acquisition by clicking the REC button button. Let the bob swing for about 10 periods. Calculate the mean and standard deviation by simply clicking on the table statistics icon (i.e. $\boxed{\Sigma}$). For a single measurement of the period the standard deviation is a reasonable measure of the uncertainty. With 10 measurements the uncertainty of the mean is $\sigma/\sqrt{10}$. (Refer to the section in this manual on errors.)

REQUIRED INVESTIGATIONS: (Error analysis required only for item 3)

1. Period *vs* Amplitude: For a pendulum of convenient length L (about 0.5 m) determine the dependence of period on angular amplitude. (Do **not** cut string to decrease L; there are clips on the strings to adjust the length.) Use several amplitudes between 5 and 50 degrees. Measuring the angle is a bit hard; to avoid parallax effects, position your eye so the two strings are aligned with each other and read the protractor. [Of course, the amplitude of the swing will decrease slowly because of friction. Keep the number of swings that you time small enough that the amplitude changes (because of friction) by less than 5 degrees during the timing. This is especially important for large amplitudes.] For each group of swings timed, record the average angular amplitude.

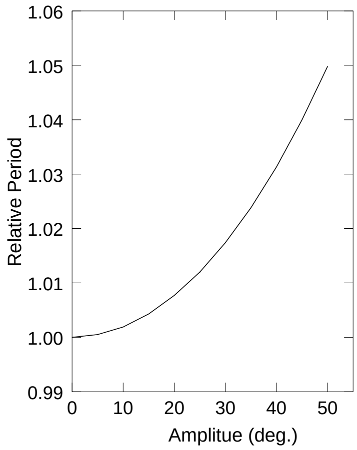
   Plot the measured period as a function of angular amplitude including a few error bars.

   The accurate formula for period as a function of amplitude θ is:

$$T = T_0 \left(1 + \frac{1}{2^2}\sin^2\frac{\theta}{2} + \frac{1(3^2)}{2^2(4^2)}\sin^4\frac{\theta}{2} + \cdots\right)$$

   where $T_0 = 2\pi\sqrt{(L/g)}$ and θ is the angular amplitude. The results follow:

θ(deg)	T/T_0	% Change
5	1.0005	0.05
10	1.0019	0.19
15	1.0043	0.43
20	1.0077	0.77
25	1.0120	1.20
30	1.0174	1.74
35	1.0238	2.38
40	1.0313	3.13
45	1.0400	4.00
50	1.0498	4.98



   Compare your plot (above) with values predicted from this table.

2. *Period vs. Length:* For an amplitude small enough that the period is almost independent of amplitude, determine the variation of period with length. (The length of the pendulum is the vertical distance from the support to the center of the bob.) Try four or more lengths from 0.20 m to 1.0 m.
   Note: Change lengths by using the two spring-loaded clamps above protractor.

   I. Plot period (T) versus length (L) and extend the curve to $L = 0$. Can you tell at a glance how T_0 depends on L?

   II. Plot T_0^2 vs L. What is the shape of this curve? What can you tell from this curve about the dependence of T_0 on L?

3. *Measurement of* **g**: With a pendulum about 1.0 m long, make a measurement of **g**, the acceleration of gravity. (See your text for proof that a simple pendulum swinging through a **small** angle has $T = 2\pi\sqrt{(L/g)}$ where T is the period, L the length and g

is the acceleration of gravity.) Take enough measurements to estimate the reliability of your period determination.

4. *Calculate the uncertainty in your determination of g.*
Note that $\Delta g = g\sqrt{(\Delta L/L)^2 + (2\Delta T/T)^2}$. (Take a look at the "Errors and Uncertainties" section of this manual.) Is the accepted value within the limits you have set? If not, can you explain it? (Consider optional items below. The string lineal density is 0.5 g/m, and $M_{\text{bob}} = 62$ g.)

OPTIONAL EXERCISES:

1. Show that the buoyant force of air increases the period to

$$T = T_0\left(1 + \frac{\rho_{\text{air}}}{2\rho_{\text{bob}}}\right)$$

where T_0 is the period in vacuum and ρ is the density. Test by swinging simultaneously two pendula of equal length but with bobs of quite different densities: aluminum, lead, and wooden pendulum balls are available. (Air resistance will also increase T a comparable amount. See Birkhoff "Hydrodynamics," p. 155.)

2. The finite mass of the string, m, decreases the period to

$$T = T_0\left(1 - \frac{m}{12M}\right)$$

where M is the mass of the bob (S.T. Epstein and M.G. Olsson, *American Journal of Physics* 45, 671, 1977). Correct your value of g for the mass of the string.

3. The finite size of a spherical bob with radius r increases the period slightly. When L is the length from support to center of the sphere, then the period becomes (see Tipler "Physics" 2nd ed. p. 346, problem 26):

$$T = T_0\sqrt{1 + \frac{2r^2}{5L^2}} \simeq T_0\left(1 + \frac{r^2}{5L^2}\right) .$$

What is the resulting percent error in your determination of "g"?

NOTE: For a comprehensive discussion of pendulum corrections needed for an accurate measurement of g to four significant figures, see R. A. Nelson and M. G. Olsson, *American Journal of Physiscs* 54, 112, (1986).

MC-8 The Physical Pendulum

Do either PART A or PART B but not both!

PART A

OBJECTIVE:

To measure the rotational inertia of a ring by swinging it as a pendulum from a point of the rim and to compare the value to that computed.

APPARATUS:

Basic equipment: Metal rings, (full, half, and quarter rings); knife edge and 2-point supports.

Computer equipment: Personal computer set to the MC-8 lab manual web-page; PASCO interface module; photogate sensor and extension jack.

INTRODUCTION:

The period of a rigid body swinging on an axis as a pendulum is (for small amplitudes $\simeq 5°$):

$$T = 2\pi\sqrt{\frac{I_0}{Mgh}}$$

where

$$\begin{aligned} T &= \text{period} \\ I_0 &= \text{rotational inertia relative to axis about which ring swings} \\ M &= \text{mass of the body} \\ g &= \text{acceleration of gravity} \\ h &= \text{distance from axis to center of mass.} \end{aligned}$$

REQUIRED INVESTIGATIONS:

1. Initiate the PASCO interface software by clicking on the telescope icon. There will be just a single table for recording the measured period.

2. Measure the period of the ring supported on the knife edge and swinging in its own plane. From the period calculate I_0. Use the parallel axis theorem to also calculate I_c, the rotational inertia about an axis through the c. of m. and perpendicular to plane of the ring. Compare this I_c with the computed value of $I_c = M(r_1^2 + r_2^2)/2$ where the r's are the inner and outer radii of the ring. Do the two I_c's agree within reasonable experimental error? Explain.

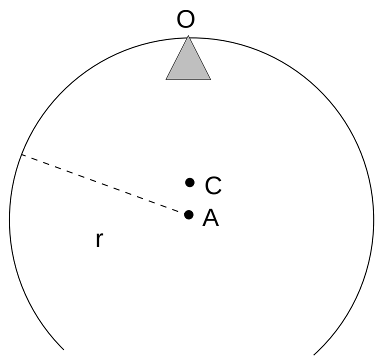
3. Measure the period of the ring when supported on the two sharp points but swinging perpendicular to its plane. From this period calculate the rotational inertia about an axis through those two points. Then use the parallel axis theorem to find the rotational inertia about a diameter of the ring. How does this rotational inertia compare with that about an axis perpendicular to the plane of the ring and through the center of mass?

OPTIONAL PROBLEM: Prove that this relationship should exist by calculating the rotational inertias about the two orthogonal axes.

4. One circular hoop has been cut in sections. Measure the period of a half-hoop when set at its midpoint on a knife edge; also the period of a quarter-hoop. (Be sure hoop sections have the same radius of curvature r).

Proof that any section of a thin hoop has the same period if oscillating in the plane of the hoop:

Let r = radius of the thin hoop
O = axis about which it swings (knife edge)
C = center of mass of the partial hoop
m = mass of the partial hoop
h = OC = distance from c. of m. to axis



Note that $I_A = mr^2$ for any partial hoop.
The parallel axis theorem then gives:

$$I_A = I_C + m(r-h)^2 \qquad \text{or} \qquad I_C = I_A - m(r-h)^2$$

Hence

$$I_C \;=\; mr^2 - mr^2 + 2mrh - mh^2 \;=\; 2mrh - mh^2 \;.$$

When one substitutes this I_C into

$$T = 2\pi\sqrt{I_0/(mgh)} \;=\; 2\pi\sqrt{(I_C + mh^2)/(mgh)}$$

one finds

$$T = 2\pi\sqrt{(2mrh - mh^2 + mh^2)/mgh} \;=\; 2\pi\sqrt{2r/g}$$

or a period independent of what fraction of a hoop is used!

OPTIONAL PROBLEM:

For a thick hoop, the above relationship does not hold exactly. Show that the thickness of the laboratory hoops accounts for the small difference between T for a whole hoop and T for a half hoop. (This is a rather difficult problem. For the older style laboratory hoops the finite thickness increases the period by $\simeq 1.8\%$; the period of the half-hoop will be $\simeq 5.2\%$ larger, and the quarter-hoop over 22% longer!)

PART B of M-8

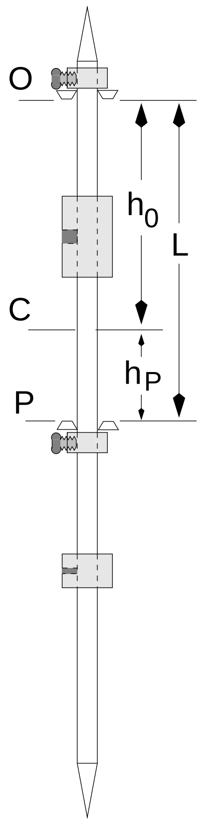
KATER'S REVERSIBLE PENDULUM

OBJECTIVE: To study conjugate centers of oscillation and to measure g accurately.

APPARATUS:

Long rod with movable weights and knife edges; bearing surface (Kater's pendulum mount); infrared photogate & support stand; computer equipment as in PART A.

INTRODUCTION:



When swung from O the period is:

$$T_0 = 2\pi\sqrt{\frac{I_0}{mgh_0}} = 2\pi\sqrt{\frac{I_C + mh_0^2}{mgh_0}}$$

When swung from P: $$T_P = 2\pi\sqrt{\frac{I_P}{mgh_P}} = 2\pi\sqrt{\frac{I_C + mh_P^2}{mgh_P}} \; .$$

By substitution one easily verifies that $T_0 = T_P$ for two arrangements:

1) the trivial solution $h_0 = h_P$, and

2) when $I_C = mh_0h_P$.

For the second case:

$$T_0 = T_P = 2\pi\sqrt{(h_0 + h_P)/g} = 2\pi\sqrt{L/g} \qquad (1)$$

where O and P are called conjugate centers of oscillation. (See note at end of experiment.) A measurement of the period (T) and distance ($L = h_0 + h_P$) between knife edges then gives an accurate value of g.

Empirically finding an L to give $T_0 = T_P$ is tedious. Instead, find an L for which $T_0 \sim T_P$ and then eliminate I_C from the first two equations above to give:

$$\frac{4\pi^2}{g} = \frac{h_0T_0^2 - h_PT_P^2}{h_0^2 - h_P^2} = \frac{T_0^2 + T_P^2}{2(h_0 + h_P)} + \frac{T_0^2 - T_P^2}{2(h_0 - h_P)} \; . \qquad (2)$$

If one chooses an asymmetric geometry for the location of the weights, one can avoid $h_O \sim h_P$. Note then that for $T_0 \sim T_P$ the first term dominates, and an accurate value of g results if one knows *accurately* $L = h_0 + h_P$, (the distance between the knife edges), and to much less accuracy the difference $h_0 - h_P$.

SUGGESTED PROCEDURE:

1. Find the approximate c. of m. of the pendulum plus asymmetrically located weights. Then set one knife edge as far from the c. of m. as feasible. This avoids $h_0 \sim h_P$ which would make the last term of Eqn. 2 large.
   NOTE: Keep this knife edge position fixed throughout the experiment.

2. Determine the period of the pendulum, T_0, when swinging from this knife edge. Keep amplitude small ($< 5°$) and use a photogate timer.

3. Calculate the length, L, of a simple pendulum to give the same period.

4. Set the second knife edge at this distance L from the first knife edge.

5. Determine the new period of the pendulum, T_P, for swinging from the second knife edge. This period T_P will *not quite equal* T_0 since moving the second knife edge has changed slightly the c. of m. and hence I_C, h_0, h_P, and also both T_0 and T_P.

6. Recalculate the length L of the equivalent simple pendulum to give this new period T_P. Then reset the knife edge accordingly.

7. Redetermine the period about the first knife edge.

8. One can continue this iterative process (6 thru 7) until the two periods are arbitrarily close to each other, and hence g is given by Eqn. 1. However this is not necessary if one accurately finds the new c. of m. (e.g. by balancing the pendulum on the knife edge for the hoops of M-8A) and then uses Eqn. 2.

9. Estimate your uncertainty in g and compare with accepted value (see M-4).

NOTE: These two conjugate centers of oscillation (O and P) exist of course for any rigid physical pendulum, e.g. *a baseball bat*:

Let one center of oscillation be where the batter grasps the bat. The conjugate center of oscillation is then called the *center of percussion* because if the ball hits the bat at this point, the blow rotates the bat about the other center of oscillation, (i.e. the batter's hands) and so the bat transmits no "sting" to the hands. However, if the ball hits very far from the *center of percussion*, the hands receive much of the blow and an unpleasant "sting" can result.

MC-9 Angular Acceleration and Rotational Energy

OBJECTIVE:

To study rotational motion resulting from constant torque. Since there are only six flywheels, groups of up to four may be necessary.

FUNDAMENTAL CONCEPTS:

1. The equation that describes the *rotational* motion of an object moving with *constant angular velocity* are completely analogous to those of linear motion with: $\theta(t) = \theta_0 + \omega \cdot t$.
   If you make a plot of θ versus t, you find that it describes a straight line. The greek letter θ_0 indicates the angular position of the object at time $t = 0$. The letter ω is the slope of the line, and is equal to the angular velocity of the object.

2. The equations that describe the *rotational* motion of an object that moves with *constant angular acceleration* are: $\theta(t) = \theta_0 + \omega_0 \cdot t + \frac{1}{2}\alpha t^2$ and $\omega(t) = \omega_0 + \alpha t$.
   The greek letter θ_0 again indicates the angular position of the object at time $t = 0$. The letter ω_0 is the the angular velocity of the object at time $t = 0$, and α (angular acceleration) is the slope of the graph of ω *vs* t.

3. It is important to stress that the natural units of angular displacement are in *radians* and NOT degrees. One full revolution of an object corresponds to 2π radians or, equivalently, $360°$. Typically units for angular velocity are in rad/sec and for angular acceleration are in rad/sec^2. Notice that the unit dimensions for equivalent dynamical variables in rotational and linear motion do not match. *A linear velocity can never be compared with a rotational velocity.* If you are unclear with this distinction ask your lab instructor for further clarification.

APPARATUS:

Basic equipment: Heavy cylindrical flywheel mounted on a low friction bearing and having a thin hub about which one winds string to hold a weight hanger; weight hanger and slotted masses.

Computer equipment: Personal computer set to the M9 lab manual web-page; PASCO interface module; photogate sensor and extension jack.
NOTE: If a flywheel needs more than six grams on the end of string to maintain constant rotational velocity, notify the instructor.

EXPERIMENT I (suggested procedures):

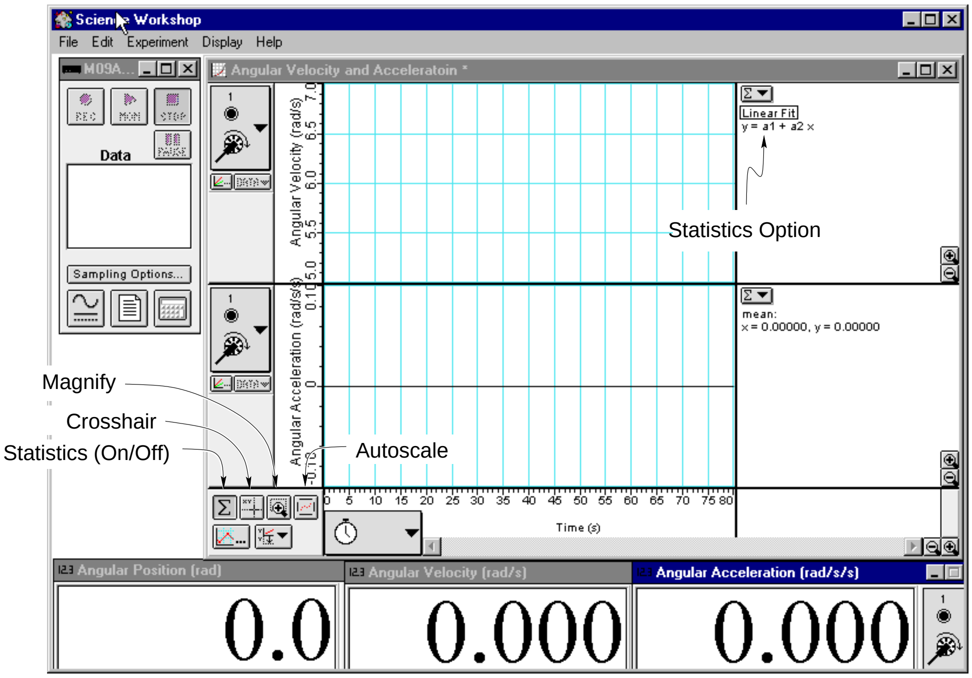
1. Make sure the PASCO interface has been turned on. If not you will have to reboot the computer. Next make sure that the phone jack connector from the photogate has been plugged into the extension wire and that the extension wire phone jack is plugged into the first position in the PASCO interface module. The computer itself is configured to measure the on/off timing between two adjacent holes on the wheel (i.e., this gives four timing measurements) and assumes that there are six holes per revolution. All other dynamical measurements are based on these input parameters.

2. Make sure the infrared photogate is properly aligned with the fly wheel so that the little red LED sensor turns off when each of the six holes drilled in the flywheel move past the photogate.

3. To initiate the PASCO interface software you will need to click on the telescope icon in the “toolkit.” The bitmap image below gives a good idea of how the display should appear. Note that, while you are able to reconfigure the display parameters, the default values that are specified on start-up should allow you to do this experiment without necessitating any changes. Three instantaneous measurements, angular position (θ), velocity (ω), and acceleration (α), are displayed simultaneously in combination with plots of the angular velocity and acceleration. Since ω is determined from the position data and α from ω the “scatter” in the data will become progressively more pronounced.

4. With NO string or weights attached to the flywheel spin the flywheel by hand so that you obtain approximately one revolution per second. Start the PASCO data acquisition by CLICKing on the REC icon. To stop it CLICK on the STOP icon. (Each data run gets its own data set in the “Data” display window. If there are any preexisting data sets you cannot reconfigure either the interface parameters or sensor inputs.)

5. Record data for about 60 revolutions of the flywheel and answer the following questions by using the cross-hair, magnify and rescaling features of the PASCO plot display. For the statistical analysis you can specify a “region of interest” by clicking to points inside the plot window. Your lab instructor can provide assistance if necessary.
   - (i) What was the total angular displacement (θ) of the wheel?
   - (ii) What were the initial and final angular velocities? Do you observe any systematic variations?
   - (iii) From these two measurements what was the average angular acceleration?
   - (iv) Does this agree with the average from acceleration plot? USE the interactive statistical analysis feature of the plot software to find the mean acceleration.
   - (v) What was the initial kinetic energy of rotation? (K.E. = $\frac{1}{2}I\omega^2$, I=$\frac{1}{2}MR^2$)
   - (vi) What was the final energy of rotation and energy loss per revolution?



6. Repeat the above experiment for 60 revolutions and qualitatively check (have one lab member count the number of revolutions by brute force) the computer calibration. Does everything seem consistent?

7. Predict how many revolutions and how long it will it take for the flywheel to slow to 75% of its initial angular velocity of 1 rev/sec. What is the change in kinetic energy?

8. Now test your prediction by performing this experiment.

EXPERIMENT II Suggested Procedures:

1. **Measuring the angular acceleration** α

   (i) Close the PASCO window and launch the next PASCO application by CLICKing on the telescope icon.

   (ii) Take a 2 m length of string and tie a multiple knot in one end so that it slides into the slot of the flywheel inner ring and cannot be easily pulled through. If necessary use a piece of tape, from the inside, to hold it more securely. On the other string end tie a loop so that either the 50 g mass hanger or a small weight may be hung.

   (iii) Find the friction correction by hanging small masses directly on the string (no hanger) until the tension "f" in the string just maintains flywheel rotation at a constant ω with the string hanging about halfway to the floor. Obtain the torque applied to the flywheel and compare the work done in one revolution ($W = \tau\ \theta$) to the energy lost in the Exp. I through the same rotation.
   Are these two values comparable?

   (iv) Replace the small masses with the 50 g hanger and, with the string fully extended and hanging vertically, roll the string up by exactly four full turns of the flywheel. The hanger should not hit the ground at any time.

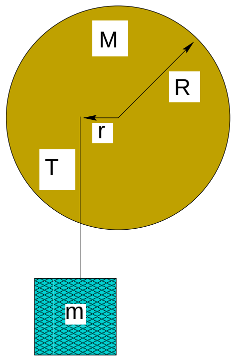
   (v) Add masses to reach exactly 400 g and start the data acquisition while releasing the 400 gm mass. Wait until the mass has both fallen and risen one time before stopping the scan. Print out one copy of each curve (three all told) by clicking on the "File" menu option and then clicking on the "Print All Displays" button. Use scissors to cut out each plot and enter it into you lab book labeling all regions of the position, velocity and acceleration plots.

   (vi) Determine the angular accelerations, α, of the wheel (falling and rising) and the energy lost (from $\Delta\theta$) in this one cycle.
   Does this compare favorably with the work done by the frictional torque?

   (vii) Repeat the experiment a few more times to ascertain the reproducibility of α.

   (viii) Repeat the last step using a net mass of 200 g.

2. **Predict** α after correcting for friction. One way to make the calculation follows:



$$\begin{aligned}
\text{Let } I &= \text{moment of inertia (calculated from } \tfrac{1}{2}MR^2) \\
a &= \text{linear acceleration of the mass} \\
m &= \text{mass hung on string (e.g. 200 or 400 grams)} \\
T &= \text{tension in the string} \\
f &= \text{tension for constant velocity} = \text{friction force} \\
r, R &= \text{radii of the hub and disc, respectively.}
\end{aligned}$$

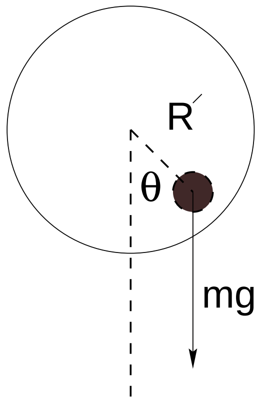
Then the torque on the wheel is $\tau = (T - f)r = I\alpha$.
But $F = ma$, so $(mg - T) = ma = mr\alpha$.

$$\text{Solving these for } \alpha \text{ gives:} \quad \alpha = \frac{(mg - f)r}{I + mr^2} \; .$$

3. **Compare the measured α's with the computed values**. Are they consistent within your estimated uncertainties?
4. By drastically increasing the mass added to the hanger deduce the maximum angular acceleration that the flywheel could receive.

EXPERIMENT III (optional):
Second Measurement of the Flywheel Rotational Inertia

1. **Unbalance the flywheel** by taping two stacked 100 g masses, m, near the outside edge of the wheel (see figure). The system then becomes a physical pendulum and will execute angular simple harmonic motion when displaced from equilibrium. Be careful that during the oscillatory motion only one hole moves across the infrared photogate. If the amplitude of swing is $< 10°$, the period, T, of the motion is to sufficient accuracy:



$$T = 2\pi\sqrt{I/\kappa}$$

where $I = I_0 + I_m$ is the total rotational inertia of the system about the axis of motion, and κ, the torsion constant, is

$$\kappa = \frac{\text{restoring torque}}{\text{angular displacement}} = \frac{mgR' \sin\theta}{\theta} \simeq mgR' \; .$$

By the parallel axis theorem, $I_m = I_c + mR'^2$ where I_c is the rotational inertia of the added mass m about its own c. of mass. Hence the rotational inertia of the flywheel, I_0, is:

$$I_0 = \left(\frac{T}{2\pi}\right)^2 mgR' - I_m.$$

2. Close the PASCO window and launch the third PASCO application window by CLICKing on the telescope icon immediately below.

3. **Measure the period of the pendulum** and deduce the rotational inertia of the flywheel, I_0. Compare with the earlier calculated value.

M-10 Power and Friction

APPARATUS: Prony brake, scale, timer, stopwatch, 15 meter tape.

Part A - POWER

1. Measure your horsepower (1 hp = 746 watts) by use of the Prony brake which is mounted on the wall. Try a slow rate that you could keep up all day and another as fast as you can turn the wheel. Figure out what data are required and how to use them. Check with your instructor to be sure you have analyzed the problem correctly. Many calculate the frictional force incorrectly.

2. Measure the horsepower you develop running up a flight of stairs.

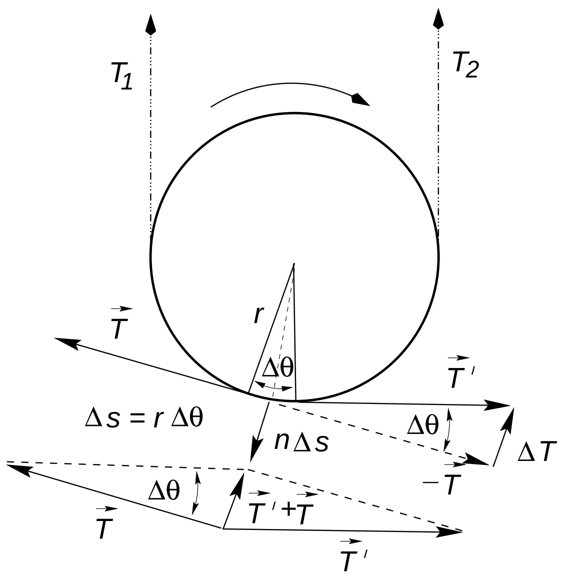
Part B - FRICTION

INTRODUCTION:

We measure the coefficient of kinetic friction, μ, between rope and wheel of the Prony brake and test whether μ is independent of velocity and of normal force. (By the way, the Prony brake is named after Gaspard Clair Francois Marie Riche de Prony, who invented it in Paris in 1821 to measure the power of engines. He was the director of the Ecole des Ponts et Chausses, a position he chose over joining Napoleon's army that invaded Egypt.)

Let **f = frictional force/unit length** and **n = normal force/unit length** along the rope. For an element Δs $= r\Delta\theta$ along the rope, let T' and T be the tensions at opposite ends of Δs. For small $\Delta\theta$, tensions $\vec{T}'$ and $\vec{T}$ will be almost antiparallel and differ in magnitude, $T' - T = \Delta T$, only by the frictional force $f\Delta s$. Thus

$$\Delta T = f\Delta s \qquad (1)$$



The reaction of the wheel must supply a normal force $n\Delta s$ sufficient to balance the vector sum $\vec{T}' + \vec{T}$, or $n\Delta s = |\vec{T}' + \vec{T}|$. But from the figure we see that $|\vec{T}' + \vec{T}| \simeq T\Delta\theta$ so

$$T\Delta\theta = n\Delta s \qquad (2)$$

Eliminating Δs between (1) and (2) gives $T\Delta\theta = n\Delta T/f = \Delta T/\mu$ since $f = \mu n$. Hence

$$\mu\Delta\theta = \frac{\Delta T}{T} \qquad \text{and} \qquad \mu d\theta = \frac{dT}{T} \quad .$$

For the Prony brake, we integrate between 0 and π:

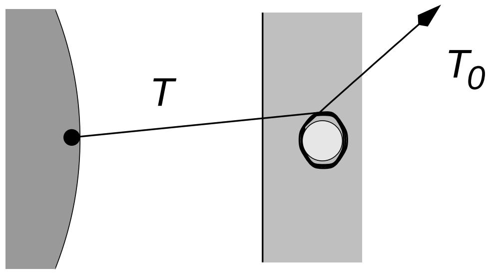
$$\mu \int_0^{\pi} d\theta = \int_{T_1}^{T_2} dT/T \quad \text{and so} \quad \mu = \frac{1}{\pi}\ell n\left(\frac{T_2}{T_1}\right)$$

.

Note #1: For the Prony brake, $\theta = \pi$. More generally, the string could go around any number of turns. Suppose it goes around x times, where x is of course not necessarily an integer. The string then wraps through a total angle $\theta = 2\pi x$. Then $dT/T = \mu d\theta$ integrates to

$$\ell n(T/T_0) = \mu\theta \quad \text{and hence} \quad T = T_0 e^{\mu\theta}$$

which holds also for a slipping string if $\mu = \mu_k$.



SUGGESTIONS FOR PART B:

1. Calculate your value of μ.
2. Test whether μ is independent of velocity by turning at different speeds and noting whether T_2 or T_1 changes appreciably.
3. Test whether μ is independent of the normal force by using a different tension setting for the brake. (Use different link setting in the chain to vary the tension).

QUESTIONS:

1. For a stationary wheel the two spring balances will not read the same if
   (a) there are zero errors in the balances,
   (b) there is friction in the wheel bearing.
   (c) the handle for turning the brake is not directly above or below the wheel's axis.

   Discuss the errors resulting from these effects.
2. How might one understand a change in μ as the normal force increases in #3 of Part B?

Note #2:

Friction arises from intermolecular forces between the atoms (or molecules) of the two surfaces, and so frictional forces are very complex. It is therefore surprising that the simple empirical relations for friction (e.g. that μ is a constant) hold as well as they do.

M-11 Young's Modulus of Elasticity and Hooke's Law

OBJECTIVE:

To study the elastic properties of piano wire under tension (Hooke's law and Young's modulus).

APPARATUS:

Frame for holding the steel wire, optical lever, telescope & support stand for measuring elongation (alternatively, dial gauges), 1 kg slotted masses, micrometer, tape measure.

INTRODUCTION:

Young's modulus M_Y is the ratio of longitudinal stress to the resultant longitudinal strain:

$$M_Y = \frac{\text{stress}}{\text{strain}} = \frac{\Delta F/A}{\Delta L/L}$$

where

$$\begin{aligned} \Delta F &= \text{longitudinal force in newtons} \\ A &= \text{area in square meters} \\ \Delta L &= \text{elongation} \\ L &= \text{length of wire undergoing the elongation (not the total length!)} \end{aligned}$$

Note that

$$\Delta F = \left\{ \frac{M_Y A}{L} \right\} \Delta L = k(\Delta L)$$

is Hooke's law where k is a constant if the elastic limit is not exceeded.

SUGGESTIONS:

1. First read about the optical lever (Appendix 3) and about parallax and focusing a telescope (Appendix 4). Also be sure that the frame holding the wire, and the stand holding the telescope are on solid bases. If you use a table, avoid leaning on it. With the telescope check the table sag resulting from leaning on it (least sag when units are near the table legs).

2. Adjust the height of the platform holding the optical lever so that the long arm of the lever is approximately horizontal.

PROCEDURE:

1. Put a load of three kilograms on the wire to straighten it.

2. Measure the successive deflections as you increase the load one kilogram at a time up to a total of 10 kg (i.e., 7 kg additional).

3. Repeat (2) but reducing the load 1 kg at a time.

4. Convert differences in scale readings to elongations (see Appendix 3).

5. Plot total elongation as abscissa against load in Newtons as ordinate. Find the average slope (i.e. $\Delta F/\Delta L = k$).

6. Measure A and L; compute Young's modulus. The value for piano wire is $\simeq 20 \times 10^{10}\ N/m^2$.

QUESTIONS:

1. Piano wire has a tensile strength (breaking stress) of 19 to 23 $\times$ 10^8 N/m^2. (The elastic limit may be about 0.7 the breaking stress). Calculate the maximum load your wire could stand. At what load would you pass the elastic limit?

   Note that the wire fails at a stress which is 100 x less than Young's modulus. Can you understand why the two values are not inconsistent? (Hint: If the stress equaled M_Y, what would be the strain?).

2. Discuss the sources of error in this experiment and estimate the reliability of your result. Is the accepted value for piano wire steel within the limits you have estimated?

3. How could you detect slipping of the wire in the chuck during the experiment?

4. How could you modify the experiment so as to detect and correct for any sagging of the support frame?

5. Poisson's ratio, $\left(\frac{-\Delta r/r}{\Delta L/L}\right)$, for steel is about 0.3.

   Could you notice the decrease in diameter of the wire in this experiment by use of a micrometer caliper? If so, try it.

   Suggested additional experiment: Determine the dependence of elongation on stretching force for a rubber band over a wide range of elongations (up to 3 or more times the unstretched length). Does it obey Hooke's law? Can you suggest reasons for its behavior?

M-12 Torsion Pendulum and Shear Modulus

OBJECTIVES: To study a torsion pendulum and to measure a shear modulus.

APPARATUS:

Wall mounted torsion pendulum (large disk plus suspension assembly); rods of different lengths, diameters and material; a ring and two cylindrical masses; timer, tally counter.

INTRODUCTION:

If a torque τ twists a rod, then the angle θ through which the rod twists is proportional to the torque if the elastic limit is not exceeded: Thus $\tau_{\text{applied}} = \kappa\theta$ where κ is the torsion constant.

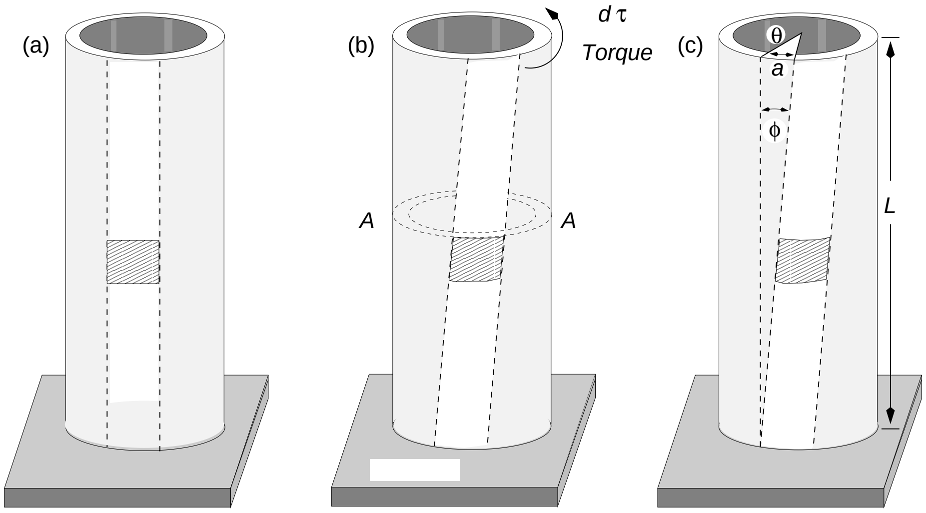


Figure 1: (a) A thin-walled hollow cylinder. (b) The cylinder is twisted by applying a torque. (c) The angle of shear ϕ and the angle of twist θ.

The equal and opposite reaction torque is

$$\tau_{\text{el}} = -\kappa\theta = I\alpha = I\frac{d^2\theta}{dt^2}\,.$$

Since this is the same mathematical form as

$$F_{\text{el}} = -kx = ma = m\frac{d^2x}{dt^2},$$

it has a similar solution: $\theta = \theta_0 \sin\omega t$ where $\omega = \sqrt{\kappa/I} = 2\pi f = 2\pi/T$. Thus the system undergoes angular simple harmonic motion, SHM, with $T = 2\pi\sqrt{I/\kappa}$ and T is independent of amplitude, (in contrast to a simple pendulum where T is only approximately independent of amplitude.) For simple geometries, one readily calculates the rotational inertia I. Hence measuring T can determine precisely the torsion constant, κ. To relate the torsion constant κ to the shear modulus M_s, consider first a tangential force dF_t applied to the end of a thin (dr thick) hollow tube. This tangential force/area is the shear stress and equals $dF_t/(2\pi r\ dr)$. The resultant shear strain is the angle ϕ (see Fig. 1(c)):

$$\phi = \frac{a}{L} = \frac{r\theta}{L} \quad .$$

Multiplying the stress by r/r gives

$$\frac{r dF_t}{2\pi r^2 dr} = \frac{d\tau}{2\pi r^2 dr} = \text{shear stress.}$$

Hence the shear modulus M_s becomes:

$$M_s = \frac{\text{shear stress}}{\text{shear strain}} = \frac{d\tau/2\pi r^2 dr}{r\theta/L} = \frac{L d\tau}{\theta(2\pi r^3)dr} \qquad \text{[tube]}$$

and $L d\tau = M_s\theta(2\pi r^3 dr)$. If one integrates from 0 to R, the tube then becomes a solid rod:

$$L\tau = M_s\theta\ 2\pi\left(\frac{R^4}{4}\right) \qquad \text{[solid rod].}$$

Thus

$$M_s = \frac{2L}{\pi R^4}\ \frac{\tau}{\theta} = \left(\frac{2L}{\pi R^4}\right)\kappa \quad .$$

SUGGESTED EXPERIMENTS: (only one of which need be performed)

1. Determine the shear modulus of one or more materials by measuring the period of a torsion pendulum with the materials as the suspension.

2. Use the result of Exp. 1 (or from a table of shear moduli) to calculate the period of a torsion pendulum with the ring (or the two cylindrical masses) placed on the disc. Compare with the measured value of the period.

3. Check the expression for shear modulus,

$$M_s = \frac{2L\tau}{\pi R^4\theta} = \frac{2L}{\pi R^4}\kappa.$$

   where $\kappa = \tau/\theta$ is the torsion constant, R and L are the radius and length of the rod, τ is the torque and θ is the angular displacement. Use rods of the same material but different lengths and diameters.

4. Determine how the period of the torsion pendulum depends on the length and diameter of the rod; also on the rotational inertia of the suspended mass. Check that the period is independent of angular amplitude (as long as one is below the elastic limit). Compare with theory.

QUESTION:

Is it reasonable to ignore the rotational inertia of the suspending rod? Check your answer by estimating the ratio of the rod's rotational inertia to that of the large disc.

M-13 Gyroscope

OBJECTIVE:

To study gyroscopes and to check the relationship between angular momentum, applied torque and rate of precession.

REFERENCES:

Don't do this experiment before reading about angular momentum and gyroscopes; see also the Mitac Instruction Manual, especially pages 1-7, experiments 1-4, pages 11-14, and Sec. 6 on "Operation".

APPARATUS: Mitac Gyroscope, 150 g and 300 g snap on slotted masses, and stroboscope.

SUGGESTIONS:

1. Read the sections of the Mitac Manual referred to above, start the gyroscope, and experiment with it. For example, predict the direction of precession when a certain torque is applied; then check it experimentally.

2. When you have become familiar with the operation of the gyroscope perform the following quantitative experiment:

   i. Calculate the rotor's angular momentum from the following data:

Outer diameter of rotor	20.0	cm
Inner diameter of rotor	15.1	cm
Thickness of rotor	2.54	cm
Rotor webb thickness	0.435	cm
Hub diameter	3.01	cm
Mass of rotor	2.73	kg
Rotor rpm	150	
(Check rotor rpm with the stroboscope)		

   ii. Place one of the supplied weights into the groove on the rotor axis so as to create a torque on the axis and cause the gyroscope to precess. From this weight and its position, calculate the torque. Predict the rate of precession by using the calculated torque and angular momentum. Check by experiment.

   iii. Ask your instructor what other quantitative measurement you should make. The experiments in the Mitac Manual are good possibilities.

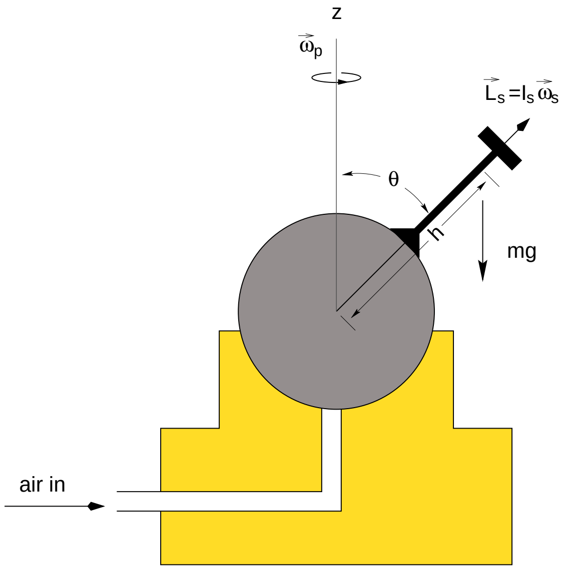
Alternative M-13 Gyroscope:

OBJECTIVE: To study gyroscopic effects especially precession and nutation.

APPARATUS: Air bearing gyroscope, torque weights, timer, stroboscope.

INTRODUCTION: Review gyroscope treatment in text (or in handout).

CAUTION: *The air bearing surfaces and torque shaft can easily suffer damage so handle carefully:* e.g. when the air pressure is on (and ball friction negligible) the *glued-on torque shaft may break off if the shaft bangs against the stationary ball holder.*

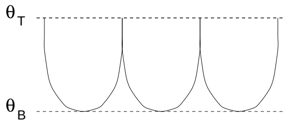


SUGGESTED STUDIES:

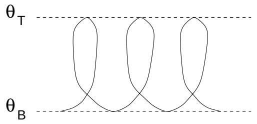
1. *Qualitative explorations:* With air flow on to minimize friction, spin ball by twisting the torque shaft with thumb and forefinger. Observe *precession* about the vertical Z axis, and observe *nutation* (torque shaft bobbing up and down) for different torque weights on the shaft and for different spin angular velocities, ω_s. Upon what factors does the precession depend?

   Observe the three basic *nutation* patterns when the total torque weight, m, has:

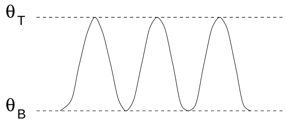
   a) no initial vertical velocity:



   b) a small initial vertical velocity:



   c) an initial small horizontal velocity:



2. *Measure the angular acceleration, α, from the air friction:* For low ω_s use a stopwatch and time the revolutions of a mark on the rotating shaft. Plot spin angular velocity, ω_s, versus time (remember to plot *average* ω_s at the middle of the time interval). Calculate α from the slope. For higher ω_s use the stroboscope. To avoid errors from subharmonics synchronizing with the mark on the shaft, start with the strobe at too

high a frequency. If the synchronization is at the n^{th} *higher harmonic*, you will see the shaft marks appear at n different places.

3. *Calculate mass of the unweighted torque shaft from observed precessional angular velocity, ω_p.* Spin the shaft at about $\omega_s \sim 2\pi$ rad/s and minimize nutation by starting the shaft with a small horizontal (or slightly upward) velocity to approximate the ω_p expected. Since the torque $\tau = (mgh)\sin\theta$ and $\omega_p = \tau/(L_s sin\theta)$, we predict ω_p is independent of θ. Check this experimentally. With spin angular momentum $L_s = I_s\omega_s$ the result is
$$\omega_p = \frac{mgh}{I_s\omega_s} \qquad \text{or} \qquad m = \frac{\omega_p\omega_s I_s}{gh}.$$
For the shaft mass $m \ll M$ of the ball, $I_s \sim (2/5)MR^2$ where R is radius of the ball. Measure the appropriate quantities and calculate m. Compare with m and h measured directly for an unattached torque shaft.

4. *Sleeping Top:* If
$$\omega_s^2 \;\geq\; 4mgh\left[I_s \;+\; mh^2\left(\frac{M}{m+M}\right)\right]/I_s^2,$$
then one can show (see handout) that the gyroscope will spin without nutation if released carefully at $\theta = 0$. You can observe the qualitative effect by starting with a higher ω_s and then gradually slowing down ω_s by touching the rotating ball symmetrically in the horizontal plane until ω_s becomes critical when the shaft will suddenly fall out of the vertical line. Calculate and check this critical ω_s.

OPTIONAL:

Nutation frequency, ω_N: One can show (see handout) that:
$$\omega_N = \frac{\omega_s I_s}{I_s + mh^2 M/(m+M)}.$$
Check this expression when $\omega_s \sim 2\pi$ rad/s. If one has time, change m and h by adding torque weights and check the new ratio of ω_N/ω_s.

Amplitude of nutation: The handout shows that the amplitude of nutation varies as $1/\mathrm{L}_s^2$. Since $L_s = I_s\omega_s$, spinning the gyroscope faster should reduce nutation amplitude at the same time the nutation frequency increases. Check this at least qualitatively. Use strobe to measure high ω_s.

MC-14–Elastic and Inelastic Collisions

MC-14a Collisions Between Rolling Carts

NOTE TO INSTRUCTORS:

This lab uses PASCO carts and avoids many of the problems of the air track. The older version of this lab that uses gliders on the air track can be found in MC-14b. The three procedures outlined below will certainly take more than the three hours allotted. Please do procedure I, then as many of the others as time allows.

OBJECTIVE:

In this experiment you will observe elastic and inelastic collisions between two PASCO carts; the carts are provided with Velcro bumpers on one end, and magnetic bumpers on the other.

THEORY:

Collisions are a common experience in our lives–the collisions of billiard balls, the collisions of two football players, the collisions of cars. Various 'natural philosophers' from Galileo onwards discussed the laws that govern such collisions. As early as the XVIIth century it was understood that collisions between hard bodies, like billiard balls, behaved differently from collisions like car wrecks, where the two masses stick together after the collision. The concept of Momentum was first introduced in mechanics by Descartes as $momentum = mass \times velocity$; he did not get it quite right however, because he thought of momentum as a **scalar** quantity. It was Leibnitz that defined momentum as a **vector** quantity:

$$\vec{p} = m \times \vec{v}.$$

Newton's IInd Law $\vec{F} = m\vec{a}$ can be rewritten

$$\vec{F} = m\frac{\Delta\vec{v}}{\Delta t};$$

now $m\Delta\vec{v}$ is the change in the quantity $\vec{p} = m\vec{v}$, so one can finally rewrite Newton's IInd law as

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

or

$$\vec{F}\Delta t = \Delta\vec{p}.$$

The quantity $\vec{F}\Delta t$ is called the **impulse** and is equal to the change in momentum.

FUNDAMENTAL CONCEPTS:

A physical quantity is said to be conserved in a process if its value does not change even though other quantities are changing. An example of a conserved quantity is energy. When a body falls freely its total energy (potential energy plus kinetic energy) remains unchanged while the position and velocity of the body change with time.

Linear momentum is conserved if the net force acting on an object is zero. This follows from the equation which relates the change in momentum to the impulse given to the object. Clearly if the force F is zero, the impulse is zero and the change in momentum is zero, hence the momentum remains constant.

This same principle becomes more useful when it is applied to an *isolated system* of objects. An isolated system is one where the only forces acting on an object *in* the system are due to the interaction with another object *within* the same system: there are no *external* forces.

A simple such system may consist of two particles, let us call them A and B, interacting with each other. The force $\vec{F}_A$ on particle A is equal and opposite to the force $\vec{F}_B$ acting on particle B: $\vec{F}_A = -\vec{F}_B$. Because the times during which the forces act are the same, it follows that the changes in momentum of the two particles are also equal and opposite, so that the total change in momentum is zero. The conservation of momentum is therefore a consequence of Newton's IIIrd Law. In collisions, provided there is no net external force on any of the bodies, the sum of the initial momenta equals the sum of the final momenta:

$$\vec{p}_1 + \vec{p}_2 = \vec{p_1}' + \vec{p_2}'$$

where the unprimed quantities refer to the velocities before the collision, and the primed quantities refer to the velocities after the collision.

For one dimensional processes the physical quantity that is conserved is linear momentum.

A collision is called totally inelastic if the two bodies stick together after colliding. The conservation of momentum for a *one dimensional* totally inelastic collision is then:

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) \cdot v'.$$

Energy is not conserved in inelastic collisions.

A collision of two bodies is called totally elastic if energy is conserved in the process. In this case the result of the collision in one dimension can be calculated by

$$(v_1 - v_2) = -(v_1' - v_2')'.$$

It is interesting to note that $(v_1 - v_2)$ and $(v_1' - v_2')$ are the relative velocities of body # 1 relative to body # 2 before and after the collision.

APPARATUS:

1. Personal Computer with monitor, keyboard, and mouse
2. PASCO dynamic track with magnetic bumpers
3. PASCO signal interface
4. Two PASCO carts with a "picket fence"
5. A set of 500 g masses
6. One meter long ruler
7. Two photogates: these devices consist of
    (a) a source which produces a narrow beam of infrared radiation
    (b) an infrared detector at the other side of the gate that senses the radiation.

When the beam between the source and detector is blocked, a red Light Emitting Diode (LED) on the top of the gate lights up; concurrently an electrical signal is sent to the Signal Interface which converts the time during which the beam was blocked into the velocity of the fence. NOTE: for the photogates to work properly, the picket fence must be on the opposite side of the cart closest to the LED.

SUGGESTIONS:

1. Measure the mass of the empty carts using the pan balance; MAKE SURE YOU PUT THE CART ON ITS SIDE ON THE PAN. Record these masses in your lab notebook. Note that the cart with the magnet bumper is somewhat heavier than the cart without magnet, you may wish to tape some masses on the lighter cart to equalize the weights.

2. Initiate the PASCO interface software in the usual way. The monitor should now look as shown in Figure 1.

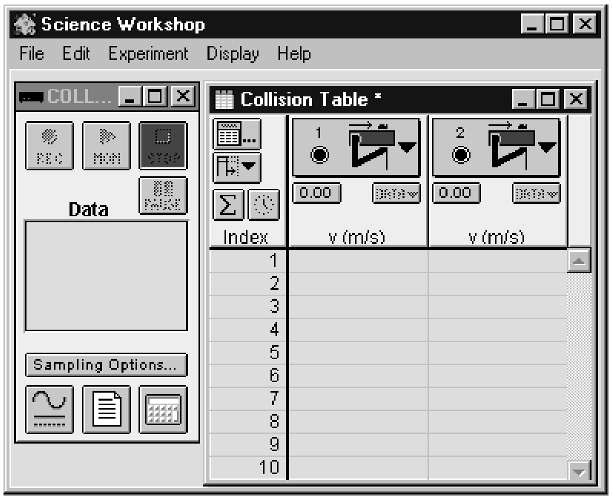


Figure 1:

On the computer monitor you see a table of velocities for each of the two photogates. As seen by a person looking at the computer monitor the photogate on the left is # 1 and the one on the right is # 2.

3. Level the track so the carts do not move to the right or the left.

4. Place the photogates 15 cm apart.

PROCEDURE I - INELASTIC COLLISION - EQUAL MASSES

1. Prepare a table like the one below:

2. Install a picket fence on one cart (on the side opposite the LED). Make sure that it is the longest black stripe on the fence that intercepts the LED beam. Put the cart at the left end of the track. Place the other cart just to the right of photogate #2 as shown below. Check that the carts do not repel each other; the Velcro hooks and loops must be able to stick together easily.

Run #	Velocity # 1 (m/s)	Velocity #1 (m/s)
1		
2		
3		

Table 1:

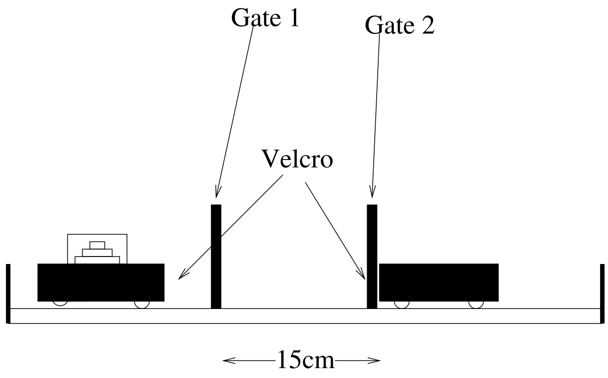


Figure 2:

3. CLK on REC. Gently push the projectile cart to the right toward the first photogate and the target cart. Catch the carts before they hit the end of the track. CLK on STOP. Record the velocity of the projectile cart before and after the collision in Table 1.

4. Take two more runs.

5. ANALYSIS OF THE DATA.

   Prepare tables as shown below in table 2 and 3.

	Car 1	Cars 1+2	
Run #	p_{ini} (kg m/s)	p_{final} (kg m/s)	$100 \times \Delta p/p_{ini}$
1			
2			
3			
Avg			

Table 2: Momentum

   Calculate the initial and final momenta and enter them in table 2.

   Calculate the initial and final kinetic energies and enter them in table 3

   Calculate and record in table 2 the percent difference in the momenta: $100 \times (p_{ini} - p_{fin})/p_{ini}$. Calculate the average percent difference in the momentum.

   Calculate and record in table 3 the percent difference in energy: $100 \times (E_{ini} - E_{fin})/E_{ini}$. Calculate the average percent difference in the energy.

Run #	Car 1 E_{ini} (J)	Cars 1+2 E_{final} (J)	$100 \times \Delta E/E_ini$
1			
2			
3			
Avg			

Table 3: Energy

1. QUESTIONS:

   Give a likely physical cause for the observed difference between initial and final momentum.

   What fraction of the energy was dissipated?

   If you could remove the physical cause you listed, what fraction of energy would you expect to find dissipated?

PROCEDURE II - ELASTIC COLLISION - EQUAL MASSES

In this experiment you will be measuring the velocities of two bodies with equal masses, before and after a totally elastic collision.

1. Place the photogates about 35cm apart with one cart just to the left of photogate #2. *Make sure the carts will repel each other.* (One cart has one end without any magnets-that end will not repel the other cart.) Place the other cart on the left end of the track. as shown in Fig 3 below. Cart # 2 should be close to gate # 2, (not as shown in the figure). Make sure that there is enough space, so that cart #2 starts moving **after** cart #1 has passed completely through gate #1.

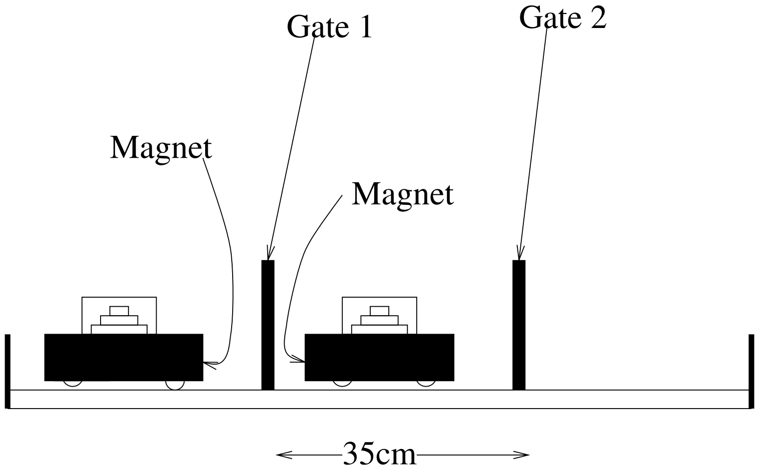


Figure 3:

2. Prepare a table like table 4.

Run #	Velocity # 1 (m/s)	Velocity #1 (m/s)
1		
2		
3		

Table 4:

3. CLK on REC. Gently push the projectile cart (the one on the left) to the right toward the first photogate and the target cart. The projectile cart should stop between the gates and the target will move through photogate # 2. Catch it before it has time to hit the end of the track; CLK on STOP. Record the velocities in table 4. Take two more runs.

4. ANALYSIS OF THE DATA

   Prepare tables like tables 5 and 6.

	Car 1	Cars 1+2	
Run #	p_1 (kg m/s)	p_2 (kg m/s)	$100 \times \Delta p/p_1$
1			
2			
3			
Avg			

Table 5: Momentum

	Car 1	Cars 1+2	
Run #	E_1 (J)	E_2 (J)	$100 \times \Delta E/E_1$
1			
2			
3			
Avg			

Table 6: Energy

Calculate the initial momentum of Car # 1 and final momentum of Car # 2 and enter them in table 5.

Calculate and record the percent difference of the momenta: $100 \times (p_1 - p_2)/p_1$. Calculate the average percent difference in the momentum.

Calculate the initial and final kinetic energies and enter them in table 6.

Calculate and record the percent difference of the energies: $100 \times (E_1 - E_2)/E_1$.

5. QUESTIONS

   What is a reason for the observed difference between p_1 and p_2?

   What fraction of the energy was dissipated?

   What fraction of the energy should have been dissipated in the absence of friction?

PROCEDURE III - ELASTIC COLLISION - UNEQUAL MASSES

1. The photogates should be about 35cm apart. Find and record the masses of two 500g masses using the triple beam balance; place them on the target cart, place this cart just to the left of photogate # 2 as shown in fig 3 (not as shown in the figure) . The carts should be placed so they will repel each other. The lighter cart is 'the projectile' and should be placed on the left end of the track.

2. Prepare a table like table 7.

Run #	V_{1ini} (m/s)	V_{1fin} (m/s)	V_{2fin} (m/s)
1			
2			
3			

Table 7:

3. CLK on REC. Gently push the projectile cart (the lighter one on the left) to the right toward the first photogate and the target cart. Try to predict what will happen. Catch the carts before they hit the end of the track. CLK on STOP. Record the velocities in Table 7

4. Take two more runs.

5. ANALYSIS OF THE DATA

   Prepare tables like tables 8 and 9.

	Car 1 ini	Car 1 fin	Car 2	
Run #	p_{1ini} (kg m/s)	p_{1fin} (kg m/s)	p_{2fin}(kg m/s)	$100 \times \Delta p/p_{1ini}$
1				
2				
3				
Avg				

Table 8: Momentum

   Calculate the initial and final momentum of each cart and enter them in table 8. Calculate the total momentum of the system p_{fin} after the collision and record it in table 8. Remember that the final momentum is the sum of the momenta of carts # 1 and # 2 after the collision.

Run #	Car 1 ini E_{1ini} (J)	Car 1 fin E_{1fin} (J)	Car 2 E_{fin} (J)	$100 \times \Delta E/E_{1ini}$
1				
2				
3				
Avg				

Table 9: Energy

Calculate the initial and final kinetic energies and enter them in table 9. Calculate the total energy of the system E_{fin} after the collision and record it in table 9. Remember that the final energy is the sum of the energies of carts # 1 and # 2 after the collision.

Calculate and record the percent difference $100 \times (p_{1ini} - p_{fin})/p_{1ini}$.

Calculate and record the percent difference $100 \times (E_{1ini} - E_{fin})/E_{1ini}$.

6. QUESTIONS:

   What is a reason for the observed difference?

   Was momentum approximately conserved?

   What fraction of the energy was dissipated?

   What fraction of the energy should have been dissipated in the absence of friction?

JAVA APPLET:

If time permits and you are interested the HTML version of the lab has a link to an applet which animates elastic collisions of two point masses.

MC-14b Air Track Collisions

OBJECTIVE: To study conservation of energy and momentum in collision.

APPARATUS:

Air track, assorted slotted masses, air supply, hose, gliders; photogates & support stands and PASCO interface and computer.

> **PRECAUTIONS:** *The soft aluminum gliders and track surfaces damage easily: Don't drop!* With the air pressure on, use a glider to *check that the track is level and free of high friction areas* (from scratches or plugged air holes). Getting good results can, at times, be surprisingly difficult in this lab. All collisions must be free from any glider contact with the rail. In general speeds which are too slow are overly influenced by residual friction and air track leveling errors. On the other hand, speeds which too high invariably cause pitch and yaw motions of the gliders which increase the likelihood of physical contact with the track. Good alignment of the needle assembly is also a necessity. You should perform a number of preliminary trials to discern which speeds work best.

SUGGESTIONS:

1. Use the large gliders whenever possible. Small ones often tilt upon impact and hence give excessive friction.
2. Turn on the air supply and experiment with the gliders. Adjust the leveling screw so that the nominal cart acceleration is minimized. Adjust the air flow so the gliders move freely without rocking side to side.
3. Make sure that the photogates are plugged into the first two phone jack inputs in the PASCO interface module. Also set the two photogates approximately 40 to 50 cm apart and so that they track the 10 cm long plate atop the glider.
4. Click on the telescope icon below to launch the PASCO data acquisition software. It should already be configured to display a two column table which will display the *speed* of the glider as it passes through the infrared photogate. (It already assumes that the plate is exactly 10 cm long and actually measures the time.)
   To start the data acquisition CLICK on the REC icon. To stop it, CLICK on the STOP icon (the upper left window is configured like a tape player). Each time a glider passes the photogate an entry will appear in the appropriate table column. NOTE: The photogates measure the speed but do NOT sense the direction of motion. You are responsible for the latter.
5. The results of this experiment are very technique sensitive. Take a single glider and practice sending it through both photogates until you are able to get reasonably equivalent speed readings from both photogates. Check the behavior by launching the glider from both ends and record, in your lab book, the speed measured in four or five satisfactory trials. Estimate the precision associated with a pair of velocity measurements and show how this will impact your momentum and kinetic energy measurements.

EXPERIMENT I:

1. Choose gliders of equal mass (or make them approximately the same by fastening weights on one). Qualitatively predict the expected outcome of the next step (i.e., item 2) before attempting the experiment.

2. With glider #1 at the end of track and glider #2 at rest near the center, give #1 a push toward glider #2.
   To help prevent confusion, stop glider #2 before it bounces back.
   As before you should perform multiple trials until you achieve consistent results and record a few of them.

   Check conservation of momentum and energy in the impact. In equation,

   $$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2,$$

   call velocities to the right positive, those to the left negative.

3. Comment on how well these two quantities are conserved and, if your results seem poorer than expected, suggest possible sources of error.

4. Devise a method for determining how elastic is a rubber band collision and record your results.

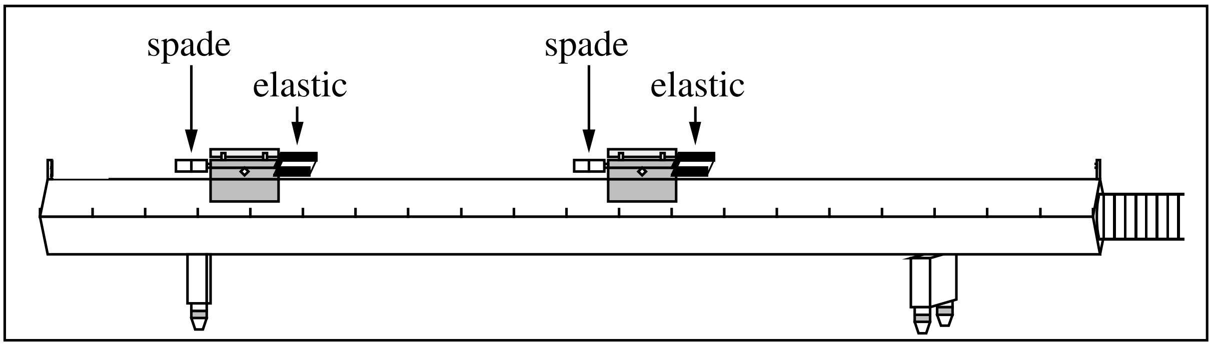


Figure 1: Sketch of the air track configuration for elastic collisions.

Suggested tabulations:

Glider:	#1	#2	Velocity Readings:	#1	#2
mass			Before impact (u)		
length			After impact (v)		

Before Impact				After Impact			
$u_1 =$	______	$u_2 =$	______	$v_1 =$	______	$v_2 =$	______
$m_1u_1 =$	______	$m_2u_2 =$	______	$m_1v_1 =$	______	$m_2v_2 =$	______
$\frac{1}{2}m_1u_1^2 =$	______	$\frac{1}{2}m_2u_2^2 =$	______	$\frac{1}{2}m_1v_1^2 =$	______	$\frac{1}{2}m_2v_2^2 =$	______

change in momentum = ______; % change in momentum = ________.

change in energy = ______; % change in energy = ________.

EXPERIMENT II:

Perform the same procedure as Exp. I (steps 1, 2 and 3) except start both gliders from opposite ends of the air track and with considerably different velocities.

EXPERIMENT III:

Repeat Exp. II but for inelastic collisions by attaching cylinders with needle and wax inserts. Note that the needle must lines up exactly with the insert or there will be significant sideways motion when the two gliders strike.

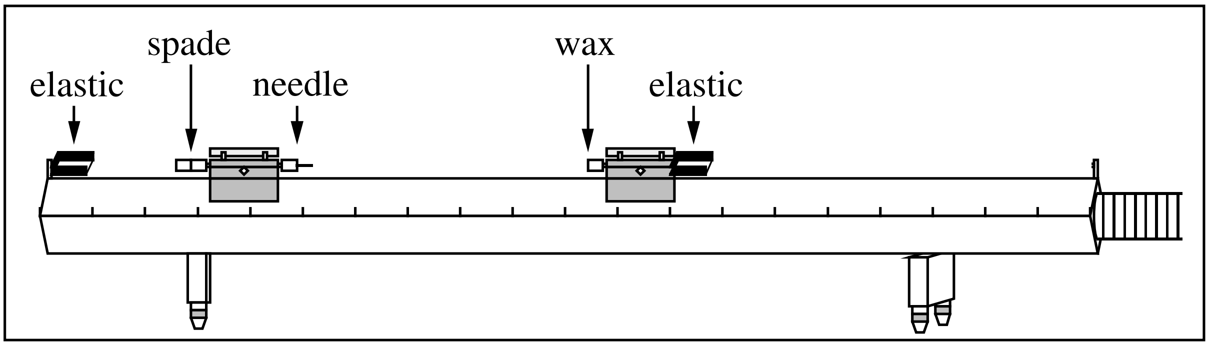


Figure 2: Sketch of the air track configuration for inelastic collisions.

EXPERIMENT IV:

Increase m_1 by adding masses a nd repeat EXP. I.

MEASUREMENT OF FRICTION:

Estimate the frictional force between the glider and track by using the velocity data recorded before Exp. I. From any net decrease in velocity you should be able to obtain the frictional force. Does this information help you understand the experimental data?

JAVA APPLET:

If time permits and you are interested the HTML version of the lab has a link to an applet which animates elastic collisions of two point masses.

MC-15–Simple Harmonic Motion and Resonance

MC-15a Simple Harmonic Motion and Resonance (Rolling Carts)

NOTE TO INSTRUCTORS: This lab uses PASCO rolling carts instead of the more problematic gliders on the air track. The older airtrack version of this lab is MC-15b.

OBJECTIVES:

1. To study the period of Simple Harmonic Motion (SHM) as a function of oscillation amplitude.
2. To demonstrate Hooke's Law, $F = -kx$, where k is the spring constant, and x is the elongation.
3. To study the period T of SHM as a function of the mass m which is oscillating, investigating $\sqrt{m/k} \propto T(= 2\pi/\omega = 1/f)$.
4. To observe the relationships between the potential and kinetic energy.

THEORY:

The restoring force (F) on an object attached to a "simple" one-dimensional spring is proportional to the displacement from equilibrium and has the form, $F = -k(x - x_0)$, where k is the spring constant (or stiffness in N/m), x_0 is the equilibrium position (i.e., no net force) and x is the position of the object. This is Hooke's Law. Remember that the simple harmonic oscillator is a good approximation to physical systems in the real world, so we want to understand it well. That's the purpose of this lab!

The expression $F = ma = -k(x - x_0)$ is a 2nd order differential equation with

$$a = \frac{d^2x'}{dt^2} = -\frac{k}{m}(x')$$

where $x' \equiv x - x_0$. The most general solution for this expression is often given as

$$x'(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$$

where $\omega_0 \equiv \sqrt{k/m}$. ω_0 is defined as the *natural frequency* for the *undamped* harmonic oscillator. A and B are arbitrary initial displacement parameters. Alternatively the solution is often specified as

$$x'(t) = C\sin(\omega_0 t + \phi) \qquad \text{or} \qquad x'(t) = C\cos(\omega_0 t + \phi)$$

where ϕ is the starting phase and C is the displacement. It is also possible to write down these solutions using complex numbers as in

$$x'(t) = A'e^{i\omega_0 t} + B'e^{-i\omega_0 t} \quad .$$

The relative merit of these equivalent expressions will become clearer in EXPTS. V and VI.

FUNDAMENTAL CONCEPTS:

1. The solution to the undamped (i.e., no frictional or drag forces) harmonic oscillator is time dependent and periodic. When the $\omega_0 t$ term varies by 2π (or one period $T = 1/f = 2\pi/\omega_0$) both the position, $x'(t+T) = x'(t)$, and velocity, $v'(t+T) = v'(t)$ return to their previous values.

2. Total energy (TE) is conserved in SHM motion. As time evolves kinetic energy (KE) is transferred to (and from) potential energy (PE). Thus at any time t:

$$TE = \text{constant} = KE(t) + PE(t) = \frac{1}{2}m[v'(t)]^2 + \frac{1}{2}k[x'(t)]^2 \quad .$$

APPARATUS:

Basic equipment: PASCO dynamic track, PASCO cart with "picket fence", PASCO cart with aluminum plate, adjustable stop, assorted masses, springs: *this experiment works best with a pair of short (10 mm unstretched length) springs*, timer, photogate & support stand, knife edge assembly.

Computer equipment: Personal computer set to the MC15a lab manual web-page; PASCO interface module; photogate sensor and extension jack, PASCO sonic position sensor, speaker with driver stem, power amplifier module.

PRECAUTIONS:

1. See MC-14a

2. **When setting up the springs, use the adjustable end stop on the track so the springs are stretched properly: don't stretch them beyond their elastic limit and don't let them sag and drag on the track**. Stretching the springs to reach the ends of the track will damage them.

3. *Keep the amplitudes small enough that a slack spring doesn't touch the track nor a stretched spring exceed its elastic limit.*

SUGGESTIONS: To measure the period of the oscillating cart:

1. Install a "picket fence" on the cart.

2. Locate the photogate so the black stripe on the fence just cuts off the LED beam (see MC-14a) when the cart is in the equilibrium position ($x - x_0 = 0$). The photogate phone jack should be in the first PASCO interface position. Choose two springs having a similar length and refer to the above precaution.

3. To initiate the PASCO interface software click the mouse on the telescope icon in the "toolkit" area below. There will be just a single table for recording the measured period.

4. Start the cart by displacing it from equilibrium and then releasing it. Then start the data acquisition by clicking the REC button button. Let the cart oscillate for about 10 periods. Calculate the mean and standard deviation by simply clicking on the statistics icon (i.e. $\boxed{\Sigma}$) on the data table.

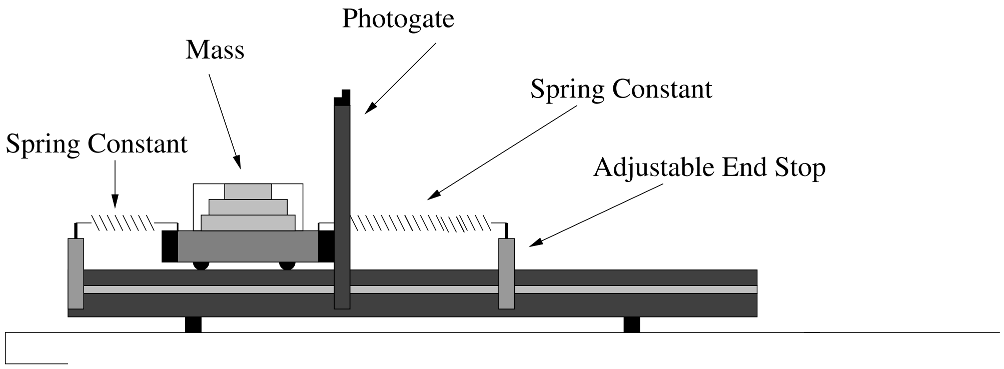


Figure 1: Sketch of the dynamic track configuration for SHM.

UNDAMPED SIMPLE HARMONIC MOTION:

1. Using sketches in your lab book, both at equilibrium and after a displacement from equilibrium, show that the effective force constant for 2 identical springs of force constant k on either side of an oscillating mass is $2k$.

2. **EXPERIMENT I:** Show experimentally that the period is independent of the amplitude. Try amplitudes of *approximately* 10 cm, 20 cm, and 30 cm. (Friction may be a problem at very small amplitudes).

3. **EXPERIMENT II:** By adding mass to the cart, study the period versus total oscillating mass. The latter must include a correction for the oscillating springs whose effective oscillating mass (see the note below) is approximately $m_s/3$ where m_s is the mass of the *two* springs. Explain, in words, how this correction may be qualitatively justified (Why not m_s or $m_s/2$?) By inspection of your T vs $[M + (m_s/3)]$ curve, what function of T might yield a straight line when plotted versus $[M + (m_s/3)]$? Prepare this plot. Calculate the effective force constant, k', (equal to $k_1 + k_2$, the two springs will actually differ slightly) of the system from the slope of this straight line graph. (Remember $T = 2\pi\sqrt{M_{\text{eff}}/k'}$ where $M_{\text{eff}} \simeq \text{M} + \text{m}_s/3$.) Estimate the uncertainty in k' by solving for k' at each cart mass and calculating the standard deviation of the mean.

   NOTE: Theoretically $m_{\text{eff}} = m/3$ only for $M/m_s = \infty$ For $M = 0$, the effective mass is $4m/\pi^2 = 0.405\,\text{m}$. However for $M/m_s = 5$, the effective mass is already $\simeq 0.336\ m_s$. See Fig. 1 of J.G. Fox and J. Makanty, *American Journal of Physics*, **38**, 98 (1970).

4. **EXPERIMENT III :** Also measure k directly by hanging the two springs vertically on a "knife edge" assembly in the laboratory. Record the stretch produced by a series of weights, *but do not exceed the elastic limit of either spring!* Graph F vs y for each spring and obtain a best-fit straight line. The slope should be the spring constant. Compare this value of k' with that obtained in part #3.

5. QUESTION: Assuming the spring constant doubles, how would T vary?

FURTHER INVESTIGATIONS OF "UNDAMPED" SIMPLE HARMONIC MOTION:

Up to this point you have characterized the SHM of a spring-mass assembly in terms of only a single parameter T (the period). One of the possible time-dependent expressions for describing the motion was

$$x'(t) = A\cos(\omega_0 t + \phi_0) \quad \text{and} \quad v'(t) = -A\omega_0 \sin(\omega_0 t + \phi_0)$$

or

$$v'(t) = \ A\omega_0 \cos(\omega_0 t + \phi_0 - \frac{\pi}{2})$$

where A is the amplitude (displacement from equilibrium), ω_0 is the *natural* angular frequency and ϕ_0 is the starting phase. Notice that the velocity has a *phase* shift of $\frac{\pi}{2}$ relative to the displacement.

Equally characteristic of SHM is the process of energy transference: kinetic energy of motion is transferred into potential energy (stored in the spring) and back again. Friction is an ever present energy loss process so that the total energy always diminishes with time.

To capture this rather rapid cyclic process we will again use the PASCO interface while replacing the photogate sensor with the sonic position sensor.

EXPERIMENT IV: Measuring the x vs t behavior:

1. Replace the cart with the picket fence by a cart with an aluminum vane attached to it. You will want to measure the mass of this new cart and predict the new natural frequency.

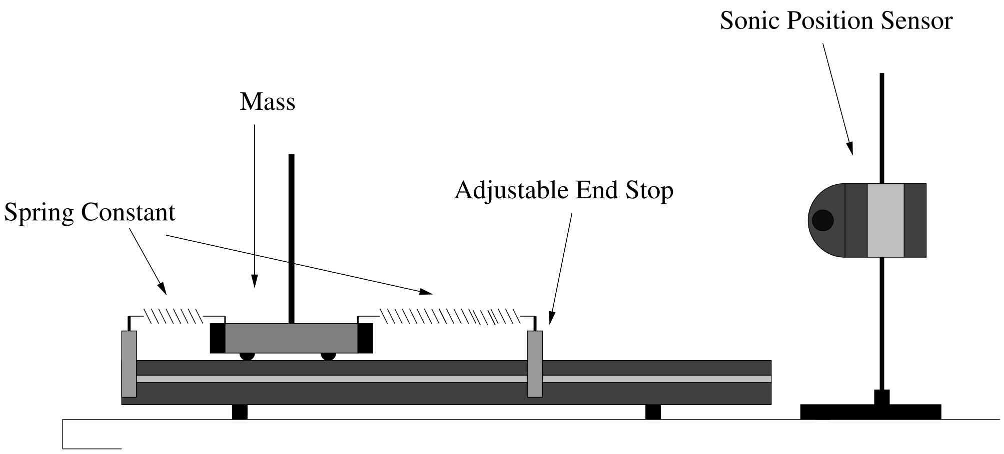


Figure 2: Sketch of the dynamic track with the sonic position sensor.

2. Place the position sensor approximately 60 cm from the vane in the direction of oscillatory motion. Make sure the yellow phone jack is in the third slot and the black phone jack is in the fourth slot. Alignment is very important so that the sensor senses only the vane and not the cart. A slight upward tilt may help (or raising the vane up slightly as well).

3. CLICK on the telescope icon below to initiate the PASCO© interface software.

4. Displace the cart approximately 20 cm from equilibrium to initiate the oscillatory motion. CLICK on the the REC button to start your data acquisition. The graph will simultaneously display both absolute position and velocity versus time.

5. Practice a few times to make sure you can obtain smoothly varying sinusoidal curves. Then run the data acquisition for just over ten cycles and use the cross-hair feature to read out the time increment for ten full cycles. How does your prediction check out? Determine the initial phase (i.e., at $t = 0$). Record the equilibrium position (x_0) as well.

6. Use the magnification option of the PASCO software to better view a single full cycle by clicking on the magnifying glass icon (in the graph window) and then select two points in the graph using a CLICK and DRAG motion of the mouse.

7. Print out (click in graph region and then type ALT, CTRL-P) or, alternatively, sketch the position and velocity curves in your lab book identifying key features in the time dependent curves. In particular identify the characteristic(s) which demonstrate the $\pi/2$ phase difference between the velocity and displacement curves.

8. Using the PASCO cross-hair option to read out the relevant time, position and velocity, make a table as below and fill in the missing entries (identify units).

time	phase($\omega_0 t + \phi_0$)	$x'(t)$	$v'(t)$	KE	PE	TE
	0					
	$\pi/4$ (45 deg)					
	$\pi/2$ (90 deg)					
	$3\pi/4$ (135 deg)					
	π (180 deg)					

9. Is the total energy a constant of the motion?

10. How much displacement amplitude and energy are lost after five full cycles? What is the approximate friction coefficient?

OPTIONAL INVESTIGATIONS OF "UNDER-DAMPED" SIMPLE HARMONIC MOTION:

In the real world friction is an ever present process. In the case of SHM friction can have a profound effect. Damping of unwanted vibrations is important in a myriad of situations. (Imagine what driving a car would be like if there were no shock absorbers!)

Introducing friction can be done by simply adding one more term in the force expression, $F_{\text{drag}} \equiv -Rv$, a drag force which is proportional to the velocity where R is the drag coefficient [units of kg m/s or N/(m/s)] . This is appropriate for motion thru a viscous fluid but it is really only a rough approximation for the frictional forces in the track. The modified force expression now becomes

$$F = ma = -kx' - Rv' \quad \text{or} \quad 0 = \frac{d^2x'}{dt^2} + \frac{R}{m}\frac{dx'}{dt} + \omega^2 x'$$

Since energy is continuously lost, solutions of this expression will be *time dependent* but NOT periodic. Adding this "simple" term dramatically complicates the process of finding appropriate solutions. The most general form of the solution is

$$x' = e^{-(R/2m)t}\left[Ae^{+\sqrt{(\frac{R^2}{4m^2}-\omega_0^2)}\ t} + Be^{-\sqrt{(\frac{R^2}{4m^2}-\omega_0^2)}\ t}\right]$$

which is quite formidable. Since the track drag is low (i.e., R is relatively small) the solution is said to be *underdamped* and oscillatory when $\omega_0^2 > R^2/4m^2$. The solution in this case becomes:

$$x' = Ce^{-(R/2m)t}\cos(\omega_1 t + D) \quad \text{where} \quad \omega_1^2 \equiv \omega_0^2 - \frac{R^2}{4m^2}$$

where ω_1 is the *natural frequency* of the *underdamped* system and C and D are the initial displacement and phase. Because the frequency is lowered, the period lengthens. This is consistent with one's intuition; drag works against oscillatory motion.

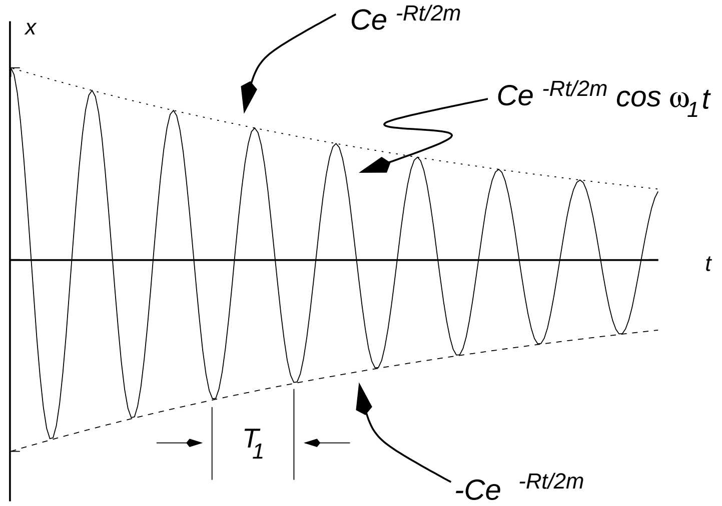


Figure 3: Sketch of underdamped harmonic motion with $\omega_0^2 > R^2/4m^2$.

EXPERIMENT V:

1. CLICK on the telescope icon below to initiate the next PASCO© interface application. In addition to the graph there will be a "two" column table displaying the time and position then the time and velocity.

2. Displace the cart approximately 20 cm from equilibrium to initiate the oscillatory motion. CLICK on the the REC button to start your data acquisition and record enough cycles to see the amplitude diminish by two thirds.

3. As the amplitude of oscillation decreases, does the period T change? Base the answer on measurements of T using the cursor in the PASCO display.

4. Select six representative times using the table (identifying where the velocity changes sign) and make a table of t vs maximum displacement. Use these points in the graphical analysis package and fit these points to the expression $C\exp-(R/2m)t$ or, in terms of the explicit analysis formula, $y = C * \exp[-B * (x - x_0)]$.

5. How good or poor is the assumption that the drag force is proportional to the velocity?

6. Sketch out an approximate curve of x vs t if R were significantly larger. Which R would be more appropriate for absorbing and dissipating a physical "shock" (and why)?

OPTIONAL INVESTIGATIONS OF RESONANCE:

One of the most important situations of the harmonic oscillator is that of FORCED, damped harmonic motion. In one-dimension the applied force is typically sinusoid and when ω approaches the natural frequency of the system (nominally ω_1 and, if $R^2/4m^2 \ll 1$, also ω_0) the energy of the driver is additively coupled to the moving mass (e.g., a glider) and resonance occurs. Resonance is a *very* important aspect of the world around us and many mechanical and electronic devices employ resonant behavior as a fundamental aspect of their operation (e.g., musical instruments, radios, televisions).

Along with the frictional drag (Rv' where R is the drag coefficient) one more force term must be added, that of the mechanical driver, with $F_{\text{driver}} = F_d\ \cos(\omega t)$. The new force expression is conventionally written as:

$$F = ma = -kx' - Rv' + F_d \cos\omega t \quad \text{or} \quad \frac{F_d}{m}\cos\omega t = \frac{d^2x'}{dt^2} + \frac{R}{m}\frac{dx'}{dt} + \omega^2 x' \ .$$

Since the system energy is lost through friction and may be gained through the driver action, solutions of this expression will be *time dependent* but with both *transient* and *steady-state* attributes. In many instances resonant systems respond so quickly that one only views the steady-state behavior. In this lab you will be able to observe BOTH the *transient* and *steady-state* processes.

As you may expect the most complete solution of this new differential equation has a rather complicated form and so is not reproduced here. Since we are interested only in resonance we can simplify the expression by assuming solutions that apply to the *underdamped case* (those with oscillatory behavior). Thus the analytic solution reduces to:

$$x' = Ce^{-(R/2m)t}\ \cos(\omega_1 t + D) + \frac{F_d}{[m^2(\omega_0^2 - \omega^2)^2 + \omega^2 R^2]^{1/2}}\cos(\omega t - \phi)$$

where the first term is the *transient* behavior, identical to that of the simple damped harmonic oscillator (described in the last section) and the second term is the *steady-state* solution. At large times t the first terms dies out exponentially so that x' is approximated by only

$$x'((R/2m)t \gg 1) = \frac{F_d}{[m^2(\omega_0^2 - \omega^2)^2 + \omega^2 R^2]^{1/2}}\cos(\omega t - \phi)$$

where ω_0 is the natural frequency of the undamped harmonic oscillator, ω is the mechanical driver frequency, R is the drag coefficient and ϕ is a measure of the phase difference between the driver motion and the cart motion.

In this lab we will only investigate the nature of the cart displacement with driver frequency (ω) in the vicinity of the resonant frequency. Thus the only relationship of interest becomes amplitude $\propto [m^2(\omega_0^2 - \omega^2)^2 + \omega^2]^{-1/2} \equiv Z^{-1/2}$. Z is a minimum when the driver frequency is set to $\omega_0^2 - R^2/(2m^2)$ or $\omega_1^2 - R^2/(4m^2)$ which is defined to be ω_2. Since $R^2/2m^2$ is small (the PASCO track is a low friction experiment) ω_0 is nearly the *same* and, in addition, x' will be sharply peaked about ω_0.

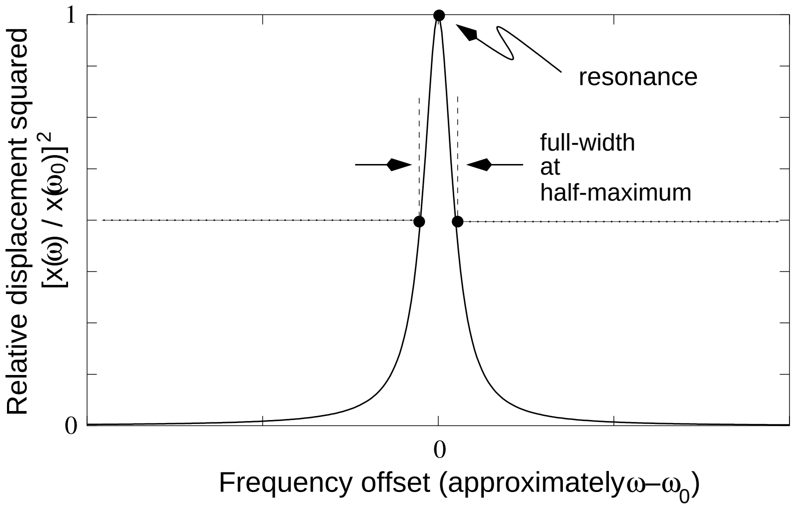


Figure 4: Sketch of the amplitude squared vs driver frequency.

SUGGESTED PROCEDURE:

1. **EXPERIMENT VI:** Use the rolling cart with the aluminum plate. Weigh the cart and predict the natural frequency for this new arrangement.

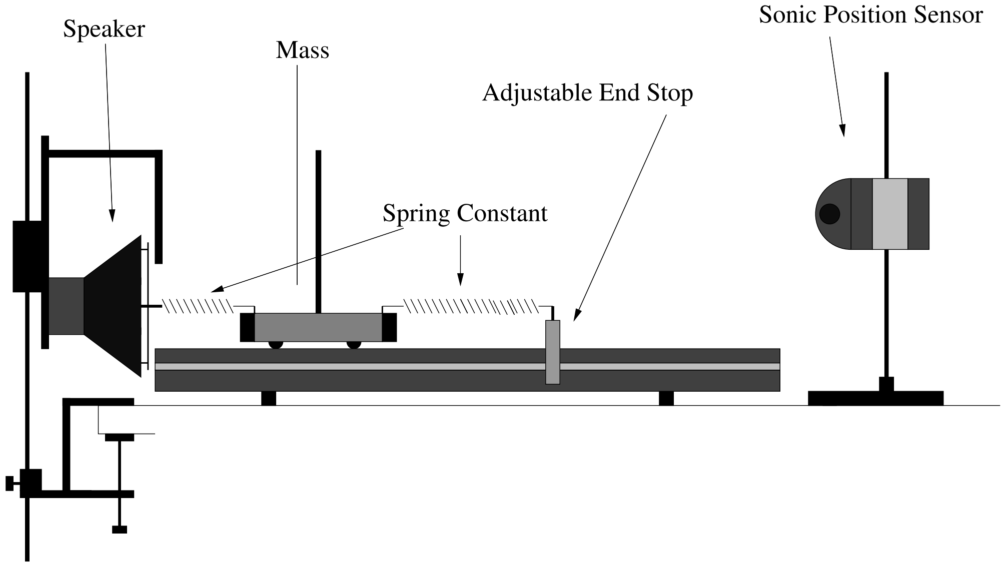


Figure 5: Sketch of the dynamic track with the position sensor and speaker.

2. Place the position sensor approximately 60 cm from the vane in the direction of oscillatory motion. Make sure the yellow phone jack is in the third slot and the black phone jack is in the fourth slot. Alignment is very important so that the sensor senses only the vane and not the cart. A slight upward tilt may help.

3. Detach the fixed spring end stop and place the speaker as shown in the figure above with the spring looped through the small slot in the speaker driver stem using the

same considerations for the spring extension as in the previous experiments.

4. Make sure the speaker power leads are plugged into the amplifier module and that its power is turned on. Also verify that the DIN-9 pin connector (from the amplifier module) is plugged into the A position in the PASCO interface module.

5. CLICK on the telescope icon below to initiate the PASCO© interface software.

6. Make sure the speaker driver window has been switched to the **Off** position. Displace the cart approximately 20 cm from equilibrium to initiate the oscillatory motion. CLICK on the the REC button to start your data acquisition. The graph will simultaneously display glider position and velocity versus time and the amplifier current (which should be zero). Practice a few times to make sure you can obtain smoothly varying sinusoidal curves.

7. Displace the cart approximately 20 cm from equilibrium to initiate the oscillatory motion. CLICK on the the REC button to start your data acquisition and record enough cycles to see the amplitude diminish by 90%. Compare your data to Fig. 3 and verify that your cart has a similar transient behavior. Determine the *natural frequency* of the system and compare to your prediction. (This frequency is actually ω_1 but it is, for this low friction set-up, nearly the same as either ω_2 or ω_0.) To observe the steady-state properties of resonance you will have to wait for times *longer* than those required for this step.

8. Moving to the PASCO Signal Generator window (shown in Fig. 6), set the amplifier frequency (which is in Hertz or cycles per second) to the closest to resonance (0.01 Hz steps) and engage the speaker but CLICKing the **Auto** icon. The driver output should use the sinusoid AC waveform and, if the overload light on the Power Amplifier flashes on, reduce the voltage setting slightly. Start the data acquisition (and speaker motion) by CLICKing on the REC button and record data until you achieve steady-state behavior. You have to wait a couple of minutes before you reach steady state.

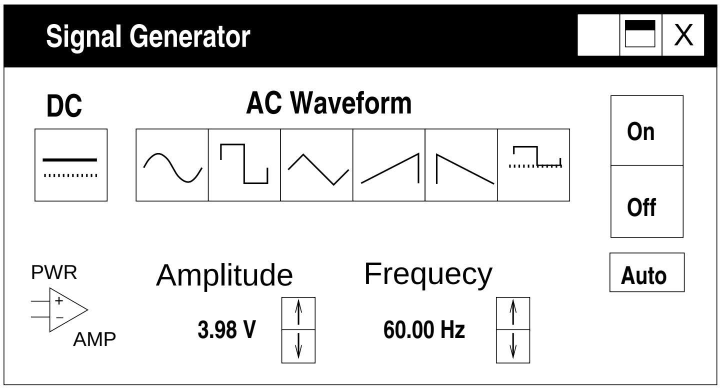


Figure 6: Sketch of the PASCO software Signal Generator window.

NOTE: The nominal step sizes for adjusting the amplifier frequecy and voltage are *very* large. To reduce the step size depress either the CTRL key (1 Hz), the ALT key (0.1 Hz), or both (0.01 Hz) while simultaneously CLICKing the Up or Down arrows.

9. Set the amplifier frequency 0.01 Hz steps above and below resonance and record data until you achieve steady-state behavior. Repeat for 0.10 Hz and (if time permits) 0.40 Hz steps. Plot out a few representative sets of data.

10. Determine the maximum steady-state displacement of the cart for each of the measured frequencies and plot the relative amplitude squared $[(x'(\omega)/x'(\omega_0))^2]$ vs frequency offset $(\omega - \omega_0)$. Estimate the full width at half maximum for this curve. This value should be equal to R/m.

11. Discuss the nature of this resonance curve. If you adjust the R/m ratio to further sharpen the resonance curve can you identify a compensating complication if you are interested in achieving steady-state behavior? If the speaker were attached to an amplifier playing audible music (nominally 20-20,000 Hz), what do you think this mass will do?

MC-15b Simple Harmonic Motion and Resonance (Air Track)

OBJECTIVES:

1. To study the period of Simple Harmonic Motion (SHM) as a function of oscillation amplitude.
2. To study the period of SHM as a function of oscillating mass.
   Expected result: Period($\equiv\ T\ =\ 2\pi/\omega\ =\ 1/f$)
   is proportional to $\sqrt{\text{Mass/Spring Constant}(k)}$
3. To demonstrate Hooke's Law, $F = -kx$
4. To observe the relationships between the potential and kinetic energy

THEORY:

The restoring force (F) on an object attached to a "simple" one-dimensional spring is proportional to the displacement from equilibrium and has the form, $F = -k(x - x_0)$, where k is the spring constant (or stiffness in N/m), x_0 is the equilibrium position (i.e., no net force) and x is the position of the object. This is Hooke's Law. Remember that the simple harmonic oscillator is a good approximation to physical systems in the real world, so we want to understand it well. That's the purpose of this lab!

The expression $F = ma = -k(x - x_0)$ is a 2nd order differential equation with

$$a = \frac{d^2x'}{dt^2} = -\frac{k}{m}(x')$$

where $x' \equiv x - x_0$. The most general solution for this expression is often given as

$$x'(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$$

where $\omega_0 \equiv \sqrt{k/m}$. ω_0 is defined as the *natural frequency* for the *undamped* harmonic oscillator. A and B are arbitrary initial displacement parameters. Alternatively the solution is often specified as

$$x'(t) = C\sin(\omega_0 t + \phi) \qquad \text{or} \qquad x'(t) = C\cos(\omega_0 t + \phi)$$

where ϕ is the starting phase and C is the displacement. It is also possible to write down these solutions using complex numbers as in

$$x'(t) = A'e^{i\omega_0 t} + B'e^{-i\omega_0 t} \quad .$$

The relative merit of these equivalent expressions will become clearer in EXPTS. V and VI.

FUNDAMENTAL CONCEPTS:

1. The solution to the undamped (i.e., no frictional or drag forces) harmonic oscillator is time dependent and periodic. When the $\omega_0 t$ term varies by 2π (or one period $T = 1/f = 2\pi/\omega_0$) both the position, $x'(t+T) = x'(t)$, and velocity, $v'(t+T) = v'(t)$ return to their previous values.

2. Total energy (TE) is conserved in SHM motion. As time evolves kinetic energy (KE) is transferred to (and from) potential energy (PE). Thus at any time t:

$$TE = \text{constant} = KE(t) + PE(t) = \frac{1}{2}m[v'(t)]^2 + \frac{1}{2}k[x'(t)]^2 \quad .$$

APPARATUS:

Basic equipment: Air track, assorted slotted masses, air supply, hose, adjustable stop, glider, springs, timer, photogate & support stand, knife edge assembly.

Computer equipment: Personal computer set to the MC15A lab manual web-page; PASCO interface module; photogate sensor and extension jack, PASCO sonic position sensor, speaker with driver stem, power amplifier module.

PRECAUTIONS:

1. See MC-14b
2. **When setting up the springs, use the adjustable end stop on the air track so the springs are stretched properly: don't stretch them beyond their elastic limit and don't let them sag and drag on the track**.
3. *Keep the amplitudes small enough that a slack spring doesn't touch the track nor a stretched spring exceed its elastic limit.*

SUGGESTIONS: To measure the period of the oscillating glider:

1. Locate the photogate so the glider just cuts off beam when glider is in the equilibrium position ($x - x_0 = 0$). The photogate phone jack should be in the first PASCO interface position. Choose two springs having a similar length and refer to the above precaution. Turn on the air supply adjust the blower speed to minimize frictional forces.
2. To initiate the PASCO interface software click the computer mouse when centered on the telescope icon in the "toolkit" area below. There will be a just a single table for recording the measured period.
3. Start the glider by displacing it from equilibrium and then releasing it. Then start the data acquisition by clicking the REC button button. Let the glider oscillate for about 10 periods. Calculated the mean and standard deviation by simply clicking on the statistics icon (i.e. $\boxed{\Sigma}$) on the data table.

UNDAMPED SIMPLE HARMONIC MOTION:

1. Using sketches in your lab book, both at equilibrium and after a displacement from equilibrium, show that the effective force constant for 2 identical springs of force constant k on either side of an oscillating mass is $2k$.
2. **EXPERIMENT I:** Show experimentally that the period is independent of the amplitude. Try amplitudes of *approximately* 10 cm, 20 cm, and 30 cm. (Friction may be a problem at very small amplitudes).

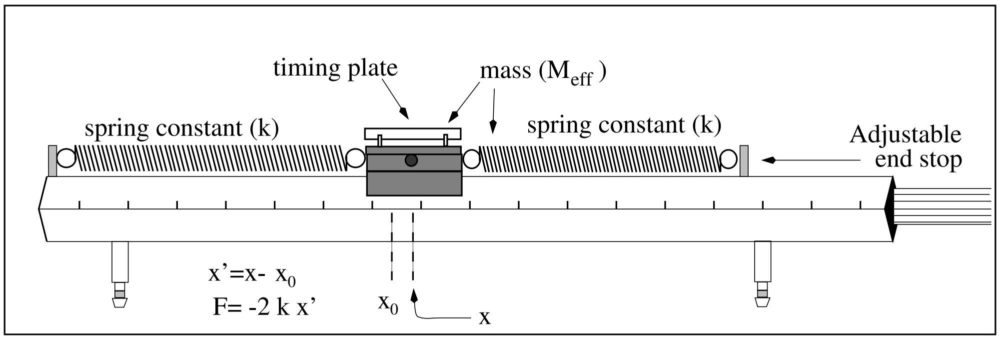


Figure 1: Sketch of the air track configuration for SHM.

3. **EXPERIMENT II:** By adding mass to the glider, study the period versus total oscillating mass. The latter must include a correction for the oscillating springs whose effective oscillating mass (see the note below) is approximately $m_s/3$ where m_s is the mass of the *two* springs. Explain, in words, how this correction may be qualitatively justified (Why not m_s or $m_s/2$?) By inspection of your T vs $[M + (m_s/3)]$ curve, what function of T might yield a straight line when plotted versus $[M + (m_s/3)]$? Prepare this plot. Calculate the effective force constant, k', (equal to $k_1 + k_2$, the two springs will actually differ slightly) of the system from the slope of this straight line graph. (Remember $T = 2\pi\sqrt{M_{\text{eff}}/k'}$ where $M_{\text{eff}} \simeq \text{M} + \text{m}_s/3$.) Estimate the uncertainty in k' by solving for k' at each glider mass and calculating the standard deviation of the mean.

   NOTE: Theoretically $m_{\text{eff}} = m/3$ only for $M/m_s = \infty$ For $M = 0$, the effective mass is $4m/\pi^2 = 0.405$ m. However for $M/m_s = 5$, the effective mass is already $\simeq 0.336\ m_s$. See Fig. 1 of J.G. Fox and J. Makanty, *American Journal of Physics*, **38**, 98 (1970).

4. **EXPERIMENT III :** Also measure k directly by hanging the two springs vertically on a "knife edge" assembly in the laboratory. Record the stretch produced by a series of weights, *but do not exceed the elastic limit of either spring!* Graph F vs y for each spring and obtain a best-fit straight line. The slope should be the spring constant. Compare this value of k' with that obtained in part #3.

5. QUESTION: Assuming the spring constant doubles, how would T vary?

FURTHER INVESTIGATIONS OF "UNDAMPED" SIMPLE HARMONIC MOTION:

Up to this point you have characterized the SHM of a spring-mass assembly in terms of only a single parameter T (the period). One of the possible time-dependent expressions for describing the motion was

$$x'(t) = A\cos(\omega_0 t + \phi_0) \quad \text{and} \quad v'(t) = -A\omega_0 \sin(\omega_0 t + \phi_0)$$

or

$$v'(t) = \ A\omega_0 \cos(\omega_0 t + \phi_0 - \frac{\pi}{2})$$

where A is the amplitude (displacement from equilibrium), ω_0 is the *natural* angular frequency and ϕ_0 is the starting phase. Notice that the velocity has a *phase* shift of

$\frac{\pi}{2}$ relative to the displacement.

Equally characteristic of SHM is the process of energy transference: kinetic energy of motion is transferred into potential energy (stored in the spring) and back again. Friction is an ever present energy loss process so that the total energy always diminishes with time.

To capture this rather rapid cyclic process we will again use the PASCO interface while replacing the photogate sensor with the sonic position sensor.

EXPERIMENT IV: Measuring the x vs t behavior:

1. Remove the 10 cm timing plate and plug the aluminum vane into the central banana plug position with the vane perpendicular to the long axis of the glider. The mass of this arrangement will now have changed slightly; predict the new natural frequency.

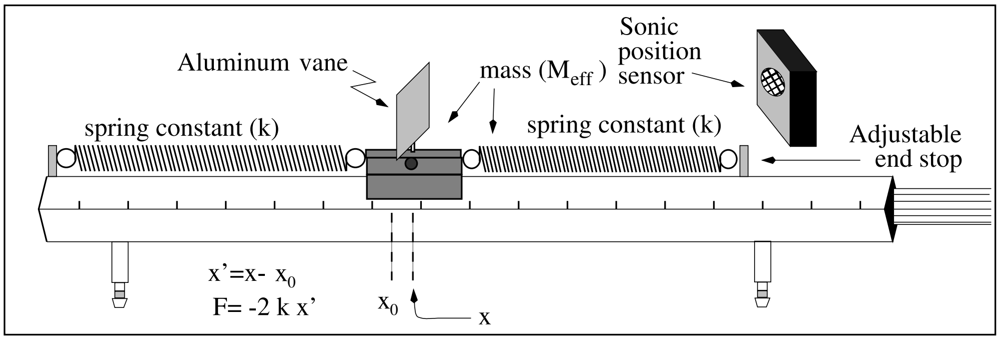


Figure 2: Sketch of the air track with the position sensor.

2. Place the position sensor approximately 60 cm from the vane in the direction of oscillatory motion. Make sure the yellow phone jack is in the third slot and the black phone jack is in the fourth slot. Alignment is very important so that the sensor senses only the vane and not the cart. A slight upward tilt may help (or raising the vane up slightly as well).

3. CLICK on the telescope icon below to initiate the PASCO© interface software.

4. Displace the cart approximately 20 cm from equilibrium to initiate the oscillatory motion. CLICK on the the REC button to start your data acquisition. The graph will simultaneously display both absolute position and velocity versus time.

5. Practice a few times to make sure you can obtain smoothly varying sinusoidal curves. Then run the data acquisition for just over ten cycles and use the cross-hair feature to read out the time increment for ten full cycles. How does your prediction check out? Determine the initial phase (i.e., at $t = 0$). Record the equilibrium position (x_0) as well.

6. Use the magnification option of the PASCO software to better view a single full cycle by clicking on the magnifying glass icon (in the graph window) and then select two points in the graph using a CLICK and DRAG motion of the mouse.

7. Print out (click in graph region and then type ALT, CTRL-P) or, alternatively, sketch the position and velocity curves in your lab book identifying key features in the time dependent curves. In particular identify the characteristic(s) which demonstrate the $\pi/2$ phase difference between the velocity and displacement curves.

8. Using the PASCO cross-hair option to read out the relevant time, position and velocity, make a table as below and fill in the missing entries (identify units).

time	phase($\omega_0 t + \phi_0$)	$x'(t)$	$v'(t)$	KE	PE	TE
	0					
	$\pi/4$ (45 deg)					
	$\pi/2$ (90 deg)					
	$3\pi/4$ (135 deg)					
	π (180 deg)					

9. Is the total energy a constant of the motion?

10. How much displacement amplitude and energy are lost after five full cycles? What is the approximate friction coefficient?

OPTIONAL INVESTIGATIONS OF "UNDER-DAMPED" SIMPLE HARMONIC MOTION:

In the real world friction is an ever present process. In the case of SHM friction friction can have a profound effect. Damping of unwanted vibrations is important in a myriad of situations. (Imagine what driving a car would be like if there were no shock absorbers!)

Introducing friction can be done by simply adding one more term in the force expression, $F_{\text{drag}} \equiv -Rv$, a drag force which is proportional to the velocity where R is the drag coefficient [units of kg m/s or N/(m/s)] . This is appropriate for motion thru a viscous fluid but it is really only a rough approximation for the frictional forces in the air track. The modified force expression now becomes

$$F = ma = -kx' - Rv' \quad \text{or} \quad 0 = \frac{d^2x'}{dt^2} + \frac{R}{m}\frac{dx'}{dt} + \omega^2 x'$$

Since energy is continually lost solutions of this expression will be *time dependent* but NOT periodic. Adding this "simple" term dramatically complicates the process of finding appropriate solutions. The most general form of the solution is

$$x' = e^{-(R/2m)t} \left[Ae^{+\sqrt{(\frac{R^2}{4m^2} - \omega_0^2)}\ t} + Be^{-\sqrt{(\frac{R^2}{4m^2} - \omega_0^2)}\ t} \right]$$

which is quite formidable. Since the air track drag is low (i.e., R is relatively small) the solution is said to be *underdamped* and oscillatory when $\omega_0^2 > R^2/4m^2$. The solution in this case becomes:

$$x' = Ce^{-(R/2m)t}\cos(\omega_1 t + D) \quad \text{where} \quad \omega_1 \equiv \omega_0^2 - \frac{R^2}{4m^2}$$

where ω_1 is the *natural frequency* of the *underdamped* system and C and D are the initial displacement and phase. Because the frequency is lowered, the period lengthens. This is consistent with one's intuition; drag works against oscillatory motion.

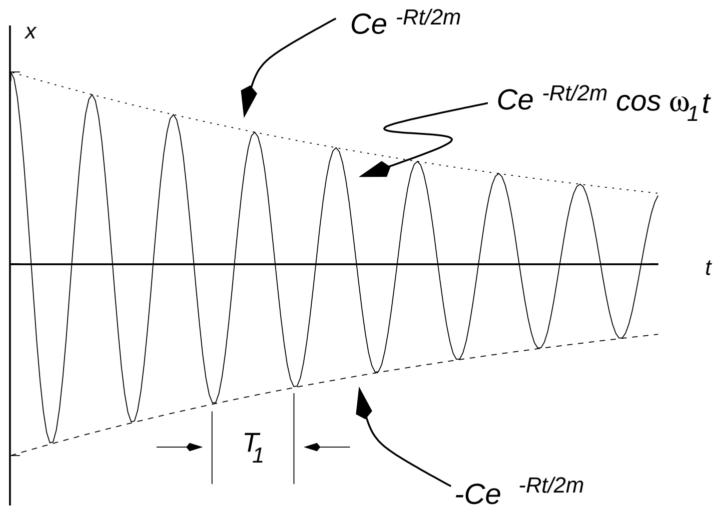


Figure 3: Sketch of underdamped harmonic motion with $\omega_0^2 > R^2/4m^2$.

EXPERIMENT V:

1. Reduce the air flow to the air track so that the amplitude of the glider motion diminishes by 50% (or so) in a few minutes. Typically the lowest blower setting will work well enough. (Make sure the other group is also ready.)

2. CLICK on the telescope icon below to initiate the next PASCO© interface application. In addition to the graph there will be a "two" column table displaying the time and position then the time and velocity.

3. Displace the cart approximately 20 cm from equilibrium to initiate the oscillatory motion. CLICK on the the REC button to start your data acquisition and record enough cycles to see the amplitude diminish by two-thirds.

4. Repeat the experiment at full blower speed for approximately the same period of time. Can you distinguish the difference between "ω_0" and ω_1 (T_0 and T_1)?
   HINT: The graph display for the position is configured to overlay the data sets. Use the magnifying glass icon feature to examine the relative phase difference at early time and late time.

5. If so, from the formula $\omega_1^2 = \omega_0^2 - R^2/4m^2$ determine R.

6. For the reduced air flow data, select six representative times using the table (identifying where the velocity changes sign) and make a table of t vs maximum displacement. Use these points in the graphical analysis package and fit these points to the expression $C \exp -(R/2m)t$ or, in terms of the explicit analysis formula, $y = C * \exp[-B * (x - x_0)]$.

7. Does this value compare favorably with the results of item 4? How good or poor is the assumption that the drag force is proportional to the velocity?

8. Sketch out an approximate curve of x vs t if R were significantly larger. Which R would be more appropriate for absorbing and dissipating a physical "shock" (and why)?

OPTIONAL INVESTIGATIONS OF RESONANCE:

One of the most important situations of the harmonic oscillator is that of FORCED, damped harmonic motion. In one-dimension the applied force is typically sinusoid and when ω approaches the natural frequency of the system (nominally ω_1 and, if $R^2/4m^2 \ll 1$, also ω_0) the energy of the driver is additively coupled to the moving mass (e.g., a glider) and resonance occurs. Resonance is a *very* important aspect of the world around us and many mechanical and electronic devices employ resonant behavior as a fundamental aspect of their operation (e.g., musical instruments, radios, televisions).

Along with the frictional drag (Rv' where R is the drag coefficient) one more force term must be added, that of the mechanical driver, with $F_{\text{driver}} = F_d \cos(\omega t)$. The new force expression is conventionally written as:

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As you may expect the most complete solution of this new differential equation has a rather complicated form and so is not reproduced here. Since we are interested only in resonance we can simplify the expression by assuming solutions that apply to the *underdamped case* (those with oscillatory behavior). Thus the analytic solution reduces to:

$$x' = Ce^{-(R/2m)t} \cos(\omega_1 t + D) + \frac{F_d}{[m^2(\omega_0^2 - \omega^2)^2 + \omega^2 R^2]^{1/2}} \cos(\omega t - \phi)$$

where the first term is the *transient* behavior, identical to that of the simple damped harmonic oscillator (described in the last section) and the second term is the *steady-state* solution. At large times t the first terms dies out exponentially so that x' is approximated by only

$$x'((R/2m)t \gg 1) = \frac{F_d}{[m^2(\omega_0^2 - \omega^2)^2 + \omega^2 R^2]^{1/2}} \cos(\omega t - \phi)$$

where ω_0 is the natural frequency of the undamped harmonic oscillator, ω is the mechanical driver frequency, R is the drag coefficient and ϕ is a measure of the phase difference between the driver motion and the glider motion.

In this lab we will only investigate the nature of the glider displacement with driver frequency (ω) in the vicinity of the resonant frequency. Thus the only relationship of interest becomes $x' \propto [m^2(\omega_0^2 - \omega^2)^2 + \omega^2]^{-1/2} \equiv Z^{-1/2}$. Z is a minimum when the driver frequency is set to $\omega_0^2 - R^2/(2m^2)$ or $\omega_1^2 - R^2/(4m^2)$ which is defined to be ω_2. Since $R^2/2m^2$ is small (the air-track is a low friction experiment) ω_0 is nearly the *same* and, in addition, x' will be sharply peaked about ω_0.

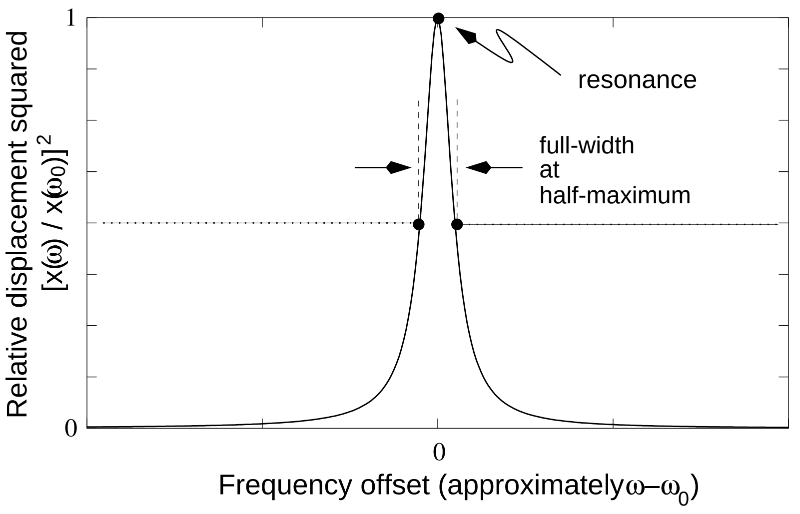


Figure 4: Sketch of the relative maximum displacement squared vs driver frequency.

SUGGESTED PROCEDURE:

1. **EXPERIMENT VI:** Make sure the 10 cm timing plate is replaced by the aluminum vane (in the center banana plug position) with the vane perpendicular to the long axis of the glider. Predict the natural frequency for this new arrangement.

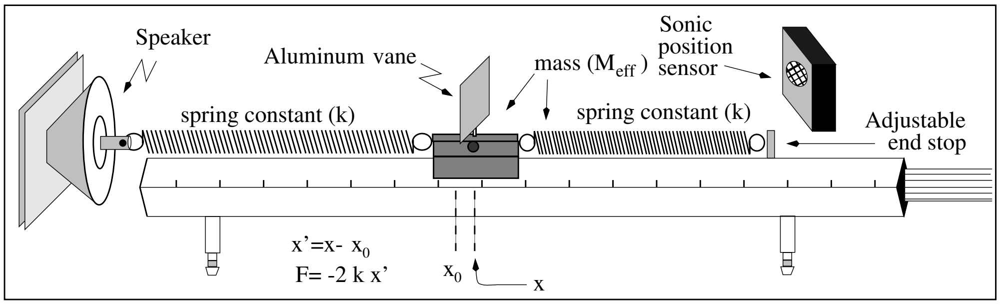


Figure 5: Sketch of the air track with the position sensor and speaker.

2. Place the position sensor approximately 60 cm from the vane in the direction of oscillatory motion. Make sure the yellow phone jack is in the third slot and the black phone jack is in the fourth slot. Alignment is very important so that the sensor senses only the vane and not the cart. A slight upward tilt may help (or raising the vane up slightly as well).

3. Detach the fixed spring end stop and place the speaker as shown in the figure above with the spring looped through the small slot in the speaker driver stem using the same considerations for the spring extension as in the previous experiments.

4. Make sure the speaker power leads are plugged into the amplifier module and that its power is turned on. Also verify that the DIN-9 pin connector (from the amplifier module) is plugged into the A position in the PASCO interface module.

5. CLICK on the telescope icon below to initiate the PASCO© interface software.

6. Make sure the speaker driver window has been switched to the **Off** position. With maximum blower airflow, displace the cart approximately 20 cm from equilibrium to initiate the oscillatory motion. CLICK on the the REC button to start your data acquisition. The graph will simultaneously display glider position and velocity versus time and the amplifier current (which should be zero). Practice a few times to make sure you can obtain smoothly varying sinusoidal curves.

7. Reduce the air flow to the air track so that the amplitude of the glider motion diminishes by 50% (or so) in a few minutes. Typically the lowest blower setting will work well enough. (Make sure the other group is also ready.)

8. Displace the cart approximately 20 cm from equilibrium to initiate the oscillatory motion. CLICK on the the REC button to start your data acquisition and record enough cycles to see the amplitude diminish by 90%. Compare your data to Fig. 3 and verify that your glider has a similar transient behavior. Determine the *natural frequency* of the system and compare to your prediction. (This frequency is actually ω_1 but it is, for this low friction set-up, nearly the same as either ω_2 or ω_0.) To observe the steady-state properties of resonance you will have to wait for times *longer* than those required for this step.

9. Moving to the PASCO Signal Generator window (shown in Fig. 6), set the amplifier frequency (which is in Hertz or cycles per second) to the closest to resonance (0.01 Hz steps) and engage the speaker but CLICKing the **Auto** icon. The driver output should use the sinusoid AC waveform and, if the overload light on the Power Amplifier flashes on, reduce the voltage setting slightly. Start the data acquisition (and speaker motion) by CLICKing on the REC button and record data until you achieve steady-state behavior.

   NOTE: The nominal step sizes for adjusting the amplifier frequecy and voltage are *very* large. To reduce the step size depress either the CTRL key (1 Hz), the ALT key (0.1 Hz), or both (0.01 Hz) while simultaneously CLICKing the Up or Down arrows.

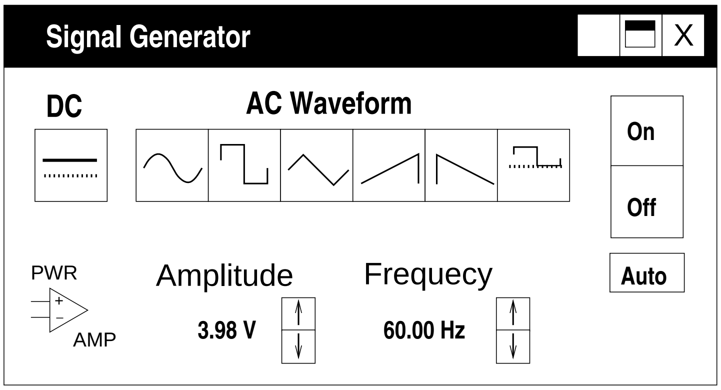


Figure 6: Sketch of the PASCO software Signal Generator window.

10. Set the amplifier frequency 0.01 Hz steps above and below resonance and record data until you achieve steady-state behavior. Repeat for 0.10 Hz and (if time permits) 0.40 Hz steps. Plot out a few representative sets of data.

11. Determine the maximum steady-state displacement of the glider for each of the measured frequencies and plot the relative amplitude squared $[(x'(\omega)/x'(\omega_0))^2]$ vs frequency offset $(\omega - \omega_0)$. Estimate the full width at half maximum for this curve. This value should be equal to R/m.

12. Discuss the nature of this resonance curve. If you adjust the R/m ratio to further sharpen the resonance curve can you identify a compensating complication if you are interested in achieving steady-state behavior? If the speaker were attached to an amplifier playing audible music (nominally 20-20,000 Hz), what do you think this the mass will do?

Heat:

HC-1 The Ideal Gas Law and Absolute Zero

OBJECTIVES:

To observe the behavior of an "ideal" gas (air) and to determine absolute zero (in Celsius).

FUNDAMENTAL CONCEPTS:

The behavior of an ideal gas under varying conditions of pressure and temperature is described by the "Ideal Gas Law":

$$PV = nRT \tag{1}$$

Where P is the pressure (in Pascals, Pa), V is the volume (in m^3), T is the temperature in Kelvins (K= C+273), n is the number of moles present (1 mole $\equiv 6.023 \times 10^{23}$), and R is the gas constant ($8.31\ J/mol \cdot K$).
This equation is a combination of two laws which were discovered previously:

(1) *Boyle*'s Law relates volume and pressure of a fixed quantity of gas at a constant temperature: $PV = constant$
(2) *Charles'* Law relates volume and temperature of a fixed quantity of gas at a constant pressure $V/T = constant$

Gay-Lussac finally combined the two laws into the ideal gas law shown in Eqn. 1.
In EXPT. I Boyle's law will be verified by varying V and P (assuming fixed T).
In EXPT. II You will vary T and V to determine the value of absolute zero.

APPARATUS:

Basic equipment: EXPT. I - Lab stand; 60 cm^3 plastic syringe attached to a plastic "quick-connect" coupling.
EXPT. II - PASCO Steam generator; water jacket container; steel Dewar flask; small stainless steel can with a volume of 98 cm^3 with a "quick release" connector; FLUKE digital thermometer and temperature probe

Computer equipment: Personal computer set to the HC1 lab manual web-page; PASCO interface module; PASCO pressure sensor (mounted on lab stand).

EXPERIMENT I: BOYLE'S LAW

1. CLICK on the telescope icon below to initiate the PASCO© interface software. The computer, monitor and PASCO interface must already be on and pressure sensor plugged into the **A** DIN connector position. (If not see your instructor.)
2. The PASCO software window should display graphs configured to display pressure *vs* volume and pressure *vs* inverse volume, a table for these three values, a panel meter for the instantaneous pressure reading.
Since there is no automated volume measurement you will have to enter these data points manually.
NOTE: The volume is recorded in *milliliters* (1 mL = 1 cm^3) and the pressure in *kilopascals* (kPa). In this case the ideal gas law ($PV = nRT$, T in Kelvin) uses $R = 8.31 \times 10^3$(kPa·cm^3)/(mol·K).

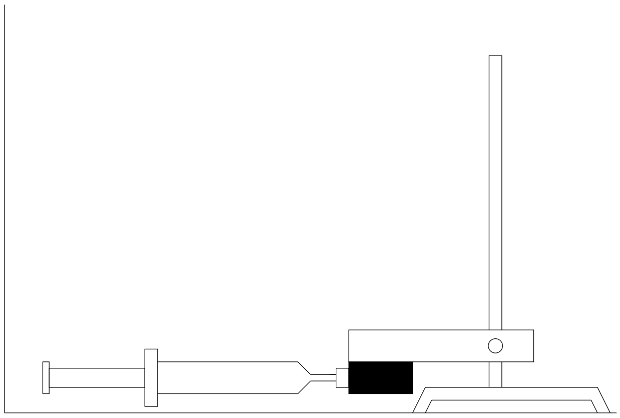


Figure 1: Sketch of pressure sensor and syringe/plunger assembly.

3. If the syringe is attached to the pressure sensor, disconnect it at the twist-lock quick connect coupling. Set the syringe plunger to the 60 cm^3 mark and then reconnect it to the pressure sensor. Make sure that the equipment looks as sketched in Fig. 1.
4. Initiate the data acquisition by clicking on the REC button in the upper left panel. A large window for keyboard entry will appear (i.e. Keyboard Sampling). There is no automated volume measurement for the plunger position so you will use the keyboard to enter data points manually.
   Slowly push the plunger in to the 50 cm^3 position (about 10 seconds) and back to the original setting while watching the panel pressure meter and record in your lab-book the readout precision of the pressure sensor.
5. Now begin the actual data logging by clicking the mouse on the numeric entry field in the Keyboard Sampling window, replacing the current value with 60.0 and then striking the *Enter* key. Data logging should be confirmed by observing a new data point on the plots and three new values in first row of the table. The PASCO software will try to *anticipate* your next volume setting in the Keyboard Sampling window. You should change this value to the appropriate value before logging your next data point.
6. Reduce the volume slowly in discrete 5 cm^3 increments while logging the data at each step (see the previous item) until you reach 20 cm^3. Then as a check, slowly increase the volume in 5 or 10 cm^3 steps (remember to log the data) until you reach 60 cm^3.
7. Stop the data acquisition and transfer your data to a table in your write-up and comment on the reproducibility of your measurements. Do your plots have the correct functional behavior? Your can refrain from printing out the graphs until your have performed a linear regression using either the PASCO graphical analysis capabilities or the Vernier GA graphical analysis package.
8. Which of the two graphs do you expect to be a straight line and why? For this "curve" what is the expected y intercept? Now fit this data to a line (using linear regression) and record the slope and intercepts with the appropriate units. Print out the respective plots and include them in your lab write-up.
9. OPTIONAL: Repeat the procedure of item 4 except move from 60 to 20 cm^3 as rapidly as possible and record the pressure data at the start and stop points. Are

the pressure readings consistent with those of the "slow" moving experiment? If not can you suggest a reason?

QUESTIONS:

I. Does the line go through the origin as expected? By how much should you change the pressure readings so that the line goes through the origin?

II. What are the possible sources of error in this experiment? For each source, try to decide what effect it might have on the experimental results.

III. Read the barometer that is located in your laboratory room, ask your instructor for help if you are having trouble. Convert the pressure from cm of Hg to kPa and compare this value with the one obtained at 60 cm^3 in step 5. Is the difference between the two readings important? Does it affect your conclusions?

IV. OPTIONAL: Assuming that there are 22.4 liters per mole of an ideal gas, how well does your observed slope agree with your expected slope?

EXPERIMENT II: IDEAL GAS LAW

In this part the Ideal Gas Law is verified by observing the pressure of a fixed volume of gas at four different temperatures: Room temperature, 100 °C, 0 °C and -197 °C. Since $PV = nRT$ and $T = T_{\text{Celsius}} + T_0$ we can rewrite this as

$$P(T_c) = \frac{nR}{V}T_c + \frac{nR}{V}T_0$$

and thereby determine the value of absolute zero (T_0) from the intercept.

SUGGESTED PROCEDURE:

1. CLICK on the telescope icon below to initiate the PASCO© interface software. The computer, monitor and PASCO interface must already be on and pressure sensor plugged into the **A** DIN connector position. (If not see your instructor.)
2. Attach the small stainless steel can (volume of 98 cm^3) as shown below to the PASCO pressure sensor using the plastic "quick release" connector. Raise the assembly up if necessary. Turn on the Fluke thermometer. Unless the stainless steel can has been recently used it should now be at room temperature.
3. Initiate the data acquisition by clicking on the REC button. There should be a single plot (temperature vs pressure), a panel meter for pressure and a keyboard entry window. Record the Fluke meter reading in the keyboard entry window (if uncertain see EXPT. I, item 5 for a more complete description). Pause the data acquisition by clicking on the PAUSE button.
4. Fill the PASCO steam generator with water and turn it on. The level of the water should be $\sim 3\ cm$ below the top, or about 1/2 – 3/4 full (Fig. 3). Turn the knob to 9 until the water boils and then turn it down somewhat.
5. As soon as the water in the steamer is boiling, raise the pressure sensor stand and then lower it so as to immerse the stainless steel can in the boiling water.

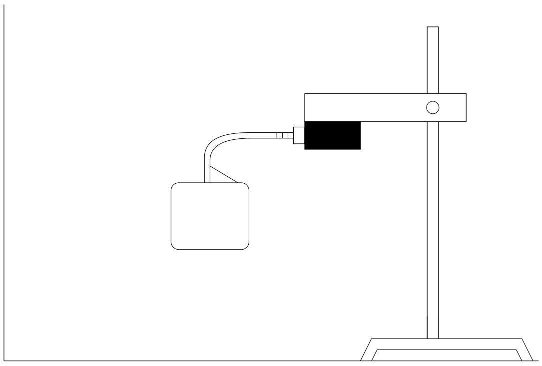


Figure 2: Sketch of pressure sensor and stainless-steel can assembly.

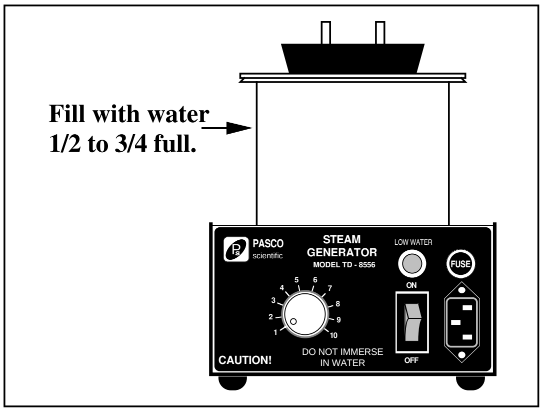


Figure 3: PASCO steam generator.

6. Wait until the water is boiling again and then put the Fluke thermometer in the boiling water. Resume the data acquisition by clicking on the REC button. Watch the pressure readout and after it is stable (with time) record the Fluke temperature in the data entry window. Turn the PASCO steam generator off and remove the small stainless steel can from the bath. Pause the data acquisition.

7. Fill the water jacket container with water and ice. The level of the water should be $\sim 5\ cm$ below the top. Repeat the last two steps for obtaining the 0 °C pressure reading. Empty and dry the stainless steel dewar.

8. Take the stainless steel dewar and ask the instructor to fill your dewar with liquid Nitrogen and repeat step 3 for the container at liquid Nitrogen temperature.

9. Stop the data acquisition and transfer your data to a table in your write-up. Do your plot have the correct functional behavior? Your can refrain from printing out the data until your have performed a linear regression using either the PASCO graphical analysis capabilities or the Vernier GA analysis package.

10. For your curve what is the expected y intercept? Now fit this data to a line (using linear regression) and record the slope and intercepts with the appropriate units. Print out the plot and include it in your lab write-up.

QUESTIONS

I. What is the percentage difference between the value you found and the accepted value for absolute zero?

II. Assuming that ice water is exactly at 0 °C and the boiling water is at 100 °C, estimate the systematic error introduced into your absolute zero measurement? Does this improve your results?

III. OPTIONAL: Qualitivity what error results from the gas in the small tube not being always at the can temperature?

IV. OPTIONAL: Does thermal expansion of the can affect your results? In what way?

H-2 Latent heat of fusion of ice

OBJECTIVE: To measure the latent heat of fusion, L_f, of ice.
APPARATUS:

Chrome plated brass calorimeter (cup), brass stirrer, water jacket for thermal ballast, digital thermocouple thermometer, ice bucket; ice (opposite 4411 Sterling), double pan balance; 400 ml glass beaker; coffee pot for hot water, selection of slotted masses.

SUGGESTED PROCEDURE:

1) Find the mass of the calorimeter plus stirrer.

2) Add ∼300 g of water at temperature T as far above jacket T as one expects the final T will be below it, (thus minimizing heat exchange with the environment). For this estimate assume that one will add 60 g of ice. **Record the mass of the water**.

3) Record the water temperature in the calorimeter each minute for several minutes while gently stirring.

4) Gently add (without splashing!) the ∼60 g of ice in one or a few pieces after carefully drying each piece with a paper towel. Continue recording the temperature each minute until five minutes after it begins a slow rise.

5) Record the final mass of calorimeter plus contents. **Deduce the mass of ice added**.

6) Plot the temperature vs time as recorded in 3) and 4).

7) From the data calculate L_f of ice. (You may neglect the heat supplied by the thermocouple type digital thermometer.)

8) Estimate quantitatively the error in L_f. [Recall that **absolute** (not relative) errors add when you add or subtract, whereas **relative** errors add when you multiply or divide.

SPECIFIC HEAT CAPACITIES
(in kcal/kg/K or cal/g/K)

water	1.00	glass	0.199
brass	0.090	Hg	0.033

H-3 Latent heat of vaporization of liquid-N_2

OBJECTIVE:

To measure the heat of vaporization of liquid nitrogen, L_v, at its boiling point (T_b = 77 K at standard atmospheric pressure).

APPARATUS:

Dewar flask; liquid nitrogen (ask for help in getting it from a large storage Dewar opposite room 4411 Sterling); aluminum cylinder on a long thread; double pan balance; calorimeter plus water jacket for thermal ballast (as in H-2a); timer; thermocouple type digital thermometer; selection of slotted masses; coffee pot for hot water.

PRECAUTIONS:

Liquid N_2 is fascinating to work with. However, keep in mind the following simple safety precautions.

1. **Never stopper a flask of liquid N_2 with an unperforated stopper.**
2. Have a perforated stopper on the Dewar throughout the experiment to prevent condensation of moisture from the air on the inside of the flask.
3. **Avoid prolonged contact of liquid N_2 with your skin.** The insulating vapor layer may disappear and severe frost-bite may result.

INTRODUCTION:

When one lowers an aluminum cylinder of mass, m_{Al}, and at room temperature, T_r, into liquid N_2 at its boiling temperature, T_b, the cylinder cools to T_b. The heat given off during this cooling, Q_{Al}, will vaporize a mass m_N of liquid N_2.

You might expect to find L_v, of the nitrogen by setting

$$m_N L_v \;=\; Q_{Al} \;=\; m_{Al} c_{Al}(T_r - T_b) \quad .$$

This method fails because c_{Al} is not constant over the $\sim$ 220°C temperature range between T_r and T_b. See figure 1.

We can avoid this difficulty by noting that Q_{Al} is also the heat needed to warm the same aluminum cylinder to from T_b to T_r. You can measure this heat by placing the cold aluminum cylinder (at temperature T_b) in a "calorimeter" that contains water and observing the change in temperature of the water, $-\Delta T$, –provided that the final temperature of the water, T_f, is room temperature, T_r. It is hard to arrange for T_f to end up exactly at room temperature, but if T_f is *close* to T_r, one can accurately correct the calorimeter data for the small additional heat term, namely $m_{Al}c_{Al}(T_r \;-\; T_f)$, since over the small $T_r \;- T_f$ interval, $c_{Al} = 0.212$ cal/g/K is constant.

SUGGESTIONS ON PROCEDURE:

1. To maximize sensitivity (i.e. to get a large temperature change in the water) use only enough water (~125-150 grams) in the calorimeter to cover the metal cylinder.

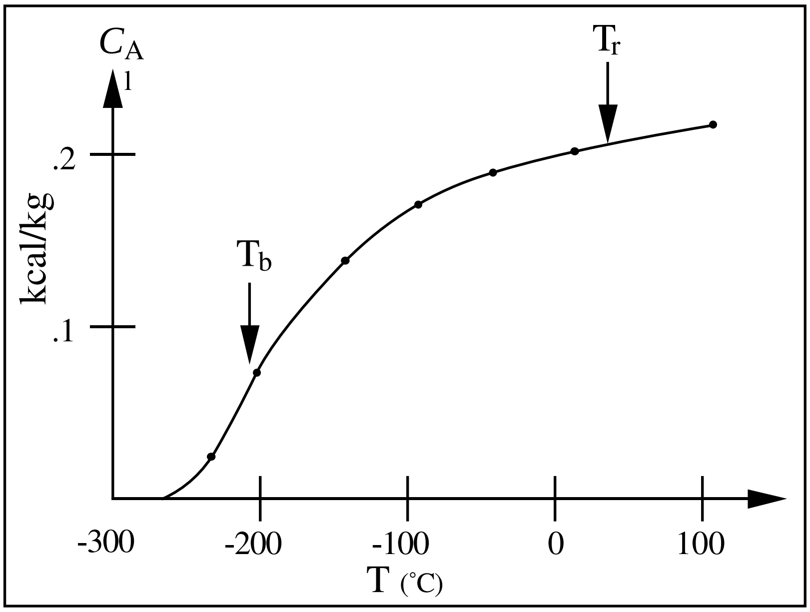


Figure 1: Specific heat capacity of Al vs. temperature.

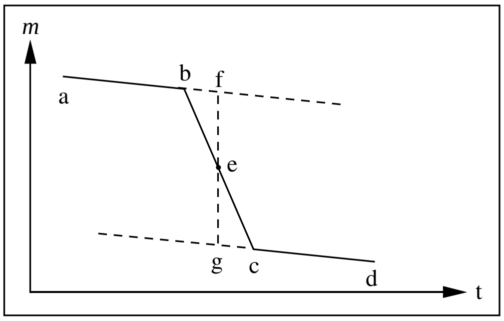
2. The calorimeter is designed to thermally isolate the water from the surroundings. The inner vessel of the calorimeter is mounted within, but thermally isolated from, a surrounding water jacket, which is close to room temperature. The isolation isn't perfect, so here will be some small amount of heat flow between the calorimeter and the jacket. To minimize the *net* heat exchange with the water jacket, you will want to make the initial water temperature as far *above* the jacket temperature as you expect it to end up *below* the jacket temperature after the water in the inner vessel has been cooled by your Al cylinder. (The jacket temperature will remain fairly constant during the experiment.) Estimate roughly the proper initial water and calorimeter temperature. [Use the measured mass of the aluminum cylinder, m_{Al}, its specific heat (0.212 kcal/kg°C) and the b.p. of liquid N_2, $T_b = 77$ K. For this rough calculation, assume that the specific heat of Al *is* constant with temperature.] Determine the water mass, m_w, and appropriate starting temperature.

3. Place the flask containing liquid nitrogen (plus perforated stopper) on one pan of a balance and record the mass each minute for 10 minutes. (Why is the mass decreasing?)

4. Record the temperature of the metal cylinder, T_R, and then lower it (by a thread) gently to the bottom of the flask. Replace the stopper (perforated) on the top of the flask and continue recording the total mass each minute until it shows a slow steady decrease.

5. Record the initial temperature, T_i, of the calorimeter, which you have chosen so cleverly in section 2. Transfer the cold metal cylinder into the calorimeter, and note the calorimeter temperature every two minutes (while gently stirring). Record the mass of the flask of liquid N_2 on alternate minutes. When the calorimeter temperature has reached a slow steady rate of change and the mass of the flask of liquid N_2 is falling at a slow steady rate, discontinue the readings.

6. Plot the mass of the flask plus nitrogen as a function of time. For the minutes that the cylinder of metal was in the flask, subtract the mass of the cylinder. See figure for a typical plot.



How much of the N_2 mass change was caused by the heat from the cylinder? If a-b and c-d were parallel, it would be the vertical distance between these lines. But c-d ordinarily has a smaller slope than a-b, possibly because the evaporation of liquid nitrogen between b and c has cooled the upper part of the flask.

Hence we use the average of the two rates of fall by drawing a vertical line through e, (the midpoint of line b-c). Then f-g estimates the mass, m_N, evaporated by the heat from the cylinder.

7. The final temperature, T_f, of the Al cylinder in the calorimeter usually will not be quite the same as the initial temperature ($= T_r$) of the cylinder before it was lowered into the liquid nitrogen.

Hence Q_{Al} will be the heat to warm the Al cylinder in the calorimeter to T_f *plus* the mass of the cylinder × (specific heat of aluminum) × $(T_r - T_f)$.

Specifically if:

L_v = latent heat of vaporization of nitrogen
m_N = mass of nitrogen evaporated by heat from the cylinder
m_{Al}, c_{Al} = mass and specific heat of Al cylinder
T_i = initial temperature of water and calorimeter
m_w, c_w = mass and specific heat of water
m_c, c_c = mass and specific heat of calorimeter and stirrer
h_t = heat capacity of immersed part of thermometer,

then, if we neglect h_t:

$$Q_{Al} = m_N L_v = (m_w c_w + m_c c_c)(T_i - T_f) + m_{Al} c_{Al}(T_r - T_f) \ .$$

Calculate L_v from the above relation. The accepted value is 47.8 kcal/kg.

OPTIONAL:

1. Calculate the **apparent** specific heat of the Al block by use of

$$c_{Al} = \frac{Q}{m_{Al}(T_r - T_b)}$$

and your data. How does your result compare with the accepted value of $c_{Al} = 0.212$ kcal/kg? Explain. (See Introduction).

2. Observe (but do not touch) the following items after immersion in liquid N_2: rubber (get a piece from the instructor), pencil eraser.

3. Pour a little liquid N_2 onto the floor. Explain the behavior of the small spheres of liquid N_2.

Sound/Waves:
SC-1 Transverse Standing Waves on a String

OBJECTIVE: To study propagation of transverse waves in a stretched string.

INTRODUCTION:

A standing wave in a string stretched between two points is equivalent to superposing two traveling waves on the string of equal frequency and amplitude, but opposite directions. The distance between nodes (points of minimum motion) is one half wavelength, ($\lambda/2$).

The wave velocity, v, for a stretched string is $v = \sqrt{F/\mu}$ where **F = tension in the string** and μ **= mass per unit length**. But v = fλ and hence

$$f = \frac{\sqrt{F/\mu}}{\lambda} \quad . \tag{1}$$

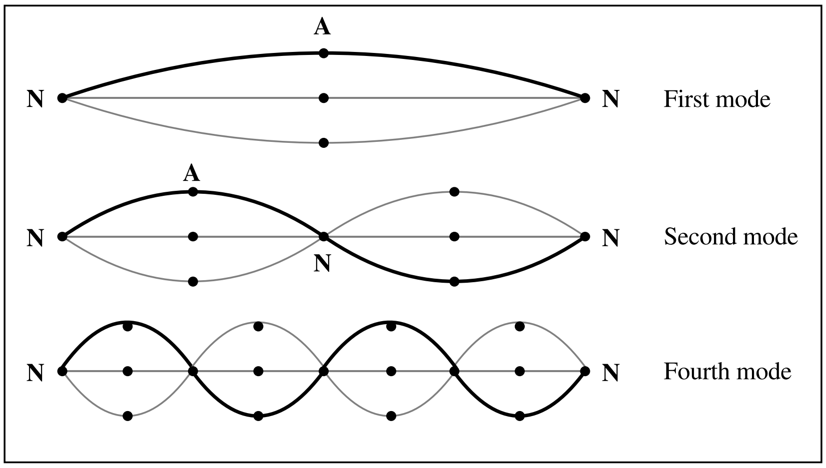


Figure 1: The Modes of a String

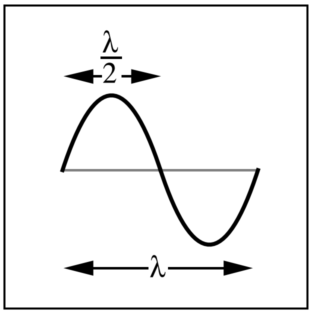


Figure 2: A close-up

PART A: Waves from a mechanical driver (i.e. a speaker)

APPARATUS:

Basic equipment: Electrically driven speaker; pulley & table clamp assembly; weight holder & selection of slotted masses; black Dacron string; electronic balance; stroboscope.

Computer equipment: Personal computer; PASCO© interface module; power amplifier module; various electrical connectors.

The set-up consists of an electrically driven speaker which sets up a standing wave in a string stretched between the speaker driver stem and a pulley. Hanging weights on the end of the string past the pulley provides the tension.

The computer is configured to generate a digitally synthesized sine wave (in volts versus time) with adjustable frequency and amplitude (max: ~10 V).
PASCO interface: This transforms the digital signal into a smooth analog signal for input into the power amplifier.

Power amplifier: The amplifier transforms the voltage sine wave single into a current suitable to drive the loudspeaker. (A few exotic speakers, often referred to as electrostatic speakers, actually utilize high voltages directly to produce sound.)

Precautions: Decrease the amplitude of the signal if the speaker makes a rattling sound, or if the red pilot light on the amplifier is lit. The generator is set to produce sine waves; **do not change the waveform**.

Note: Although the speaker is intended to excite string vibrations only in a plane, the resultant motion often includes a rotation of this plane. This arises from nonlinear effects since the string tension cannot remain constant under the finite amplitude of displacement. [See Elliot, **Am. J Phys.** 50, 1148, (1982)]. Other oscillatory effects arise from coupling to resonant vibrations of the string between pulley and the weight holder; hence keep this length short.

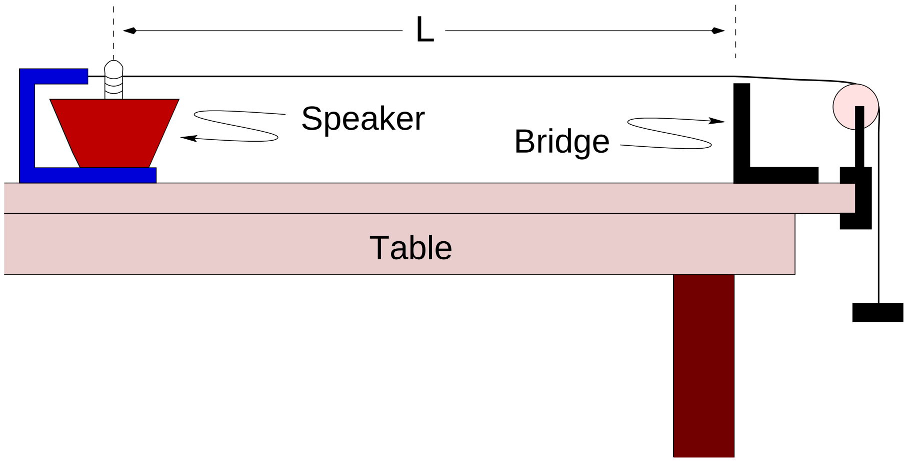


Figure 3: The apparatus

SUGGESTED EXPERIMENTS:

PROCEDURE I: Checking Equation (1)

1. Place the sheet of paper provided on the table; this will make it easier to see the vibration of the string. Measure accurately the distance, L, between the bridge and the pin of the speaker using the two meter ruler; record this in your lab notebook. Click on the LAUNCH EXPERIMENT icon (i.e., the telescope), from the on-line lab manual. The computer monitor will appear as shown in Fig. 4.

2. You will see that the computer is set to produce a 60 Hz sine wave with an amplitude of 3 V. To start the string vibrating **CLICK** the "ON" button.

3. **CLICK** on the up/down arrow in order to change the amplitude or the frequency of the signal although this produces rather large steps.
   In order to produce other step increments see the table below or simply double click right on the display number itself to use numeric input.

4. At 60 Hz check eqn. 1 by first calculating the necessary string tension to produce a standing wave in the third or forth mode. Weigh the string to get μ. Your instructor will provide you with a one meter length of string. (Dacron 30# has $\sim$ 0.283 g/m.)

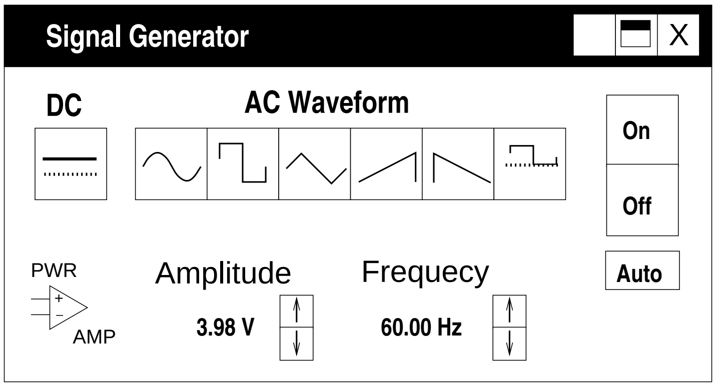


Figure 4: The PASCO display

Keyboard	Change in f	Amp. Change
Shift key	100 Hz	about 2.00 V
No key	10 Hz	about 1.00 V
Ctrl key	1 Hz	about 0.50 V
Alt key	0.10 Hz	about 0.10 V
Alt + Ctrl keys	0.01 Hz	about 0.02 V

Table 10: Controlling the Signal Generator

Note that the hanger itself has a 50 g mass so it may not be easy to access the forth mode (depending on L).

Check your results by adjusting the string tension by increasing/decreasing the weight to find the tension which results in the largest amplitude vibrations. How do the two values (calculated and measured) compare?

5. Now put a 200 g mass on the mass hanger and restart the signal generator. Record the total mass and tension in your lab book.

6. Adjust the frequency so that the amplitude of the oscillation is at its maximum by changing the frequency in 1 Hz steps. This is best done as follows: First decrease the frequency until the amplitude of the string is very small.
   Then increase the frequency in 1 Hz steps, observe that the amplitude first increases and then decreases. Record the best frequency f_2 in your table.

7. Change the frequency to observe the third mode. Find and record the best frequency (using 10 Hz steps at first may be faster).

8. Find and record the frequency of the higher modes.

9. OPTIONAL: Check the frequency f of the string in its 2nd mode with the stroboscope. Note that the stroboscope is calibrated in **RPM** or cycles per minute, NOT Hz (cycles per second). You should find a value close to 70 Hz.

ANALYSIS:

1. Divide the various frequencies f_n by n and enter the values in a table. Calculate the average value of f_n/n; this is the expected value of the frequency of the first mode.

2. Calculate the velocity of propagation on the string using the appropriate equation.

3. Calculate the mass per unit length of the string. How do the two values for the string mass per unit length compare?

PROCEDURE II: f_n vs string tension

In this section you will investigate the dependence of the resonant frequency of a string as a function of the applied tension.

1. Choose six masses between 100 gm and 1 kg and enter the values in the data table.
2. Determine the resonant frequency of the second mode of the string under these different tensions and record your results. (Hint: increasing the mass by a factor of two increases f_n by nominally a factor of $\sqrt{2}$.)
3. Plot a graph of frequency versus mass, m, and include the zero value.
4. Plot a graph of frequency versus $\sqrt{m}$ and again include the zero value.

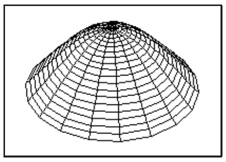
QUESTIONS:

1. Which of the two graphs can be fitted with a straight line? A parabola? Why?
2. From the slope of the graph having the linear relationship obtain the mass per unit length of the string and compare to your previous result.

PART B: "Virtual" waves on a drum head

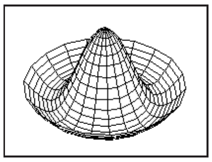
PROCEDURE III: (If time permits)

Vibrations of a circular drum head In this section you will examine, via a virtual demonstration, the vibrational modes of a two dimensional drum head.



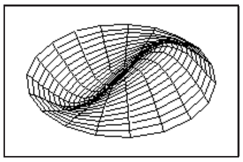
The [0,1] mode.

1. Click on the icon at left to down-load and initiate the MPEG movie viewer to observe the "first" mode.
2. Use the replay and step frame functions to view the motion.
3. Where is the displacement at a maximum? Always at a minimum?



The [0,2] mode.

1. Click on the icon at left to down-load and initiate the MPEG movie viewer to observe the first of the two "second" modes.
2. Use the replay and step frame functions to view the motion.
3. Where is the displacement at a maximum? Always at a minimum?



The [1,1] mode.

1. Click on the icon at left to down-load and initiate the MPEG movie viewer to observe the second of the two "second" modes.
2. Use the replay and step frame functions to view the motion.
3. Where is the displacement at a maximum? Always at a minimum?

S-2 Velocity of Sound in Air

OBJECTIVE:

To calculate the velocity of sound from measurement of the wavelength in air for sound of a certain frequency.

APPARATUS:

Resonance tube with arrangement for varying water level (use only distilled water); rubber tipped hammer; tuning fork; Hg thermometer.

INTRODUCTION:

For a closed tube, resonance occurs at tube lengths of an odd multiple of one-fourth wavelength, i.e. at $\lambda/4$, $3\lambda/4$, $5\lambda/4$ etc.

SUGGESTIONS:

1. Find the positions of the water level in the tube for the first three of these resonances. Use these readings to calculate the velocity of sound, **v**. Initially have enough water that you can raise the level above the first resonance position. The tuning fork frequency is on the fork.

   Since the effective end of the resonance tube is not at the tube's end, **do not use the position of the tube's top** in your calculations, but rather **take differences between the other readings**.

2. Since v = $v = \sqrt{\gamma RT/M}$, correct your value of **v** to 0°C (T = 273.16 K) and compare with that accepted for **dry** air at 0°C: 331.29 ± .07 m/s, [Wong, *J. Acoust. Soc. Am.*, **79**, 1559, (1986)]. For humid air see 3. below.

3. How should the v.p. of water, P_w, in the tube affect the measured velocity? An estimate is not difficult from v = $\sqrt{\gamma RT/M}$. Use an average γ/M:

   $$< \gamma/M > \ = \ [(\gamma_a/M_a)P_a \ + \ (\gamma_w/M_w)P_w]\,(P_a + P_w)$$

   where $\gamma_{air} = 1.40, \gamma_w = 1.33; P_a$ is the partial pressure of air and P_w is the v.p. of water; $M_a \sim 29$ kg and $M_w = 18$ kg. Hence

   $$v_{dry} \ \cong \ v_{humid}\sqrt{(}\gamma_a/M_a)/ < \gamma/M >$$

   OPTIONAL: Humidity changes will affect tuning of what musical instruments?

4. What effect does atmospheric pressure have on the velocity of sound in dry air? (Assume air at these pressures is an ideal gas.)

5. Viscosity and heat conduction in the tube may reduce **v** by ∼0.1%. See N. Feather, "The Physics of Vibrations and Waves", Edinburgh Univ. Press, (1961), p. 110-120; this reference also has a delightful historical account (including Newton's famous goof).

Electricity & Magnetism:
E-1 Electrostatics

PART I. THE ELECTROSCOPE

OBJECTIVE: To use an electroscope to study electrostatic phenomena.

APPARATUS:

A wooden box containing: aluminum-leaf electroscope, insulated hollow sphere proofball, hard rubber rod, rabbit fur, lucite rod, silk cloth.

INTRODUCTION:

In this lab you will use a deceptively simple device, an electroscope, to study the nature of charge. The electroscope's primary working parts are two connected conducting foil leaves. Charge appears in two forms, positive and negative, and like charges repel. At this point the only meaningful distinction between a conductor and an insulator is that a conductor allows charge to flow (in analogy to water in a pipe) whereas insulators do not. Thus as charge is placed on (or near) the conducting electroscope knob at top, the like charge in the electroscope assembly will redistribute so as to move as far away from charge on the rod and electroscope as physically possible. Since the hanging foil leaves of the electroscope are extremely lightweight the repulsive force between the like charges on each leaf is sufficient to force the leaves apart; the greater the charge the greater the angular displacement. In the schematic below we demonstrate the physical response.

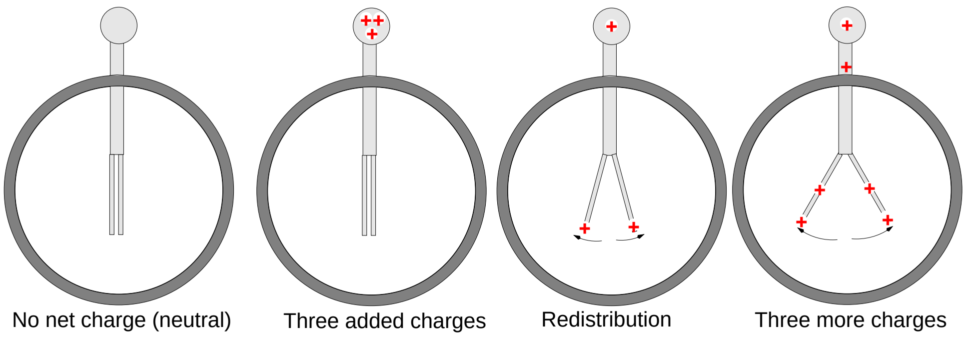


Figure 1: *Qualitative* schematic of electroscope response.

Preliminary Question: Will the charge in the electroscope assembly redistribute itself if a charged object is brought close to (but does not touch) the conducting knob? If so, will the net charge on the knob be of the same or opposite sign? What will be the sign of the charge on the foil leaves be?

Suggestions and precautions

1. In humid air insulators may adsorb enough moisture that charges leak off rapidly. If so, dry all insulators with a heat gun.

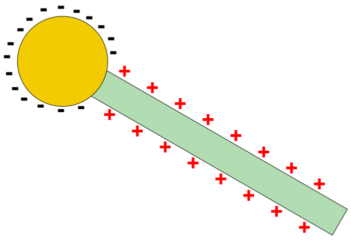
2. In very dry or cold weather the humidity is so low that clothing, table tops, etc. become good insulators. Their large surface charges may influence nearby unshielded instruments. If so, ask your instructor for help; e.g. use grounded foil to shield

against them. Also remove any clinging loose bits of fiber (e.g. silk or rabbit fur) that may disturb results.

3. The fragile leaves of the electroscope may tear if charged too heavily. **Do not disassemble electroscope** to attempt repair: see your instructor.

4. To remove charges on the glass windows of the electroscope, lightly rub your hands over the windows while grounding your body.

5. Charges on the insulating handle of a proofball can cause serious measuring errors. Test the handle by grasping the ball with one hand (while the other hand touches the electroscope ground), and then bring parts of the insulating handle close to the electroscope knob. If the leaves move, the handle is charged.

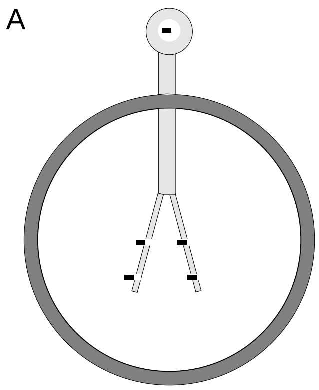


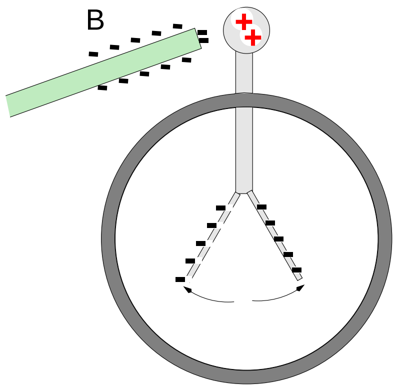
   To discharge it, hold the handle in a source of ionized air. The charged insulator will attract ions of the opposite sign until it is neutral. An open flame is a simple source of both positive and negative ions. The heated air convects these ions upward so **hold the insulator at least 10 cm above open flame to avoid heat damage to the insulator**. Avoid unnecessary handling of insulators because handling may impair their insulating capability. (Perspiration is a salt solution which is a conductor.)

6. Ground the electroscope *case* to a grounding jack near the electrical outlet. Leave it grounded throughout the experiment.

EXPERIMENTS:

1. Charge a rubber rod negatively (-) by rubbing with fur, and transfer some of the charge to the electroscope leaves by touching the rod to the electroscope knob. See diagram A. Note what happens when a (-) charged rod approaches the knob (diagram B). Without grounding the electroscope, also observe and diagram what happens when a lucite rod rubbed with silk approaches the knob.





2. **Charging by induction** (First discharge the electroscope by touching the knob and case simultaneously.) To **charge by induction** make the leaves diverge by bringing up a (-) charged rod: keeping the rod fixed, **ground the knob** to the case, **break the ground** and then **remove the rod**. Explain with diagrams each step of the process. Show by diagrams what happens when a (-) charge is brought near the knob; a positive (+) charge. Explain each step briefly.

3. In the following give the proofball a charge of known sign by contact with a charged lucite or rubber rod. **Use diagrams to explain and record results**.

   a. Connect the hollow conductor to electroscope knob by fine wire. Discharge them. Charge proofball by touching[1] it to a charged rubber or lucite rod; then introduce it into the hollow conductor but without contacting the conductor. Ground the hollow conductor by touching it. Note behavior of electroscope. Break the ground and remove proofball. Test the sign of charge on the electroscope.

   b. Repeat part a, but now ground the hollow conductor by touching it **only on the inside** with your finger or a short conductor. Explain.

PART II. Use of ELECTROMETER (a sensitive voltmeter)

OBJECTIVE:

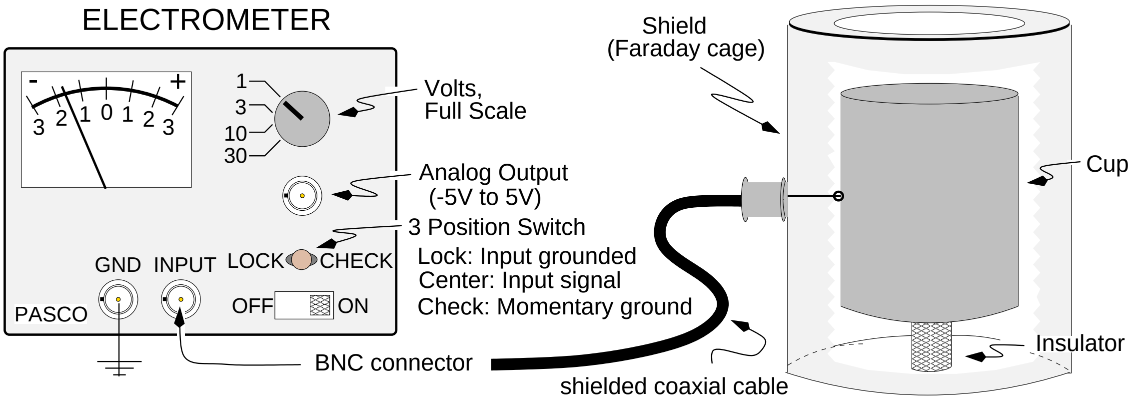
To measure potential (voltage) differences (i.e. the work/charge to move a small test charge between two conductors) and thus, in an indirect fashion, charges.

Preliminary Questions:

1. Does the electroscope allow you to discern the absolute sign of a charge?
2. If you are given a spherical conductor where does the excess charge reside?

APPARATUS:

PASCO electrometer & power supply; two hollow conducting spheres, one open hollow sphere; two charge producing paddles (white & blue), one aluminum paddle; proofball; insulated cup and shield: **do not carry the cup and shield by the top cover** (it may separate causing the cup to drop and become damaged); heat gun; alcohol lamp.



INTRODUCTION:

The electrometer reads directly the difference of potential (in volts) between the ground and input terminal. The electrometer's input resistance ($\sim 10^{14}\ \Omega$) is so high that negligible current flows and hence discharging effects are negligible. Typical handheld voltmeters have resistances five to six orders of magnitude *smaller*. (For

[1]Avoid rubbing which may result in charge separation of opposite sign (depending on the proofball metal and type of insulator: "triboelectric series", see exp #6 of Part II).

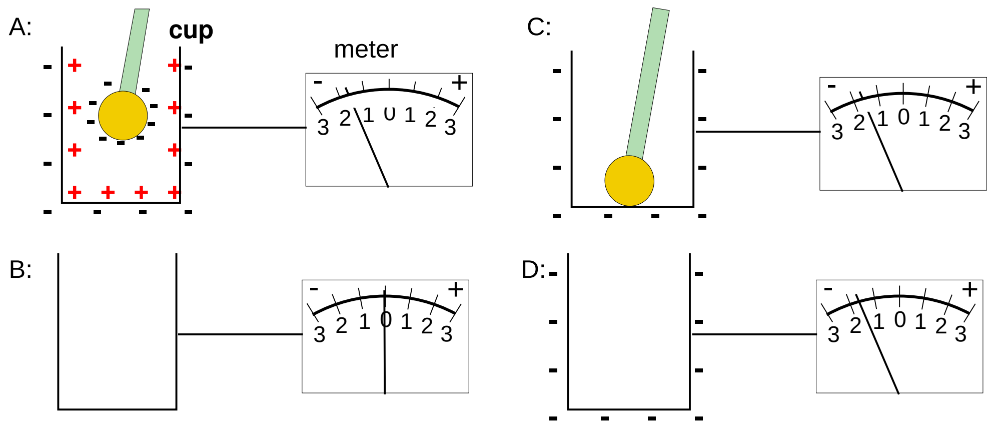
the circuit "layout," see your instructor.) From the several voltage ranges, choose one that gives a large but less than full scale deflection. Accuracy is about ± 3% of full scale. Connect the insulated cup and shield to the electrometer as shown in the sketch. The electrometer measures the potential difference between the cup and grounded shield. *This potential is proportional to the quantity of charge on the cup if the surrounding aluminum can furnishes a perfect shield from external charged objects.*

PRECAUTIONS: These are needed for reliable measurements:

1. Large static charges (common in dry weather) if applied to electrometer input may damage the sensitive input **f**ield **e**ffect **t**ransistor ("FET"). Minimize this possibility by *keeping the electrometer input grounded via the SHORT switch during the initial hook up and when you are done with the experiment.* Also use a banana plug lead to connect the GND terminal of the electrometer to the ground jack near an electrical outlet. In otherwords:
   Keep the Input switch in the "Short before making connections" position whenever there is nothing connected to the input or while you are making a connection. (In older models, this switch may be labled "lock." It is the left position in either case.) After the connection is made, you put the switch in the Input position. If you need to get rid of charge that may have been collected, you can move the switch to either of the short positions, the "momentary" positionusually being more convenient.
2. If your clothing or hair has a net charge, the electrometer reading may change if you move around. Hence, during a given measurement, change position as little as possible and ground yourself (e.g. touch the grounded shield all the time during a measurement or connect a bare wire from your body to ground. Note that grounding yourself is usually a *bad* idea when working with electronics, because of the danger of electrocution. Be careful not to touch any voltage sources while you are grounded.)
3. To remove all charge from the cup, switch the electrometer momentarily to the CHECK position. (This connects the electrometer terminals to each other so that any charge flows from/to ground). If the meter does not read zero, notify your instructor. adjust to zero
4. Always discharge paddles and cup before starting an experiment. To test if an object is charged, put it into the cup and see whether the electrometer deflects. Conductors discharge easily by touching them to a grounded conductor. To discharge an insulator, you must create sufficient ions in the surrounding air. The insulator will then attract ions of the opposite charge until all charge is neutralized. An open flame is a simple source of ionized air; the ions in the flame convect upward with the hot gas. **To avoid damage to the insulator, keep it at least 10 cm above the flame**!

SUGGESTED EXPERIMENTS:

1. Rub the blue and white paddles together and determine which paddle has a net positive charge and which has a net negative charge. Explain the following experiments with diagrams showing charge distributions, e.g. appropriate sample diagrams for this experiment are:
2. Discharge both the blue and the white paddles. Gently rub the white and the blue surfaces of the paddles together. Then:



a) Hold one of them near the bottom of the cup, but don't let it touch.
b) Take the paddle out.
c) Put it back in, touching the cup this time.
d) Remove the paddle.

Record the electrometer reading after each step. Explain the results. Was there any charge left on the paddle at the end?

3. **Charging by induction:** Momentarily ground the cup. Charge one of the paddles, then:

   a) Place paddle in cup without **touching**.
   b) Again momentarily ground the cup.
   c) Remove the paddle.

   Note the reading after each step. Explain what happened at each step.

4. Start with both paddles discharged. Then rub them together. Measure the charge on each. Explain. Compare the amount of charge produced by rubbing two paddles of the **same** kind (borrow one from another group), or by rubbing a metal paddle on a metal paddle, etc. (Surface "dirt" on one paddle may mean you are rubbing dissimilar paddles. Sometimes cleaning the paddles with alcohol makes a big difference.)

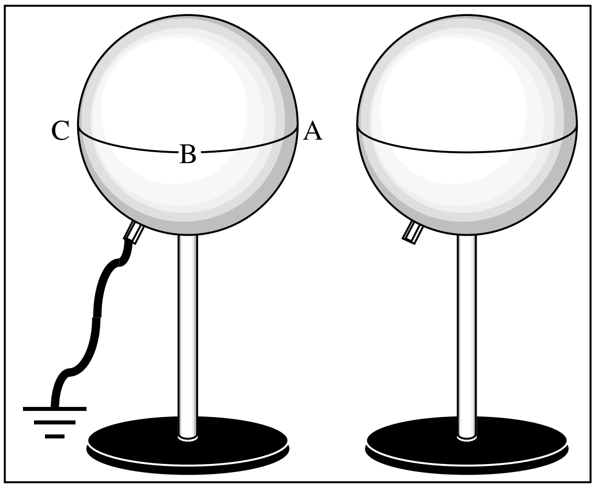
5. Discharge the cup and the blue and white paddles. What happens if you place the paddles inside the cup (without touching the cup) and,

   a) you charge them by rubbing while they both are inside the cup?
   b) Take out one paddle?
   c) Put it back in?
   d) Take out the other?
   e) Take them both out?
      Observe electrometer reading after each step. Explain.

6. QUESTION: Do only insulators acquire charge by rubbing? Rub the aluminum paddle on the white (or blue) paddle. Determine sign of effect. Arrange the white, blue and aluminum paddles in a series such that the rubbed one with a positive charge is always higher ("triboelectric" series). Tribology is the study of friction.

NOTE: In the remaining parts you may measure charge and still avoid spurious effects from charges on the insulating handles if you will touch the charged proofball (or paddle) to the bottom of the cup and then remove it from the cup before reading the electrometer. But remember to discharge the cup (by momentarily grounding) before taking the next reading! However, if the potential of the insulating handle is too large (e.g. way off the least sensitive scale), one can still get spurious effects from leakages.

7. Instead of frictional charging, use the Pasco power supply. Ground the black binding post and connect the **green** binding post to the hollow sphere with a copper wire. Adjust the voltage to $\cong$ 500V. **The switch must be in the correct position or no voltage results even though the meter may read**! Discharge the proofball and test that it (and handle!) have zero charge; then touch it to the outside of the hollow sphere and measure its charge. Do the same experiment but touch inside the sphere. What do you conclude?



8. Connect the power supply to an isolated solid metal sphere (shown in the next figure). Measure the relative charge density σ at various points of the sphere (such as A, B and C). Charge density σ is ΔQ/ΔA where ΔQ is the charge on a small element ΔA of the surface. Since it is not practical to remove a piece of the surface, we place the aluminum paddle **flat** against the surface and measure the charge on the paddle after it is removed. This measurement gives a number only approximately proportional to the charge density (why?) but does give a good idea of the relative charge distribution.

   OPTIONAL: Repeat step 8 but with paddle perpendicular to the surface instead of flat. Can you explain the differences?

9. Discharge the second solid sphere when it is far from the charged sphere. Then move it within a few centimeters of the charged sphere. Explore the charge distribution on both spheres with the paddle. Observe carefully the polarity of charge on different parts of the second sphere.

10. Does a grounded conductor necessarily have no net charge? Momentarily ground the second sphere when it is near the first one, then move it away and see whether it is charged. What is the polarity? Explain.

11. How much deflection does one get if one applies 1000V to the aluminum leaf electroscope? Compare the sensitivity to that of the electrometer (but do **not** connect 1000V to the electrometer or to anything connected to the electrometer e.g. the cup.)

12. Set the electrometer switch in the LOCK position.

EC-2 Electric Fields

OBJECTIVES:

To develop an intuitive understanding of relationship between electric fields and equipotential contours and physically map electric fields in two dimensions.

APPARATUS:

Pasco power supply, (use only 0-30 V DC range); PASCO electrometer plus coaxial cables, leads & two test probes; graphitized paper on which are drawn conducting electrode configurations; field plotting board; carbon paper; pen with silver conductive ink (*use cap when not in use!*) shared between 4 experiments; white paper.

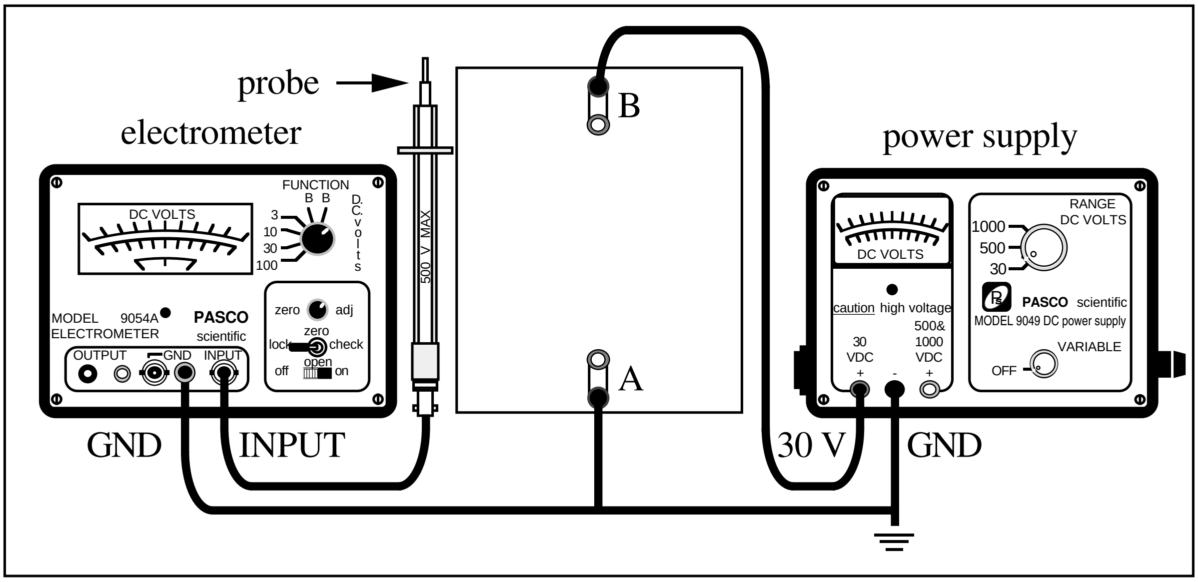


Figure 1: The apparatus.

Preliminary Questions:

1. Do electric fields extend through a vacuum?
2. Do electric fields extend through the interior of an insulator?
3. Do electric fields extend through the interior of a conductor?

INTRODUCTION:

To map electric fields in a plane near charged conductors we use a potential difference to produce a small current in a uniform material of high resistivity which surrounds the conductors. One locates equipotential lines between the conductors by means of probes connected to an electrometer. The lines of the electric field are perpendicular to the equipotential lines. The planar resistance material is a graphite treated piece of paper. The Pasco 0-30 V supply furnishes the current.

For the conductors we need a paint with resistance negligible compared to that of the graphitized paper. A suspension of silver flakes in a carrier which evaporates after painting, is excellent but expensive. Next best is a suspension of very fine copper or nickel flakes; we have mainly used nickel. The suspending fluid, an insulator, evaporates so slowly that recently painted electrodes are not good conductors unless special precautions and procedures are followed:

PRECAUTIONS:

1) *Use adequate ventilation. Avoid inhaling the vapor and avoid contact of the ink with your skin.*
2) Before using, shake the pen vigorously to disperse the particle matter suspended in the ink. Avoid thick layers. *USE CAP THE PEN WHEN NOT IN USE.*
3) Allow 3-5 minutes for the ink to dry (at room temperature: a heat gun will speed the drying, but can easily overheat the surface causing blistering if the layer is thick).

GENERAL METHODOLOGY:

1) Place white paper on the field plotting board, then a piece of carbon paper, and finally the graphitized paper with the conducting electrodes on the top. You may use samples already prepared or draw your own.
2) Record the shape of the electrodes on the white paper by tracing with a hard pencil or a ball point pen.
3) With 24 volts across the terminals of the plotting board (as in Fig. 1) check the quality of your painted electrodes by using the electrometer probe to find whether there is an appreciable potential drop **along** the painted electrode when you have 24 volts **between** the electrodes.

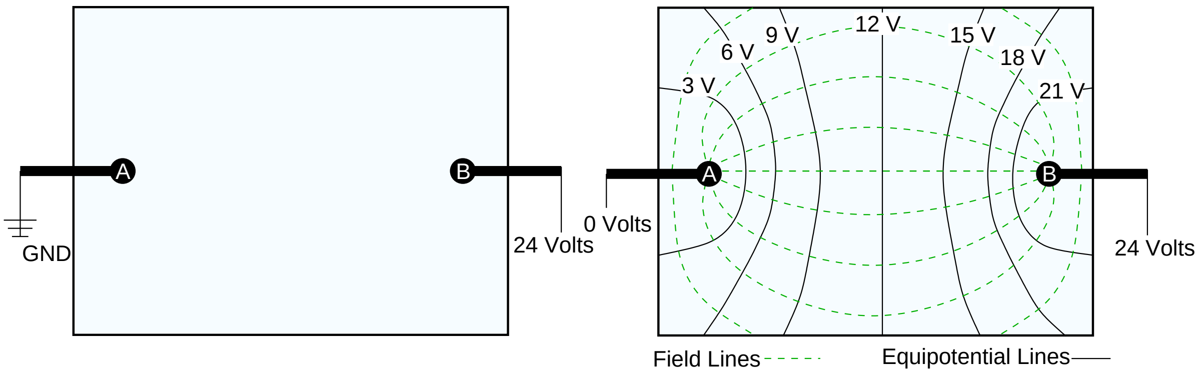


Figure 2: Electrode placement that qualitatively approximates a charge dipole configuration.

4) With 24 volts connected as in Figure 1, ground the electrometer to the power supply ground. For the other electrometer lead use the sharp steel probe. Check the electrometer zero, and then move the sharp steel probe along the graphitized paper from the ground terminal A toward B until the electrometer reads +3 volts. (Hold the probe only by the insulated base, and avoid grounding the graphitized paper with your hand or you may get nonsensical results.) Press down on the sharp probe to record this location as a dot on the underlying white paper.

   Map the equipotential line which corresponds to +3 volts by exploring (with the movable probe) the neighborhood of the first recorded point. Make your recorded points close enough together that you can later draw a smooth line through the points: the points need to be close together only when the direction of the equipotential changes rapidly.

NOTE: Your results, in terms of the actual voltages may vary somewhat from the idealized sketch of Fig. 1 because of contact resistance, leakage currents and other losses. To compensate you can measure the voltage at points very close to the A and B electrodes to find values V_A and V_B respectively and then choosing voltage steps of $(V_B - V_A)/8$.

5) Map other equipotentials: 6 volts, 9 volts,... 21 volts.

6) After you have drawn the equipotentials on the white sheet, dot in a few electric field lines (seven or so) as demonstrated in the right panel of Figure 1 (remembering that electric field lines are always perpendicular to equipotential contours.) latex

ALTERNATIVE METHOD: (OPTIONAL)

For one of your electrode configurations, replace the electrometer ground lead by another coaxial cable connected to a second sharp probe. Place one exploring probe about midway between electrodes A and B. Use the other probe to find positions on the paper which give zero reading on the electrometer. Since these are electric field lines as for example Fig. 1. obviously points of the same potential, they can be connected to map an equipotential line. Map three other equipotential lines approximately equally spaced on each side of the first. This method permits use of the electrometer's most sensitive voltage range and hence allows very precise location of the equipotential points. However, it does not give the potential of the equipotential line.

(NOTE: Since most existing lab electrometers are referenced to ground this only works if your DC power supply 0 V output terminal itself is not grounded.)

SUGGESTIONS:

Map equipotential lines and electric field lines for several of the configurations shown in Figs. 2 or 3 (or create your own test design). The distances are merely suggestions. Return to your instructor the graphitized paper with painted electrodes.

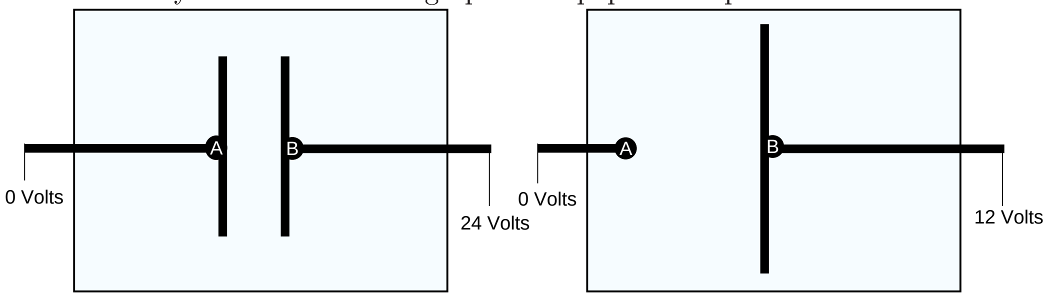


Figure 3: Electrode configurations for parallel plates (left) and a "half" dipole (right).

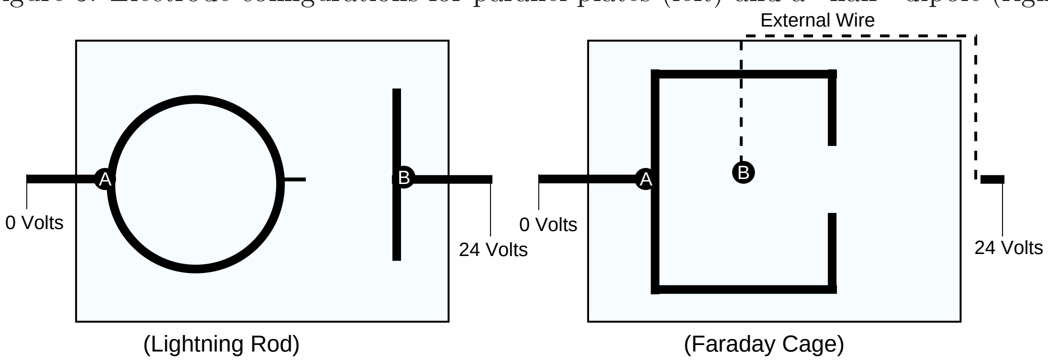


Figure 4: Two other illustrative electrode configurations.

In each case map about seven equipotential lines as continuous lines and dot in seven to nine electric field lines as dotted lines. Make clear which is which.

Turn off the electrometer when you finish mapping and *place* the three position switch in the grounded LOCK position.

QUESTIONS:

1. Explain in a sentence or two how you mapped the equipotential lines.

2. Pick two adjacent equipotential contours in two different electrode configurations and calculate the mean magnitude of the electric field strength where it is largest.

3. Where in Fig. 3(a) do you expect the electric field strength to be strongest? Weakest? Why do experts recommend that if you are caught outside during a thunder shower and cannot obtain shelter that you find a low spot and curl up your body? Note: Air "breaks down" (i.e., changes from an insulator to a conductor) at an electric field strength of approximately 5000 V/cm.

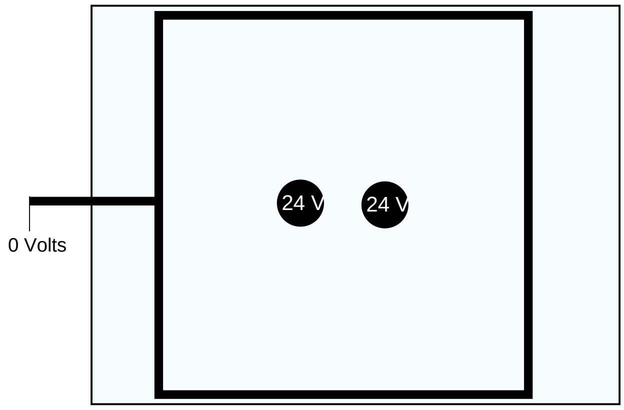
4. For Fig. 3(b) do you expect there to be appreciable electric field strength outside the confines of the box? If using the web-version of the lab manual click below on the virtual Faraday Cage demo to study this configuration in more detail.

5. Why must electric field lines be perpendicular to equipotential lines? Hint: Consider the relationship between electric potential and electric field:

$$V_b \ - \ V_a \ = \ - \int_a^b \vec{E} \cdot d\vec{l}$$

   and note that $V_b = V_a$ for any points b and a on the equipotential.)

6. For the sketch below see if you can sketch out the electric field lines. Afterwards check your answer (web-version only) by clicking below on the Like Charge Pair demo.



7. If you are you using the web-version of the lab manual, click on the button below to randomly place five unknown charges in a small region of space. Use any information available to determine the relative strength and sign of these five charges.

EC-3 Capacitors and RC-decay

OBJECTIVES:

I. Experiment with a parallel-plate capacitor to understand the behavior of capacitors.

II. Scrutinize how well the parallel-plate model describes real capacitors.

III. Experiment with the charge and voltage on parallel and series capacitors.

Preliminary Questions:

1. A capacitor plate holds a given charge Q. Why is the voltage of this plate small when a grounded plate is near, but large (for the same Q) when it is alone?
2. If the charge Q on a capacitor is doubled, what is the change in the voltage across the capacitor? (Remember that Q is the magnitude of the charge on each plate, both positive and negative.)
3. How could you double the positive charge on one capacitor plate without changing the negative charge on the other plate?

APPARATUS:

Conventional equipment: Parallel plate capacitor; Pasco electrometer & power supply; commercial capacitors & resistors on circuit board; aluminum paddle; low capacitance lead, insulated cup and shield; coaxial lead & test probe; digital multimeter (DMM); heat gun.

Computer equipment: Computer, monitor, keyboard, mouse; PASCO interface module; PASCO voltage test leads (plug into PASCO input A).

PRECAUTIONS: These are needed for reliable measurements:

1. The output BNC from the electrometer is calibrated to give 5 V when the meter reads full scale. This means you have to convert the voltage you read from the output in order to get the correct numerical value for the voltage at the input.
2. The input BNC from the electrometer has the outer conductor (the shield) connected to ground (this is true for all BNC connectors unless indicated otherwise on the connector.) This means that you run the risk of discharging your capacitors when you make measurements. Be careful with the polarity when you make measurements.
3. Large static charges (common in dry weather) if applied to electrometer input may damage the sensitive input **f**ield **e**ffect **t**ransistor ("FET"). Minimize this possibility by *keeping the electrometer input grounded via the SHORT switch during the initial hook up and when you are done with the experiment.* Also use a banana plug lead to connect the GND terminal of the electrometer to the ground jack near an electrical outlet. In otherwords: **Keep the Input switch in the "Short before making connections" position whenever there is nothing connected to the input or while you are making a connection.** (In older models, this switch may be labled "lock." It is the left position in either case.) After the connection is made, you put the switch in the Input position. If you need to get rid of charge that may have been collected, you can move the switch to either of the short positions, the "momentary" position usually being more convenient.

4. If your clothing or hair has a net charge, the electrometer reading may change if you move around. Hence, during a given measurement, change position as little as possible and ground yourself (e.g. touch the grounded shield all the time during a measurement or connect a bare wire from your body to ground. Note that grounding yourself is usually a *bad* idea when working with electronics, because of the danger of electrocution. Be careful not to touch any voltage sources while you are grounded.)
5. To remove all charge from the cup, switch the electrometer momentarily to the CHECK position. (This connects the electrometer terminals to each other so that any charge flows from/to ground). If the meter does not read zero, notify your instructor.
6. Always discharge paddles and cup before starting an experiment. To test if an object is charged, put it into the cup and see whether the electrometer deflects. Conductors discharge easily by touching them to a grounded conductor. To discharge an insulator, you must create sufficient ions in the surrounding air. The insulator will then attract ions of the opposite charge until all charge is neutralized. An open flame is a simple source of ionized air; the ions in the flame convect upward with the hot gas. **To avoid damage to the insulator, keep it at least 10 cm above the flame**!

INTRODUCTION:

A capacitor consists of two electrodes separated by an insulator. An electrode is just a fancy word for a piece of metal that can be connected to a voltage or current source. In part I you will study a parallel plate capacitor consisting of two circular metal plates separated by air. In Part II you will study simple circuits and the charging behavior of capacitors using commercial capacitors. These are made from two long strips of aluminum foil separated from each other by a thin sheet of plastic, all rolled into a cylinder.

Part I: THE PARALLEL PLATE CAPACITOR

EXPERIMENT A: Potential Difference *vs* Separation (for fixed charge)

INTRODUCTION:

For a fixed charge the voltage of a conductor, i.e., the potential difference between that conductor and ground, depends on what bodies are nearby. If you charge a parallel plate capacitor and then increase the plate spacing–leaving Q unchanged–you will find that the potential difference increases.

Q1.1) How do you reconcile this with the fact that Q = CV remains constant? (Two good approaches are either using what you have learned about C, or using what you have learned about $\vec{E}$ for capacitors and the integral $\int \vec{E} \cdot d\vec{l}$).

Q1.2) **Preliminary calculation:** Assuming air has a dielectric constant $\kappa = 1$ estimate how many excess electrons exist on one plate of the capacitor in front of you when V is set to 30 V and the plates are separated by 1cm.

EXPERIMENT A: Potential Difference *vs* Separation for Fixed Charge on a Capacitor

1) Referring to Fig. 1, connect the electrometer across the capacitor, but use the special low capacitance lead and a separate ground instead of the shielded coaxial cable. Ignore the sonic sensor shown on the right hand side of Fig. 1.

   Use the movable plate as the grounded one, and turn the apparatus so the fixed plate faces away from you. The ground removes any new excess charge that might accidentally come in contact with the plate, in order to keep the voltage the same. As a result, it provides some shielding of the system from charges on your hand or clothing. Also, with one plate at ground, you will only have to touch the other plate with the supply in order to charge the capacitor in step 2) below.

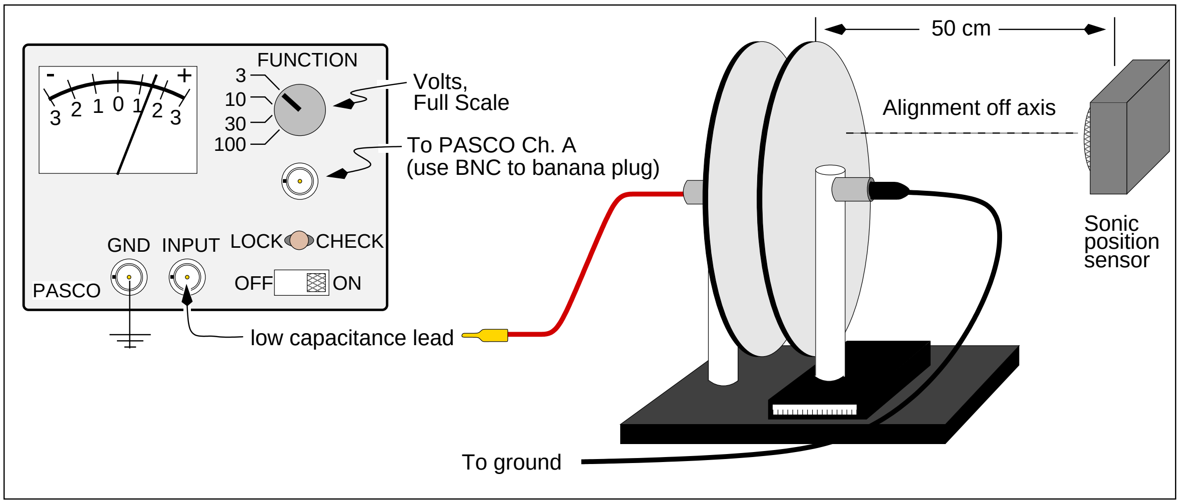


Figure 1

2) Start with an initial plate separation of 1 cm. This is large enough to keep charge from leaking across the spacers on dry days. Use the 30 V scale and output of the PASCO DC power supply. Connect the negative terminal to the ground terminal of the electrometer. (*Standby switch must be in proper position or no voltage results even though meter reads).* Set the output of the DC power supply to 5 Volts using the Electrometer (not the meter on the supply; it is not as accurate. Watch out, the Electrometer can acquire and keep a voltage bias because of it's very high input impedance...how do you avoid this before setting the supply output?) Charge the capacitor to 5 V by touching the appropriate plate with the positive voltage supply lead.

   Now, change the plate spacing and observe the change of the voltage across the capacitor. Record your qualitative answer in your lab book, and then record readings of the voltage for different plate spacings. Does it make sense to make small changes to d when the plates are close? How about when they are far appart? Span 10 cm in your measurements. Of course, *zero* on the cm scale will *not* be zero separation). Devise and report a way to make sure no charge has leaked off or been acquired by the plates during your measurements.

   NOTE: In dry weather stray static charge on your body can adversely affect the charge on the parallel plate capacitor. Keep body movement to a *minimum.* There is an optional shielding screen which you may place in front of your body to minimize this effect. In addition there is an optional extension handle that attaches to the moving plate which will increase your arm to plate distance.

In humid weather the charge may leak too rapidly off the plate to get reasonableresults. Use the heat gun to *gently* warm up the parallel plate capacitor and eliminate some moisture.

3.) Analysis:

3.a) Plot the voltage on the capacitor vs. the distance d between the plates using the graphical analysis for windows software GAX. Click on "title" and enter your name, to avoid confusion at the printer. Double-click on the graph, go to "column appearance," and change them to V and d.

Q 1.3 Which parts of the plot are consistent with our model of a parallel plate capacitor, and which are not? Make sure only the graph is selected (highlighted), and print your plot by printing "selected display." Show your answer on the plot.

Q 1.4 Is the deviation from ideal behavior what you expect from what you know about capacitance?

Now, create two new columns in GAX from your existing data by making a column of 1/V and a column of 1/d. Plot 1/V vs. 1/d.

Q1.5 If your capacitor were behaving as a single ideal capacitor, your data would follow a curve $1/V = (\epsilon A/Q) \times 1/d$. Derive this formula.

Q1.7 This formula will pass through the origin when d gets large (1/d goes to zero). What do you think is causing your data not to go through the origin? (Hint: if the capacitance of your cables is important, then it adds in parallel to the capacitance of the plates: C(plates) + C(cables) = Q/V.)

Q1.8 Why do we need the low capacitance lead?)

EXPERIMENT B: (OPTIONAL after completing PART II)

Surface Charge Distribution on a Parallel Plate Capacitor
(at fixed Potential Difference)

SUGGESTIONS:

1) Ground yourself, the electrometer and the movable plate of the capacitor. Turn the parallel plate capacitor so that you are behind the movable plate. Set up electrometer and cup as in **E1** but not close to the capacitor. (Why?)

2) *Connect 500 volts to the fixed plate.* Do NOT apply this voltage to electrometer directly.

3) Use the aluminum paddle (as in experiment **E1**) to probe the charge density on the capacitor's surfaces, and then use the electrometer and cup to measure the charge on the paddle. To avoid spurious effects from charges on the paddle's handle, touch the paddle to the bottom inside of the cup and remove the paddle before taking the reading.

4) Record the relative charge density (sign and magnitude) on both the inner and outer surfaces of the two plates for three radial positions: center, halfway out, near edge of plate. Use plate separations of 2.5, 5, and 10 cm.

QUESTIONS for Experiment B:

1) Why are measurements for separations < 2.5 cm not very meaningful?

2) How does relative charge density, σ, inside and outside the capacitor depend on plate spacing? On distance from center of the plate? Explain.

EXPERIMENT C: (OPTIONAL)

For a fixed spot inside the capacitor, find how σ varies with voltage.

Part II:
CAPACITORS IN PARALLEL, IN SERIES, AND CONNECTED TO RESISTORS

SUGGESTIONS:

1) *Use the electrometer to test voltages in Part II: A and B experiments*–**review the Electrometer Precautions!**. Use the lucite circuit board containing different capacitors and resistors. Although a push-button switch (plus connectors) permits applying 30 V momentarily to any capacitor, you may prefer just to touch the voltage supply leads directly to the capacitor being charged.

EXPERIMENT A: CAPACITORS IN SERIES

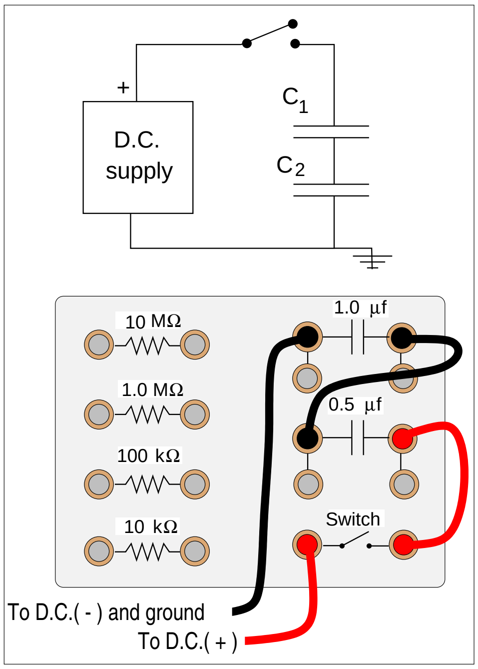
1) Discharge all capacitors first by momentarily shorting leads with a banana plug connector.

2) Connect 20 V across two capacitors in SERIES as the circuit and schematic at shows. (The schematic is only a *suggested* configuration.)

3) Calculate the potential difference expected across each capacitor.

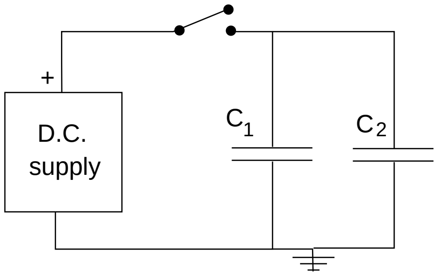
4) Now measure the voltage across the individual capacitors.

   OPTIONAL: If that's too easy, try this: charge $C_1 = 0.5\,\mu$F to 30 V; discharge $C_2 = 1.0\,\mu$F and *then* connect 10 V across $C_1 + C_2$. Now measure the voltage across C_1 and across C_2. Is this consistent with the idea of charge conservation in the region between the two capacitors?



EXPERIMENT B: CAPACITORS IN PARALLEL

1) Discharge $C_2 = 1.0\,\mu$F and remove from circuit.
2) Charge $C_1 = 0.5\,$F to 20 V and then disconnect the D.C. supply.
3) Measure voltage on C_1.
4) Connect C_2 in PARALLEL to C_1 and measure the final voltage across the capacitors.
5) Calculate the voltage you would expect.



EXPERIMENT C: DISCHARGING OF CAPACITOR THROUGH A RESISTOR

In this experiment you will use the electroscope and computer interface to observe the discharging of a capacitor through a resistor. So far you have only observed the "steady state" or DC behavior of capacitors. Now you will charge the capacitor to a given voltage and then remove the voltage supply. Because the two plates of the capacitor are acutally connected through the resistor, the charge on the capacitor will "drain," or move to the opposite plate through the resistor. You will measure the voltage drop across the capacitor, which tells you the amount of charge Q remaining on the capacitor. This is related to the flow of charge, or current I by the following relations as a function of time t during the discharge:

$$\frac{V(t)}{R} = I = -\frac{dQ(t)}{dt} \quad \text{and} \quad Q(t) = CV(t).$$

Solving the above equation gives the relationx: $V = V_0 e^{-t/\tau}$ where $\tau \equiv RC$ is the *time constant* for a particular circuit. R is the value of the resistor, which is measured in Ohms, or Ω. Note that the input resistance of the Electrometer is much larger than the values of R you will be using, so the charge flow into the electrometer is negligable.

SUGGESTED PROCEDURE:

1) Connect the 10^7 *ohm* resistor (marked as 10 $Mohm$ or 10 MΩ) across the large capacitor (1.0 μF) and set up the electrometer to measure the voltage on the capacitor. Fig. 4 shows the nominal circuit configuration and a possible wiring diagram for the circuit. Depressing the switch connects the power supply to the circuit, which will rapidly charge the capacitor, and releasing the switch will initiate the discharge.
2) Connect 30 V across the capacitor and observe the voltage V_C on the capacitor as a function of time after you disconnect the 30 volts and switch in the resistance R. Qualitatively describe the discharge behavior observed by watching electrometer display.
3) After making sure the PASCO interface is connected to the electrometer output (through Channel A), CLICK on the "Launch RC III" icon below to initiate the PASCO interface software. There should be a panel, a table and a graphing display for V *vs* t.
4) CLICK the REC button, charge the capacitor, and record data while the capacitor is discharging. CLICK on the STOP button. Record six (or so) evenly spaced data points into your lab book.

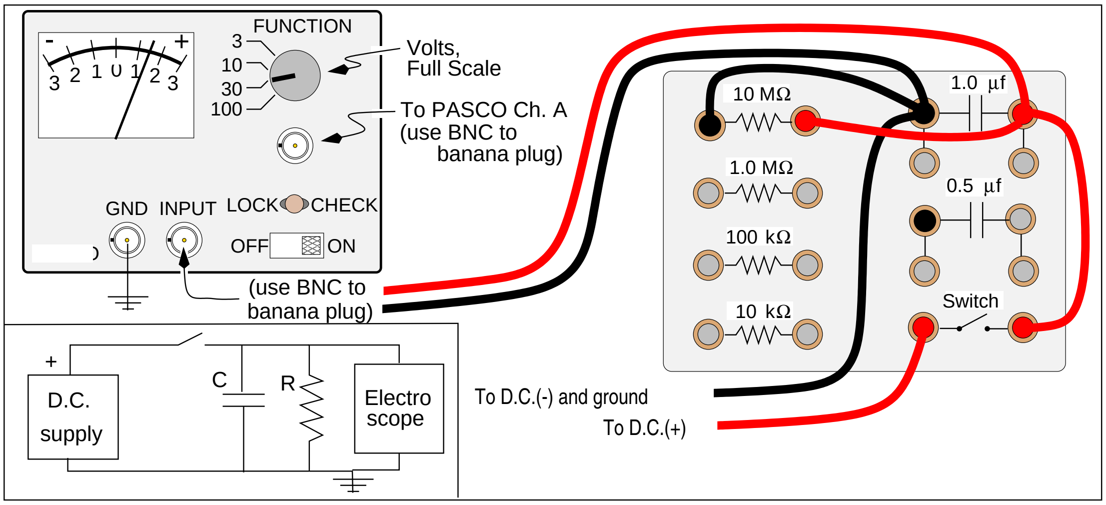


Figure 4

5) Move the leads to the 1 $M\Omega$ resistor and repeat the experiment. (Configure the interface to show multiple data sets.)

6) Move the leads to the 100 $k\Omega$ resistor and repeat the experiment.

QUESTIONS:

1) Do the curves have the expected functional behavior?

2) By moving the curser over the inital voltage and time, and then over the voltage at $1/e$ of the inital value, compare the nominal product $R \times C$ to the time required for the voltage to drop to $1/e$ of the initial value ($e = 2.72\ldots$). You may need to adjust the sampling rate to check this for the $100\,\mathrm{k}\Omega$ resistor.

E-6 Measurement of Charge to Mass Ratio (e/m) for Electrons

OBJECTIVES:

To observe magnetic deflection of electrons, at fixed energy, in a uniform magnetic field and then use this information to obtain e/m, the electron charge/mass ratio.

APPARATUS:

Sargent-Welch e/m equipment; power supplies with built-in digital meters; dip needle and compass; black cardboard and black cloth.

INTRODUCTION

Your e/m vacuum tube contains a number of features for producing and visualizing a thin uniform electron beam. Refer to Figs. 2 & 3 for schematic detais. Passing current through a wire filament (F) causes it to become hot and if the hot filament is in close proximity to a higher potential electrical element (the anode C) some of the electrons will accelerate through the vacuum towards the anode. To allow a narrow beam of electron to leave the vacinity of the anode a thin slit, S, has been cut in the anode cylinder.

Normally the electron beam is invisible to your eye. However the e/m tube also contains a saturated vapor pressure of Hg. When electrons, with energies in excess of 10.4 eV collide with Hg atoms, some atoms become ionized and some become excited. (These electrons are, of course, now permenently lost from the beam). Most of the Hg atoms/ions quickly recombine and/or de-excite to emit a bluish light before moving appreciably from the collision center. Hence the bluish light marks the path taken by the electron beam.

The potential difference V between the filament and anode (Fig. 2 and 3) accelerates electrons thermionically emitted by the filament. Those electrons travelling toward the slit S emerge with a velocity v given by

$$Ve = \frac{1}{2}mv^2 \tag{1}$$

provided that the thermal energy at emission is small compared to Ve.

Preliminary Questions:

1. What is purpose of the Helmholtz coil (read through the appropriate section first)?
2. What aspect of the electron trajectory will indicate that it is undergoing uniform circular motion?

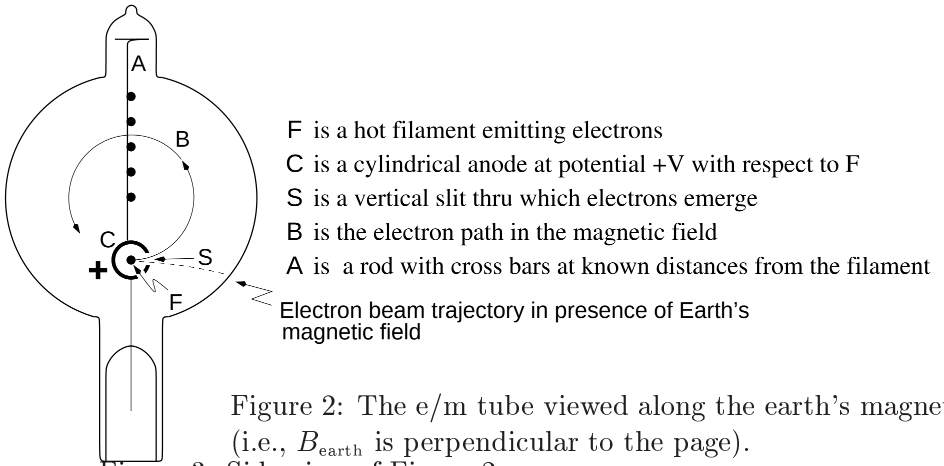
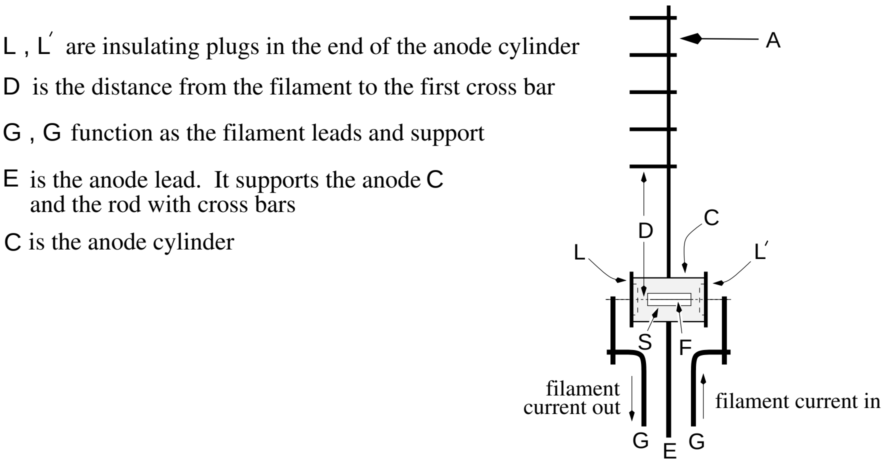


Figure 2: The e/m tube viewed along the earth's magnetic field (i.e., B_{earth} is perpendicular to the page).

Figure 3: Side view of Figure 2.



If the tube is properly oriented, the velocity of the emerging electrons is perpendicular to the magnetic field. Hence the magnetic force vector $\vec{F}$, which is described as a cross product $\vec{F} = e\,(\vec{v} \times \vec{B})$, supplies the centripetal force $\frac{mv^2}{r}$ for a circular path of radius r. Since $\vec{v}$ is perpendicular to $\vec{B}$

$$evB = \frac{mv^2}{r} \tag{2}$$

Eliminating v between (1) and (2) gives

$$e/m = \frac{2V}{B^2 r^2} \tag{3}$$

Our digital voltmeter measures accurately the accelerating voltage V. The radius of curvature, r or $D/2$, is half the distance between the filament F and one of the cross bars attached to rod A. The cross bar positions are as follows:

Crossbar No.	Distance to Filament	Radius of Beam Path
1	0.065 meter	0.0325 m
2	0.078 meter	0.039 m
3	0.090 meter	0.045 m
4	0.103 meter	0.0515 m
5	0.115 meter	0.0575 m

To find e/m we still need to know the magnetic field strength B. Instead of measuring B, we calculate it from the dimensions of the Helmholtz coils and the measured coil current, I, needed to bend the beam so that it hits a given cross bar.

Helmholtz coils consist of two identical coaxial coils which are separated as in Fig. 4 by a distance R equal to the radius of either coil. They are useful because near the center ($X = R/2$) the field is nearly uniform over a large volume. (Most textbooks assign this proof as a problem. Hint: Find how dB/dX changes with X for a single coil at $X = R/2$, etc.)

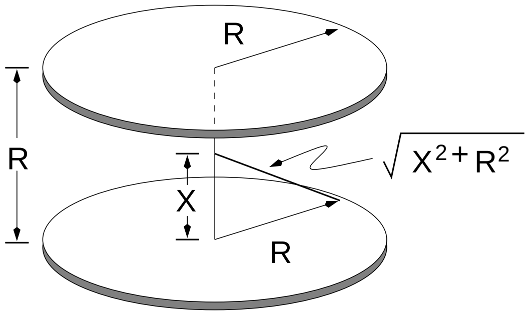


Figure 4: Helmholtz coil geometry.

The field at $X = R/2$ is in fact: [2]

$$B = \frac{8}{\sqrt{125}} \left(\frac{N\mu_0 I}{R} \right) \text{teslas} \tag{4}$$

as one easily sees by adding the axial fields $B(x)$ from each of the single coils:

$$B(X) = \frac{N\mu_0 I R^2}{2\,[R^2 \;+\; X^2]^{3/2}}.$$

Thus when X = R/2, then

$$B \;=\; 2B(X) \;=\; \frac{N\mu_0 I^2 R^2}{[R^2 \;+\; R^2/4]^{3/2}} \;=\; \frac{N\mu_0 I}{R\,\left[\frac{5}{4}\right]^{3/2}} \;=\; \frac{8}{\sqrt{125}} \left(\frac{N\mu_0 I}{R} \right).$$

In Eqn. 4,

N = number of turns on each coil (72 for these coils)
I = current through each coil in amperes
R = mean radius of the coils in meters (approximately 0.33 m but varies slightly from unit to unit)
μ_0 = $4\pi \times\ 10^{-7}$ tesla meter/ampere.

Finally if we substitute (4) into (3) we obtain

$$e/m \;=\; \left(2.47 \;\times\; 10^{12}\; \frac{R^2}{N^2} \right) \frac{V}{I^2 r^2} \;\; \text{coulombs/kg} \;. \tag{5}$$

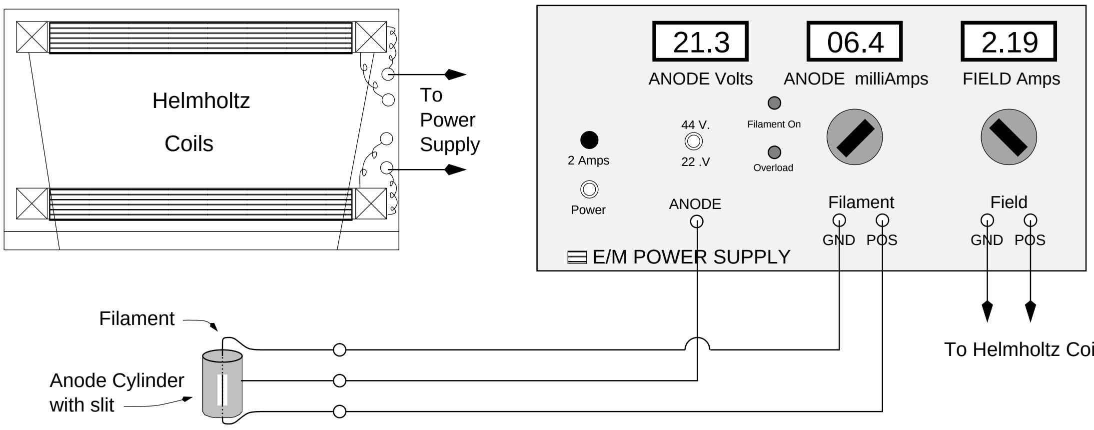


Figure 5: Basic wiring layout.

CIRCUIT DIAGRAM: Fig. 5 should help you hook up the components. The current must have the same direction in both sets of Helmholtz coils.

SUGGESTIONED PROCEDURE:

1. Set the axis of the Helmholtz coils along the local direction of the earth's field, B_e, as determined with a compass and dip needle. The axis should point towards the geographic north but at an angle about 60 degrees from the horizontal. Thus the axis should point deep under Canada. Put the long axis of the e/m tube in the north-south direction and with the cross bars up.

> CAUTION: Nearby ferromagnetic material (e.g. steel in the power supply, the table and in the walls) can alter the local direction of B_e. As long as the field direction does not change during the experiment, there should be no problem.

2. Set the filament supply knob to zero before turning the power on. A red light comes on when excessive filament current or a too rapid increase trips a protective relay. Reset by putting the filament control to zero and slowly turning it up.

3. Since e/m depends on V/I^2, one needs high quality meters for V and I. Our digital meters have an accuracy of $\pm$ one digit in the last displayed digit.

4. Start with $\sim 22\ V$ between filament and anode. With the room dark, *gradually* turn up the filament control until the beam is visible. To make the beam more visible use black cloth and black cardboard to block out stray light: place the black cardboard **inside** the Helmholtz coils and view from the top. If the protection circuit trips and a red light comes on, reset by zeroing the filament control; then turn it up **slowly**. When the beam appears, adjust the filament control to give 5-10 mA of anode current. **Do not exceed 15 mA!** With no current through the Helmholtz coils, the earth's magnetic field should slightly curve the beam (like the dotted line in Fig. 2.)

[2] The actual B at the maximum electron orbit is only $\sim 0.5\%$ less. See Price, "Electron trajectory in an e/m experiment", *Am. J. Phys.* **55**, 18, (1987)

5. *Correction for the earth's magnetic field:* Increase I through the Helmholtz coils until the beam deflects. If the sense of deflection is the same as that from the earth's field (i.e. when $I = 0$), reverse the leads to the Helmholtz coils; the curvature should then be like B in Fig. 2. The field from the coils then opposes the earth's field; hence adjust the coil current until the beam path is straight. Since light travels in straight lines, the beam should then hit the glass at the center of the area illuminated by light from the filament.

   This field current which just cancels the earth's field, we will call I_e. Clearly the I for Eqn. 5 must have I_e subtracted: Let the correct field current for crossbar n be I_n, and let I'_n be the measured current to bend the beam around so that the outside sharp edge of the beam hits the center of the n^{th} crossbar. Hence $I_n = I'_n$ - I_e is the correct current to bend the beam to the appropriate radius, r_n.

MORE ACCURATE METHODS TO ELIMINATE THE EFFECT OF THE EARTH'S FIELD:

Uncertainty in I_e can dominate the uncertainty in e/m, so consider using the following more accurate methods to eliminate the effect of the earth's field:

i. Take two readings of coil currents at $n = 3$, I'_3 and I''_3: one with the earth's field opposing that of the Helmholtz coils, the other with it aiding. To accomplish this, first (with the cross bars up) measure I'_3 (as described above); then rotate just the e/m tube 180° about its *long* axis.

> CAUTION: Always rotate the tube in the sense to *REDUCE* the *TWIST* in the filament and anode leads. *Otherwise one can twist the leads off.*

Next reverse the current in the Helmholtz coil and adjust it so that the beam again hits the same cross bar. Call this reading I''_3. Then

$$I'_3 \ - \ I''_3 \ = \ 2I_e.$$

ii. Or better yet, for each cross bar record an I'_n and I''_n reading. The average

$$\frac{I'_n \ + \ I''_n}{2} \ = \ I_n$$

is the current required if the earth's field were absent. You may find it easier to measure I'_n for $n = 1$ to 5 and then rotate the e/m tube just once to measure I''_n.

MEASUREMENTS AND ANALYSIS: (If time is short, measure only two cross bars at each voltage.)

1. Make two determinations of e/m for each cross bar.

2. Repeat for an accelerating voltage of $\sim$ 44 volts. (Note I_e should remain the same and one can measure it more precisely at the lower accelerating voltage.)

   SUGGESTED DATA TABULATION: {If you use the alternative method (ii. above), replace I_e by I''_n and $I_n = I'_n - I_e$ by $I_n = \frac{1}{2}(I'_n + I''_n)$.}

Volts	bar	radius	I_e	I'_n	$I_n = I'_n - I_e$	e/m

3. Estimate the reliability of your value of e/m and compare to 1.76×10^{11} C/kg.

OPTIONAL:

Note that the electron beam expands in a fan shape after leaving the slit. Interchange the filament leads and note how the fan shape flips to the other side of the beam.

Explanation: The fan shape deflection arises from the filament current's magnetic field acting on the beam. Our filament supply purposely uses unfiltered half wave rectification of 60 Hz AC so that for half of the cycle no current flows thru the filament but which is still hot enough to emit sufficient electrons. The electrons emitted in this half cycle constitute the sharp beam edge which we use for measurement purposes. This trick not only avoids deflection effects for this part of the beam, but also avoids the uncertainty in electron energy arising from electrons being emitted from points of different potential along the filament. (During this half cycle of no filament current the filament is $\sim$ an equipotential.) The fan shape deflection relates to electrons emitted during the other half cycle when filament current flows. [See: F.C. Peterson, *Am. J. Phys.* **51**, 320, (1983)]

EC-7 Magnetism

EC-7a Lenz's Law

OBJECTIVES:

1. To observe the eddy currents induced by the magnetic field in a moving conductor.
2. To follow how the eddy current intensity depends on the shape of the conductor.

YOU NEED TO KNOW: You must understand and be able to use the Right Hand Rule in order to determine the direction and sense of the forces involved.

APPARATUS:

A Permanent Magnet; four different paddle pendulums; two small red and green magnets.

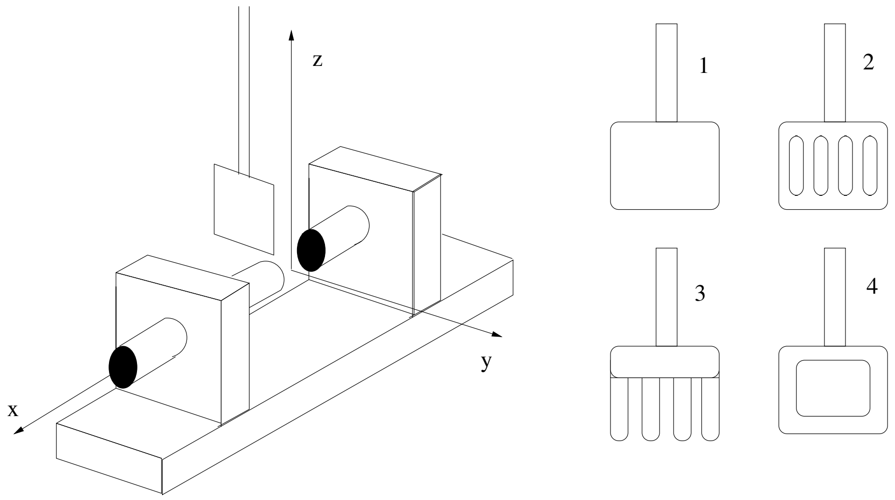


Figure 1: The magnet and the paddle pendulum

PROCEDURE I: (10 min)

1. Using one of the small magnets determine the direction of the magnetic field between the jaws of the permanent magnet.
2. Observe and describe the behavior of the four pendulums when dropped between the jaws of the permanent magnet.
3. Reverse the magnet. Observe and describe the behavior of two pendulums when dropped between the jaws of the permanent magnet.
4. Set pendulum #1 between the jaws of the magnet. Move the magnet rapidly in a direction parallel to the pendulum plate i.e. *without touching* jaws. Describe what happens.

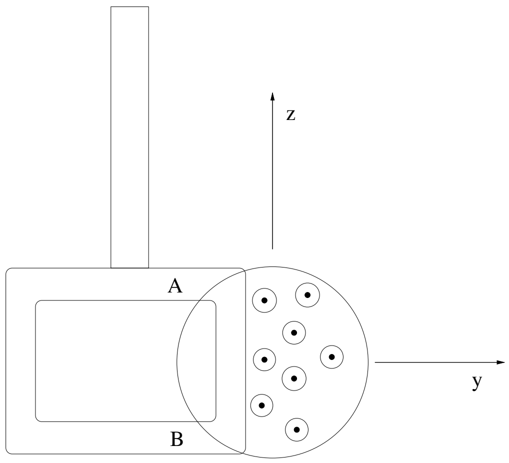


Figure 2: Paddle moves in the $+y$ direction

QUESTIONS:

Q1 Use the right hand rule to determine the motion of electrons at the moment when pendulum # 4 enters the magnetic field region. Is it from A to B? Or from B to A?

Q2 What is the direction of the current flowing in the pendulum? Is it from A to B? Or from B to A?

Q3 Use the right hand rule to determine the direction of the force exerted on the current that flows in the arm of the pendulum by the magnetic field. Is it along the positive or the negative y axis?

EC-7b Induction - Dropping Magnet

OBJECTIVE:

To show that a moving magnet induces an *emf* within a coil of wire.

INTRODUCTION:

After discovering the phenomenon of induction using two coils wrapped around an iron ring, Faraday was able to show that plunging a magnet into a coil of wire also generated a momentary induced current. Faraday did not use the concept of *Changing Flux* instead he thought of the *'lines of force'*, and concluded that when these lines move across a wire an *emf* is induced. Can you see the correspondence of this concept with the concept of flux? Think about this.

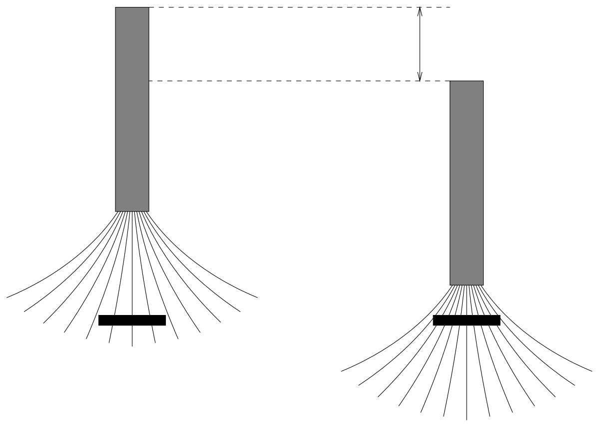


Figure 3: A moving magnet - lines move across a wire

EXPERIMENT:

You will be repeating Faraday's experiment by observing the *emf* induced in a coil by a fast moving magnet, and you will check that the *emf* you observe obeys the 'right hand rule'. If you have the time you can see the effect of changing the speed and the polarity of the magnet.

> *YOU NEED TO KNOW:* Lenz's Law and the use of the right hand rule. A varying magnetic field induces a current that opposes the magnetic field change.

EQUIPMENT:

⇒ PC and PASCO interface.

⇒ A voltage sensor is connected to the coil and to port A of the interface. Be sure the red wire is connected to the top of the coil. In this way a positive voltage corresponds to a current in the direction of the arrow on the top of the coil.

⇒ A stand holding a plastic tube and a coil.

⇒ A long bar magnet, this magnet has a small cut at one end, that is the North pole.

⇒ A pair of red and green bar magnets

> PRECAUTION: The long bar magnet is very fragile! DO NOT DROP IT ON THE FLOOR!.

PROCEDURE I: (15 min)

1. Click on the Launch EC-7B icon below (web version) to initiate the PASCO software window.
2. Set the distance $d_1 \sim 15\ cm$ between the top of the plastic tube and the top of the coil. Record this in your lab notebook. Be sure that the red terminal of the voltage sensor is plugged on the top plug of the coil.
3. The long bar magnet has a narrow cut at one end, this is the North pole of the magnet.
4. Hold the long bar magnet at the top of the tube; the end with the cut should be down, and just one centimeter or so inside the tube.
5. DCLK on RECORD and drop the magnet immediately after.
6. to see the whole graph, and measure the heights h_1 and h_2 of the two peaks using the cursor cross-hairs (5).
7. Slide the tube upwards, remove the rubber stopper and the magnet, replace the rubber stopper at the bottom of the plastic tube, and slide the tube down again.
8. Print the graph on printer.

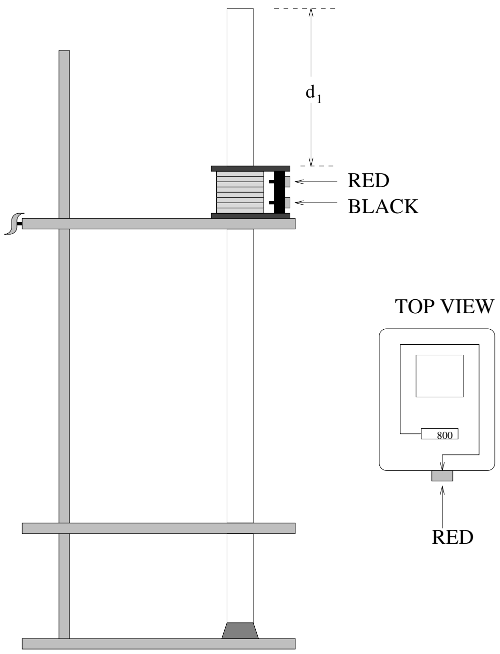


Figure 4: The setup

QUESTIONS: (10 min)

Q1 The graph you obtained has a fall, a rise, a very short flat part, then a rise and finally a fall. Explain what is happening at these various times.

Q2 Explain why the two voltage peaks you measured in 1.6 are not the same size. Can you give an approximately quantitative justification for the ratio h_1/h_2?

Q3 Examine the direction of the winding in the coil; verify that the directions of the peaks is what you would expect using the right hand rule (curled fingers = current; thumb = magnetic field).

PROCEDURE II: (10 min)

Repeat procedure I with the magnet inverted (the South Pole at the bottom).

QUESTION:

Q1 Explain in what way(s) and why the new graph is different.

PROCEDURE III: (20 min)

1. Move the coil further down on the tube making the new distance d_2 at least about two and a half times larger than d_1. Record this new distance d_2.
2. Repeat procedure I. Measure and record the heights h'_1 and h'_2 of the new peaks.

QUESTIONS:

Q1 Explain the reason for the difference between this graph, and the one obtained in procedure I.

Q2 Can you give an approximate quantitative explanation for the ratio h'_1/h_1? HINT: remember Physics 103 formula for free fall $v = \sqrt{2gx}$.

EC-7c Induction - Test Coil

OBJECTIVES:

1. To observe the *emf* induced in a small detecting coil by the varying magnetic field produced by the varying current in a separate large coil.
2. To follow how how this *emf* depends on the shape, and the frequency of the inducing current. The computer will produce a voltage varying with time in the form of a triangular waveform with amplitude $V_{max} = 3\ V$ and a frequency $f = 10\ Hz$.

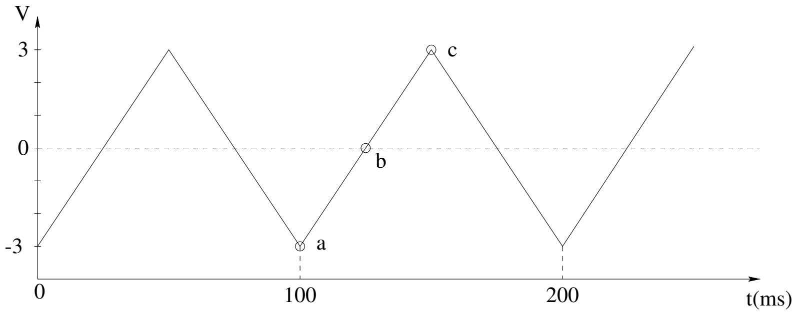


Figure 5: The triangular waveform

INTRODUCTION:

The applied voltage varies linearly from $V_{min} = -3\ V$ to $V_{max} = 3\ V$ in 50 ms. The *slope* of the voltage curve is therefore $\Delta V/\Delta t = 120\ V/s$.

The resistance of the large coil is $R \simeq 7\Omega$ so current flowing in the large coil will vary linearly from $I_{min} = -3/7 = -0.43\ A$ to $I_{max} = 3/7 = +0.43\ A$ in 50 ms, Correspondingly $\Delta I/\Delta t = 17.2\ A/s$. The strength of the magnetic field produced by the large coil is proportional the the voltage applied on it, and therefore the rate of change $\Delta B/\Delta t$ is proportional to the *slope* of the curve that shows the voltage versus time.

> *YOU NEED TO KNOW:* The *emf* induced in a coil is then $\mathcal{E} = -NA\Delta B/\Delta t$ where N is the number of turns in the small coil and A its cross-sectional area.

EQUIPMENT:

⇒ PC and interface.

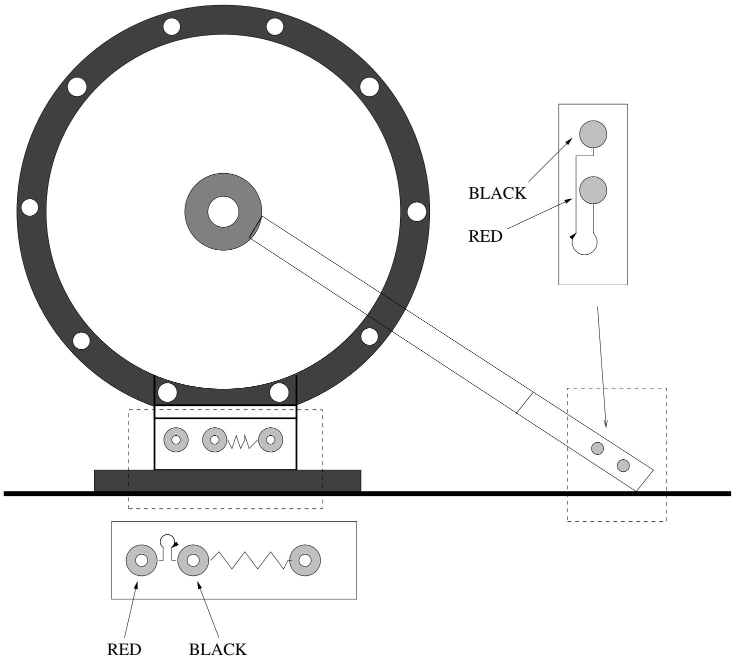
⇒ A computer controlled power amplifier.

⇒ A stand holding a small detecting coil.

⇒ A large coil with an ohmic resistance $R = 7\Omega$. The radius of the coil is $r = 0.1$ m.

PROCEDURE I: (10 min)

1. Click on the Launch EC-7C icon below (web version) to initiate PASCO software window. The monitor should now show a scope window, and a window for the control of the signal generator; CLK on the frequency, backspace and enter a frequency of 10 Hz. CLK on the triangular waveform. DCLK anywhere on the face of the scope and enter 1 V for the trigger level.
2. CLK on the amplitude, backspace and enter 3 V.
3. Turn on the signal generator by CLKing on the ON button then DCLK on RECORD, you now see the voltage induced in the small coil together with the one applied on the large coil
4. Adjust the size of the waves you see in the scope window using the size controls on the side of the scope window, then measure the height h_1 of the square wave using the cursor (5). Record this height.



QUESTIONS

1. Explain why the induced voltage is a square wave.
2. Observe the direction of the winding in the large coil. What is the direction of the magnetic field during the rising part of the triangular wave? Explain your thoughts.
3. Observe the direction of the winding in the small coil. What is the direction of the induced emf during the rising part of the triangular wave? Explain your thoughts.

PROCEDURE II:

Move the small detecting coil 5 centimeters away from the large coil, measure the amplitude of the induced voltage. Repeat at distances of 10 and 20 centimeters. Plot a graph.

QUESTION

How does the strength of the magnetic field depend on distance? $\sim 1/r$?$\sim 1/r^2$? $\sim 1/r^3$?

PROCEDURE III:

1. Replace the detecting coil to its original position at the center of the large coil. Decrease the amplitude of the triangular wave to 2 V.
2. Measure the height of the square wave. record this value.

QUESTIONS

Q1 Did the slope of the triangular wave change? By how much?
Q2 Is the change in amplitude of the square wave what you would expect? Why?

PROCEDURE IV:

1. Repeat procedure III with a frequency of 15 Hz.
2. Compare the result with those of procedure I and III

PROCEDURE V:

1. Loosen slightly the clamp holding the small coil; rotate the coil 90° so that it is in the horizontal plane. Observe the resulting induced voltage.
2. Rotate the coil another 90° so that it is in the vertical plane again. Observe the resulting induced voltage. Compare the result with procedure III.

EC-7d Induction - Faraday Discovery

INTRODUCTION:

Michael Faraday (1791-1907) was a bookbinder journeyman (English for apprentice). He read some of the books he was supposed to bind, and became interested in 'natural philosophy' (the term science did not exist in the XVIIIth century).
In 1813 he applied for a job as a technician in the laboratories of the Royal Laboratories to professor Davy, he got the job. Some people say that this was the greatest discovery Davy ever made! He worked all his life in the laboratories, starting as a technician, became Director of the Laboratories, was elected to The Royal Society of London, and became a Professor at the Royal Institution. Some of you will be pleased to know that he knew no math, and never used math in his researches.

Faraday performed many kinds of experiments, in chemistry, optics, and metallurgy, but perhaps the most important experiments were on electricity and magnetism. Having learned of Oersted's experiments on the magnetic fields produced by an electric current, he wondered if the reverse could be true: perhaps magnetic fields could in turn produce an electric current. His first experiments were unsuccessful, until he realized that it was the *change* in magnetic field that produced a *momentary* current.

His first experiments were similar to the one you will be doing today: two separate coils were wound on an iron ring, the current in the primary coil was interrupted, and the momentary current in the secondary was observed.

The explanation of this effect is, as you know, that the change in current in the primary circuit produces a change in the magnetic field that exists inside the secondary coil, the electromotive force in the induced secondary is then

$$\mathcal{E} = -AN \cdot (\Delta B/\Delta t)$$

where N is the number of turns, and A is the cross-sectional area of the secondary coil.

OBJECTIVES:

To observe the induced momentary *emf* that appears in a secondary circuit when the current in the primary circuit is turned on, or interrupted. This is the original Faraday discovery: induction of a current by a varying magnetic field.

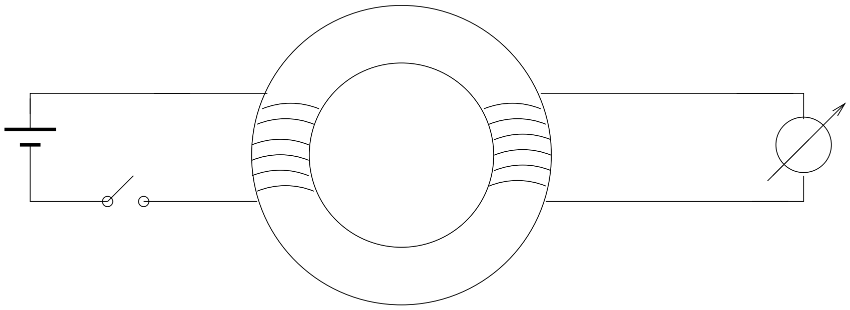


Figure 7: Faraday's experiment

In procedure II you will observe that induced momentary emf appears also in the primary circuit itself when the current is turned on, or interrupted; you will be observing the self induction of a coil. Faraday used a switch to interrupt or start the current as shown in Fig 1; you will use the computer to do the same job: the computer will produce a square wave, the voltage will change abruptly from 0 volts (current off = switch open) to a value of about 4 Volts (current on = switch closed) as shown below.

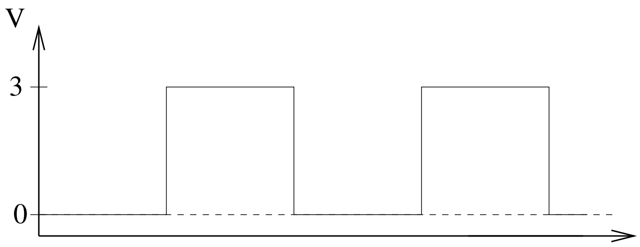


Figure 8: The square wave

EQUIPMENT:

⇒ PC and interface, with the power amplifier plugged in port A.

⇒ Two voltage sensors are plugged in the interface. The voltage sensor plugged in the B port measures the voltage across the external resistor; the voltage sensor plugged in the C port measures the voltage across the coil.

⇒ Computer Controlled Power Amplifier.

⇒ A pair of nested coils. The outer one is the 'primary'. It has about 2600 turns of fine wire and has a resistance of $\sim$ 95 Ω. The primary coil is insulated from the inner secondary coil, an iron bar fits inside the inner coil.

⇒ A plug board and a 100 Ω resistor.

PROCEDURE I:

1. Click on the Launch EC-7D icon below (web version) to initiate the PASCO software window.
2. Check that the power amplifier is on, (the yellow pilot light labeled 'Power' should be on and the red pilot light light labeled 'Distorting' on the power amplifier should off).
3. Check that the amplitude is $\sim$ 4 Volts, and the frequency is $\sim$ 10 Hz. CLK on the ON button of the Signal Generator, then DCLK on RECORD. DCLK on STOP after a second.

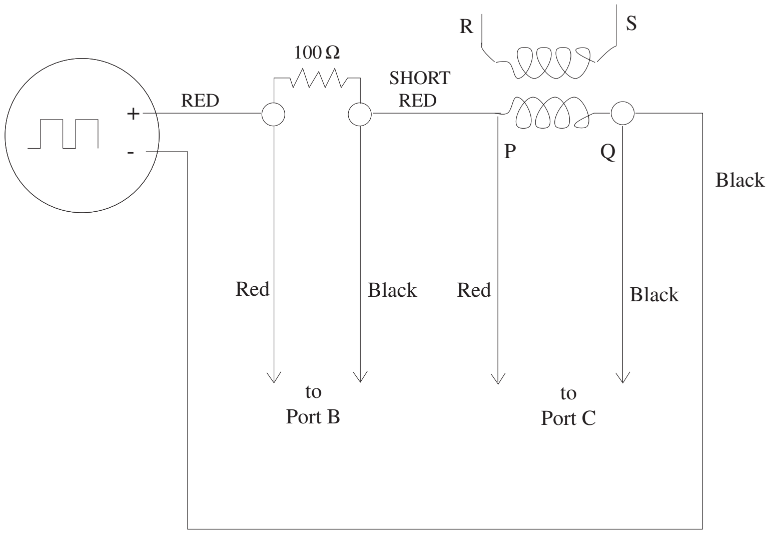


Figure 9: The schematic setup

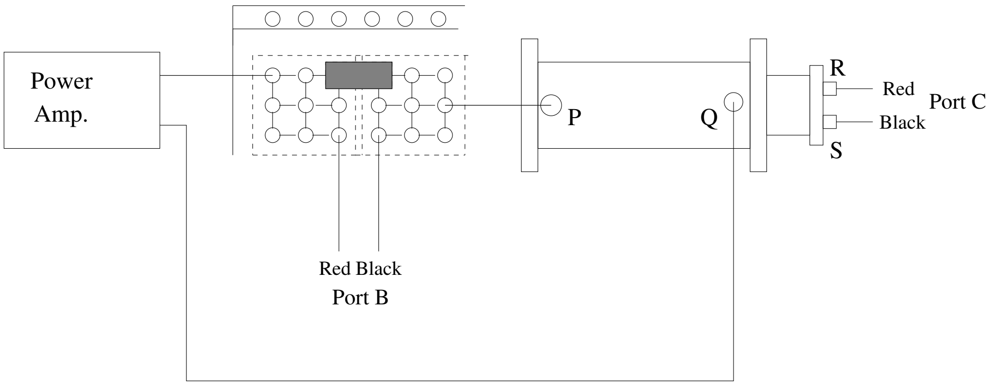


Figure 10: The physical setup

4. The Monitor now displays two curves:
   - one curve (curve A) looks like a square wave, but not quite; it shows the voltage across the external 100 Ω resistor, and therefore tells you the current flowing in the coil.
   -the other curve (curve B) shows the induced voltage across the secondary coil
5. CLK on GRAPH SETUP (8) then print the graph of one of the peaks [instructions in PASCO Interface and Computer Primer section (refer to Appendix F.)]

   QUESTIONS:

   Q1 The shape of curve A is not quite a 'square wave'. Why not? HINT: this is an LR circuit with a time constant $\tau = L/R$
   Q2 Are the peaks of voltage across the secondary coil, (curve B) that correspond to current on vs. current off about the same size? Explain.
   Q3 Are the peaks of voltage across the secondary coil, that correspond to current on vs. current off of the same polarity? Explain.

PROCEDURE II: (15 min)

1. Remove the voltage sensor banana pins from the terminals R and S of the secondary coil and insert them into the terminals P and Q of the primary coil, without disconnecting these terminals from the power supply.
2. CLK on the ON button of the Signal Generator, then DCLK on RECORD, DCLK on STOP after a second.
3. The Monitor now displays two curves:
   - one curve (curve A) looks like a square wave, but not quite; it shows the voltage across the external 100 Ω resistor, and therefore tells you how the current flowing through the primary coil varies with time.
   -the other curve (curve B) shows the voltage across the primary coil.
   The two curves look very similar: they should have about the same height, and should rise and fall at the same times.

   If curve B is inverted relative to curve A, exchange the banana pins in terminals P and Q and repeat item 1.2 of this procedure.
4. CLK on GRAPH SETUP (8) then print the graph of one of the peaks [instructions in PASCO Interface and Computer Primer section (refer to Appendix F.)]

   QUESTIONS

   Q1 The shape of curve A is not quite a 'square wave', why not? HINT: this is an LR circuit with a time constant $\tau = L/R$
   Q2 The curve that describes the voltage across the primary coil, (curve B) is similar to curve A. Why? HINT: think of two approximately equal resistors in series.
   Q3 The curve that describes the voltage across the primary coil, (curve B) is similar to curve A, but it shows 'spikes'. Why? HINT: the voltage across this coil is due to two factors, R and L.

E-8 Cathode Ray Oscilloscope and Differential Amplifiers

OBJECTIVES: To learn the basic operation of an oscilloscope in order to observe the voltage *vs* time relationships of basic electrical signals.
Preliminary Question:

- What signal (voltage or otherwise) *vs* time displays do you have in your house?

APPARATUS:

Dual-trace oscilloscope plus manual, signal generator plus frequency counter, digital multimeter, (DMM); circuit plug board & component kit; differential amplifier.

Part A – Operation of the Oscilloscope
(Leader, Model 1021, or Hitachi, Model V-202F)

INTRODUCTION:

The oscilloscope is basically a graph-displaying device – it draws a graph of an electrical signal. In most applications the graph shows how signals change over time: the vertical (Y) axis represents *voltage* and the horizontal (X) axis represents *time*. The intensity or brightness of the display is sometimes called the Z axis. This simple graph can tell you many things about a electrical signal. Here are just a few, you can ...

- determine the time and voltage values of a signal.
- measure the frequency of an oscillating signal.
- see the "moving parts" of a circuit represented by the signal.
- tell how often a particular portion of the signal is occurring relative to other portions.
- find out how much of a signal is direct current (DC) or alternating current (AC).
- tell how much of the signal is noise and whether the noise is changing with time.

Your dual trace analog oscilloscopes have circuits which can alternately display sweep signals from two different input channels (Y_1 and Y_2). Unfortunately this instrument can address only a single point on the display window at one instant in time. However the screen is coated with a phosphor coating which whose image persists for a short time. When this property is combined with the finite time resolution of the eye one can observe "simultaneous" dual traces.

CAUTION: Avoid placing signal generator on top of scope.

[Stray magnetic fields from power supplies can interfere with the cathode ray tube (i.e., the display or CRT) operation and create a noisy (fuzzy) "trace."]

If you are unfamiliar with the various control knobs and switches you should read through this section carefully.

SUGGESTED PROCEDURE (refer to Fig. 1):

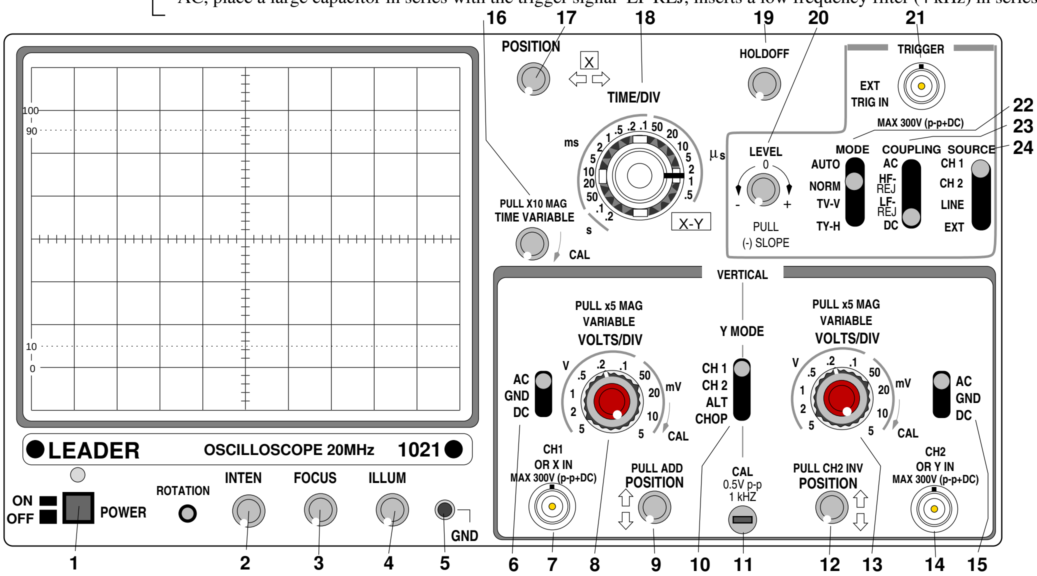
1. Using a BNC cable and the sinusoidal wave signal output from the function generator (1kHz, 1V rms) connect the signal to the CH1 input (# 7) oscilloscope (or *scope*). Note that each input, CH1 and CH2, has a three position switch (#6 or #15) for either grounding the input (GND), coupling the signal through a capacitor (AC), or a direct connection (DC). Using the AC input (*use this position*) removes all DC components of the input signal.

HORIZONTAL X SWEEP

16 TIME VARIABLE provides a countinously variable adjustment of the sweep rate.
16 PULL x10 MAG switch expands the horizonal deflection by a factor of ten
17 X-POSITION control adjusts the horizontal positon of the CRT trace
18 TIME/DIV switch selects the horizontal sweep rate or X-Y operation (fully clockwise position)
NOTE: TIME/DIV setting is accurate only when the TIME VARIABLE (16) control is turned fully clockwise to the calibration (CAL) positon.
19 HOLDOFF control may be used for complex signal when a delay of the trigger signal is desired; normally fully counterclockwise

TRIGGER INFO

20 LEVEL control uses the SOURCE(24) signal, voltage and slope (dV/dt), to decide when to sweep (trigger) the trace
21 EXT TRIG IN connector is for providing an external trigger signal (only valid when SOURCE(24) is set to EXT)
22 MODE switch sets the triggering mode:
AUTO automatically sweeps the trace when the trigger signal drops below 25 Hz.
NORM sweeps the trace horizontally when a trigger signal is given
TV-H and TV-V are for video signals
23 COUPLING switch selects the trigger signal coupling
DC, no modification of trigger signal
AC, place a large capacitor in series with the trigger signal
24 SOURCE switch selects the trigger signal source
HF REJ, inserts a high frequency filter (4 kHz) in series
LF REJ, inserts a low frequency filter (4 kHz) in series



1 POWER switch, push in to turn power on or off, POWER lamp lights when on.
2 INTEN control, adjusts bright ness of the CRT display
3 FOCUS control, adjust CRT focus to obtain maximum trace sharpness
4 ILLUM control, adjusts scale illumination for better viewing
5 GND access to instrument grounds

VERTICAL Y SWEEP

TWO inputs may viewed simultaneously, CH1 (channel 1) and CH2 (channel 2), referenced to ground.
6 and 15 switches select the coupling for the CH 1 (7) and CH 2 (14) inputs
DC, no modification AC, place a capacitor in series GND connect input to ground (signal is bypassed)
7 CH 1 (or X IN) connector is the signal for triggering (23) and for displaying (10)
14 CH 2 (or Y IN) connector is the signal for triggering (23) and for displaying (10)
8 and 13 VOLT/DIV knob set the vertical scaling for the CRT trace
8 and 13 inner control provides a continuously variable vertical scaling
NOTE: For proper calibration these must be in the fully clockwise position
9 and 12 POSITION controls adjust the vertical position for CH1 and CH2 respectively.
10 V MODE switch selects which inputs (CH1 and./or CH2) are displayed.

Figure 1

2. Turn on the scope (#1)and note the *four* main groups of controls:

    i. those associated with the *intensity* and *focus* of the trace
    ii. those controlling *vertical deflection* (#6 thru 15).
    iii. those controlling *horizontal deflection* (#16 thru 19).
    iv. those associated with initiating the trace movement, TRIGGER,(#20 thru 24).

3. Nominal starting positions: (If you have trouble finding the beam consult your instructor.)

LELEL(21)	0 (or so)	VMODE(10)	CH1
MODE(22)	NORM	VOLT/DIV	0.2 V
COUPLING(23)	DC	RED knobs	fully CW (to CAL)
SOURCE(24)	CH1	AC/GND/DC	AC
HOLDOFF(19)	fully CCW	POSITIONs (9,12,17)	centered
TIME VARIABLE(18)	fully CW	INTEN/FOCUS/ILLUM	centered
TIME/DIV(16)	0.2 ms	9,12 and 16	pushed in

Additional desciption of oscilloscope controls

A: The INTENSITY knob controls the trace brightness[3] by varying the potential of a control grid or an aperture near the electron emitting cathode. The FOCUS adjusts the trace sharpness by varying the potential on the focusing anode. POSITION knobs move the traces up and down, and also right and left.

B: The VOLTS/DIV and red VARIABLE knobs provide coarse and fine adjustment of the pattern height by varying the amplification of the signals from the input terminals. To read the *correct* volts/division shown on the scales, the red variable knobs *must* be in the CAL (calibrated) position (fully clockwise).

The V MODE lever permits display of either or both channels; for both use either ALT for alternate sweeps, or CHOP which switches channels at a 250 kHz rate. (Hitachi's DUAL automatically switches from ALT to CHOP at a TIME/DIV setting = 1 ms/DIV).

Except for the Hitachi, pulling out the CH1 POSITION control (PULL ADD) adds both channels (CH1 + CH2), and pulling out the CH2 POSITION control (PULL INV) inverts its signal (-CH2). Hence using both PULL ADD and PULL INV gives the *difference* of the two channels (CH1 + -CH2). (For the Hitachi use the ADD or DIFF on the vertical MODE lever).

C: The TIME/DIV and VARIABLE or SWP/VAR knobs give coarse and fine control of the trace horizontal sweep motion. Again for the TIME/DIV scales to be *correct*, set the VARIABLE (or SWP/VAR) knob at the CAL position (fully clockwise).

Once the trigger requirements have been satisfied, the scope sweeps the trace horizontally from left to right at a uniform rate (set by the TIME/DIV knob), and then quickly returns the trace to the start position. These trigger circuits are useful for synchronizing the sweep start with some common feature of the input signal although the signal itself may not be perfectly periodic (e.g. your heartbeat). (see Fig. 2).

[3]CAUTION: a stationary intense spot may damage the screen

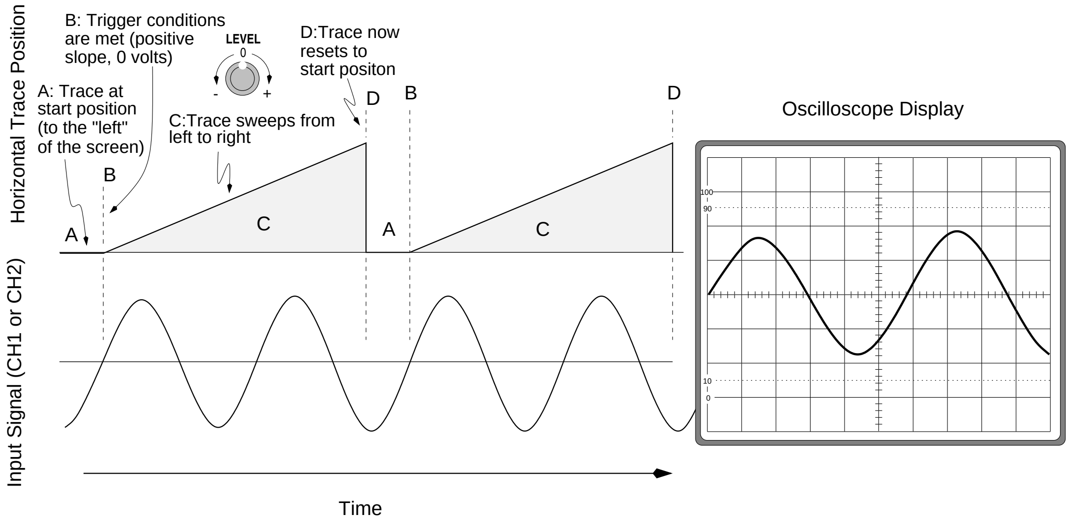


Figure 2

D: The TIME/DIV knob, when set to the $\boxed{\text{X-Y}}$ position, disconnects the internal horizontal sweep generator and uses the CH1 X input to specify the horizontal sweep; CH2 is still connected for vertical displacements and thus becomes Y. (For Hitachi also set MODE lever to CH 2 X-Y). The resultant X-Y pattern forms a Lissajous figure.

E: Other triggering options:

- **To sweep automatically:** Set trigger MODE lever (22) to AUTO.
- **To trigger and sweep only on a large signal:** Set trigger *MODE* lever (or Hitachi horizontal MODE) to NORM and set SOURCE lever to the appropriate CH1 or CH2; then adjust trigger LEVEL knob (20) to desired amplitude.
- The trigger COUPLING lever can also couple the trigger input signal directly (DC); through a large capacitance, (AC); or (except Hitachi) through a filter (HF REJ or LF REJ) which can reject high or low frequencies: > 4 kHz or < 4 kHz.
- The trigger SOURCE lever has options of triggering at 60 Hz (LINE) or by using an external trigger (EXT) signal.

Part B - Suggested Oscilloscope Experiments

1. *Observe both sine and square wave signals of various frequencies.* Experiment with setting to learn the effect of different controls, (e.g. automatic *vs* triggered sweeps). For the square waves look both at low frequencies and at maximum frequencies. Notice any effects of changing the input coupling (#6 or 15) (switch back and forth from AC to DC). Sketch the appearance of “square” waves at high and low frequencies. If possible, explain the differences.

   i. Verify the calibration of the scope’s Y scaling by using the 0.5 volt peak to peak square wave signal from the calibration terminal (CAL or #11). To connect this signal to the scope, just touch the exposed calibration terminal with the center gold pin of the *bayonet coaxial* (BNC) connector on the input cable.

ii. Use a digital multimeter (DMM) to *measure* both the *sine* wave and *square* wave voltage output of the signal generator. Avoid very high frequencies since capacitative loading may then be a problem when the signal generator is also connected to the scope.

iii. Compare the DMM readings (which are r.m.s. or "effective" voltages) with the *peak-to-peak* voltages reading directly off the scope screen. Are they consistent? Remember the voltage amplitude is one-half the peak to peak voltage and the rms voltage is

$$\frac{\text{peak voltage}}{\sqrt{2}} = \frac{\text{peak-to-peak voltage}}{2.818} \quad !$$

2. OPTIONAL: Check the calibration of the scope's X (horizontal) scaling by using the TIME/DIV reading to measure the period for one wavelength of the signal. Compare the deduced frequency with signal generator scale setting and/or that of a frequency meter.

OPTIONAL:

Lissajous Figures: When two periodic motions at right angles combine, the resulting pattern is generally very complex. However, sine waves whose frequencies are in small integer ratios give simple stationary patterns. For example see Fig. 3.

To produce *Lissajous* figures on the scope, set the TIME/DIV control to $\boxed{\text{X-Y}}$. Connect the 60 Hz terminal to the CH1 X input. To the CH2 Y input apply a sine wave input whose frequency bears a simple ratio with respect to 60 Hz. (For the Hitachi, also set the vertical MODE lever to CH 2, X-Y.)

A Lissajous pattern should result. Adjust amplitudes and positions to locate the figure properly on the screen. The frequency scale on the signal generator may not be accurate, but the 60 Hz power line is an excellent frequency standard.
Use Lissajous figures to calibrate your signal generator at 120 Hz and 180 Hz.

Observe and explain patterns with the frequency ratios of Fig. 3. Also try a ratio 2:3.

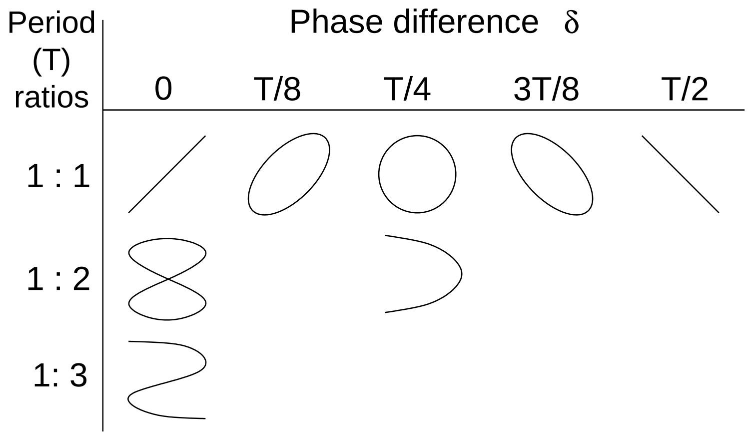


Figure 3: Examples of Lissajous figures.

Part C – Differential Amplifiers

INTRODUCTION:

Amplifiers are devices which usually increase the amplitude of the output signal compared to the input signal. The customary symbol for an amplifier is a triangular shape:

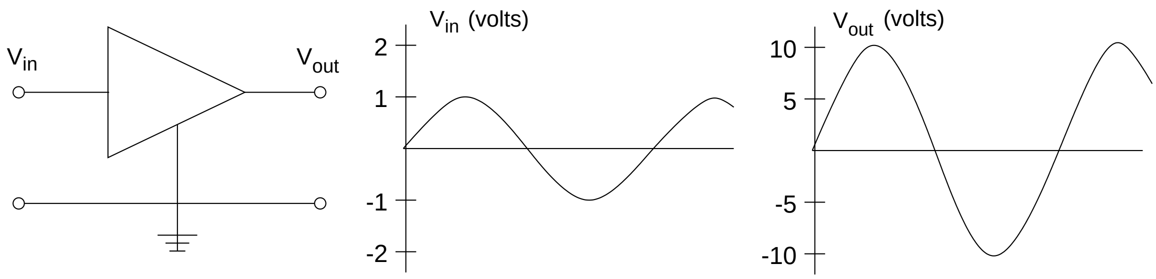


Figure 4

In the sine wave example above, the gain is 10 and the input signal is between an input terminal and ground. The internal oscilloscope amplifiers for channels Y_1 and Y_2 are of this type and have a common ground. Because of this common ground, one has problems in using a scope to examine **simultaneously** voltages across individual circuit elements that are in series (see below, also E9 lab).

We can avoid these problems by interposing "differential amplifiers" which have two inputs V' and V'', (neither at ground), and which amplify only the voltage difference $(V' - V'')$. Symbolically:

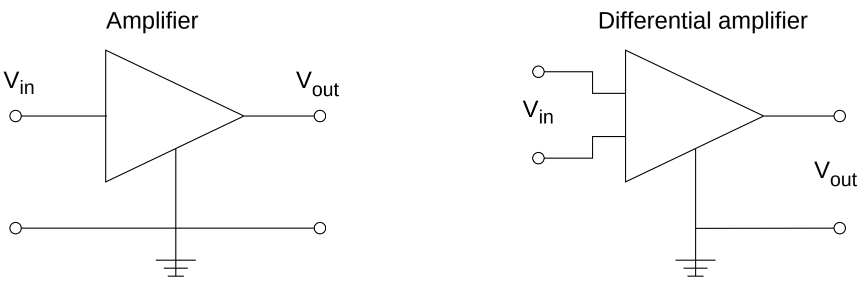


Figure 5

Note now that the ground of the output signal is independent of any input ground.

SUGGESTED EXPERIMENTS:

1. *Become familiar with the circuit plug board (Fig. 6) and the banana type plug-in components.* Note there are 24 groups with 9 socket plug-in holes in each group. While the 9 plug-in holes in each group are electrically connected together, each of the 24 groups is isolated unless connected by plug-in components. Two isolated metal bus bars (with banana plug-in holes) on the sides may facilitate common connections for some circuits.

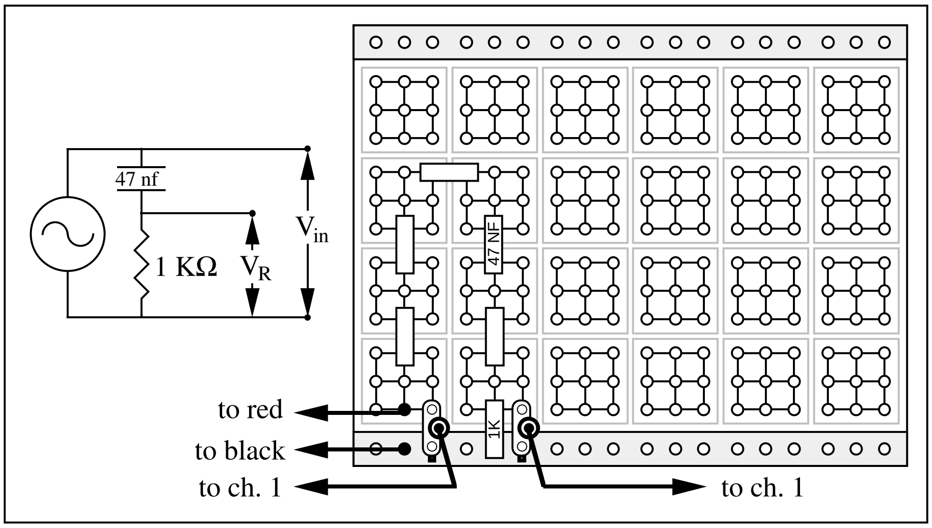
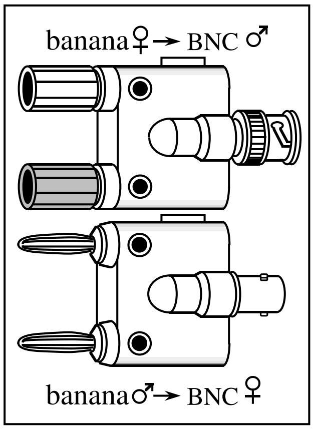


Figure 6



The plug-in components (metal bridges, resistors, capacitors, inductors, or other circuit elements) will only connect between adjacent groups of the 9 (already connected) plug-in holes or from the edge metal bus bars to an adjacent 9 hole group. We also provide coaxial cable to banana plug-in connectors: these facilitate using signal generators, scopes, etc. with the circuit plug-in board. *Be sure to notice the raised retangular protuberance on one side of the connector.* This indicates the connector ground bar side.

As a first configuration use the RC series circuit sketched in Fig. 6 as a guide using the scope to observe simultaneously the output of the AC signal generator and the signal across the resistor. How do the relative $V_{\rm peak}$-to-$V_{\rm peak}$ amplitudes compare at low, medium and high frequencies. It may help to know that the reactance X_C of a capacitor is $X_C = 1/\omega C$, the impedence Z is $Z = \sqrt{R^2 + X_C^2}$ and medium frequencies are given by $X_C \approx R$. Qualitatively explain the behavior.

Do to voltage peaks always occur at the same time?

2. The *phase shift* ϕ between the current (monitored by the voltage drop across the resistor) and the voltage has the relation $\tan\phi = -X_C/R = V_C/V_R$. To observe this behavior you must simultaneously monitor the signal across the resistor and capacitors. Unfortunately of these components does reference to ground; a necessary aspect of your scope inputs. (Which one?) To compensate for this short coming a module containing dual differential amplifiers is provided.

   Connect the resistor and capacitor signal to the differential amplifiers as shown in the figures below. Connect the outputs of the differential amplifiers to the scope

CH 1 and CH 2 inputs, and set both gains to 1.

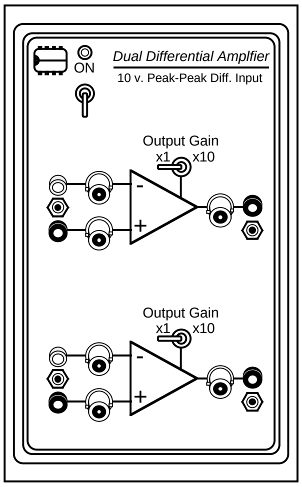


Figure 7: Dual differential amplifier module.

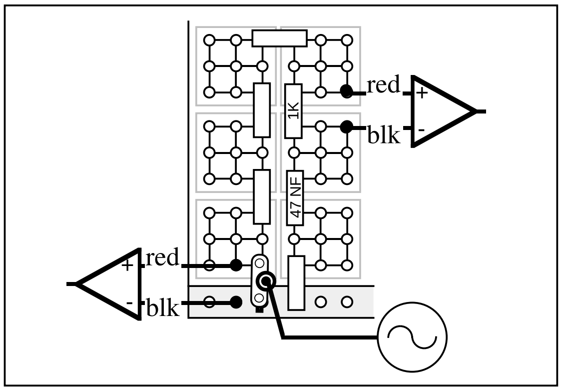


Figure 8: Inputs to differential amplifiers.

Sketch the oscilloscope display when $X_C = R$ and show how you can use this information to obtain the phase shift. Compare your result to that of the $\tan\phi$ formula.

3. Replace the capacitor with the diode and use the differential amplifiers to now observe the action of a silicon diode on an A.C. signal.

NOTE: Silicon diodes (Fig. 9) are junctions between silicon semiconductors with different type doping: n-type with an impurity which easily donates electrons, e.g. phosphorus (P) in silicon;[4] and p-type with an "acceptor" impurity, e.g. aluminium (Al) in silicon[5]. The junction between the two types becomes a very non-linear device which if subjected to an electric field acts almost as a one way valve to current flow. The diode symbol has the arrow pointed in the direction of conventional easy current flow (from p-type to n-type) though actual electron flow may be in the opposite direction.

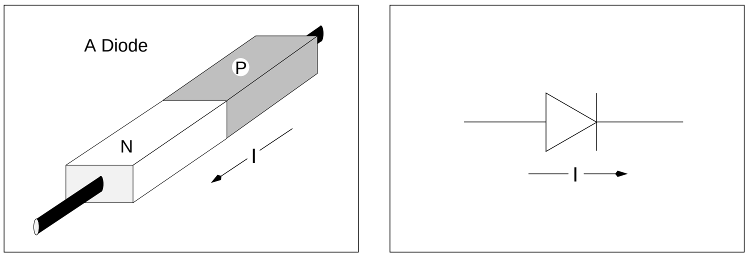


Figure 9

SUGGESTED EXPERIMENT:

Set up the circuit plug board as shown in Fig. 10. Connect Y_1 and Y_2 to the two scope inputs.

[4]As one might expect since silicon has valence 4, phosphorus valence 5, and aluminum valence 3

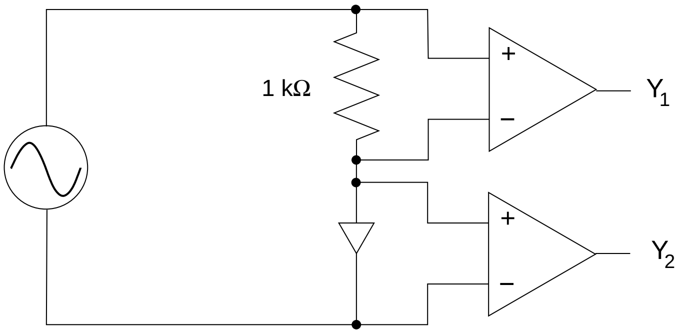


Figure 10

Vary the amplitude of the signal generator input and sketch the resultant waveforms across the resistor and across the diode.

E-9 A.C. Circuits

OBJECTIVE:

To study voltage and phase relationships in A.C. circuits

APPARATUS:

Dual trace oscilloscope & manual; differential amplifiers, circuit plug board & component kit; signal generator & frequency counter; digital multimeter (DMM).

Part A: **SERIES A.C. CIRCUITS**

INTRODUCTION:

Overview: Remember that "rms" value of the AC voltage or current means the "root-mean-square" and this value is useful even when the voltage or current is changing but not as perfect sine waves. For example, the voltage used to provide power for most lights and appliances is roughly $V_{rms} = 110$ Volts. Some higher power appliances such as stoves use $V_{rms} = 220$ Volts. Although the voltage and current may often be negative, the rms values are always positive. Similarly, the impedance Z is always positive. Sometimes the word "effective" is used instead of "rms" but it means the same thing. For a pure sine wave voltage, the peak voltage and rms voltage are related by $V_{rms} = \frac{V_{\text{peak}}}{\sqrt{2}}$. Thus the peak to peak voltage *swing* of a 110 volt AC wire is $V_{\text{peak}} = 110 \times \sqrt{2}$ volts ≈ 155 volts!

The rms concept allows us to describe any AC voltage as having a particular *rms* voltage and a particular *phase*.

In this experiment we will use the subscript "rms" for some measurements and will use "V" or "v" or "I" or "i" without this subscript when referring to *instantaneous* values of the voltage or current. Thus, even if two rms voltages are equal, ($V_{rms} = v_{rms}$), V may *not* be equal to v since V and v may have different phases.

The impedance Z (in ohms) of any part of a circuit is the ratio of the rms voltage across that part and the rms current though that part: Because impedance is defined as a ratio of voltage/current, impedance is measured in ohms. If $\omega = 2\pi f$ and we measure L and C in henries and farads, respectively, then it can be shown that the impedance X_R of an resistance R is R and the impedance X_L (in ohms) of an inductance L is $X_L = \omega L = 2\pi f L$. and the impedance X_C (in ohms) of a capacitance C is $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$. It can also be shown that the impedance Z (in ohms) of an RLC series circuit (shown in Fig. 1) is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

where, X_L and X_C are the impedances of the individual inductor and capacitor.

The impedance Z of the RLC series circuit is a *minimum* for $X_L = X_C$. The frequency for which this occurs is the *resonant frequency*. At this frequency, f_r, the current thru R is maximum, but the voltage $V_{rms\ LC}$ across the LC series combination is a minimum and in fact would be zero if the inductor had no resistance. Hence one can search for f_r by varying the frequency and looking either

1) for a *maximum* $V_{rms\ R}$ signal, or 2) a *minimum* $V_{rms\ LC}$ signal.
A search for the minimum has a practical advantage that near the resonant frequency, f_r, one can increase enormously the detection sensitivity by going to maximum signal generator amplitude and also by going to higher scope gain.

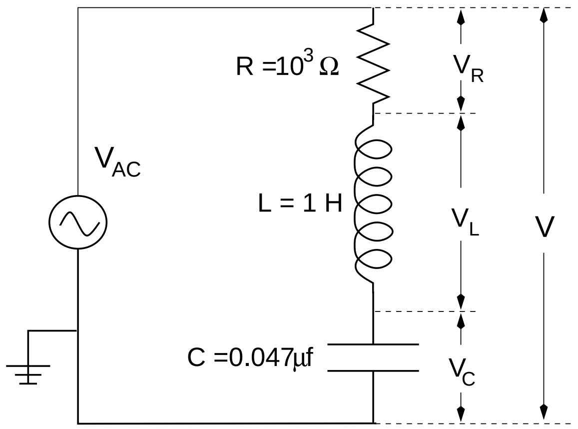


Figure 1: A series LRC circuit.

Our dual trace scope allows the simultaneous observation of two voltages. If we use differential amplifiers (E-8) to avoid ground problems, then we can compare the signal across the resistor (displayed on one trace) with the signal across other circuit elements (displayed on the other trace). Since across a resistor the voltage and current are in phase, and since the current throughout a series circuit is the same, we can observe the relative phase relationship between the current and any other measured voltage.

SUGGESTIONED PROCEDURE:

1. Hook up the circuit plug board as in Fig. 1. (The signal generator supplies V_{rms} (or V_{AC}.)
2. Use the digital multimeter (DMM) to measure V_{rms}, $\mathrm{V}_{rms\ R}$, $\mathrm{V}_{rms\ L}$, and $\mathrm{V}_{rms\ C}$ at frequencies near 400, 700, and 1000 Hz. Is $V_{rms} = \mathrm{V}_{rms\ R} + \mathrm{V}_{rms\ L} + \mathrm{V}_{rms\ C}$? Explain.
3. Calculate the frequency for which resonance should occur.
4. Use the scope to search for the *resonant frequency* f_r by the following techniques and also estimate the uncertainty $\Delta \mathrm{f}_r$ in each measurement. Use the trigger output of the signal generator to trigger the scope. (For the Krohn-Hite signal generator, use the "TTL" output. This output gives a "rectangular" signal, snapping between zero and +3 volts, which is suitable for digital circuitry such as computers and counters.)
   (i) As one varies f, observe the *maximum* in the *current* I_{rms} *thru R*. (Since $I_{rms} \propto \mathrm{V}_{rms\ R}$, use a differential amplifier with scope channel Y_2 to look at $\mathrm{V}_{rms\ R}$).
   (ii) Observe the *minimum* in the *voltage* $V_{rms\ LC}$ *across the LC combination.* (Use the other differential amplifier to input $\mathrm{V}_{rms\ LC}$ to scope channel Y_1. To improve sensitivity, turn up signal amplitude and/or scope gain).
   (iii) *Produce a Lissajous figure* using $\mathrm{V}_{rms\ R}$ and $\mathrm{V}_{rms\ LC}$ as the X and Y inputs. (Use the X-Y setting of the TIME/CM or TIME/DIV knob and set mode lever

to X-Y). At resonance the usual elliptical pattern becomes a straight line as $V_{rms\ LC}$ goes to zero. Again turn up the gains to improve sensitivity.

5. Compare the calculated f_r with your observed values. Is there agreement?

6. *Phase relations:* Use scope channel 1 (plus a differential amplifier) to observe the total voltage $V_{rms\ RLC}$ across the RLC combination. Use scope channel 2 (plus differential amplifier) to observe $V_{rms\ R}$. Fig. 2 shows the phasor relationships.

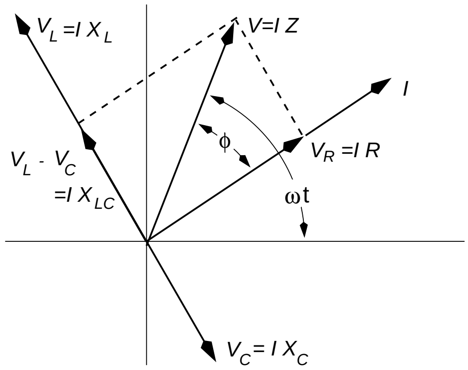


Figure 2

(i) Set the signal generator near f_r. At resonance the two signals should be in phase. Why? (If at $f = f_r$ the signals are 180° out of phase, the input polarity is wrong on one differential amplifier; hence interchange the input leads on *one* of the differential amplifiers.)

Find the resonant frequency by this method. Adjusting gain and position controls so the signals nearly overlap will help.

(ii) With $f = f_r$, short out the capacitor by a jumper, (e.g. with metal bridge connector). Note what happens. Unshort capacitor and then short the inductor. Explain results in terms of Fig. 2. Does current lead or lag the applied voltage in each case?

(iii) Remove the shorts and try $f < f_r$ and $f > f_r$. Explain phase behavior.

OPTIONAL:

A quality factor Q (so named because the energy loss ΔU per cycle of energy U stored in oscillating LC elements is inverse to the circuit's Q) is

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}.$$

Also Q measures the sharpness of a resonance. Rewrite Z in terms of Q:

$$Z = R\left[1 + Q^2\left(\frac{f}{f_r} - \frac{f_r}{f}\right)^2\right]^{1/2}.$$

Note that large Q means Z will assume large values when f differs slightly from f_r. Hence sharp resonant effects occur. Calculate Q for your circuit. What $\Delta Z/Z$ results for $\Delta f/f_r = 0.1$? (Large Q occurs for R small, but the resulting resonant current may be excessive. To achieve sharp tuning, radio circuits may have Q's ~ 100).

Part B: **PARALLEL A.C. Circuits**

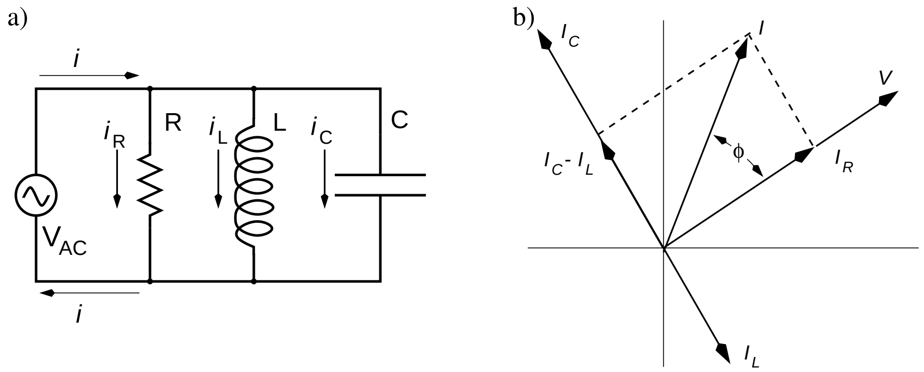


Figure 3: A parallel LRC circuit.

INTRODUCTION:

The parallel RLC circuit is very common in radio and TV circuits. The circuit analysis follows from Kirchhoff's laws, but is not in many beginning texts. The common terminal voltage V is across each element (see Fig. 3a), and the instantaneous current i is the algebraic sum of the instantaneous i_R, i_L, and i_C.

Since i_R is in phase with V, but i_L lags V by 90°, and i_C leads V by 90°, we can describe the situation by the rotating vectors in Fig. 3b where I is the vector sum of I_R, I_L, and I_C. Hence from Fig. 3b

$$I_{rms} = \sqrt{I_{rms\ R}^2 + (I_{rms\ C} - I_{rms\ L})^2}$$

where $I_{rms\ R} = \frac{V_{rms}}{R}$, $I_{rms\ C} = \frac{V_{rms}}{X_C}$, and $I_{rms\ L} = \frac{V_{rms}}{X_L}$. Substituting these gives

$$I_{rms} = V_{rms} \cdot \sqrt{\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2}.$$

Resonance still occurs for $X_L = X_C$ but the total current, I_{rms}, is then just the current thru R and thus is a MINIMUM (in contrast to the series resonant case).

Since V is the same for all the parallel elements, the **relevant phase differences are between the currents**. To measure the total current I_{rms} and the phase between it and the voltage V_{rms}, we will insert a 22 kΩ sampling resistor R_S in series with the signal generator. See Figure 4.

Measure the voltage $V_{rms\ S}$ across the sampling resistor R_S by connecting the resistor ends to scope channel 2 via a differential amplifier. Since the voltage and current are in phase across a resistor, this signal $V_{rms\ S}$ is proportional to the total current I_{rms}.

We use channel 1 and the other differential amplifier to view the common voltage V across all the parallel elements. At resonant frequency f_r the currents from L and C will cancel since they are of equal magnitude but (always) 180° out of phase. Hence at f_r the total current I will be just that thru R, i.e. $I = I_R$, and V and V_S (or I) will be in phase.

The smaller the sampling resistor R_S, the less disturbance its voltage drop $V_{rms\ S}$ will have on the net voltage V_{rms} applied across the parallel circuit.

However distortion in the generated sine wave may bother when R_S is small and large currents flow. Our suggested $R_S = 22$ kΩ is a compromise. Even so you may see a factor 2 change in V_{rms} as f varies; however we can compensate by adjusting the signal generator amplitude to keep V_{rms} constant as the frequency changes.

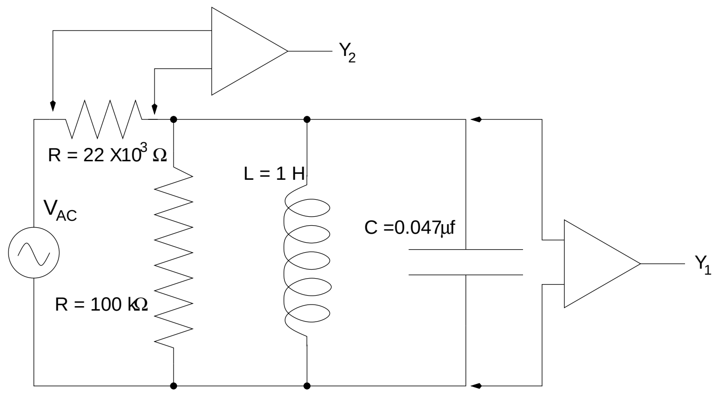


Figure 4: A parallel LRC test circuit.

SUGGESTED PROCEDURE:

1. Hook up the circuit plug board as in Fig. 4 and again trigger the scope externally from the signal generator. (We chose a large $R = 100$ kΩ circuit resistance to limit the resonant current and thus enhance sensitivity.)

2. Set the frequency near f_r found in Part A: SERIES RESONANCE. Adjust the signal generator amplitude to give a 1 volt scope signal (p to p) for V in channel 1.

3. Vary the frequency f on both sides of f_r by about 300 Hz. Adjust the signal generator amplitude to keep V_{rms} (i.e. Y_1) constant as f and $V_{rms\ S}$ change. Since $I_{rms} \propto V_{rms\ S}$, plot $V_{rms\ S}$ versus f. Note the resonant minimum in I_{rms} at f_r.

   OPTIONAL: Replace the 22 kΩ R_S with a 1 kΩ resistor. Repeat step #3 and note the increased distortion of V_S at f_r, but the relative constancy now of the voltage V, (channel 1), across the circuit elements.

4. Starting with $f = f_r$ observe how the *phase relations* between I (which is in phase with V_S) and V change as one goes to $f > f_r$ and $f < f_r$. (Again if at $f = f_r$, the two signals are *not* in phase but are 180° out of phase, interchange the input leads to one of the differential amplifiers).

5. *Use the Lissajous figure technique* to find f_r: namely, with the horizontal deflection (TIME/CM or TIME/DIV) knob at X-Y (and the mode lever at X-Y) use V and V_S as the X-Y inputs. As V_S (or I) goes thru its minimum, the elliptical pattern collapses to a straight line. Again one can increase gains to improve sensitivity.

6. Move the scope trace for V near to the top of the scope screen.

7. Move the scope trace for I near to the bottom of the scope screen.

8. Then the ratio of $\frac{\text{Magnitude of upper trace}}{\text{Magnitude of lower trace}}$ is proportional to the impedance Z.

9. Notice that, as you change the frequency, Z given by this ratio of traces, goes through a maximum at the resonant frequency.

E-10 Transistors

OBJECTIVE:

To experiment with a transistor and demonstrate its basic operation.

APPARATUS:

An **npn** power transistor; dual trace oscilloscope & manual; signal generator & frequency counter; power supply (±15 V fixed/±9 V variable); circuit plug board & component kit; two digital multimeters (DMM); differential amplifiers.

INTRODUCTION:

Our junction transistor (Fig. 1) is like two back to back *np* diodes (see E-8 Part C, #3). Hence there are two possibilities, an *npn* transistor and a *pnp* transistor. Our *npn* transistor has a central p-type layer (the *Base*) between two n-type layers (the *Emitter* and *Collector*). There are other type transistors which we will not discuss, (e.g. a *MOSFET:* Metal-Oxide Semiconductor-Field-Effect-Transistor).

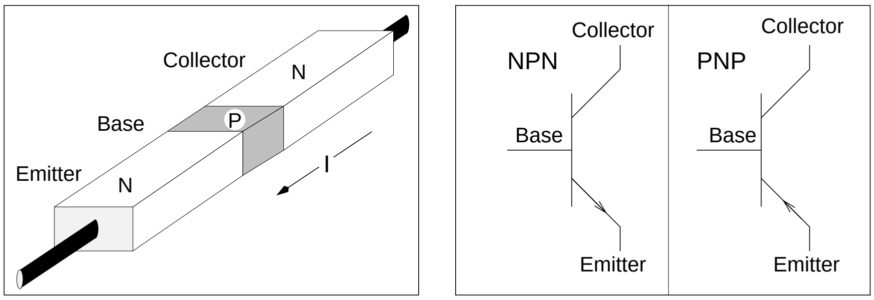


Figure 1

For an *npn* transistor the *collector is positive relative to the emitter.* The base-emitter circuit acts like a diode and is normally conducting (i.e. forward-biased). The base-collector circuit also acts as a diode but is normally non-conducting (reverse biased) if no current flows in the base-emitter circuit. However when current flows in the base-emitter circuit, the high concentration gradient of carriers in the very thin base gives an appreciable diffusion current to the reversed biased collector. The resulting collector current I_c depends on the base current I_b. We write $I_c = \beta I_b$ where β is the current amplification factor.

PRECAUTIONS: Although our power transistor is fairly indestructible, its characteristics are temperature sensitive. Hence avoid exceeding the voltages suggested; also leave the power supply off when not making measurements; read currents and voltages to two significant figures only.

SUGGESTED EXPERIMENTS:

1. *CURRENT Amplification:* Measure β for various emitter to collector voltages, V_{ec}. Set up the circuit as in Fig. 2.

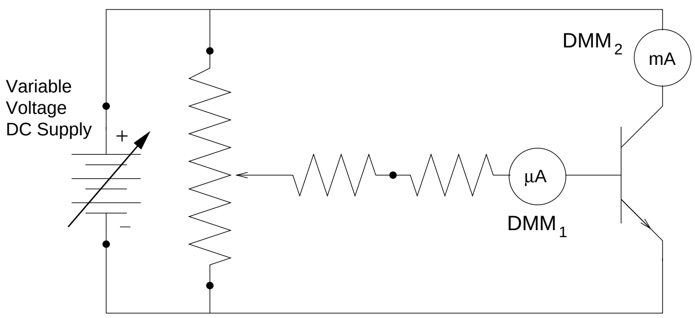


Figure 2

Start with the voltage divider completely counter-clockwise so that the emitter-base voltage V_{eb} is a minimum. With the variable power supply set to 3 V, record the I_b measured on DMM_1 and the I_c on DMM_2

Repeat the readings for ten reasonably spaced (higher) settings of the voltage divider (i.e. higher V_{eb} and hence higher I_b.

Calculate $\beta(= I_c/I_b)$ and plot it against I_b. Over what range of I_b is β reasonably constant? Repeat the above but with emitter to collector voltage V_{ec} now at 7 V.

2. (Optional) *MEASUREMENT OF I_c vs V_{eb}:* Remove the multimeter DMM_1 (used as a microammeter) from the circuit of Fig. 2 and use it instead as a voltmeter to measure the emitter to base voltage V_{eb}. With the variable power supply set to give 7 V for V_{ec}, use the voltage divider to vary V_{eb}. Read and record both V_{eb} and I_c for 10 reasonably spaced values of V_{eb}. Plot I_c vs V_{eb}. The results are very similar to a diode curve (as expected since I_c is proportional to I_b over the region where β is a constant).

3. *VOLTAGE Amplification:* By adding a large load resistor in series with the collector, one can convert the current amplification β observed earlier into a voltage amplification. Hook up the circuit plug board as in Fig. 3.

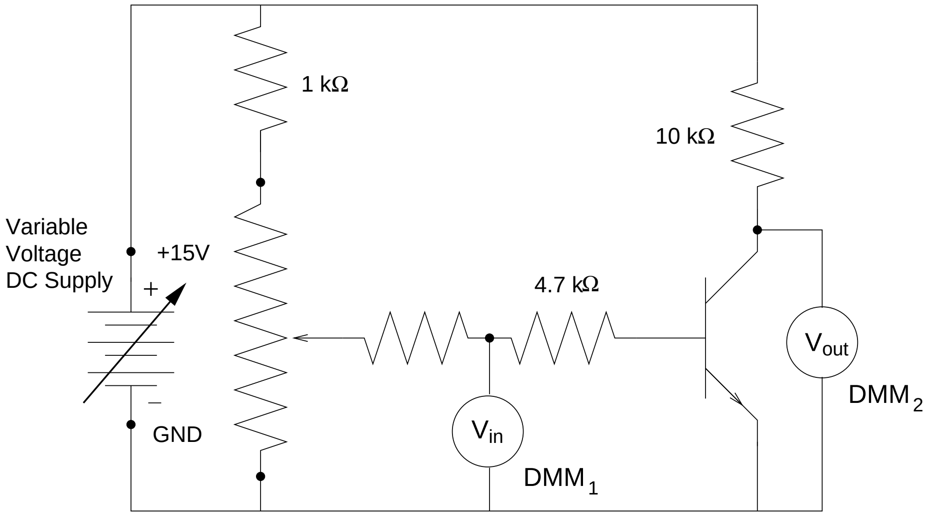


Figure 3

The input voltage to the transistor V_{in} includes the drop across the 4.7 kΩ protective resistor. The voltage gain is G $= \Delta V_{out}/\Delta V_{in}$. Record input voltages V_{in} and output voltages $V_{\text{out}}(= V_{ec})$ for ten reasonably spaced values of V_{in} from 0 to 5 volts.

Graph V_{out} vs V_{in} and calculate the voltage gain G at the steeply changing part of your graph. The value of V_{in} at the center of this region we call the "operating voltage" of the amplifier. How would the gain change if the load resistance was 2.2 kΩ instead of 10 kΩ?

4. *Distortion effects when amplifier is overdriven:* Set up the circuit plug board as in Fig. 4. Connect Y_1 and Y_2 to the scope. To connect the signal generator to the board, use the BNC to banana plug adapter and remember that the side with the bump goes to ground.

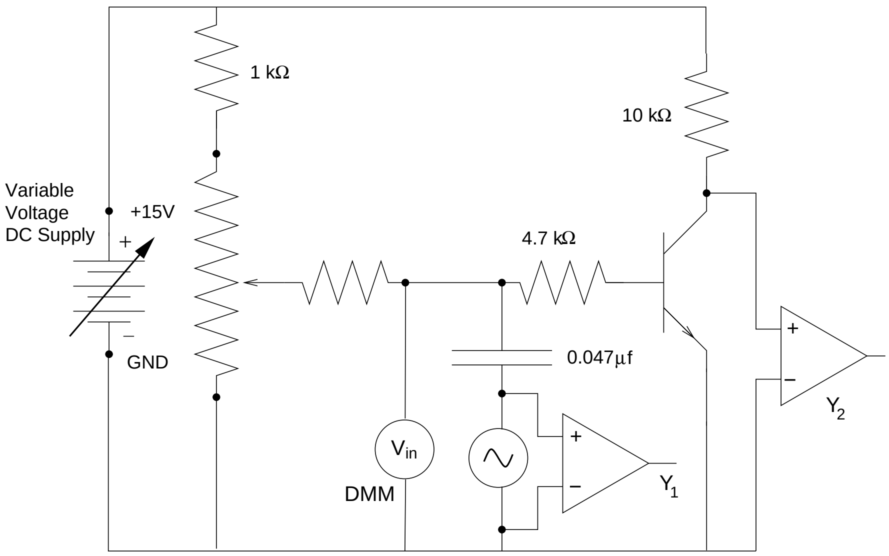


Figure 4

Adjust the voltage divider until the V_{in} is close to the "operating voltage" found in #3.

Vary (and record) the amplitude of the input signal from small values to those which overdrive the amplifier and produce considerable distortion in the output Y_2.

Observe and record the effect of changing V_{in} to values outside of the operating range (where β is constant).

E-11 Electromagnetic Radiation

OBJECTIVES:

To study microwave electromagnetic radiation; to measure the wavelength, λ, in air and in a wave guide; to observe reflection, refraction, polarization, interference and diffraction.

NOTE: If this experiment precedes L-1 and L-8, first read appropriate material on diffraction, interference, polarization, etc.

APPARATUS:

Klystron oscillator kit with wave guide and manual; horns, detector and various auxiliary equipment.

INTRODUCTION:

Our source of microwaves is a klystron amplifier and oscillator which is a device where electrons can produce oscillations at frequencies large compared to those possible from lumped LC components. High velocity electrons pass through a gap in an electromagnetic cavity. Their electric fields excite resonate cavity oscillations whose electric fields then tend to bunch the electrons at the resonate frequency. (See Fig. 1). These velocity modulated bunches in passing through gaps in a similar resonator excite the second cavity.

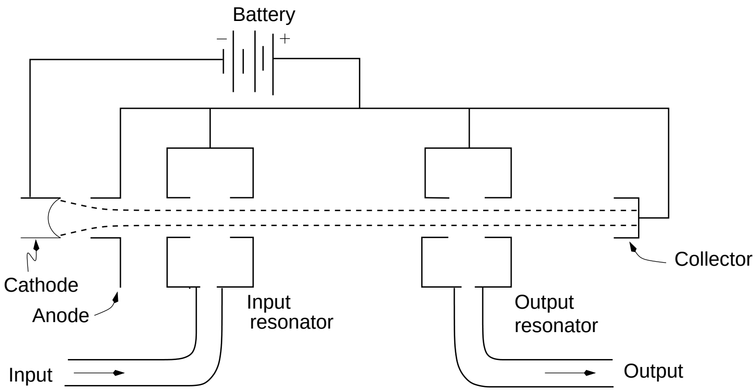


Figure 1: Schematic of klystron amplifier.

If part of this generated power feeds back into the first cavity, microwave oscillations can result. A reflex klystron oscillator (Fig. 2) uses a negative repeller to return the original velocity modulated electrons back through the first resonating cavity.

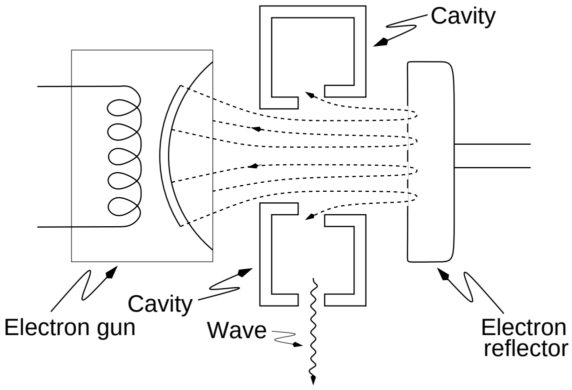


Figure 2

SUGGESTED EXPERIMENTS:

1. Follow instruction manual to assemble and operate the equipment.
2. To measure λ in air and in the wave guide, set up standing waves by placing a metal plate reflector about 30-40 cm in front of the transmitting horn.
3. Note reflection by a sheet of metal at various angles.
4. Place the metal plate with a 5 cm slot in front of the transmitting horn. Measure the diffraction pattern at a distance of about two meters from the slot. Compare with theory.
5. Make a pair of slots one cm wide and 6 cm apart in front of the transmitter. Measure the interference pattern two meters away.
6. Use the wire grill to study the polarization of the radiation.

   Describe and discuss what you observe.
7. Check the refraction of the radiation by a piece of plastic.
8. Study the Bragg reflection from arrays of bearing balls imbedded in plastic foam.
9. Study the transmission of the radiation through wood, plastic, metal, metal screen, sponge, wet sponge, etc.

Light:
LC-1 Diffraction and Interference

OBJECTIVES:

To observe diffraction and interference and to measure the wavelength, λ, of laser light.

APPARATUS:

Optical bench; He-Ne laser; slit of variable width; thickness gauge; slide of 4 pair of electroformed double slits; short focal length lens; small opaque ball tipped pin; small circular holes in thin brass; screen; Pasco Interface with light and rotation sensors, mounting bracket, light aperature module; traveling microscope; interference demonstrations: optical flats (2), Newton's rings, 18 mm gauge blocks (2), interferometer & Na lamp.

INTRODUCTION:

The He-Ne laser provides plane light waves of wavelength 633 nm. The waves in a cross section of the beam are in phase (i.e. the light is "coherent"). Any finite plane wave will spread by diffraction. The spreading is rapid if the beam is narrow, e.g. after passing the narrow slit in Part I below. If we illuminate two closely spaced narrow slits (narrow relative to the slit to slit spacing) by the same laser beam, the two spreading beams will overlap and interfere: Part II.

CAUTION: The laser is a very bright source. Do not allow the laser beam to enter the eye and do not point the beam at anyone!

Experiment I: Single Slit Diffraction:

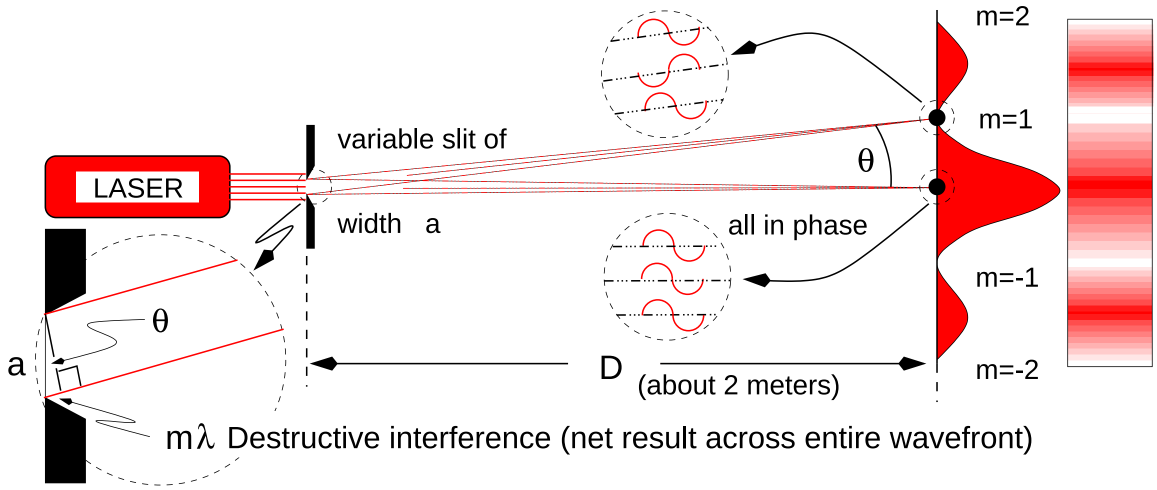


Figure 1: Schematic of single slit diffraction

THEORY:

The angular separation in *radians* of the first minimum from the center of the pattern is

$$\theta = \lambda/a$$

where a is the width of the slit. (For your derivation you may refer to the text and remember that for small θ, $\sin\theta \cong \theta$).

SUGGESTED PROCEDURE:

1. Mount the laser and variable width slit on the optical bench as shown in Fig. 1. Observe the pattern on the supplied white screen and qualitatively explain in your lab book that the first minimum occurs as shown in Fig. 1 at $m = 1$.

   QUESTIONS:

   Q1. Qualitatively, how does the pattern vary as the slit is closed?

   Q2. In "real" life when would you begin to be concerned about the appearence of diffraction? Identify at least one (if not more) example of diffraction in your daily life.

   Q3. Qualitatively, how does the pattern vary as λ decreases (from red to blue-violet)?
   Click on the Single Slit Diffraction link immediately below (web-version) to test your prediction.

2. To enable more *quantitative* observations the PASCO interface module has been configured to provide an Intensity vs. Linear position of the light sensor plot and table. Launch the experiment by CLICKing on the telescope icon below (web-version only). After the PASCO experiment window pops up, start the data acquisition by CLICKing once (or twice) on the REC button and move the combined light and rotary motion sensor in the lateral direction gently and smoothly by hand. Pratice starting on one side of the diffraction pattern and move smoothly towards the other.

3. Configuring and aligning the optical components is very important! In particular the laser alignment and apperature settings are VERY important for getting good results.
   There are two aspects for configuring the light sensor: aperature size and detector gain. Using a larger aperature lets more light reach the detector. How does this affect the image profile? A larger gain increases the sensor output but can peg the output at maximum and increase the noise. *Adjust and record the setting which give you reasonably good results.* Ask your instructor for a brief demostration if you are at all uncertain.

4. Use the thickness gauge to set a slit width, $a \cong 0.1$ mm. It is important that your slit is parallel. What would happen if the slits were not parallel to one another?

5. Using the cross-hair feature of the PASCO graph display, measure θ (from y/D) for m=2, 1, 0, -1, -2 plot m vs θ. Does this form a straight line? From the slope and known value of a, calculate λ.

6. Estimate uncertainties and then compare with the accepted value of 633 nm. Are there any obvious sources of systematic error?

Experiment II: Multiple Slit Interference

For double slit interference, shown in Fig. 2, the distance y to the mth bright fringe from the midpoint 0 is

$$y = D\tan\theta \cong D\theta \qquad \text{and} \qquad \theta \cong m\lambda/d$$

Hence

$$\lambda \;=\; yd/mD,$$

(For more information refer to the text and recall that for small θ, $\theta \cong \sin\theta \cong \tan\theta$.)

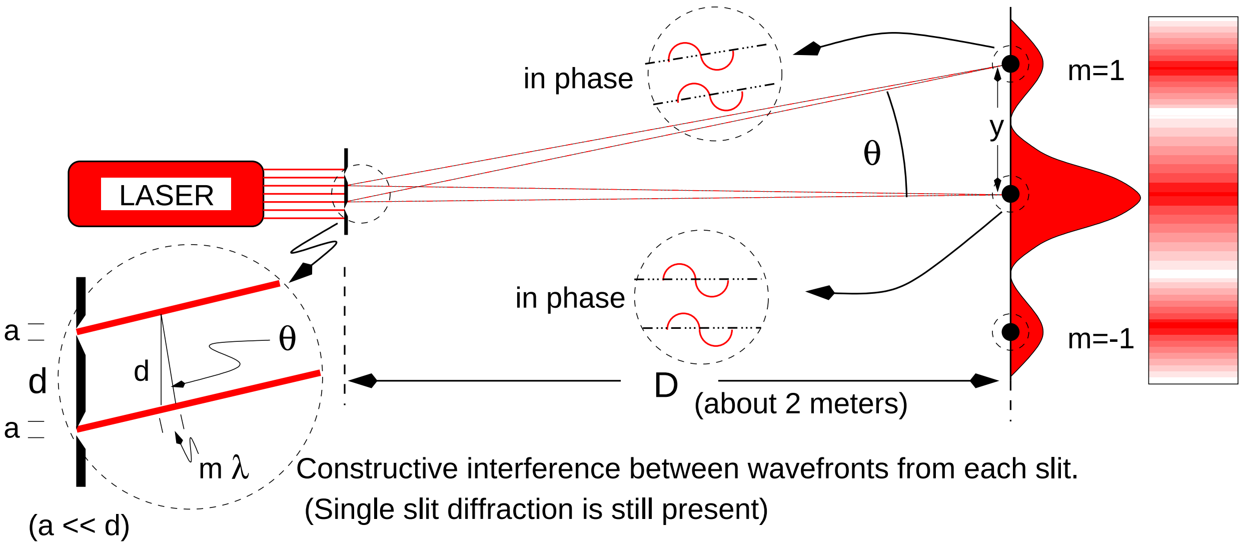


Figure 2: Schematic of double (or multiple) slit interference.

SUGGESTED PROCEDURE:

1. Mount the slide containing the four electroformed slits in the slit holder and illuminate one of the slit pairs with the laser. Two of the slit pairs have a center to center slit spacing of 0.25 mm and the other two have a 0.50 mm spacing. The slit pairs with the same spacing differ only in the width of each slit: one has slit widths of 0.04 mm and the other has slit widths of 0.08 mm.

2. Note the difference in pattern and spacing of the resulting interference fringes on the wall. Neither slit width is negligible compared to the wavelength so both diffraction and interference effects are apparent in all patterns, and the interference fringes are contained within the diffraction envelope. Observe the interference pattern for all four electroformed slit pairs.

3. Choose one pattern for careful measurements and measure the fringe separations at a number maxima (m= -1, 0, 1, etc.) and calculate λ using the same general procedure as experiment I. The separations marked on the slits are only nominal. If time permits use the traveling microscope (Appendix A.) on page 203 to measure the slit separations.

4. Estimate quantitatively your uncertainties. Compare with the accepted value.

Experiment III: Fresnel Bright Spot and Other Interference Patterns

INTRODUCTION:

The Fresnel bright spot (also called Poisson's bright spot) is a bright spot in the center of the shadow cast by every circular obstacle in the path of a plane wave. The effect is implicit in Fresnel's representation of a coherent plane wave by half-period zones (e.g. see Shortley and Williams, "Elements of Physics" 5th ed. p. 737 or, if this is the web-version, see this example at http://www.physics.ucla.edu/ dauger/fresnel/PoissonAragoStory.html), but was first pointed out by Poisson (a disbeliever in the wave theory) in an attempt to ridicule Fresnel's wave theory. Arago subsequently showed experimentally that the spot existed. You, too, may demonstrate the spot:

1. Replace the slit support rod with the rod containing the PASCO Slit Accessory wheel. Rotate and adjust the assembly to illuminate the circular obstacle in the laser beam.
2. To obtain a large enough coherent plane wave it may be necessary to diverge the laser beam by placing a short focal length lens directly in front of the laser.
3. Carefully align laser plus lens and obstacle to center the shadow along the beam axis. The coherently illuminated area (the plane wave) should be several times the area of the obstacle. Proper alignment may help significantly in "cleaning up" the image.

OPTIONAL:

Diffraction from a small circular opening: By analogy with the Fresnel bright spot in a circular shadow, you might expect a dark spot centered in the image of a circular opening. In this case the situation is more complex: the on-axis Fresnel bright spot in the shadow results from superposition of a large number of higher Fresnel half-period zones whereas the light from a small circular aperture comes only from a few low Fresnel half-period zones whose superposition on axis may result in either a dark or bright spot: dark for an even number of zones, bright for an odd.

Test this using the 2 circular aperatures on the PASCO Slit Accessory wheel. Illuminate the hole with the diverged laser beam in the same manner as described above. By varying the hole size and/or distance from hole to screen you may change the small number of half-period zones contributing and hence see the central point on axis as either a bright or a dark spot, e.g. see Fig. 13 Shortley and Williams, p. 740. Again careful alignment is important to obtain a clean symmetric image.

If time permits you may find it interesting to view the four addition two-dimensional diffraction patterns.

DEMONSTRATIONS ON THE DISPLAY TABLE:

A. Interference between optical flats, gauge blocks, etc.
B. Newton's rings
C. Michelson's interferometer

LC-2 Mirrors and Lenses

OBJECTIVES:

To study image formation and focal lengths of mirrors and lenses.

APPARATUS:

Optical bench; optical components (10) kit: lenses & mirrors (5), telescope, illuminated arrow light source & 12V power supply, sharply pointed rod, desk lamp, white card or screen, plane mirror.
NOTE: *the objects on the optical bench slides are not necessarily centered with respect to the index on the slide.*

INTRODUCTION:

We will design, test and then measure mirror and lens assemblies by several techniques. In all instances it will be possible to use a simple interactive Java applet to perform a virtual pre-lab exercise. For the physical set-ups it will require testing location of image by absence of *parallax*, and others require *focusing a telescope for parallel rays*. For both techniques see Appendix 4. Please master this material: your instructor is available for help.

Experiment I. Radius of curvature and focal length of a concave mirror:
VIRTUAL PRE-LAB:

1. If using the web-lab manual launch the virtual application by clicking on the Convex Mirror Application button below.
2. After clicking on "Start Me" you should observe a concave mirror, an object and its image. On the lower left corner is a cursor position readout.
3. With respect to the figure below and the mirror equation. Find the focal length.
4. Adjust the object to get $p = q$, and obtain the focal length again.
5. Move the object so that $p = f$. Do you see an image?
6. Move the object so that $p < f$. Is there an image? Is it real or virtual? Is the image height smaller or larger than the object?

   The Mirror Equation is given by:

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{R} = \frac{1}{f}$$

where f is the focal length and R is the mirror radius curvature. Note that when the object and image are equally distant from the mirror, $p = q = R$. You can use this condition to get an approximate value for R and f.

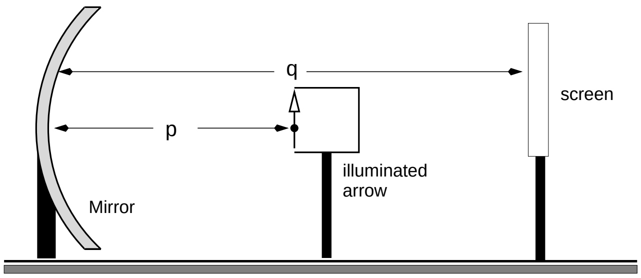


Figure 1: Layout of concave lens experiment.

SUGGESTED PROCEDURE (Note: The mirror holder contains a concave and a convex mirror on opposite sides. Be sure you use the *concave* mirror!)

I. By using object and image distances:

1. Resolve the image of the illuminated arrow formed by the concave mirror on the white card. Experiment with varying object distances, p, until you able to follow how the image distance, q, varies with the object distance.
2. Measure the image distance at several different positions of the illuminated arrow object. From each pair of conjugate object and image positions calculate f for the mirror. Make a table of the results in your lab book, but leave space to compare with the results of II. and III. below.

II. Obtaining $f = R/2$ by imaging at center of curvature ($q = p = R$): Replace the illuminated arrow and screen with the rod and place the rod at R. If the tip of the rod is at the center of curvature, a real inverted image will appear just above it with the tip and its image coinciding. An absence of parallax (see appendix D) between the tip and image is the most sensitive test for coincidence. When the *parallax vanishes*, the distance between the tip and the mirror is the radius of curvature R. From several determinations of R, (switching roles with your lab partners estimate the reliability of your results.)

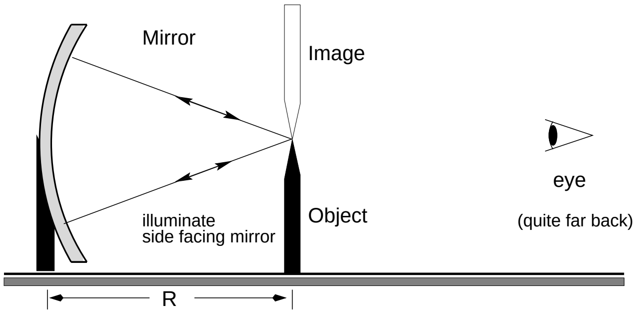


Figure 2: Object and image at center of curvature, R.

III. By placing object at the principal focus ($p = f$): If the tip of the rod is at f, rays from the tip will reflect from the mirror as a *parallel* bundle and give a sharp image of the tip in a telescope *focused for parallel rays* (see appendix D). Keep the distance between tip and telescope small or the reflected light may *miss* the telescope. The best test for locating the focal point is *absence of parallax between the tip's image and the cross hairs* in the telescope, but the telescope *must* already be focused for parallel rays (e.g., focus on something out the window). From several settings estimate the reliability of your f measurement.

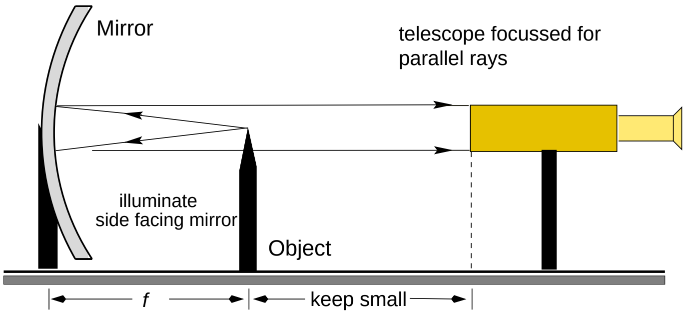


Figure 3: Object at f, image at infinity.

Experiment II: LENSES

VIRTUAL PRE-LAB:

1. If using the web-lab manual launch the virtual application by clicking on the Converging Lens Application button below.
2. After clicking on "Start Me" you should observe a convex lens, an object and its image. On the lower left corner is a cursor position readout.
3. With respect to the figure below and the lens equation. Find the focal length.

4. Move the object so that its inverted image has the same height. Find the focal length at this point.
5. Move the object so that $p = f$. Do you see an image?
6. Move the object so that $p < f$. Is there an image? Is it real or virtual? Is the image height smaller or larger than the object? What is the sign of q?

---

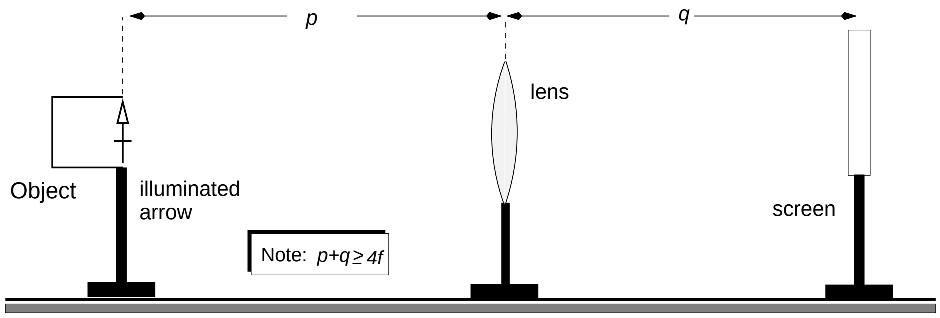
1. *Converging lens with short focal length*:

   From the thin lens formula

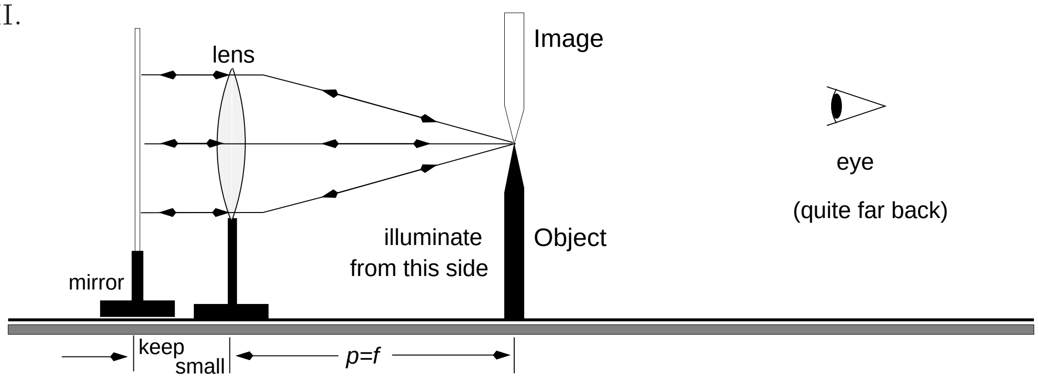
$$\frac{1}{\text{object distance}} + \frac{1}{\text{image distance}} = \frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

   Measure f using the three methods diagramed below (by adjustments in the object distance). Compare the results from the various methods.

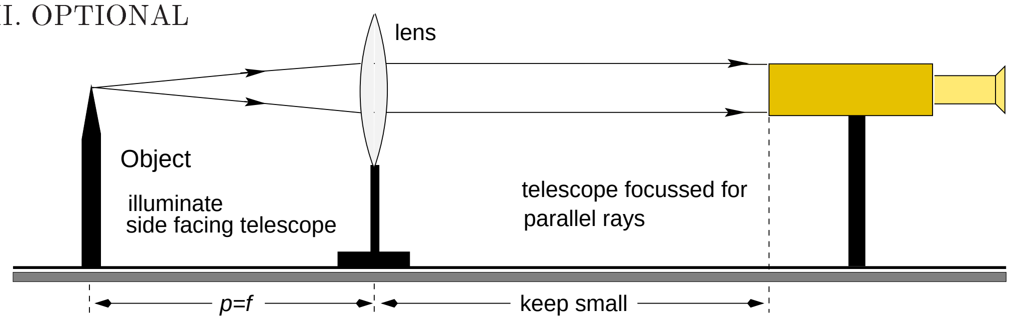
   I.



   II.



   III. OPTIONAL

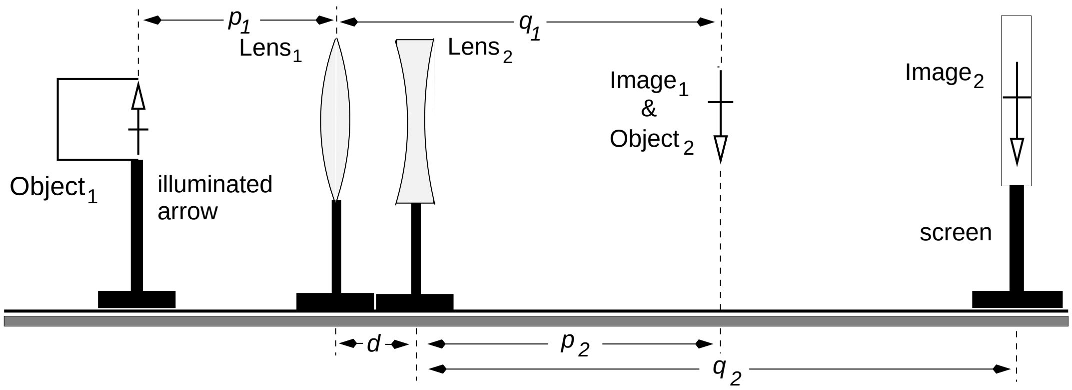


2. *Focal length of a diverging lens*:
   Since the image is always virtual it is necessary to combine it with a converging lens and configured so that the final image is real.

VIRTUAL PRE-LAB:

1. If using the web-lab manual launch the virtual application by clicking on the Diverging Lens Application button below.
2. After clicking on "Start Me" you should observe a concave lens, an object and its virtual image. On the lower left corner is a cursor position readout.
3. With respect to lens equation and starting positions, find the focal length. Can you find an object distant in which the image height and object height are the same?
4. Now restart the simulation with a combination of a convex and concave lens by clicking on the "Add 2nd lens" link.
5. This 2nd lens will render a final image which is real. The image with a "1" by it is that of just the convex lens while the image with a "2" next to it is that of the pair. Move the object and alter its height, if necessary, so that the first image has the relationship $q_1 \approx 2p_2$. (In this simulation $|f_2| = 3|f_1|$ and qualitatively resembles the figure shown below.)
6. Now repeat the last step with $|f_2| = |f_1|$ by clicking on the "Another 2nd lens" link. Finally reset to the "Another 2nd lens" and then click on the *convex* lens and drag it past the diverging lens.

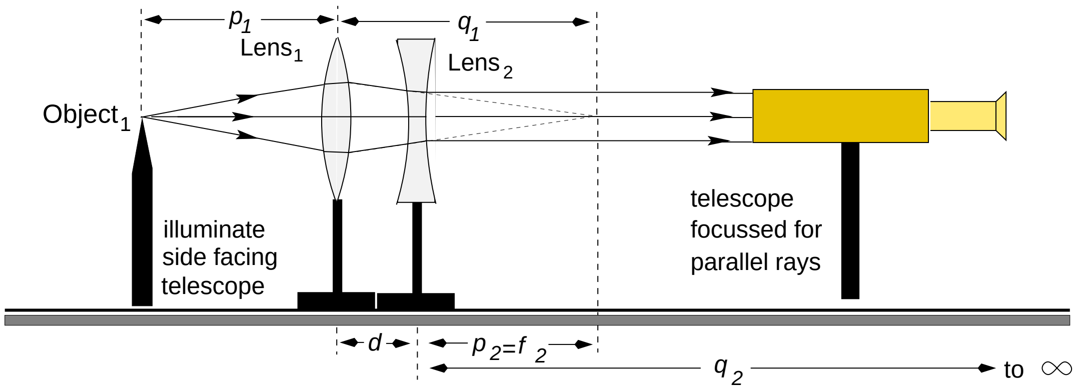
I. Use the set-up sketched below. First adjust Lens_1 so that with Lens_2 removed, $q_1 \sim 2p_1$. Measure q_1, insert the diverging Lens_2 reasonably close to Lens_1 and then locate the new image distance q_2. From $p_2 = d - q_1$ (a virtual object) and q_2 calculate f_2.



II. OPTIONAL: Find f_2 by this more sensitive method: With the pointed rod as object now detect the image by a telescope focused for parallel rays. (As shown in the following figure.)

If using the web-lab manual, launch the virtual application by clicking on the Dual Lens Test Application button below to obverse a point source in action.

As a last step you will move the "point" source object (i.e. vary p_1 and hence q_1) until a sharp image appears in the telescope with *no parallax* relative to the cross hairs. (This occurs only if parallel rays leave the diverging lens.) The image from the converging lens at q_1 serves as a virtual object for the concave lens, Lens_2. After properly adjusting p_1 this virtual object will be at the focal point of the diverging lens, Lens_2. Obtain q_1 by setting p_1 and using the known value of f_1. (Make sure that q_1 is large enough to accommodate $d + |f_2|$). Now look for the image in the telescope and adjust p_1. Note that since $q_2 = \infty$, $f_2 = p_2$ and $p_2 = d - q_1$.



OPTIONAL

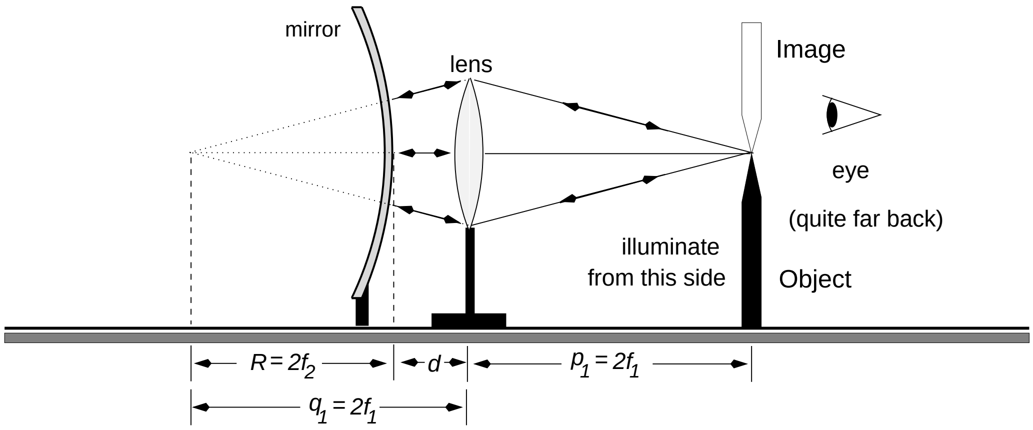
Experiment III: Focal Length of a Convex Mirror

Once again, since convex mirrors give virtualy images it is necessary to study the mirror in combination with a converging lens.

Measure the focal length of the *convex* mirror by combining it with a converging lens. Set the pointed rod at twice the focal length of the lens ($p_1 = 2f_1$). Next adjust the mirror position until the inverted image position shows no parallax with the object. Then $R = d - 2f_1$ and

$$f_{\text{mirror}} = f_2 = R/2.$$

Of course the lens must have $f_1 > f_2$, so use a long focal length lens.



L-3 Optical Instruments

OBJECTIVE:

To construct a number of optical instruments and measure magnification.

APPARATUS:

Identical to the of L-2 plus white board, achromatic doublets, Ramsden eyepiece.

SUGGESTION: In constructing an optical instrument catch the real image formed by the first lens on paper. This location can tell where the next lens goes; e.g. for a telescope or microscope place the next lens slightly less than its focal length beyond the first image.

EXPERIMENTS:

1. *Inverting Telescope*

A. First measure the f of a weak converging lens ($25 \leq f \leq 50$ cm) and of a strong converging lens ($f \sim 5$ cm). The first lens (f large) typically has the virtue of intercepting and focusing a fairly large area of light while the second lens can be positioned to give a large virtual image (and hence magnification.) Notice that if the first image, $q_1$is at a point $p_2 > f_2$ with respect to the second lens you get a real, erect second image. If you are using the web-based lab manual and desire a demonstration, click on the Inverting Telescope Demo button immediately below.

Use you will use these two lenses to construct an astronomical (inverting) telescope. As an object, use the white board and scale while illuminated by a bright light. As a procedure for measuring the magnification, vary the space between the objective L_1 and the eyepiece L_2 until the virtual image I_2 of scale S seen through L_2 lies in the plane of S (i.e. $p_1 + d = |q_2|$). As a sensitive test of this there should be no parallax between the scale as seen directly by your eye 2 and the scale image seen by eye 1 thru the telescope as shown in Fig. 2. Generally a person will adjust the eyepiece so that the virtual image appears at the distance of most distinct vision ($\sim$25 cm).

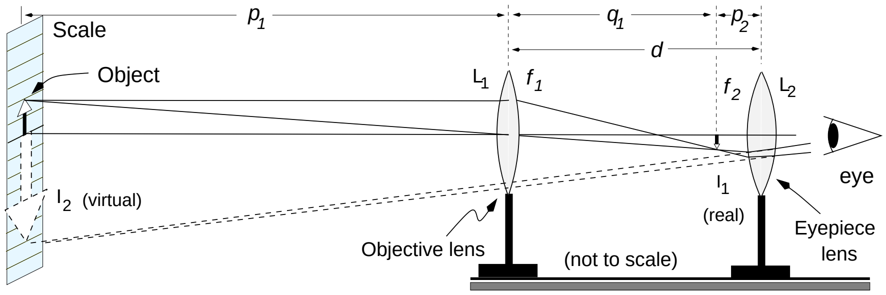


Figure 1: The inverting telescope (not drawn to scale).

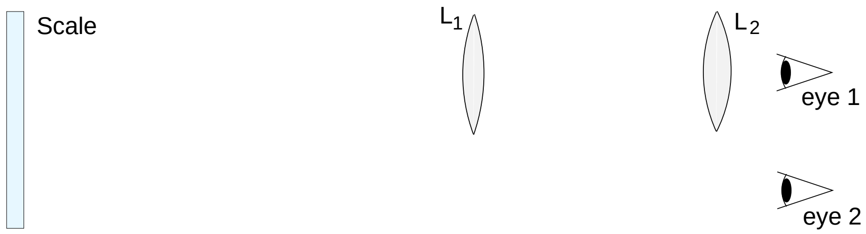


Figure 2: View scale with one eye, then the other.

B. Adjust the telescope direction until these images superimpose (as in Fig. 3). The number of divisions on the scale as viewed directly (by eye 2) which fall in one division of the image as seen through the telescope (by eye 1) is clearly the magnifying power of the telescope.
Compare your measured magnification M with that calculated for an astronomical telescope, namely $M = f_1/f_2$ where f_1 is the focal length of the objective and f_2 that of the eye piece. Why are the two different? Calculate the magnification from the actual measured object and image distances. Compare with the measured M.

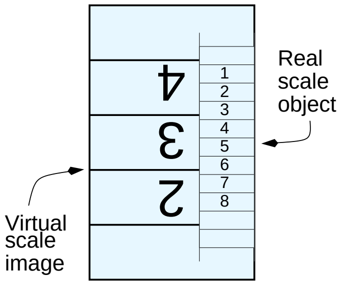


Figure 3:

2. *Erecting telescope*

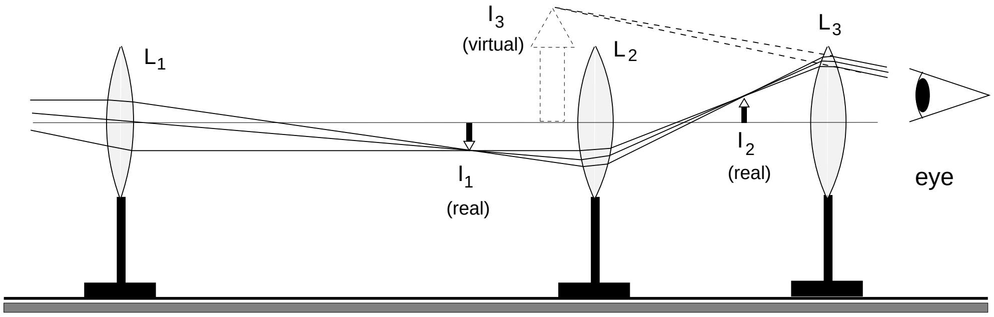


Figure 4: Configuration of a three lens erecting telescope.

A. Measure the focal length, f, of a second strong converging lens and use it as the inverting lens L_3 (Fig. 4) to form an erecting telescope. L_2 must be adjusted to give a real image, I_2. Notice that varying the space between I_1 and the inverting lens L_3 changes the magnification. If you are using the web-based lab manual and desire a demonstration, click on the Erecting Telescope Demo button immediately below. In this simulation your are now able to readjust both the positions and focal lengths!

B. What position of the inverting lens makes the shortest possible telescope, (i.e. it makes the distance between I_1 and I_2 a minimum)?

3. *Simple microscope or magnifier*: Experiment with one of the strong converging lenses as a magnifier until you can make the virtual image appear at any distance you choose: vary the object distance from zero to the focal length. The parallax test described in Part 1 will enable you to locate the virtual image. When the parallax between the object and the virtual image (seen through the lens) vanishes, the object and virtual image are in the same plane.

4. *Compound microscope:* Use the two strong converging lenses to form a compound microscope.

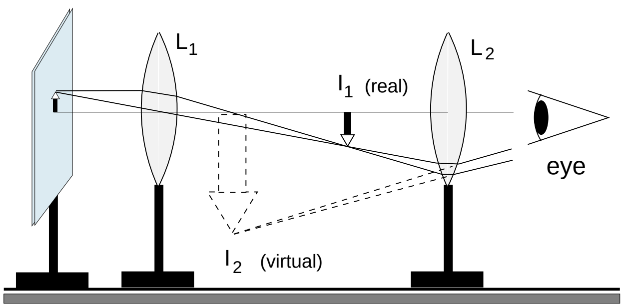


Figure 5: Setup for compound microscope.

5. *Galilean telescope (or opera glass)*: Make use of the weak converging lens and a strong diverging lens.

OPTIONAL:

1. *Ramsden eyepiece*

   A. Ask the instructor for the holder containing a Ramsden eyepiece, a crosshair, a diaphragm, and an objective lens. See Fig. 6.

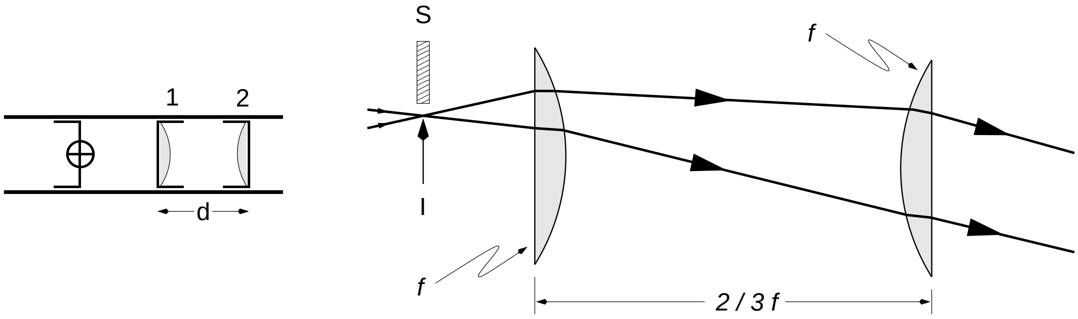


Figure 6: Construction of a Ramsden eyepiece.

The Ramsden eyepiece consists of lenses 1 and 2, plano-convex lenses of $f =$ 85 mm. Spherical aberration is less than for a single lens because the ray bending is spread over four surfaces. It is a minimum if the curved surfaces face one another as shown. (In addition if the two lenses were their focal length apart, chromatic aberration would be a minimum. But, since dirt specks on lens 1 then would be in focus, lenses 1 and 2 are usually set $2f/3$ apart). The tube also contains an objective lens to convert the tube into a telescope or microscope.

   B. Use the Ramsden eyepiece with $d = 2f/3$ in the telescope and microscope. Note improvement in image over that of a single eyepiece lens. Adjust eyepiece for a sharp image of the cross hairs. Then move the objective lens until the image shows no parallax with respect to the cross hairs.

2. *Huygens eyepiece*: A common eyepiece in which the image (and hence cross hair) is within the eyepiece. It also employs two plano-convex lenses but both convex surfaces are toward the incident light. Minimum lateral chromatic aberration exists for a separation of $(f_1 + f_2)/2$. Usually $f_1 \sim 3f_2$.

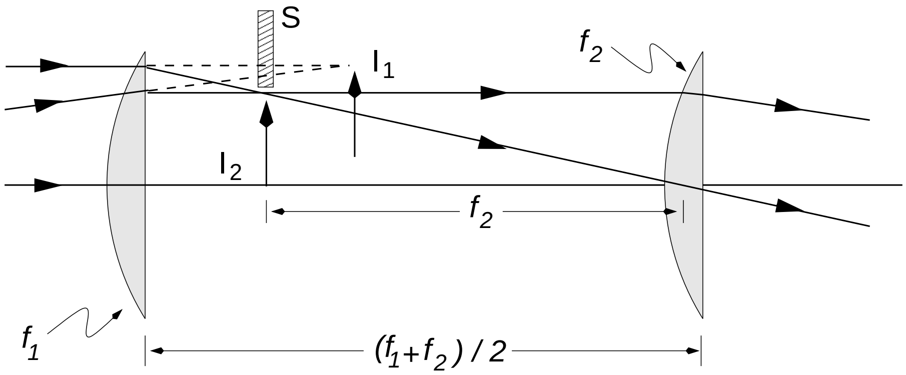


Figure 7: Construction of a Huygens eyepiece.

3. *Eye ring.*

   A. Point the astronomical telescope at a lamp and explore with a screen the illumination back of the eyepiece. [Or remove diaphragm from telescope or microscope of optional (1)].

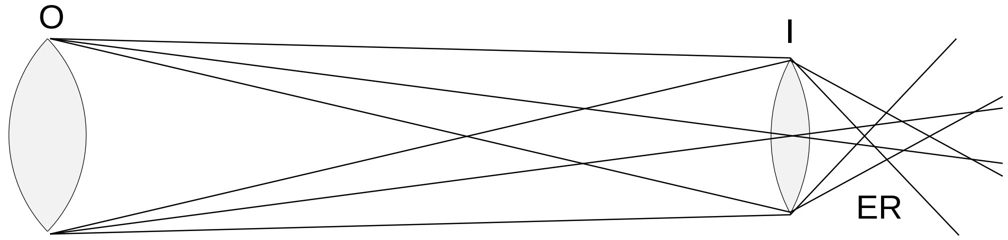


Figure 8: Position of eye ring.

   The light coming through the telescope has a minimum cross section at ER. This area ER is the image of the objective formed by the eyepiece. Obviously all light which gets through the telescope must go through this image, and thus ER is the best spot to place the pupil of the eye. Hence the name "eye ring".

   Note that if one places a diaphragm with aperture appropriately larger than the eye ring a little ahead of the eye ring, then an observer looking through the aperture would position the pupil of her/his eye on the eye ring.

   B. Using the Ramsden eyepiece, place the diaphragm at the appropriate distance in front of the eye ring. Note the improvement in the ease of observing through it, especially for the compound microscope.

4. OPTIONAL telescope: Use an achromatic objective lens, of long focal length and a commercial Ramsden eyepiece (from instructor). Note the quality of the image.

5. OPTIONAL microscope: Use a short focal length achromatic objective and a commercial Ramsden eyepiece (from instructor). Note the quality of the image.

   It is interesting that one does not need an achromatic eyepiece because the virtual images for red and blue, though at different distances (e.g., red at 2 m, blue at $\sim \infty$), subtend almost the same angle at eye. As a result the eye doesn't notice the aberration.

L-4 The optics of the eye and resolving power

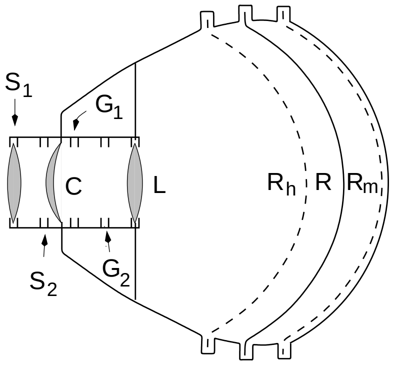
OBJECTIVES:

To study eye optics and how resolving power depends on aperture.

APPARATUS:

PART I: Eye model, (fill with distilled water to simulate the aqueous & vitreous humors); light source, lens set, 6" plastic ruler.

PART II: (2) Optical benches, Na lamps, transparent mm slides, vernier calipers, L-2 telescopes; (1) 15 meter tape.



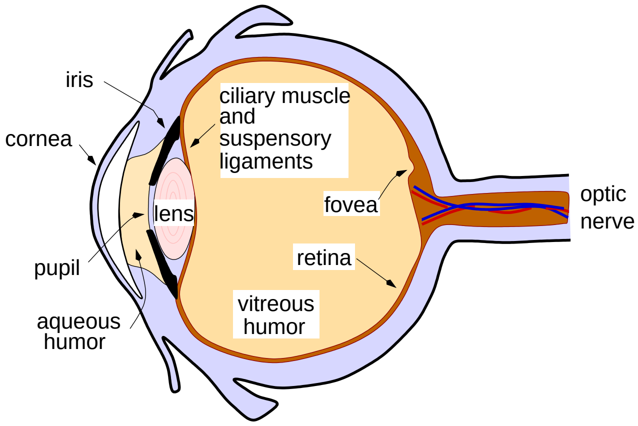


Figure 1: Various schematics for the eye

You may fix the moveable retina (cylindrical) in one of three positions:

R for the normal eye,
R_h for the hypermetropic (farsighted) eye,
R_m for the myopic (nearsighted) eye.

(An eye retina is actually spherical, but for small images the results are similar.)

The eye model has a **fixed cornea C** and racks for other lenses.

A lamp box serves as source for most of the experiment.

Set of lenses: (Strength in diopters = 1/f where f is in meters.)

	Diopters	**Focal lengths in air**
1. Converging spherical	+ 7.	0.14 m
2. Converging spherical	+20.	0.05 m
3. Converging spherical	+ 2.00	0.50 m
4. Diverging spherical	- 1.75	-0.57 m
5. Diverging cylindrical	- 5.50	-0.182 m
6. Converging cylindrical	+ 1.75	0.57 m
7. Diaphragm with small hole	.	.

PART I - EXPERIMENTS WITH THE EYE MODEL

1. ACCOMMODATION: With the retina in the normal position, point the eye model at a window or other bright object 4 to 5 meters away. Insert the +7 diopter lens in the water at the inside mount farthest from the cornea. An image of the bright object should be in focus on the retina.

   Next use as object the lamp box and place it about 30 cm from the cornea. The image is blurred until one replaces the +7 diopter lens by the +20 diopter lens. This change illustrates accommodation. Eye muscles make the lens thicker for close vision. (Under relaxed conditions the ligaments supporting the lens are in radial tension and hence thin the lens. The ciliary muscle contracts against these ligaments allowing the lens to thicken for viewing near objects. See Fig. 1.)

2. NEAR and FARSIGHTEDNESS (MYOPIA and HYPERMETROPIA): With lamp box at 30 cm and the 20 diopter lens at L, place the retina in the position R_m. Decide which lens placed in slot S_1 or S_2 will bring the image into focus. Try it.

   Make the eye *farsighted* by moving the retina to R_h, and pick the proper lens to place in front of the eye to bring the image into focus.

   Focus again on a window 4 to 5 meters away when the +7 diopter lens is in the *normal* position L and the retina in position R.

   Make the eye *nearsighted* by moving the retina to R_m. Decide which lens will bring the image into focus. Try it.

   Remove the correcting lens; the image again blurs. At S_2 place the diaphragm with hole and note image improvement. With sufficient light try a smaller hole (e.g. a hole thru masking tape on the diaphragm). Explain the effect of the reduced aperture.

3. ASTIGMATISM: With the +20 diopter lens at L in the eye and the retina in the normal position, adjust the distance to the lamp box to get a sharp image on the retina. Then produce astigmatism by placing the strong diverging cylindrical lens in mount G_1. (In the human eye astigmatism generally results from a cornea with different curvatures in the horizontal and vertical directions.) Turn this lens and note the effect on the image.

   Correct this defect by placing the converging cylindrical lens in front of the cornea and turning it to the position for which the image becomes sharp. Note the directions of the axes of the two cylindrical lenses. (*They are not necessarily aligned with the tab.*)

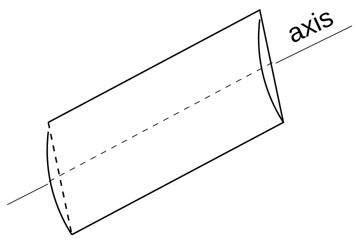


Figure 2: Cylindrical lens.

4. COMPOUND DEFECTS: Combine tests 2 and 3, i.e. make a nearsighted astigmatic eye. Correct by using both a cylindrical and spherical lens in front of the cornea. How is this double correction managed in actual spectacles?

5. CATARACTS and LENS REMOVAL: A cataract results from a cloudy lens. A cataract operation involves removal of the lens. An implanted plastic lens or a strong convergent lens in front of the eye restores vision. Remove the eye lens (L) of the model and place the +7 diopter lens in front of the cornea. A clear image reappears for nearby objects.

6. Action of a SIMPLE MAGNIFIER: With the lamp box at 35 cm, the +20 diopter lens and the retina in the normal position, measure the size of the image. Now place the +7 diopter lens in front of the cornea and move the lamp box toward the eye until the image becomes clear. Record the increased size of the image. With the lamp box at 35 cm, the 20 diopter lens and the retina in the normal position, note the size of the image. Now make the eye nearsighted and bring the lamp box toward the eye until the image becomes clear. Note the increased size of the image in the nearsighted eye.

NOTE: For a real eye and lens the index of refraction, μ, of the lens varies from 1.42 at its nucleus to 1.36 at its edges, and these values are not so different from those of the adjoining aqueous and vitreous humor ($\mu = 1.330$ and $\mu = 1.337$). Hence the real eye lens is very weak and is primarily for fine focus adjustment. Most of the refraction in the eye actually occurs at the cornea where μ changes from 1.0 (in air) to 1.37 in the cornea.

PART II - RESOLVING POWER

Before doing this part, read in your text about resolving power.

SUGGESTED EXPERIMENTS:

1. Use as a source a slide with mm divisions and illuminated by a sodium discharge lamp. The wavelength of the sodium yellow light is 589.3 nm.
2. Back away from the slide until the mm lines become indistinct. Then advance far enough so that they are just clearly visible; this procedure insures that eye defects or the density of rods and cones do not (for this measurement) limit the resolving power.

   Observing now with only one eye, place a vernier caliper[5] in front of the eye pupil with the caliper jaws parallel to the mm marks and open about 3 mm. Slowly close the jaws until the mm marks disappear. Note the separation of the jaws, w, and measure the distance, D, to the slide from the jaws.

   Compare your measurements of the angular separation α of the mm marks at your observing distance,

$$\alpha = \frac{1 \text{ mm}}{D \text{ (in mm)}},$$

   with the expression $\theta = \lambda/w$ where θ is the angular limit of resolution, (Rayleigh's criterion).
3. Mount a telescope so its objective is at the same distance from the slide. Repeat the experiment with the caliper in front of the objective.[6] How do the results compare to your measurement without the telescope?
4. Move the telescope ≥ 4 m from the slide and repeat the experiment.
5. Turn the vernier jaws perpendicular to the mm lines and repeat experiment #1. Comment on the result.

QUESTIONS:

1. Would an eye without any defects of vision have better resolving power at dusk or in bright sunlight? Explain.
2. Discuss how a contact lens functions. Would a "soft" contact lens correct well for astigmatism?

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[5]In all the following be sure you are not looking through the hole at one end of the jaws. It is advisable to cover the hole with masking tape.

[6]Be sure no light enters the telescope except through the slit jaws. (You may need masking tape on the caliper jaws to prevent this from happening.)

L-5 Spectrometer and the H Balmer Series

NOTE: Consult your instructor for assignment of parts suitable for a single lab, e.g. do Part I, II (A or B), Part IV or Part V).

OBJECTIVES:

To become familiar with a precision spectrometer and some of its uses.

APPARATUS:

Precision spectrometer; mercury and hydrogen discharge tubes; prism; diffraction grating; light source for Gaussian eyepiece & its 12V power supply (L-2); ring stand & achromatic lens.

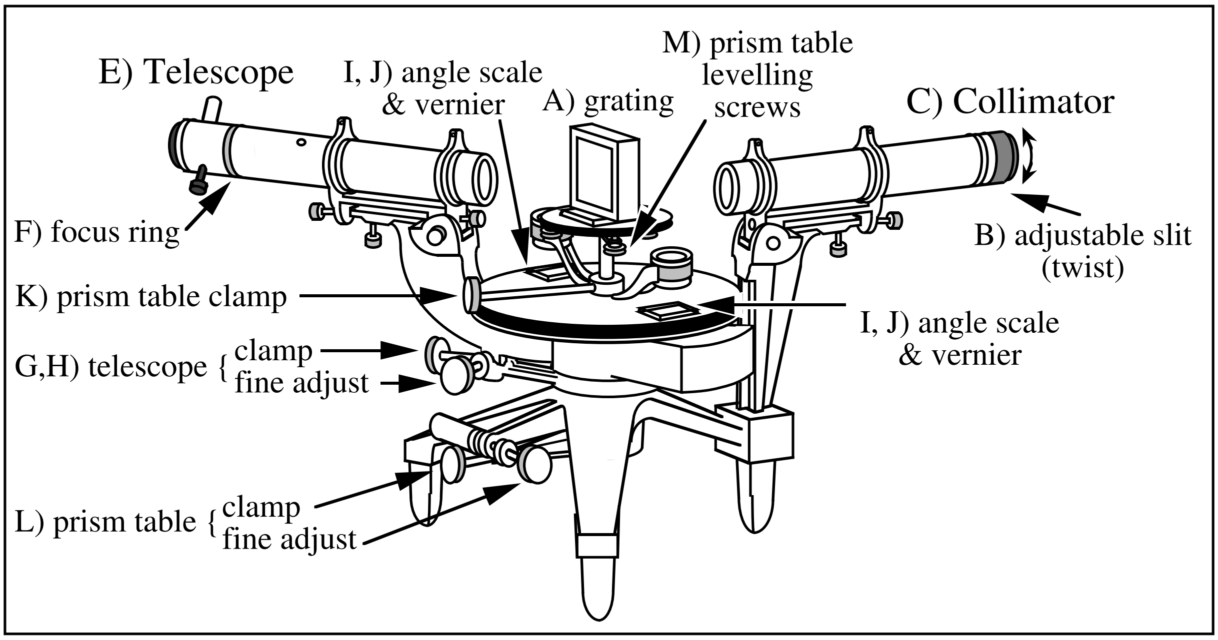


Figure 1: The spectrometer (side view).

INTRODUCTION and ADJUSTMENTS

PART I - A

1. Become familiar with the clamping and fine adjustment controls for telescope and prism table angles. **Never force a motion - you may damage the instrument. If at the end of a fine adjustment, loosen and reclamp!** The prism table clamp sets table elevation: no fine adjustment is necessary.
2. Note that 180° apart are two **angle scale reading ports and verniers, I and J**. The verniers have 30 divisions per half degree. Hence each division is $(1/30)\times(1/2) = (1/60)^\circ$ or one minute ($'$) of arc.
3. The **knurled ring about the collimator, B,** controls the **slit opening**.
4. Staff have already focussed the collimator for parallel light and have set collimator and telescope $\perp$ to spectrometer's rotation axis.

REMINDER ON READING A VERNIER

1) Position and tighten the telescope.
2) Move the magnifying glass to see the scale marking
3) Look where the zero mark on the upper scale is located. The degree reading is the firstu *full* degree mark to the *left* of the zero (210° in the top figure; 229° in the bottom figure).
4) Now make the *minute* reading: if the zero mark on the upper scale comes *after* a $\frac{1}{2}$ degree mark (the short lines on the lower scale) then start at $30'$, otherwise at $0'$.
5) Next look for the two lines which match up *best* between the two scales. Read the number of the appropriate line on the upper scale and add it to the zero mark is just past a $\frac{1}{2}$ degree mark so we begin with $30'$. The $15'$ mark is the one that lines up best so we get $210°15'$ for the top reading and $229°146'$ for the bottom one.
6) Now convert to decimal degrees, in this case $15/60 = 0.25$, so $\theta = 210.25°$. latex

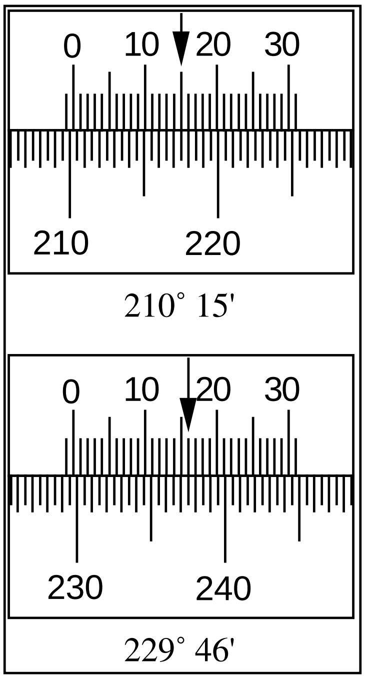
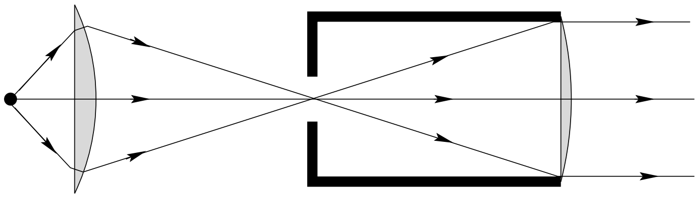


Figure 2: The verniers

PART I - B: FOCUS THE TELESCOPE FOR PARALLEL RAYS

1. i) Slide telescope eyepiece in or out until crosshairs are in sharp
   ii) Sight telescope at a distant object (thru an open window); then focus telescope by rotating its focus ring (Fig. 1) until the object's clear image falls on the cross hairs. The test for proper focus is absence of parallax between the image and cross hairs (appendix 4). The telescope, now focused for parallel rays, will stay so as long as the focus ring is unmoved; but one may still adjust the eyepiece to suit the observer.
2. An alternative method to focus for parallel light is to use the Gaussian eyepiece + light source as described below in PART II B.
3. With collimator and telescope both properly focused one should get a sharp image of the slit and no parallax between slit image and cross hairs. If you still get parallax, recheck your telescope focus and/or consult instructor.
4. *Optimizing light thru the collimator*: A properly located short focal length lens can gather a large fraction of the light from a source and redirect it



thru the collimator thereby facilitating detection of weak spectral lines. For strong lines it permits narrowing the slit width and thus improving resolution. Use such a lens (mounted on a ring stand) to find an arrangement which fills the collimator with light.

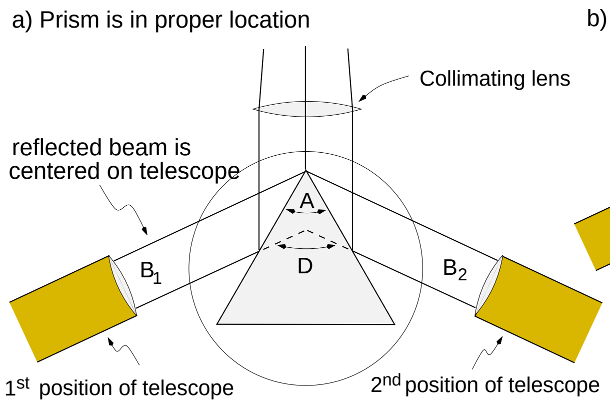


Figure 3: Proper, a), and improper, b) prism locations.

PART II-A: MEASUREMENT OF PRISM ANGLE

1. Mount the prism (in holder) on the dowel pins (in the prism table) so that the prism is far enough from the collimator that beams B_1 and B_2 are centered with respect to the spectrometer axis; otherwise much of the reflected light may miss the telescope. Turn the table so that the apex angle A (Fig. 3) splits the beam into nearly equal parts.
2. Using the *prism table levelling screws* adjust the plane of the prism table so it is close to horizontal.
3. With the prism table clamped at the proper elevation, set the telescope to receive beam B_1 and form an image of the slit on the intersection of the telescope cross hairs. **Record both VERNIER readings (in minutes)**. Average the two vernier readings (to eliminate any systematic error from misalignment of the circle scale with respect to bearing axis), and add the result to *one* of the angle scale readings. *Note that the angle scale reads in half-degree units and the vernier in minutes (***not** *in decimal degrees*!). The zero position on the vernier determines the angle to the nearest half-degree (30 minutes) and the vernier reads the number of minutes past the half-degree mark.
4. Then with telescope in the 2nd position, set on the slit image. The change in reading of the *same* angle scale should be the angle D. (See Fig. 3.) *Be sure you don't get the angle scales mixed up and subtract the first reading of angle scale 1 from a reading of angle scale 2.* Also some students incorrectly handle the subtraction when the scale passes through 360° or 0°.
5. OPTIONAL: Try to prove that angle D is twice the prism angle A.

PART II-B: ALTERNATE method for prism angle measurement using Gaussian eyepiece

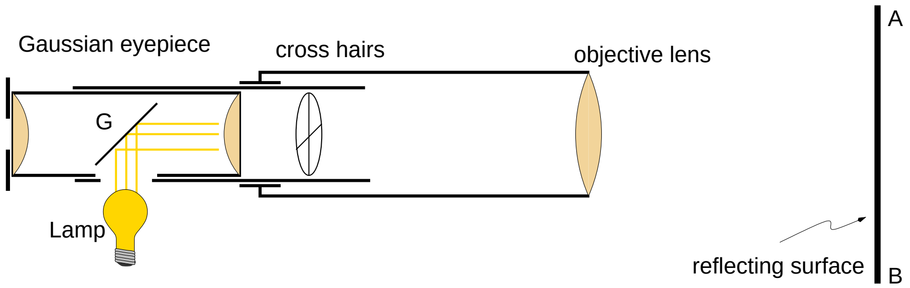


Figure 4: Spectrometer telescope

Introduction: The *Gaussian eyepiece* (Fig. 4) has a partially reflecting glass plate G set at 45° to telescope axis so that light from the lamp reflects down the telescope tube, past the cross hairs and out the objective. If in front of the objective you place a reflecting surface *perpendicular* ($\perp$) to the telescope axis, the light will reflect back into the objective and form a real image of the cross hairs. If the cross hairs are in the focal plane of the objective, their reflected image will form in the same plane. Thus both the cross hairs and the image will be in focus through the eyepiece. When one orients the reflecting surface so that the image coincides with the cross hairs, the reflector is accurately $\perp$ to telescope axis.

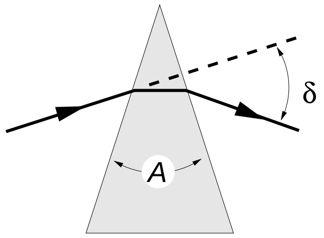
Thus the Gaussian eyepiece permits both focusing the telescope for parallel rays and setting a reflecting surface $\perp$ to the telescope axis.

1. To *focus the telescope for parallel light*:
   a) Rotate the Gaussian eyepiece to open the hole between reflecting plate G and lamp. Adjust eyepiece to give a sharp image of the cross hairs, but **don't turn the eyepiece to block the light hole**. Next, with prism on the table, adjust the prism table screws to make the table nearly horizontal.
   b) Now rotate the prism table until the prism face is approximately $\perp$ to telescope axis. Clamp the telescope and mount a light on the Gaussian eyepiece. (**Check that the light hole is still open**). With no illumination on the prism except that from the telescope, next rotate the prism table back and forth a few degrees until maximum reflected light appears in the eyepiece. Clamp the table in this position, and then focus the telescope (by turning the focus ring) until the reflected image of the cross hairs appears and shows no parallax with respect to the cross hairs. The telescope is now focused for parallel rays. Fine adjustments of the prism table leveling screws may help the images coincide and thus set telescope axis accurately $\perp$ to the reflecting surface. See Part VI, Sec. 3.
2. To find the prism angle set the telescope $\perp$ to first one prism face and then the other. The angle between these two positions is the supplement of the prism angle. See Part IIA for detail about angle readings.

PART III: INDEX OF REFRACTION

Introduction: When the path thru a prism is symmetric, the deviation is a minimum. At this angle of minimum deviation the index of refraction, n, is

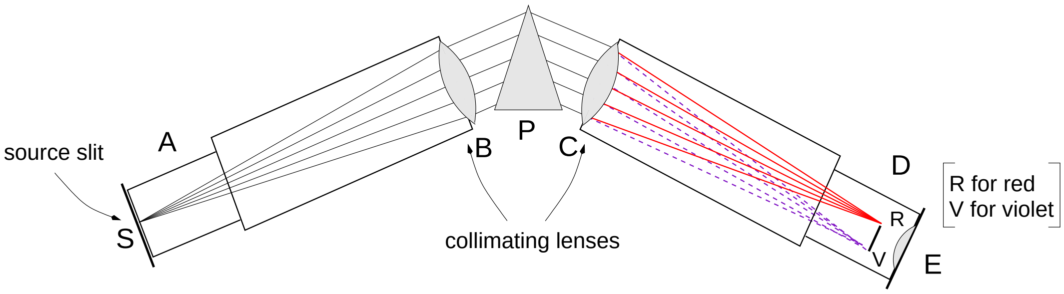
$$n = \frac{\sin\left[(A + d)/\delta\right]}{\sin\left(A/2\right)}$$



where A = angle of prism
δ = angle of minimum deviation
n = refractive index of the prism.

Devise your own methodology and determine the prism's refractive index for one or more lines of the Hg spectrum.

PART IV: CALIBRATION OF PRISM SPECTROSCOPE



1. Set the prism for minimum deviation for a green line in the Hg spectrum.
2. Determine the angle readings for the yellow, green, blue-violet, and deep-violet Hg lines. Repeat this for the red line from a hydrogen discharge.
3. Plot telescope angle vs accepted λ's for these lines.
4. For the same setting of the prism table, find the angles for the blue-green and two violet lines of the hydrogen spectrum.
5. Use your calibration curve to determine λ of the blue-green and violet lines in the atomic hydrogen spectrum. Compare results to accepted values.

OPTIONAL: The calibration curve is very non-linear. Since a more linear plot facilitates interpolation of an unknown λ, try plotting deviation *vs* $1/\lambda^2$.

TABLE OF WAVELENGTHS:

Mercury lines				Hydrogen lines		
	λ (in nm)		$(1/\lambda^2)$		λ (in nm)	$(1/\lambda^2)$
Yellow$_1$	579.0 ⌉	(unresolved)	$\times 10^6\,(\text{nm})^{-2}$			$\times 10^6\,(\text{nm})^{-2}$
Yellow$_2$	577.0 ⌋	Avg. = 578	2.993	Red	656.3	2.322
Green	546.1		3.353	Blue-green	486.1	4.232
Blue	496.0*		4.065			
	491.6*		4.138	Violet	434.0	5.309
Blue	435.8		5.265			
-violet	434.7*			Deep Violet	410.2	5.943
Deep-	433.9*					
violet	407.8*		6.013	NOTE: * implies		
	404.7*		6.106	usually faint		

PART V: DIFFRACTION GRATING

CAUTION: Do not touch grating surface under any circumstance!

1. *Grating constant (number of lines per cm)*: The lines/cm marked on these replica gratings is only approximate because the plastic often changes dimensions when it is stripped from the master grating. To achieve quantitative results calibration of the grating constant is desireable and can be performed using the known λ, 546.1 nm, of the Hg green line.

   CALIBRATION PROCEDURE:

   a) Align telescope with collimator so the slit image falls exactly on the vertical cross hair.
   b) Mount grating on the dowel pins in the prism table. Adjust and clamp the table so the grating is approximately $\perp$ to telescope-collimator axis.
   c) Then adjust the table so that the front grating face is accurately $\perp$ to telescope. Use the Gaussian eyepiece (see Part II - B) for the final adjustment: the reflected image of the cross hairs should fall on the cross hairs. (Two reflected images can result if the sides of the grating glass are not parallel. If so, test which image comes from the grating side by wetting the other side.) Clamp prism table in this position and record telescope direction.
   d) Next set telescope on the first and second orders of the Hg green line. Use both $\pm$ angles. The $\pm$ angles should of course agree. Use them and $\lambda_{\text{green}} =$ 546.1 nm to calculate the grating spacing d from

   $$n\lambda = d \sin\theta.$$

2. *Measurement of the wavelengths in the Balmer series in hydrogen*:

   a) Use the calibrated grating spectroscope to measure λ for three or four atomic hydrogen spectral lines.
   b) Compare these λ's to those from the Balmer formula

   $$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right] \text{ where } R = 10,967,758 \text{ m}^{-1}$$

   or calculate the frequencies of these lines and compare to

   $$f = cR\left[\frac{1}{2^2} - \frac{1}{n^2}\right] \text{ where } R = 10,967,758 \text{ m}^{-1}.$$

   c) Use Planck's relation, $\Delta E = hf$, to calculate the energies in eV of the photons for each frequency and, therefore, the energy change in the hydrogen atom associated with each frequency.
   d) Assuming that each observed ΔE leaves hydrogen in the $n = 2$ state, show the observed transitions on a hydrogen energy level diagram (drawn to scale; consult textbook).

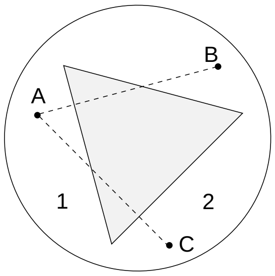
PART VI: ADJUSTMENT OF A SPECTROMETER

NOTE: Not to be performed without permission of the instructor

1. Use Gaussian eyepiece and prism face method to focus telescope for parallel rays. If necessary adjust prism table to make reflected image of the cross hairs coincide with the cross hairs.

2. Remove prism from the table and align telescope and collimator approximately. Focus the collimator until there is no parallax between the image of the slit and the cross hairs. Adjust the ring on the collimator focusing sleeve so that the collimator is in focus with the "V" in the slot. Clamp the telescope in line with the collimator so that the image of the slit (vertical) falls on the vertical cross hairs. Now rotate the slit through 90° as permitted by the "V" projection and slot. Adjust the level of the telescope so that the image of the slit falls on the horizontal cross hair. The axes of telescope and collimator are now in line but not necessarily perpendicular to the axis of rotation of the instrument.

3. Replace the prism on the prism table and clamp it relative to the adjusting screws as shown. (Line AB $\perp$ face 1. Line AC $\perp$ face 2. Thus screw B will adjust face 1 without disturbing face 2, and screw C will adjust face 2 without disturbing face 1.

   Bring the telescope to within $< 90°$ of the collimator and use a face of the prism to reflect light from the collimator down the telescope tube. Adjust the prism face so that the center of the slit image falls on the intersection of the cross hairs. The prism face is now parallel to the instrument axis.

4. Set the telescope $\perp$ to the adjusted prism face (use Gaussian eyepiece method). Return to part 3 setup and adjust the collimator so that the center of the slit image falls on the cross hair's intersection. Telescope and collimator are now properly focused and are *perp* to the instrument axis.

5. To adjust the second prism face $\perp$ to the axis of the telescope (again use the Gaussian eyepiece method). Turn only the proper screw on the prism table: otherwise one disturbs the adjustment of the first face. Recheck the first face and, if necessary, readjust it. Continue this process until both faces are parallel to the axis of the instrument. Turn collimator slit to vertical position. The spectrometer should now be in adjustment.

L-8 Polarization

OBJECTIVE: To study polarization and double refraction of light.
APPARATUS:

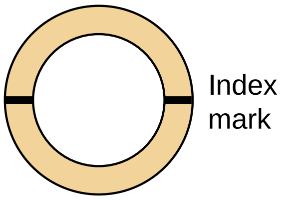
Polarization kit plus a polariscope; Brewster angle assembly (laser + mount on M-2 force table, slotted pin & black piece of plastic).

SUGGESTED EXPERIMENTS:

1. *Polarization by ABSORPTION*: Observe a light source (desk lamp) thru each of two polaroids, and then thru both together. Rotate one of the two about the line of sight. Explain what occurs.

   Orient the two polaroids of the *polariscope* so no light is transmitted. Then introduce a third polaroid between the first two and rotate it about the line of sight. Explain what happens.
   HINT: there will be an amplitude component $E\cos\theta$ in the direction of the transmission axis.

2. *Polarization by REFLECTION from a dielectric*: Observe thru the small polaroid (the one with the rim index marks) the light reflected from the black plastic (see figure at right). Rotate the polaroid about the line of sight. Note position of the index marks (i.e. the E vector direction) on polaroid when the transmitted reflected light is a minimum.



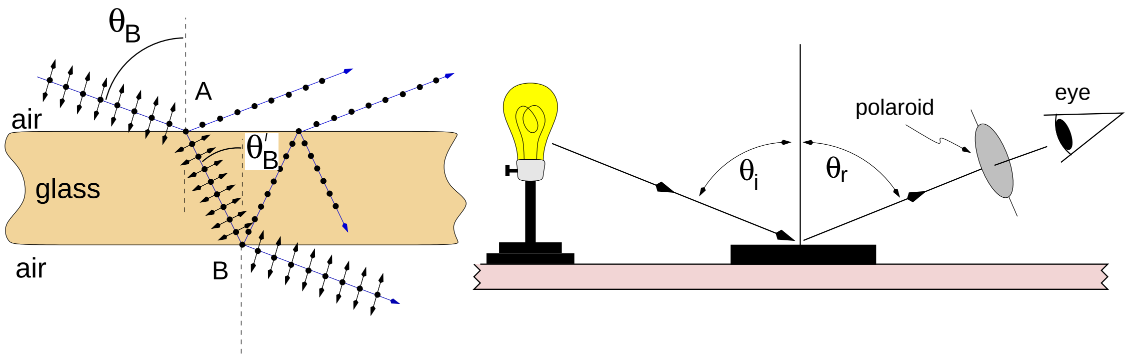


Figure 1: Reflection and transmision of light at Brewster's angle.

Now find an incident angle θ_i (and reflection angle θ_r) for which this minimum is zero. Then θ_i is *Brewster's angle*, θ_B, and $\tan\theta_B = n_2/n_1$. Estimate θ_B roughly. What is the refractive index n_2 of the plastic?

> *Alternatively* for greater accuracy use a laser plus polaroid to prepare a beam polarized in the plane containing the incident ray and the normal to the reflecting surface. *The reflected beam then vanishes at Brewster's angle.* Mount laser plus polaroid on the M-2 force table and point it toward a dielectric (plastic or microscope slide) in the slot of the central pin. Rotate the sample until the reflected beam (on the wall) disappears. A thread from the center pin to this point on the wall will help locate the angles on the M-2 table.

Observe any polarization of light reflected from painted or varnished surfaces, floor tile, *clean metal* (e.g. aluminum foil) etc.

3. *Polarization by SCATTERING*: Try to detect the polarization of skylight: **one needs a blue sky**! Explain (use a diagram) in what direction relative to sunlight should one look to see this polarization best.

4. *Polarization by a DOUBLY REFRACTING CRYSTAL*: Uniaxial doubly refracting crystals (e.g. calcite) are highly anisotropic and, as such, contain an axis of symmetry (the optic axis) for which the velocity of light depends on whether the E-field vector (polarization plane) is $\perp$ to this axis (the ordinary "o" ray) or parallel to the optic axis (the extraordinary "e" ray). In the principal plane (i.e. a plane containing the optic axis but $\perp$ to the surface) the new wave front of the ordinary ray involves the usual spherically expanding wavelets, but to construct the front for the extraordinary ray requires ellipsoidally expanding wavelets where the two axes of the ellipse are proportional to the velocities of the parallel and $\perp$ vibrations to the optic axis.

   Calcite ($CaCO_3$) has a large velocity differences between the o and e rays, and its optic axis is the direction at equal angles with the crystal edges at the obtuse corners.

   Observe through the calcite crystal a dot on a piece of paper. Rotate the crystal about the line of sight. Describe the behavior of the two images of the dot. Introduce a polaroid between your eye and the crystal. Note how the appearance of the two images changes as you rotate the polaroid about the line of sight. Explain.

NOTE: For the remaining experiments use the POLARISCOPE. The *polaroid next to the light is the "POLARIZER," the other is the "ANALYZER."* Place the samples to be studied on top of the polarizer.

5. *CIRCULAR POLARIZATION*: A thin doubly refracting crystal cut so that the optic axis is parallel to a surface constitutes a retardation plate. The difference in velocity of the o and e rays then gives a phase difference between the emerging orthogonally polarized rays. If the phase difference is 90°, the retardation is $\lambda/4$: hence a "*quarter wave plate*".

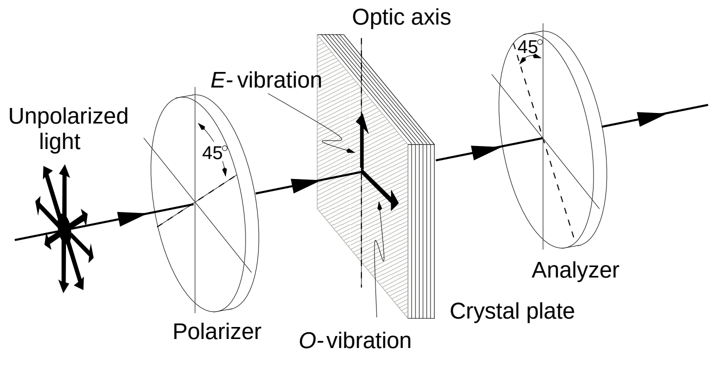


Figure 2: Obtaining circularly polarized light from unpolarized light

Set the analyzer of the polariscope to transmit no light. Then place the $\lambda/4$ plate between the polarizer and analyzer and rotate the $\lambda/4$ plate so that extinction again results. You will find two such extinction positions 90° apart. Why?
HINT: When the optic axis is aligned with the polarization direction of the incoming plane polarized wave, will there be any ordinary ray amplitude? Then when you turn the plate (and thus the optic axis) thru 90°, will there be any extraordinary ray amplitude.

Rotate the $\lambda/4$ plate to a position halfway between the two extinction positions; thus the optic axis will make a 45° angle to the incident plane of polarization. Explain why the plate now looks bright. Leaving the $\lambda/4$ plate in this position, rotate the analyzer through 2π radians. Why does the brightness stay approximately constant*?
HINT: Recall the conditions for a circular Lissajous figure (see E-8 Part B, Fig. 3).

* If it doesn't, repeat the preceding operations. You may not have set the $\lambda/4$ plate close enough to the halfway or 45° position. Also remember that a $\lambda/4$ plate is exactly $\lambda/4$ only for a single λ.

6. TWO $\lambda/4$ PLATES: Place a $\lambda/4$ plate on the polarizer so as to produce circularly polarized light. Place a second $\lambda/4$ plate on top of the first and similarly oriented. Observe and explain what happens when you now rotate the analyzer. Turn the top $\lambda/4$ through 90° about the line of sight. Observe and explain what happens now when you rotate the analzyer.

7. COLOR EFFECTS: If the plate is thick enough for retardations of several (but not too many) wavelengths, *color effects* may result. For a constant refractive index:

$$\lambda_{\text{red}} \sim 660 \text{ nm} \sim \frac{3}{2}\,(\lambda_{\text{blue}} \sim 440 \text{ nm}).$$

Thus a $\lambda/2$ plate for red is a $3\lambda/4$ plate for blue. [In general, a n($\lambda/2$) plate for red $\sim 3n(\lambda/4)$ plate for blue where n is an odd integer.] Hence for white light such a plate at 45° between parallel polaroids would completely extinguish the red but pass the circularly polarized blue light. The resultant color is complementary to that removed. The intensity (hue and saturation) will of course vary with the relative orientation of polarizer, the retardation plate, and the analyzer. As n becomes large the range of λ extinguished narrows. Hence that passed is more nearly white.

   a) Place thin mica on the polarizer (with analyzer crossed). Try various mica orientations. Explain.
   b) Repeat a) for the mounted specimen of cellophane tape.

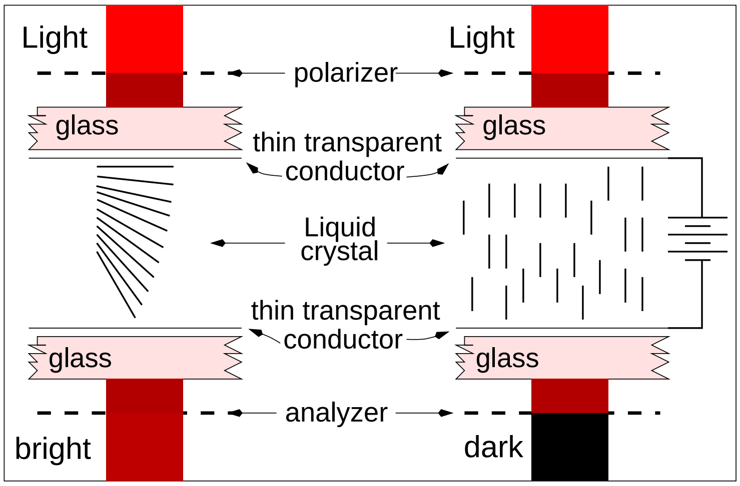
8. PHOTO-ELASTIC EFFECTS: Strained isotropic materials become doubly-refracting.

   With the polariscope set for extinction:

   a) Insert and flex the U-shaped lexan (a polycarbonate resin) sample.
   b) Insert a microscope slide vertically. View the long edge and stress the slide by *gently* flexing it. Note result. (Glass blowers use polarized light to test for residual glass strains).

9. *LCD APPLICATIONS*: Watches and calculators often use LIQUID CRYSTAL DISPLAYS (LCD) in which *nematic* liquids (thread like molecules arranged nearly parallel to each other) can become optically active, rotate the plane of polarization and thus permit light to pass thru crossed polaroids. A reflector after the analyzer returns the light through the cell.

   To produce the optical rotation: two glass plates, which have been rubbed in one direction to produce invisible scratches, attach aligned *nematic* molecules. If a few



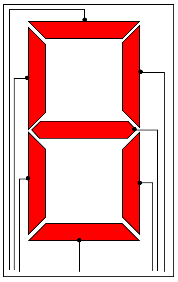


Figure 3: Construction of a liquid crystal display

micron thick *nematic* liquid separates the plates, and if one plate is rotated relative to the other, then the helical arrangement of the *nematic* liquid produces a rotation of the plane of polarization.

To extinquish the light one applies a voltage between the transparent (but conducting) tin-oxide coated glass surfaces. A sufficient voltage gradient will align the dipole moments of the molecules and thus destroy the optical rotatory power. Voltages applied to segments of the seven segment pattern (see figure) permit display of any numeral 0 to

The power consumption of such an LCD is almost negligible.

OPTIONAL QUESTIONS:

1. How experimentally can one tell whether a light beam is unpolarized? Plane polarized? Circularly polarized?

2. How could you tell whether an object is a gray plastic, a polarizing sheet, a $\lambda/4$ plate or a $\lambda/2$ plate?

3. How could you change right circularly polarized light to left circularly polarized light?

4. Why does the flexed lexan show colors but not the microscope slide?

L-9 Wavelength Measurement and Holograms

OBJECTIVES:

1. To measure the wavelength of laser light by using a steel rule as a diffraction grating; to observe and understand holograms.

> *CAUTION:* The laser is a very bright source. Do not allow the laser beam to enter the eye and do not point the beam at anyone!

APPARATUS:

He-Ne laser; steel rule; tripod stand & angle clamp; holograms; mercury arc; diverging lenses; 15 meter tape.

Part I - Wavelength Measurement

INTRODUCTION:

Specular reflection of the laser beam from a steel ruler gives a zero order image. In Fig. 1 the distance h_0 and D determine θ_r, the angle of reflection: $\theta_r = \tan^{-1}(D/h_0)$ and hence also θ_i, the angle of incidence.

The markings on the steel ruler form a coarse reflection grating which also yields higher order diffracted images. The angle of diffraction for the n^{th} order is

$$\delta_n = \tan^{-1}(D/h_n) \quad .$$

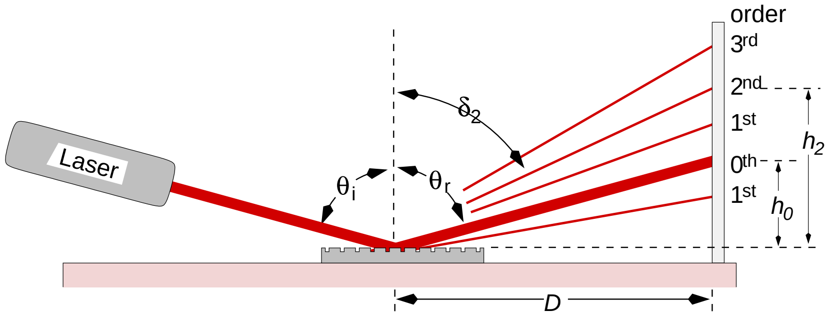


Figure 1: Schematic of diffraction from ruler markings.

Let d be the separation of lines on the steel ruler. For the n^{th} order the path difference, Δ, between waves from successive ruler lines is $n\lambda$. From Fig. 2, $\Delta =$ AC $-$ BE. But AC $= d\cos(90° - \theta_i) = d\sin\theta_i$ and BE $= d\cos(90° - \delta_n) = d\sin\delta_n$. Hence

$$\Delta = d\ (\sin\theta_i - \sin\delta_n) = n\lambda \quad .$$

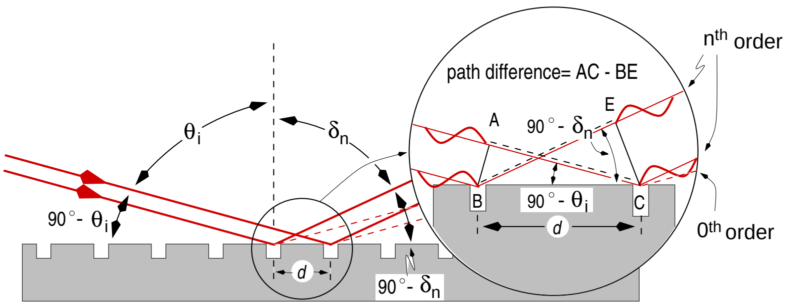


Figure 2: Geometry required for constructive interference.

QUESTION: To minimize uncertainty in λ should you measure a high or low order diffracted image?

EXPERIMENT:

Locate the zero order spectrum (specular reflection) by moving ruler sideways so beam misses the small (diffracting) divisions. Return ruler so one has the high order diffraction images. Make all the relevant measurements. Calculate λ and compare with the accepted $\lambda = 633$ nm. For the error estimate first find the uncertainty in i and δ_n (by estimating reasonable ranges of D, h_0 and h_n). Then find the corresponding uncertainties in sin i and in $\sin\delta_n$ (and hence in λ).

Part II - HOLOGRAMS

1. *To observe a hologram:* For this use a high intensity mercury arc with a green color filter. Have the room reasonably dark and stand about 3 meters in front of the mercury arc source. Look through the hologram but not directly at the source. The images are virtual and three-dimensional.

   Alternatively, diverge a laser beam to illuminate the hologram and view it as in Fig. 3.

   DO NOT ALLOW the UNDIFFUSED LASER BEAM TO ENTER THE EYE!

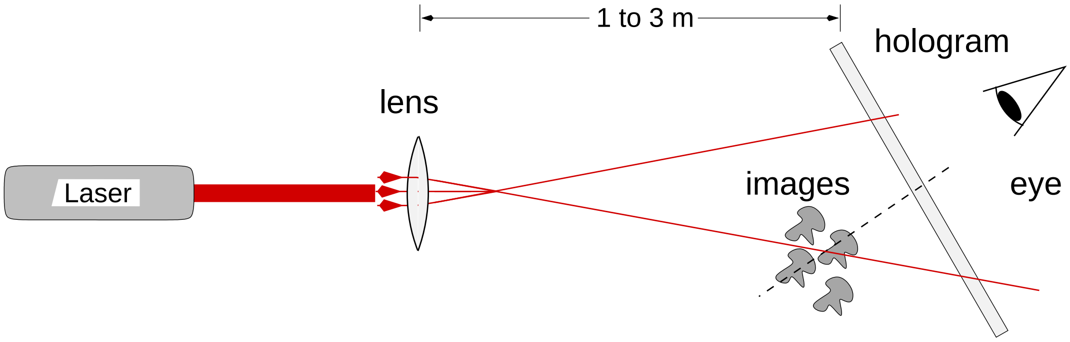


Figure 3:

   OPTIONAL: Simultaneously illuminate the hologram with a diverged red laser light and a green light from the Hg arc. Note the larger red image.

2. *An explanation of the HOLOGRAM:* To understand a hologram image, consider in Fig. 4 the formation for the simplest possible object, a small particle at O_1. The glass plate B partially reflects the plane waves from the laser and partially transmits them to illuminate the object.

   The reflected plane waves plus the spherical waves scattered from O_1 meet in the photograhic plate P and produce an interference pattern. The spacing d between regions of constructive interference is $d = \lambda/\sin\theta$ (see in Fig. 4 the enlargement of the region around I). This interference results in lines with spacing d in the photographic plate and constitute a diffraction grating of spacing d. Note that θ and d vary with position on the plate. This diffraction grating of variable spacing constitutes the hologram for our small particle at O_1.

   If we illuminate this grating with plane waves traveling in the direction BD, the same direction as the reflected plane waves (from the beam splitter B) used to to form

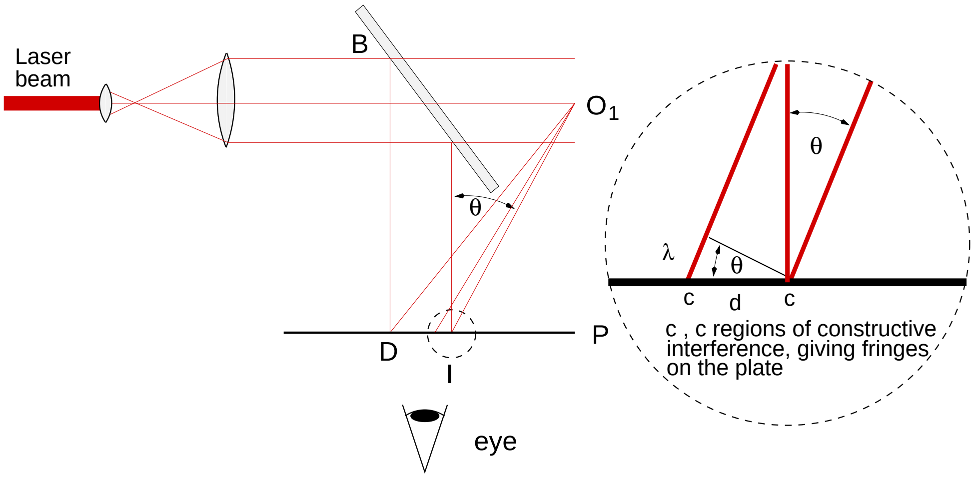


Figure 4:

the hologram, we observe constructive interference (a first order spectrum) in the direction given by the regular grating equation $\sin\theta = \lambda/d$: namely the direction of the original wave from O_1 which formed the hologram. Because d varies from place to place on the plate, no matter where we look through the plate the light in the first order image comes from the direction O_1. Thus the spherical wave fronts originally from O_1 are reconstructed, and an observer at E sees a virtual image of O_1 at O_1.

A similar treatment holds for a second scattering point O_2, and the brightness of the image point will depend on the contrast in the interference pattern which in turn depends on the brightness of the object point. The image of an extended object thus forms point by point, and the image is three dimensional because the virtual image points coincide with the corresponding object points in space. If one views the hologram with longer λ light, all angles increase and a magnified image results.

A first order diffraction beam also appears on the other side of the normal to the hologram, and the diffracted light makes the same angle with the normal to the hologram as the original waves from O_1. Thus a real image of O_1 forms at O_1' (Fig. 5) and similarly for all other object points. This real image can be caught on a screen, one plane in focus at a time.

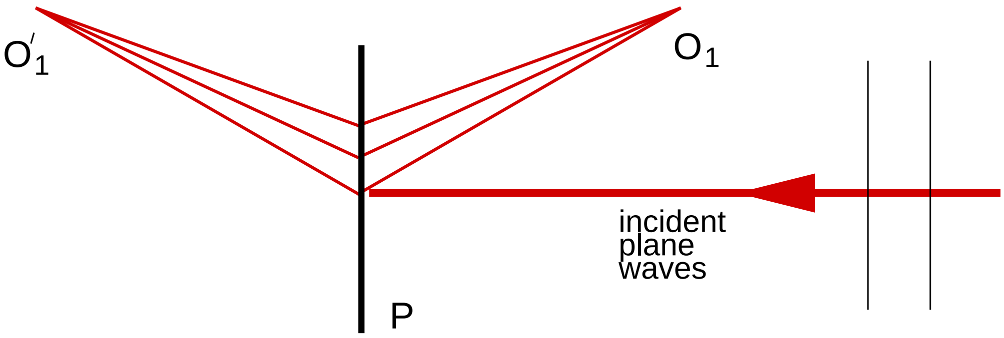


Figure 5:

Modern Physics:

MPC-1 Radiation & its Interaction with Matter

MPC-1a Random Events - Counting Statistics

OBJECTIVES:

To study and measure the occurrence of random events. This in preparation for study of the absorption of radiation by matter.

INTRODUCTION:

There are many type of events that are controlled by pure random chance: the number of babies born on a particular day of the week, the number of fender benders in Madison a year (assuming that the number of cars in Madison does not change), the number of people (out of one thousand) that feel better after taking a particular kind of pain reliever or the number of murders in a year in New York City.

In all these examples one discovers that, if one repeats the observation, the new number will differ from the preceding one. More importantly the number found can not be predicted using the knowledge of a preceding observation nor can it be used to predict the following one. For example the number of babies born on a particular Monday will usually be different from the number born on the succeeding Monday. Similarly, the number of patients (out of a different one thousand) that feel better after taking a particular kind of pain reliever, or the number of murders in the following year in New York City will be unpredictably different.

EXAMPLE I: AN IMAGINARY EXPERIMENT

The average number of births per day is $\overline{N} = 16$. Don't worry about the numbers in the last two columns, we will discuss them in the next section. However the mysterious value of σ calculated above tells one how far from the average $\overline{N}$ one can expect a single observation to lie.

Table I: Number of babies born on Monday in Hospital 'X'

Week	Date	Births N	$\Delta N_i = N_1 - \overline{N}$	ΔN_i^2
1	Mon Jan 4	15	-1	1
2	Mon Jan 11	15	-1	1
3	Mon Jan 18	13	-3	9
4	Mon Jan 25	19	3	9
5	Mon Feb 1	16	0	0
6	Mon Feb 8	18	2	4
7	Mon Feb 15	16	0	0
8	Mon Feb 22	14	-2	4
9	Mon Mar 1	24	8	64
10	Mon Mar 8	10	-6	36
Sum:		$\Sigma N_i = 160$	$\sum \Delta N_i^2 = 128$	
Average:		$\overline{N} = 160/10 = 16$	$\sum \Delta N_i^2/10 = 12.8$	
Width of Distribution: $\sigma = \sqrt{12.8} = 3.58$				

Making a 'histogram' of the above data gives⇒

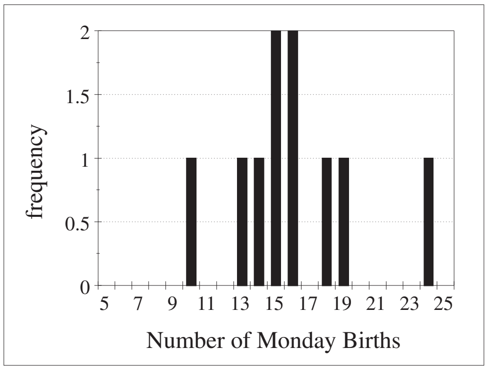


Fig. 1: Distribution of Monday births

What does the figure tell you?

1. It shows the 'distribution' of the number of births, i.e. it shows the 'frequency' of a certain number of births: there were two days with fifteen births, and two days with sixteen births, there were no days with less than ten or more than twenty-four births.
2. It shows that days with a number of births close to the average occur more frequently than days in which the number of births is very different from the average.

EXAMPLE II

You are in the street when a short shower produces just a few drops. You see the marks of the drops on the pavement, the pavement is divided in regular 2 m^2 squares. Having nothing to do, you count the drop marks in fifteen of those squares. Your observations are listed below.

You conclude that the average number of drops on each pavement square is $\overline{N} = 39.2$, but you would not be surprised to find some squares with 30 or 50 drop marks.

Table II

Counts	$\Delta N_i = N_i - \overline{N}$	$\Delta N_i^{\,2}$
$N_1 = 27$	-12.2	148.84
$N_2 = 44$	4.8	23.04
$N_3 = 41$	1.8	3.24
$N_4 = 40$	0.8	0.64
$N_5 = 50$	10.8	116.64
$N_6 = 38$	-1.2	1.44
$N_7 = 39$	-0.2	0.04
$N_8 = 37$	-2.2	4.84
$N_9 = 29$	-10.2	104.04
$N_{10} = 42$	2.8	7.84
$N_{11} = 46$	6.8	46.24
$N_{12} = 39$	-0.2	0.04
$N_{13} = 36$	-3.2	10.24
$N_{14} = 42$	2.8	7.84
$N_{15} = 38$	-1.2	1.44
	$\Sigma N_i = 588$	$\Sigma \Delta N_i^{\,2} = 476.4$
	$\overline{N} = 588/15 = 39.2$	$\sigma = \sqrt{476.4/15} = 5.6$

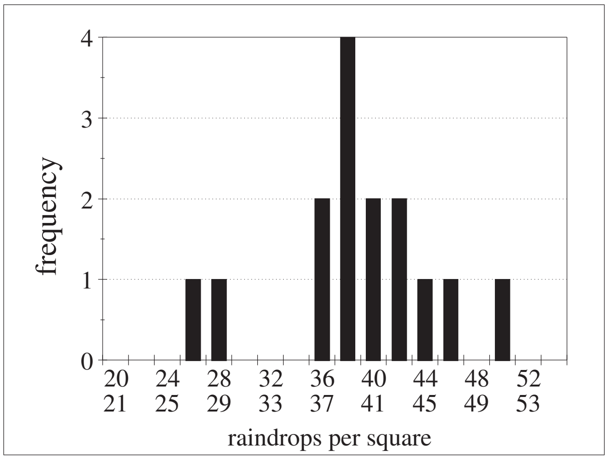


Fig. 2: The rain drop data

Again the value of σ calculated above tells how far from the average $\overline{N}$ one can expect a single observation to be.

You may argue that the value of σ in the two examples was obtained using many observations; how can one know its value if one does only one observation?

In other words, if I make only one observation, for instance if I find that the number of drops in a single square is $N = 35$, what can I say about how far from the unknown but true value $\overline{N}$ it is likely to be?

REQUIRED KNOWLEDGE:

Statistical theory predicts that if one records the counts of m different experiments: $N_1,\ N_2,\ N_3,\ N_4,\ .....\ N_m$ and one calculates the average of these $\overline{N} = (\Sigma_1^m N_i)/m$ the counts will distribute themselves around the average value. Counts close to the average will occur frequently, counts very different from the average will occur infrequently.

If one makes a histogram of the number of times a certain count appears one finds a bell shaped curve called a "Gaussian distribution" centered on the average.
The histogram below looks a lot better than the ones you have seen above; this is so because it shows the distribution of many more observations.

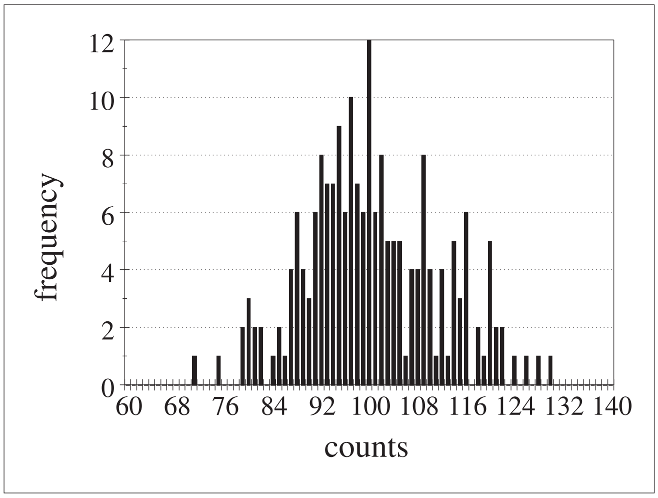


Fig 3: A Gaussian distribution

The histogram shows that counts of about 100 are most frequent, and that counts of 70, or 130 are much less likely to occur; we can think of the "width" of the curve i.e. the range of the counts that occur most frequently is about 20.

THE STANDARD DEVIATION

Statistical theory predicts that the width of the curve, the "standard deviation" of the distribution is defined as:

$$\sigma = \sqrt{\overline{N}}$$

The standard deviation σ is a measure of how wide the curve is; about 1/3 of the counts will lie *outside* the interval $\overline{N} - \sigma$ to $\overline{N} + \sigma$.
Only about 1/20 of the counts will lie *outside* the interval $\overline{N} - 2\sigma$ to $\overline{N} + 2\sigma$. The value of σ is calculated automatically by your computer, so you do not really have to worry about it. However here goes the formula, you may skip this if you wish.

$$\sigma^2 = \left(\sum_{i=1}^{m} (N_i - \overline{N})^2 \right) / m$$

EXPERIMENT

Instead of collecting data from the hospital, from the department of transportation or the NYPD, of the kind shown in the examples above, we will make our own homemade random distribution using the random decay of long lived radioactive nuclei. You will study the statistics of counting. You will find *how* the precision with which one can measure a decay rate R depends on the total number of counts, which in this case is proportional to the time interval during which you observe the radiation.
The rate R of the disintegration of the Radioactive sample is obtained by allowing the G-M counter to detect the radiation emitted for a length of time t, and dividing the number N of observed counts by the time: $R = N/t$.
If you were to repeat the experiment, for the same time interval t, the number of counts would almost certainly *not* be the same because of the random nature of radioactive decay. How accurately would you then know the *real* value of the rate? Intuition tells you that a rate measured with a long time interval t is going to be more reliable than one taken with a short time interval. But even so just how reliable would either of these be? A better way to ask this question is:
If I repeat the experiment, and obtain a different new rate, how big, on the average do I expect the difference between the two rates to be?

EQUIPMENT

⇒ Geiger-Muller counter† and stand. This device detects the (β or γ) radiation from the radioactive sources.

† The Geiger Muller tube was first developed by Hans Geiger(1882-1945) who collaborated with Sir Rutherford at Manchester in the early work on radioactivity which later led to the discovery of the atomic nucleus by Rutherford. The counter was then perfected by Muller in 1928 and is now known as the

Geiger-Muller (GM) counter. The counter consists of a fine wire along the axis of a gas filled metal tube. The wire is made about 500 V positive with respect to the tube. When ionizing radiation, such as γ rays or β particles, enters the counter it breaks up (ionizes) a few gas atoms releasing electrons which are rapidly accelerated toward the positive wire. These electrons collide with other gas molecules, releasing new electrons. Finally, an avalanche of many millions of electrons reaches the wire and produces a voltage pulse large enough to be detected and counted. The whole process can be initiated by a single α, β or γ ray entering the counter.

The Geiger counter is mounted vertically in a stand.

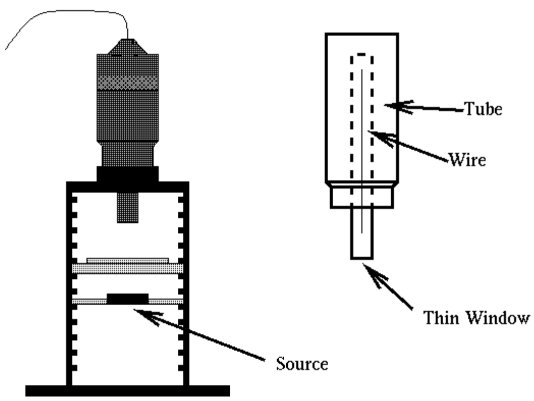


Figure 4: The Geiger-Muller counter and stand

⇒ Pasco Interface, receives the impulses from the GM counter and transfers data to the PC.

⇒ A radioactive γ source.

⇒ A platform that can be inserted into slots in the GM counter stand. This varies the source to counter distance.

> *PRECAUTIONS:* The Geiger counter has a very thin window to permit the entry of β radiation. The window is protected by a plastic cap. In this laboratory exercise you will use γ emitting Cobalt sources, the cap should be left *on*.

PROCEDURE I:

In order to obtain statistics fairly rapidly for different time intervals you will be divided into three groups of students, each group will use different time intervals. You will then compare results at the end.

1. Click on the Launch NC-1A icon below (web version) to initiate the PASCO software window. The screen should show three windows: a 'setup window', (the one with R... on the head bar), a table window and a graph window.
2. CLK on 'Sampling Options' in the setup window. Groups #1 and #2 should set the time to 301 seconds, group #3 should set the time to 337 seconds.
3. Maximize the 'setup window', DCLK on the yellow icon in it.
⇒Group #1 should set the count time period to 3 seconds, and will therefore observe the count in 100 3 second intervals.
⇒Group #2 should set the count time period to 12 seconds and will therefore observe the count in 25 12 second intervals.

$\Rightarrow$Group #3 should set the count time period to 48 seconds and will therefore observe the count in 7 48 second intervals.
All groups will compare results at the end.

4. Put **two** Cobalt sources on top of each other in slot #2 (the second slit counting from the top).
   Prepare a table as shown below; you will enter your results, and those of other groups in this table.
5. CLK on RECORD, wait until the computer has stopped taking data. Enter the mean ($\overline{N}$), and the standard deviation in Table III.
6 Now you may wish to see your data in a graphical form. In order to make a histogram of your data, 'select' the data by dragging the cursor down on the data table, the list of your counts should now be black.

TABLE III

$\overline{N}$ mean	σ std. dev.	$\sqrt{N}$	$100 \times \frac{\sigma}{N}$ rel. unc.	$100 \times \frac{\sigma - \sqrt{N}}{\sqrt{N}}$

7. CLK on 'edit', then CLK on 'copy'. Your data has been copied to the 'desk top', you cannot see it, but you can use it in the next program. Now iconize the Science Workshop Window. The whole window disappears, and an icon for it appears in the tool-bar at the bottom of the screen.
8. DCLK on the 'GA' icon, CLK OK, CLK on the box marked X in the table, it should change color, CLK on 'edit', CLK on 'paste data'. The data you had obtained in the Geiger experiment, and that you copied to the invisible desk top, should now be in the table of the Graphical Analysis window.
9. CLK on WINDOW, move down to REPLACE WINDOW, then move sideways to HISTOGRAM. Chose appropriate LIMITS and BIN SIZE, CLK OK. Voila' you have a histogram of your data. Maximize, then CLK on FILE, then CLK on 'print', CLK on 'selected display'.

ANALYSIS OF THE DATA

1. Enter the data obtained by the other groups in Table II
2. Calculate the value of $\sqrt{N}$ and enter it in column 3 of the table.
3. Calculate the value of $100 \times \frac{\sigma}{N}$ and enter it in column 4 of the table.
4. Plot a graph of the relative uncertainty in the determination of the disintegration rate $100 \times \sigma/\overline{N}$ versus $\overline{N}$.
5. If you have time calculate the value of $100 \times \frac{\sigma - \sqrt{N}}{\sqrt{N}}$ and enter it in column 5 of the table.
6. Plot a graph of the percent error $100 \times \frac{\sigma - \sqrt{N}}{\sqrt{N}}$ versus $\sqrt{N}$.

QUESTIONS

Q1 Does the precision in the measurement of the decay rate depend on on the number of counts? How?

Q2 A pharmaceutical firm tests a new medication by comparing a group of patients who were given the medicine with a group that received a placebo. The results are the following:
of 1000 patients who were given the medicine 750 were cured.
of 1000 patients who were given the placebo 600 were cured.
Is the new medicine effective? Why?
How would your opinion change, if the numbers were 100, 75 and 60?

MPC-1b Absorption of Radiation

INTRODUCTION:

Many atomic nuclei are radioactive. This means that they spontaneously decay, a nucleus of a given atomic number Z (the number of protons in the nucleus, and the number of electrons buzzing around it) and mass number A (the number of protons plus the number of neutrons) transmutes into a nucleus with a different atomic number Z' or a different mass number A' or both. In this process they emit α particles (helium nuclei), β particles (fast moving electrons), or γ rays (energetic, penetrating photons). The instant in time at which an individual nucleus will decay cannot be predicted, this is a random event, but if one observes a large collection of N identical radioactive nuclei, the number ΔN that decays in a short time interval Δt is given by

$$\Delta N \approx -N\frac{\Delta t}{\tau}, \tag{1}$$

where τ is the "mean life" of the radioactive nucleus; it is the time necessary for 63% of the original number of nuclei to decay.

If one starts with N_0 nuclei at time $t = 0$ then the number of nuclei left after a time t is:

$$N = N_0 e^{-t/\tau}$$

The rate R at which nuclei decay is therefore: $R = -\Delta N/\Delta t \approx \frac{N}{\tau} = (N_0/\tau)e^{-t/\tau}$

If the mean life τ is very long compared with the duration of an experiment the rate R at which the nuclei decay is practically constant:

$$\Delta N/\Delta t \approx N_0/\tau \tag{2}$$

Often the life of a radioactive nucleus is expressed in terms of a "half life" $\tau_{1/2}$ which is the time necessary for 50% of the nuclei to decay; $\tau_{1/2} = 0.69\tau$. The above expression is an approximation valid when the time interval Δt is much shorter than τ, as is true in this experiment.

> *NOTE TO THE INSTRUCTOR:* You should remove the plastic cap from the GM counter for the students studying the absorption of β radiation. Please be sure to replace the cap when the lab is finished; or else it will get lost!

EXPERIMENT

You will detect and study the absorption in different materials of the decay products of two nuclei, Co^{60} whose half life is 5.3 years and which emits γ rays of energy about 1.3 MeV, and $T\ell^{204}$ whose half life is 3.8 years and which emits β particles of maximum energy about 0.75 MeV. The β particles have a continuous energy spectrum from zero to the maximum. You will find that γ rays and β particle are partially absorbed as they traverse a thickness of material, and that the absorption depends mostly in the number of electrons per square centimeter in the absorber.

In order to do all this fairly rapidly different groups of students will do different parts and compare their results at the end.

IMPORTANT:

When a beam of N_0 particles crosses a layer of absorber of thickness $t\ mg/cm^2$ the number of particles that emerge is given by:

$$N = N_0 e^{-t/\lambda}$$

This formula is similar to the one at the beginning of this writeup:
the number of particles absorbed in a small thickness Δt is

$$\Delta N \approx -N\frac{\Delta t}{\lambda}$$

Here λ is the "absorption thickness" of the material, and is it is the thickness necessary for 63% of the original number of nuclei to absorbed.

The data has an exponential form; if one takes the logarithm of the number of particles one obtains a linear expression, which is easier to analyze:

$$\ln N = \ln N_0 - \frac{t}{\lambda}$$

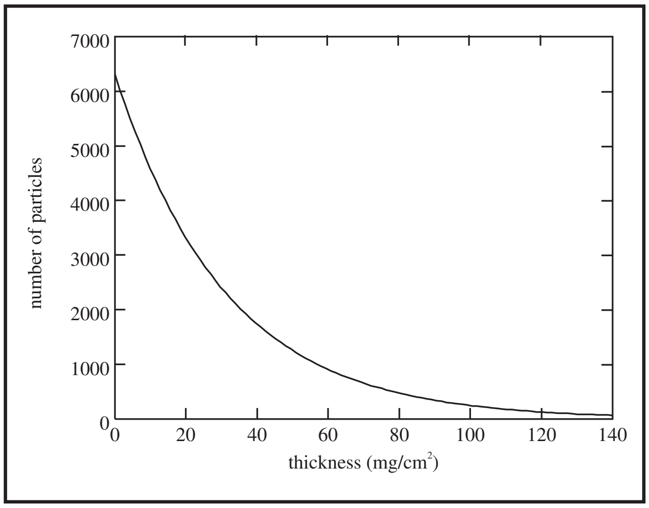


Figure 1: Number of particles vs thickness.

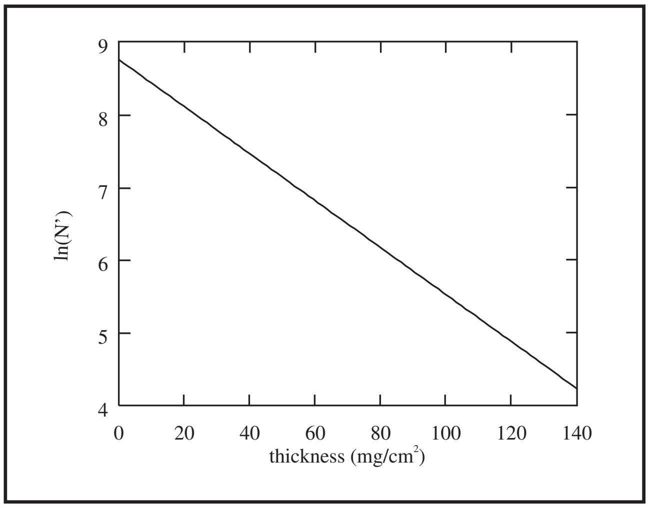


Figure 2: $\ln N$ vs. thickness on log-linear paper. The slope of the line is equal to $-1/\lambda$

EXPERIMENT

⇒ Geiger-Muller counter† mounted vertically and stand. This device detects the (β or γ) radiation from the radioactive sources.

† The Geiger Muller tube was first developed by Hans Geiger(1882-1945) who collaborated with Sir Rutherford at Manchester in the early work on radioactivity which later led to the discovery of the atomic nucleus by Rutherford. The counter was then perfected by Muller in 1928 and is now known as the Geiger-Muller (GM) counter. The counter consists of a fine wire along the axis of a gas filled metal tube. The wire is made about $500\,V$ positive with respect to the tube. When ionizing radiation, such as γ rays or β particles, enters the counter it breaks up (ionizes) a few gas atoms releasing electrons which are rapidly accelerated toward the positive wire. These electrons collide with other gas molecules, releasing new electrons. Finally, an avalanche of many millions of electrons reaches the wire and produces a voltage pulse large enough to be detected and counted. The whole process can be initiated by a single α, β or γ ray entering the counter.

The Geiger counter is mounted vertically in a stand.

⇒ PASCO Interface, receives the impulses from the GM counter and transfers data to PC

⇒ Two radioactive (β and γ sources).

⇒ Lead, Aluminum and Poly absorbers.

⇒ A platform with multiple slots. This permits the radioactive source to be placed at various distances from the counter.

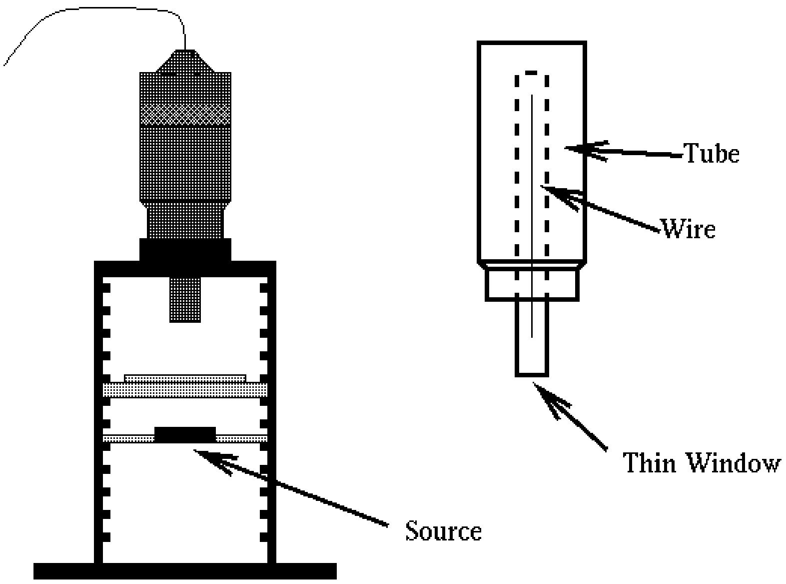


Figure 3: The Geiger-Muller counter and stand

PRECAUTIONS: The Geiger counter has a very thin window to permit the entry of the radiation under study; the window is protected by a plastic cap. Groups working on β radiation should ask their TA to remove it carefully. Do not poke anything up toward the counter. The window is very fragile, if it is broken the counter will not function, and can not be repaired.

PROCEDURE I: (45 min)

In this procedure you will do the work in separate groups and then discuss your results with the different groups. You will observe the absorption of β and γ radiation in Lead, Aluminum and Polyethylene: γ and β radiations behave very differently in traversing absorbers; because of this your instructions will depend on the source you are studying.

- γ rays are energetic photons, when a γ ray interacts with an atom, the γ undergoes 'Compton Scattering' and is effectively eliminated from the beam.
- β rays are energetic electrons, when an electron traverses matter, it knocks low energy electrons out of the atom, in this way loses energy and eventually stops.

Because of this the work will be divided among different groups:

Group # 1 $\Rightarrow$ Poly β : Group # 2 $\Rightarrow$ Pb γ : Group # 3 $\Rightarrow$ Al γ

NOTE: *All Groups must make a - Background measurement -*

Click on the Launch NC-1B icon below (web version) to initiate the PASCO software window. The screen should show three windows: a 'setup window', (the one with G... on the head bar), a table window and a graph window.

1. Remove the sources from the vicinity of the GM counter, set them at least 1 m away.
2. CLK on 'Sampling options'. The 'stop condition' should be set to 101 seconds for group #1, and to 301 seconds for groups #2 and #3.
3. Maximize the 'setup window', DCLK on the yellow icon in it, and the set the counts per time period to 10 seconds. The 10 s interval is not really needed, but this way you can see the thing is working, without waiting the full five minutes.
4. DCLK on record, CLK on the statistics icon (Σ) in the data table. Multiply the 'mean' by the number of time periods (10 for group #1, 30 for groups #2 and #3) to obtain the total number of counts. This is how you will measure the total number of counts for all data runs. Record your result in Table I.

   These counts are mostly due to a radiation called Cosmic Rays: energetic protons and other nuclei produced somewhere in the galaxy (probably supernovæ or pulsars) that impinge on the upper reaches of the earth's atmosphere. These particles interact in the atmosphere and produce secondary particles that reach sea level, cross the roof of the laboratory and finally cross your GM tube. They are mostly electrons, positrons and muons. Their number is small $\sim 10^{-2}/cm^2 \cdot s$ but not negligible. This background rate must be subtracted from all subsequent counting rate determinations for the γ or β sources.

Table I

Absorber name	t	Total Counts	$\sqrt{N}$	$N' =$	$\ln N'$
	mg/cm^2	N		$= N - N_{bgr}$	
Background	$0\ mg/cm^2$				

γ Pb Group

5. $\Rightarrow$ Place two Co sources on top of each other in slot # 5 (counting from the top), CLK RECORD, wait until the computer stops recording and enter the result in Table I with absorber thickness $t = 0\ mg/cm^2$.
6. Place various lead absorbers in the slots between the source and the counter. Record the data in table I. Repeat with different absorbers. (Use one 'T' absorber, then two 'T' absorbers and then three'T' absorbers together, for maximum absorption.) Record your data.
   NOTE: The standard deviation shown by the computer is *not* the error in the rate measurement because it refers to the number of time intervals during which the data was taken. The error in the rate measurement depends on the total number of counts, it is $\sigma = \sqrt{N}$.

γ Al Group

7. $\Rightarrow$ Same as step 5, but put the double source in slot # 6, CLK RECORD, wait until the computer stops recording and enter the result in table I with absorber thickness $t = 0\ mg/cm^2$.
8. Place one Aluminum 'P' absorber in the slots between the source and the counter. Then put two, three and four 'P' absorbers in the slots. Record the data in table I.

   ADDITIONAL PRECAUTIONS FOR THE BETA PARTICLES EXPERIMENT: The absorbers for this part are extremely thin. *Please treat them carefully* !!!

β Poly Group

9. $\Rightarrow$ Check that the plastic cap has been removed from the GM counter. Place the source in slot #4 with the side without writing facing toward the GM counter. CLK RECORD, wait until the computer stops recording and enter the result in table I with absorber thickness $t = 0\ mg/cm^2$.
10. Place various Poly absorbers in the slots between the source and the counter. Use a 'C', a 'D', two 'D's, an 'E' and a 'C' together, an 'F', and an 'F' and a 'D' together. Record the data in table I. Repeat with different absorbers. Record your data in Table I.

ANALYSIS OF THE DATA

Plot the the natural logarithm of the counts N versus the absorber thickness t. Draw a straight line through the experimental points, and find the corresponding absorption length λ by measuring the slope of the line.

QUESTIONS

Q1 Does the absorption of γ rays depend on the Z of the absorber?

Q2 Why are β rays so easily absorbed?

Appendices

A. Precision Measurement Devices

Vernier Calipers:

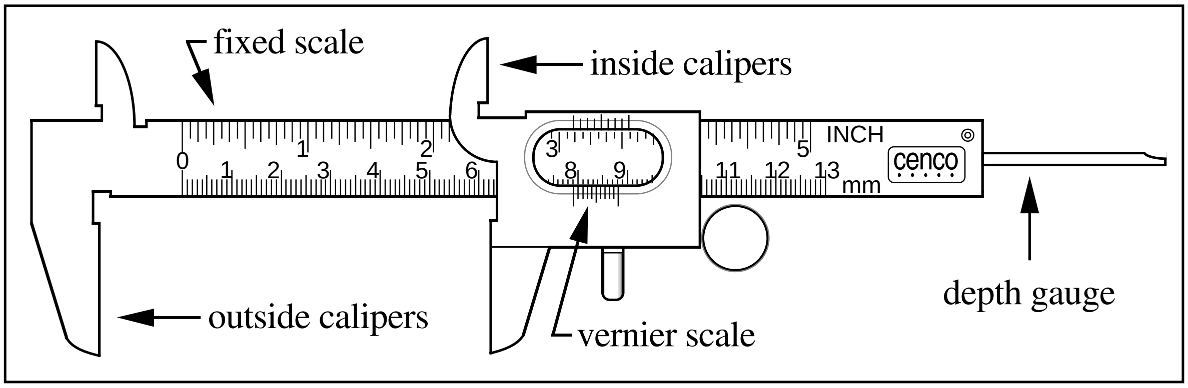


Figure 1: The vernier caliper

A Vernier consists of a fixed scale and a moving *vernier* scale. In a metric vernier the fixed scale is marked in centimeters and millimeters, the vernier scale is nine millimeters long, and is divided into ten parts each 0.9 millimeters long. The distances of each line from the first are therefore 0.9, 1.8, 2.7, $\ldots$,mm or generally: $d_i = 0.9 \times i$, where d_i is the distance between the zero line and the i^{th} line of the vernier scale. If the vernier caliper is closed, so that the two jaws touch each other, the zero of the fixed scale should coincide with the zero of the vernier scale. Opening the jaws 0.03 cm = 0.3 mm will cause the fourth line (the *three* line which is a distance of 2.7 mm from the zero line of the of the vernier scale) to coincide with the 3 mm line of the fixed scale as shown below.

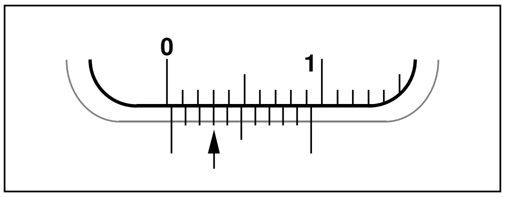


Figure 2: The vernier reads 0.03 cm

Below is another example of vernier reading; the arrow shows which mark on the vernier scale is being used.

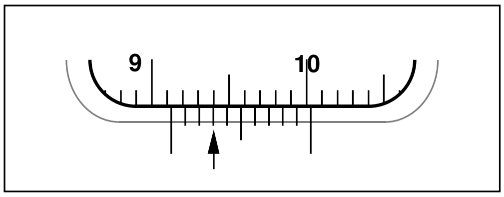


Figure 3: The vernier reads 9.13 cm

EXERCISES:

A. Close the vernier and observe that the first vernier mark coincides with the zero of the centimeter scale.

B. Open the jaws of the vernier very slowly and observe how the different vernier marks coincide successively with the millimeter marks on the fixed scale: the first mark coincides with the 1 mm mark on the fixed scale; then the second mark coincides with the 2 mm mark on the fixed scale; then the third mark coincides with the 3 mm mark on the fixed scale and so on.

C. Estimate the dimension of an object using a meter stick and then Use the vernier caliper to measure the dimension precisely.

D. In the four examples of Fig. 4 determine the actual reading.

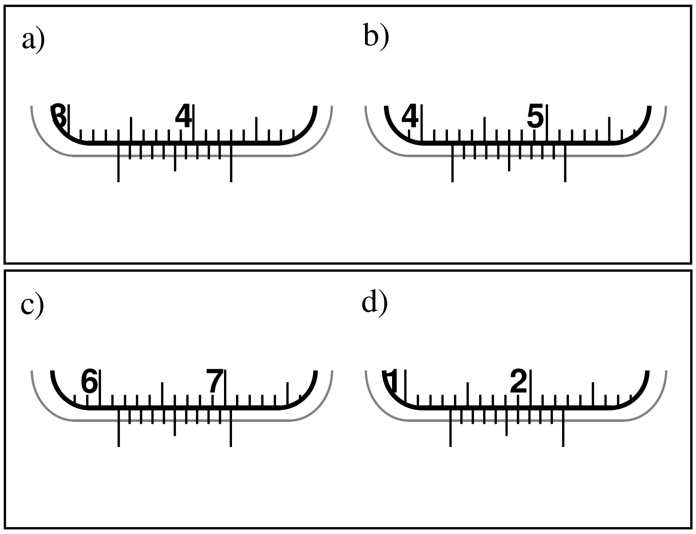


Figure 4: Test cases

Micrometer:

A micrometer can measure distances with more precision than a vernier caliper. The micrometer has a 0.5 mm pitch screw, this means that you read millimeters and half millimeters along the barrel. The sleeve is divided into 50 divisions corresponding to one hundredth of a millimeter (0.01 mm) or 10 μ each. The vernier scale on the micrometer barrel has ten divisions, marked from 2 to 10 in steps of two. The "zero" line is not marked '0', but is longer than the others. The vernier allows you to read to the nearest thousandth of a millimeter, *i.e.*, to the nearest micron (0.001 $mm = 1\ \mu$).

Precaution: Great care must be taken in using the micrometer caliper; A ratchet knob is provided for closing the caliper on the object being measured without exerting too much force. Treat the micrometer with care, **ALWAYS** close the calipers using the ratchet knob, this prevents tightening the screw too strongly. Closing the calipers too hard damages the precision screw.

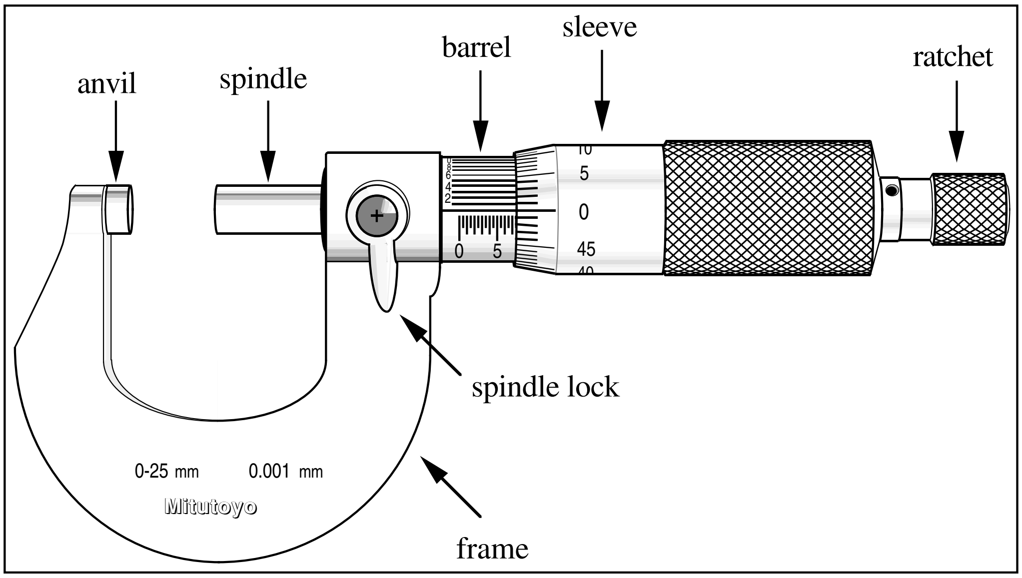


Figure 5: The micrometer calipers

Below are two examples of micrometer reading; the arrow shows which mark on the vernier scale is being used.

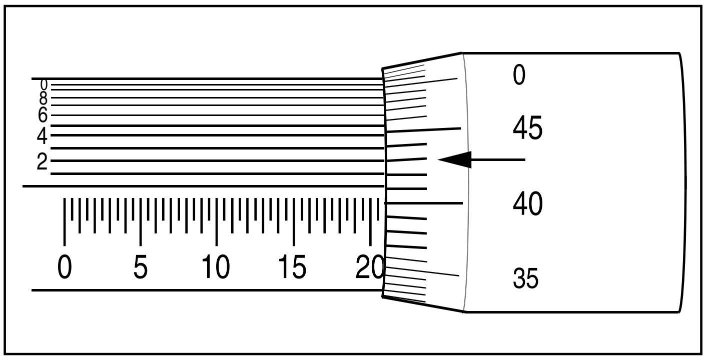


Figure 6: The micrometer reads 20.912 mm

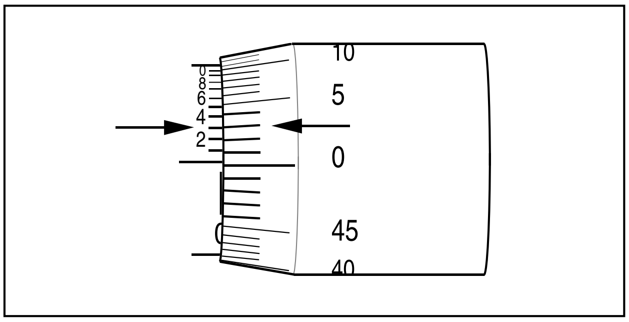


Figure 7: The micrometer reads 3 μ

In Fig. 7 the zero line on the barrel is barely visible, and the vernier reads 0.003 mm = 3 μ; the zero error is $\epsilon_0 = 3\mu$.

A negative zero error, as shown below requires a moment of thought.

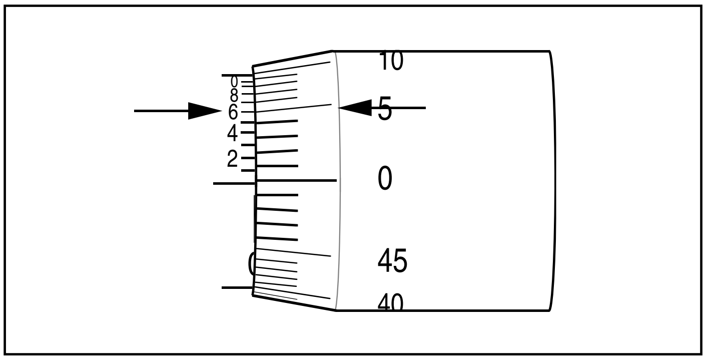


Figure 8: The micrometer reads -4 μ

In Fig. 8 the zero line on the barrel of the micrometer is obscured by the sleeve, (the "zero" line on the sleeve is above the "zero" line on the barrel) this corresponds to a reading of -0.5 mm; the vernier reads 0.496 mm the zero error is then $\epsilon_0 = -0.5 + 0.496 = -0.004 mm \ = -4\ \mu$.

B. The Travelling Microscope

The sliding carriage of the traveling microscope rides on carefully machined ways, pushed by a nut under the carriage which rides on the micrometer screw. The nut must not fit tightly on the screw or it will bind; hence there is always some slack built into the mechanism.

When the nut is being pulled to the right (dial being turned toward larger numbers), the screw threads will press against the threads in the nut as shown in Fig. 1, with the screw threads in contact with the back side of the threads on the nut. When the direction of turning reverses, the screw threads then push on the front side of the nut threads.

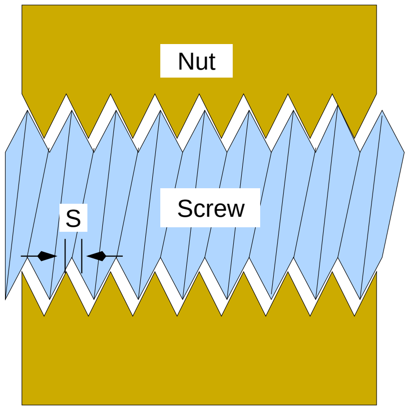


Figure 1: Blacklash in screw mechanism.

For the microscope set initially on the same line for both directions of motion, the readings will differ by distance S, the backlash (slop) in the mechanism.

One way to avoid trouble with this slack is always to make settings after turning the screw more than the slack in one direction, say the direction of increasing readings. If one overshoots on a reading, go back by more than the slack and then turn forward again. The screw will then always press on the same side of the nut and no error arises.

A much better experimental technique is to take readings both ways. Suppose one wants to measure the distance between two lines, 1 and 2. Call the reading turned toward larger readings on line 1, D_1 and when turning in the reverse direction, R_1; similarly for D_2 and R_2. Then the distance between the lines will be $D_2 - D_1$ and also $R_2 - R_1$ so that one has immediately two independent readings to compare. More important, $D_1 - R_1$ is the slack in the mechanism; it should equal $D_2 - R_2$ and should be the same for all pairs of readings. If $D - R$ changes by more than the experimental error in setting, you know immediately you have made a blunder in either setting or reading and can immediately repeat the measurement. The constancy of D - R is actually an excellent measure of the uncertainty in the measurements you are taking.

C. The Optical Lever

An optical lever is a convenient device to magnify a small displacement and thus to make possible an accurate measurement of the displacement. Experiment M-11, Young's modulus, uses an optical lever to magnify the extension of a wire produced by a series of different loads.

The plate P carries a mirror M. The mirror mount has two points resting in a fixed groove, F, and at the other end has a single point resting on the object whose displacement one is measuring. Raising the object through a distance ΔL will tilt the mirror through an angle θ or $\Delta L/d$ radians (approximately) but will turn the light beam through an angle 2θ.

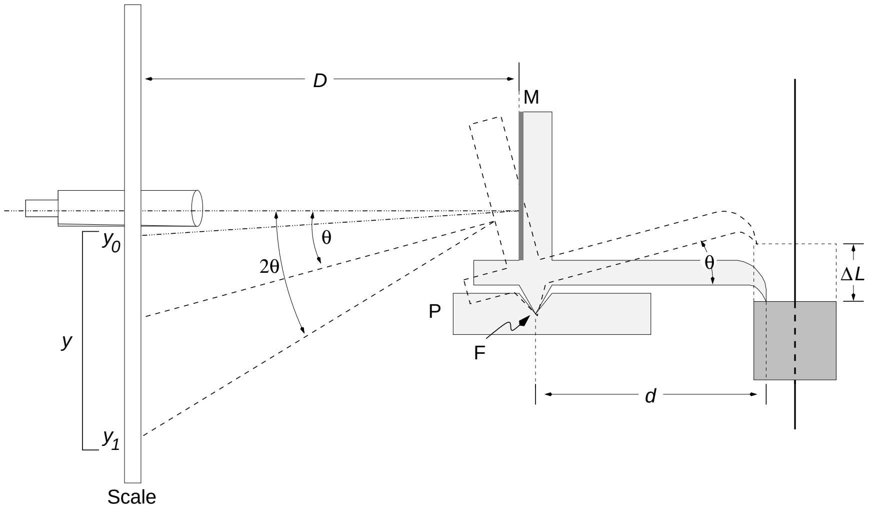


Figure 1: Schematic of the optical lever.

Hence

$$\theta = \frac{\Delta L}{d} \sim \frac{\frac{1}{2}(y_1 - y_0)}{D}$$

if θ is small so that $\theta \sim \tan\theta$. Therefore

$$2\theta = \frac{2\Delta L}{d} = \frac{y_1 - y_0}{D}, \text{ and } \Delta l = (y_1 - y_0)\left[\frac{d}{2D}\right] \quad .$$

Note that with the telescope nearly perpendicular to the scale at the beginning then y_0 is close to the telescope, and the difference between two elongations ($\Delta L_2 - \Delta L_1$) is very accurately given by

$$\Delta L_2 - \Delta L_1 = \frac{y_2\ d}{2D} - \frac{y_1\ d}{2D} = \frac{(y_2 - y_1)d}{2D}$$

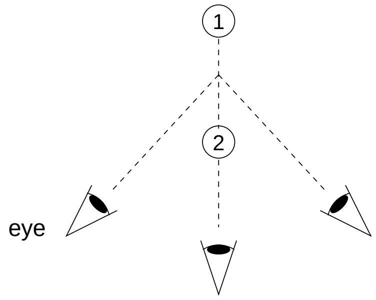
where y_i is the scale reading. This relation holds so long as 2θ is small enough that $\tan 2\theta \sim 2\theta$.

D. PARALLAX and Notes on using a Telescope

1. *PARALLAX*:

To do quantitative work in optics one must understand *parallax* and how it may be eliminated. PARALLAX is defined as *apparent motion of an object caused by actual motion of the observer.*

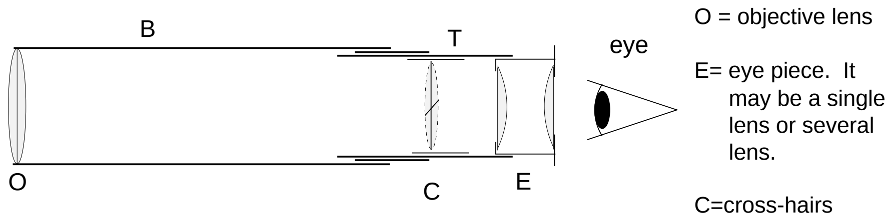
When the observer's eye is in position 1, objects 1 and 2 are in line and may appear to coincide. If the eye is moved to the left to position two, object 1 (i.e. O_1) appears to move to the left with respect to object 2, (i.e. O_2). If the eye moves to the right to position 3, object 1 (O_1) appears to move to the right with respect to object 2 (O_2).



As object 1 moves toward position 2 along the dotted line, its apparent displacement with respect to object 2 caused by motion of the eye from 2 to 3 gets smaller until it vanishes when O_1 and O_2 coincide. When O_1 gets closer to the eye than O_2, the direction of its apparent displacement reverses for the same eye motion. In short, the object farthest from the eye apparently moves in the same direction as the eye. Try this with two fingers.

Note that if O_1 is an image and O_2 a cross hair, the absence of parallax shows that the cross hairs are in the plane of the image.

2. *Focusing a Telescope for Parallel Rays*:



The eyepiece E slides back and forth in the tube T and one should first adjust the eyepiece to give a clear image of the cross hairs. Then move the tube T back and forth in the barrel B until the image of a distant object, formed by the objective O, falls on the plane of the cross hairs. The test for this is the absence of parallax between the cross hairs and image.

The rays from a distant object are nearly parallel. For viewing a distant object, use an open window if the window glass is not accurately plane. Otherwise poor image formation may result. You can check by trying it both ways. At night use a distant object in the hallway.

The telescope, now focused for parallel rays, will stay so as long as the distance between O and C is unchanged. One may still adjust the eyepiece position to suit the observer.

3. *Finding an Image in a Telescope*:

If you have trouble finding an image in a telescope, locate the image first with your unaided eye, and then pull the telescope in front of your eye, aligning it with your line of sight. With high magnification it is difficult to find the image in the telescope because the alignment must be nearly perfect before the image appears in the field of view. The eye has a rather large field of view so that the image will be visible over a range of positions.

E. Notes on Radiation Dosage, Dosimetry, and the Radon Problem

INTRODUCTION:

The biological effects of radiation arise from the absorption of the radiant energy to produce heat, electronic excitation and/or ionization.

Radio, T.V., microwave, visible light, u.v. light, x-rays, γ-rays are all electromagnetic radiation and differ only in wavelength. Electromagnetic radiation may have both beneficial and harmful effects: e.g. u.v. light absorbed by the skin can supply needed vitamin D, but excess u.v. radiation accounts for much skin cancer. X-rays and γ-rays are more penetrating and so can affect tissue below the skin.

Besides electromagnetic radiation one has high velocity charged (and neutral) particles. From *naturally* radioactive materials, the charged particles are either high speed electrons (β's, beta rays) or alpha particles (α's, the nuclei of helium atoms). Both types are rather easily stopped by a small thickness of matter e.g. ~ 1.8 mm Al stops 1.17 MeV β's from RaE, and ~ 0.06 mm Al stops 5.3 Mev α's from Po. Hence natural radioactivities are normally of little concern unless the parent nucleus has been inhaled or ingested in the body. The biological effects of neutral particles (e.g. neutrons and neutrinos) from naturally radioactive materials are normally negligible.

Radioactivity unit: 1 becquerel (Bq) = 1 disintegration/s $\cong$ 27 pCi (picoCurie)

Radiation dose is the radiant energy absorbed per unit mass.

Dose UNITS: 0.01 J of radiation absorbed/kg of mass = 1 rad
1 J of radiation absorbed/kg of mass = 1 gray (abbreviated **Gy**)

Dose Equivalent: includes the long term *relative biological effects of different types of radiation.* The original unit was the **rem** (**r**ad **e**quivalent **m**an), but the now recommended S.I. unit is the **sievert** (abbreviated Sv) with:

$$1 \text{ Sv} = 100 \text{ rem} = 10^5 \text{ mrem}$$

and so: 1 mSv = 100 mrem

Federal laws on permissible doses are:

1. For workers, $<$ 50 mSv/year for a whole body dose, but employer must follow the ALARA (**A**s **L**ow **A**s **R**esonably **A**chievable) principle. For the hands alone, 750 mSv/yr are allowed.
2. For the general population $<$ 5 mSv/yr whole body dose.

RADIATION SOURCES:

Besides the sun's u.v. radiation, the natural environment contributes an unavoidable dose equivalent to ~ 1.3 mSv/yr plus a variable dose from inhaled radon which often is several times larger: see **The Radon Problem** (below). **Hence the average natural background radiation dose is $\sim$ 3 mSv/yr**. About .25 mSv/yr of this dose comes from internal radioactivities in the body (chiefly ^{40}K which constitutes 0.0119% of natural K and has $T_{1/2} \sim 10^9$ years). The rest comes from external natural radioactivities in the earth (chiefly decay chains of uranium and thorium, $T_{1/2} \sim 4 \times 10^9$ years and $T_{1/2} \sim 10^{10}$ years) and from cosmic rays. The cosmic ray contribution increases with altitude and is $\sim$ 30 mSv/yr at 40,000 feet elevation (jet airplane altitudes).

Brick and stone houses often have larger backgrounds. Living in Denver (elevation 5200 ft.) contributes an additional $\sim$ 0.7 mSv/yr. In the Kerala region of India and the Espirito Santo region of Brazil, natural sources give $\sim$ 30 mSv/yr with no obvious abnormality resulting to the indigenous population so the Federal regulations seem very conservative for the general population.

Man-made radiation exposure averages $\sim$ .7 mSv/yr and comes almost exclusively from medical and dental x-rays. A single dental x-ray may involve 7 mSv to the skin. Exposures from nuclear power generating stations are nearly zero. In fact, per KWH of electricity generated, the radioactivity released from coal fired plants is often high compared to that permitted from nuclear plants since many coals contain appreciable uranium and/or thorium plus the equilibrium decay products from these long-lived radioactive nuclei.

For perspective on radiation exposure, Prof. Cameron of the UW Medical Physics Dept. suggests translating doses into a natural unit, the **BERT** defined as the **B**ackground **E**quivalent **R**adiation **T**ime. Thus a **BERT equal to 1 yr would correspond to 3 mSv** (see above discussion on average background dose).

THE RADON PROBLEM:

Radon (an inert gas) from decay of naturally occurring U and Th in the earth continually diffuses into the atmosphere and may cause $\sim$ 10,000 lung cancer cases per year in the U.S. The radon content of outdoor air 1 meter above ground typically gives 4 to 15 becquerels/m^3 The health effects come mainly from inhalation of ^{222}Rn (from U) since this radon isotope has a $T_{1/2}$ of 3.82 days whereas the thorium radon isotope (^{220}Rn) has $T_{1/2}$ of only 56 seconds. The indoor air concentration of radon ($\sim$ 50 Bq/m^3) varies perhaps a factor of a thousand from location to location, depending upon the U content and physical characteristics of the soil, moisture content, building construction, winds, etc. (A house in Maine had a record $\sim$ 160,000 Bq/m^3!)

A radon concentration of 50 Bq/m^3 may result in an annual dose equivalent to bronchial epithelium (site of most radiation induced lung cancer) of $\sim$ 2.5 mSv/yr. Perhaps 25% of Wisconsin houses have concentrations $>$ 150 Bq/m^3 which is the EPA guideline where action should be taken in a few years since the lung cancer risk may be comparable to smoking 3 to 10 cigarettes/day. (M.S. Blumenthal, **Wisconsin Medical Journal**, Vol. 87, May 1988, p.17) In fact 2% of U.S. homes have radon concentrations $>$ 300 Bq/m^3 and occupants should take action to reduce the concentration since they may be receiving an effective dose of $>\sim$ 16 mSv/yr. (By contrast the EPA limit for off-site exposure from nuclear reactors or from nuclear waste depositories is only .25 mSv/yr!) (Bodansky, **Physics and Society** 16, No. 4, p. 6, 1987).

If a house has high radon levels, then the radon ingress usually is from air infiltration from soil beneath the house. Natural convection in the house (chimney effect) tends to pull in the radon contaminated air. A solution is to drive pipes through the basement floor and connect them to fans exhausting to the outside air. See A. V. Nero Jr. "The Indoor Radon Story", **Technology Review** Vol.89, No. 1, p. 28, (1986); also A. Nero, "Earth, Air, Radon and Home", **Physics Today**, Vol 42, No. 4, p.32, (1989).

A good general reference on the subject is **"Radon and its Decay Products in Indoor Air"** edited by William Nazaroff and A.V. Nero, Jr., John Wiley & Sons, 1988. Ground water supplies often contain high concentrations of radon from uranium decay in the aquifers. The radon concentration in public ground water supplies averages $\sim$ 5000 Bq/m^3, and is much higher in some of the New England states. The health hazard is apparently not from drinking the water, but from the water's contribution to the indoor radon air problem: perhaps $\sim$ 5 Bq/m^3. Private wells often have high concentrations. Storage or aeration of the water provides effective control of the hazard.

OTHER ISSUES:

Is a Small Amount of Radiation Healthy - The Hormesis Effect

The following is from RADIATION DOSIMETRY by Prof. John R. Cameron, Department of Medical Physics, UW, Madison (1989):

> "Studies on nuclear workers often show that they have less cancer than other member of the population and even of other workers with similar jobs. This is usually explained as the 'healthy worker' effect. That is, for reasons not understood, radiation work attract healthy workers. An alternate explanation which is rarely mentioned is the possibility that a small amount of radiation is good for you. This is referred to as the 'hormesis' effect. Since humans and all of our ancestors evolved in a sea of natural radiation, it is possible that mutations have occurred that produce the hormesis effect. Animal experiments have demonstrated the hormesis effect. Rats exposed to increased radiation have a longer survival than their controls."

F. PASCO© Interface and Computer Primer

INTRODUCTION:

The PHYS207/208 laboratories now utilize a Web-browser based display format in combination, when necessary, computer controlled data acquisition interface (typically the PASCO CI-700 or 750). Various sensors are plugged into either digital I/O (phone jack style inputs 1 to 4) or analog I/O ports (DIN-9 style inputs A, B and C). To aid in the data acquisition and analysis PASCO module also requires use of a special purpose software package which can be easily reconfigured for the particular need of a experiment. In general all experiment starting configuration will be preset and launched through a Web-browser button at the appropriate place in the lab.

THE MOUSE

CLICKING: Most of the operations of your computer are controlled by locating the cursor on the appropriate symbol (icon) and by clicking (CLICK) or double clicking (DCLICK) the *left* button of the mouse.

If the operation you have to perform requires clicking the right button this will be shown by CLICK-R or DCLICK-R. Double clicking means pressing the mouse button twice in rapid sequence *without moving the mouse.* The image of an hour glass appears momentarily indicating that the computer is loading the program, that is, getting ready to do what you requested. It will not do this if you moved the mouse while double clicking.

WINDOWS

The monitor usually displays various "windows" with a title bar. If you CLICK anywhere inside the window, the title bar turns blue, and the window is "active" (i.e. the computer will respond to any clicks on the "icons" on the border of the window).

BASIC OPTIONS:

I. CLICK on the head bar to "drag" the window to a different position.
II. Enlarge the window by placing the cursor on the corner, a diagonal arrow will appear, then CLICK and drag to change the size of the window.

Depending on which experiment you are performing you will see various windows. These will be discusses separately.

ICONS

The icons that appear at the top right of <u>all</u> windows are (see Fig. 1):

1. EXIT: The window is removed permanently.
2. RESIZE: The size is changed from large to small, or vice-versa.
3. ICONIZE: The window disappears. Its title and icons 1, 2 and 3 appear at the bottom of the screen.

   THE GRAPH WINDOW ICONS: At the bottom left corner of the graph window you will find six icons:

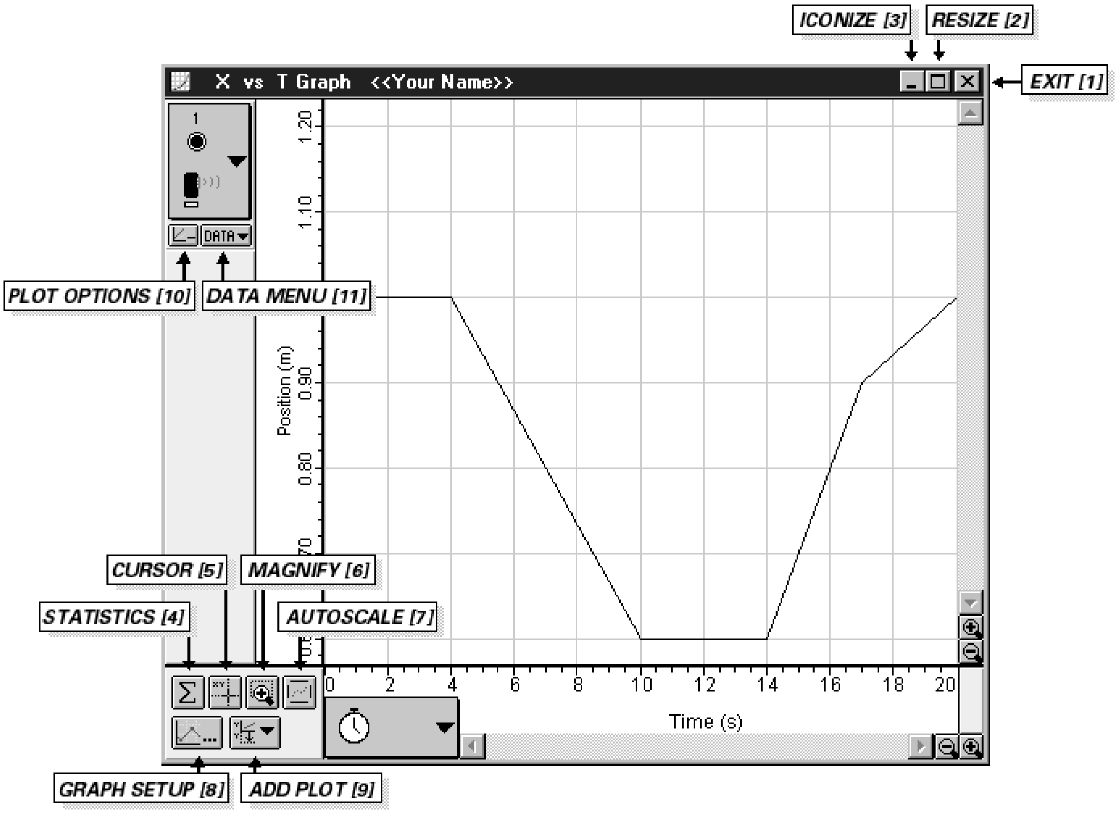


Figure 1: The Graph Window

4. STATISTICS shows/hides statistics for all existing graphs. You must first select the area of the graph you want to analyze by dragging the cursor (CLICK and hold) diagonally across the graph to generate a rectangle that encloses the area chosen. A new box will appear that allows you to select the type of fitting that is suitable to the data. The meaning of "fitting" is discussed separately below.

5. CURSOR: Displays cross hairs which can be moved about using the mouse. Simultaneously the coordinates of the cursor position are displayed and this enables the coordinates of graph features to be measured.

6. MAGNIFY: Enlarges any part of a graph. To select the area of the graph you want to enlarge by first CLICKing this icon and then CLICK a point on the graph using a drag motion (click and hold) diagonally across the graph. This will generate a rectangle that selects the area chosen.

7. AUTOSCALE: Scales the y-axis and the x-axis so that all the data is shown

8. GRAPH SETUP: This icon allows for adding your name to the title of the window, select plot (e.g. connected points, select grid).

9. ADD PLOT: Add another graph; presents a menu of available channels.

At the top *left* corner of the graph window you find two more icons:

10. PLOT DATA: Displays various data options.

11. DATA MENU: Displays the list of all the data sets recorded. CLICK on the one you want to display; a check mark will appear and the corresponding graph will be displayed. CLICK on the one you do not want to display, the check mark will disappear.

THE SCALE OF EACH AXIS:

In order to change the scale of an axis CLICK on any number on that axis; a menu will appear allowing you to enter the minimum and maximum values of the scale, CLICK at the end of the number you want to change, backspace, type the new number, CLICK "OK" in the appropriate box.
An alternative way of changing the scale of a graph is to CLICK on the ⊕ or ⊖ at the bottom right corner of the graph window.

THE SETUP WINDOW ICONS:

Usually you will find the setup window in its "minimized" form as shown below:

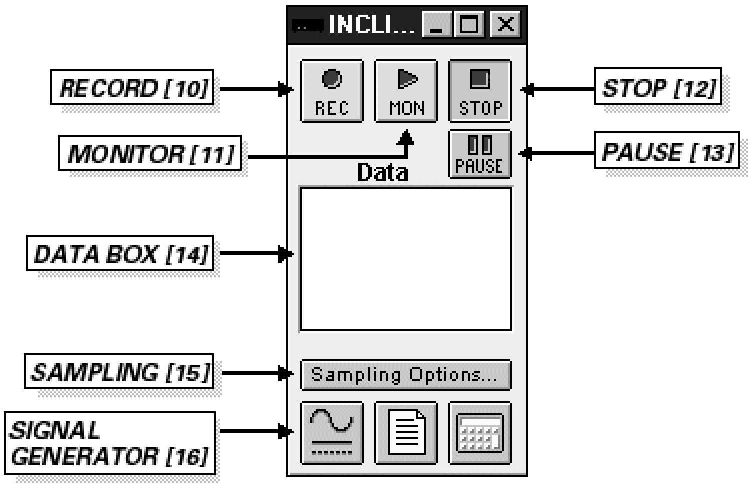


Figure 2: The Setup Window minimized.

10. RECORD: DCLICK and the computer begins taking data. The data is stored, and can be recalled.
11. MONITOR: DCLICK on this and the computer begins taking data. The data is not stored, and it disappears at the end of the run. This allows you to view what the data will look like when you RECORD.
12. STOP: Stops the run, the data is stored. The run cannot be restarted.
13. PAUSE: Stops taking data till you CLICK again and the run is restarted.
14. DATA BOX: This area contains the color coded list of all the runs. Delete a run by clicking on it, and using the back arrow at the top right corner of the keyboard.
15. SAMPLING: Allows for terminating a data run after a predeterminate time.
16. SIGNAL GENERATOR: Starts a built-in signal generator (e.g. sine or square waves) and allows control of both frequency and amplitude.

THE TABLE WINDOW ICONS:

17. STATISTICS: Calculates displays the minimum and maximum value of the data, displays also the mean, and the standard deviation.
NOTE: Subsets of the full data set can be analyzed by using the mouse and highlighting (through a CLICK and drag motion) the rows of interest.
18. TIME INDEX: Alter the display to include time at which data was recorded.
19. DIGITS SETUP: Resets the number of significant digits displayed.
20. DATA MENU: Allows access to data of preceding runs.

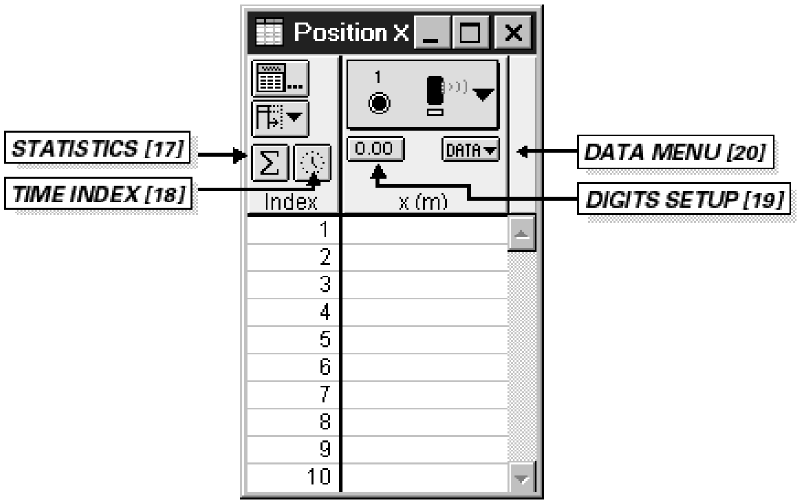


Figure 3: The Table Window

THE STATISTICS WINDOW ICONS:

21. STATISTICS: the fitting menu is displayed; a mathematical expression that approximates the data or other option (e.g. histogram data, calculate the derivate of the curve) may be chosen.

    "Fitting" or regression is the procedure used to find a mathematical expression that matches the function form of the data being analyzed (linear, exponential, etc.).

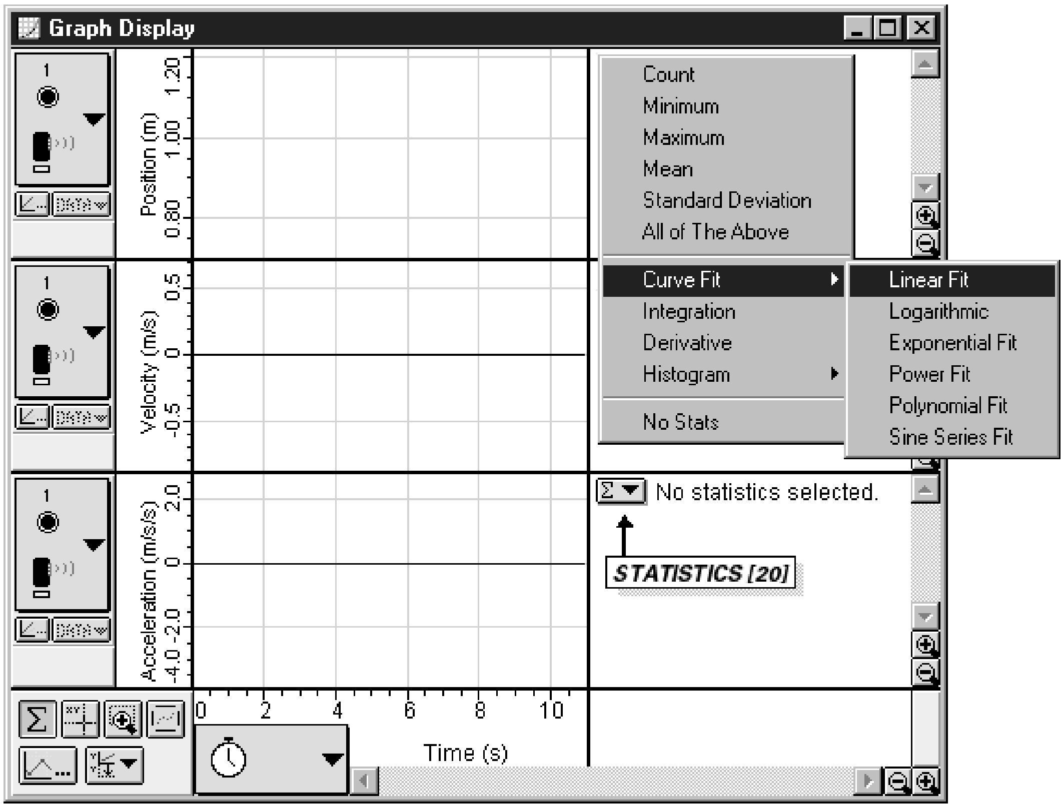


Figure 4: The Statistics Window and Menu