Physics 201/207 Lab Manual

Mechanics, Heat, Sound/Waves

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July 20, 2009

NOTE: M=Mechanics, H=Heat, S=Sound/Waves, C=Computerized Experiment

Contents

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Forward

Spring, 2005

This version is only modestly changed from the previous versions. We are gradually revising the manual to improve the clarity and interest of the activities. In particular the dynamic nature of web materials and the change of venue (from Sterling to Chamberlin Hall) has required a number of cosmetic and operational changes. In particular the PASCO computer interface and software have been upgraded from Scientific Workshop to DataStudio.

M.J. Winokur

In reference to the 1997 edition

Much has changed since the implementation of the first edition and a major overhaul was very much in need. In particular, the rapid introduction of the computer into the educational arena has drastically and irreversibly changed the way in which information is acquired, analyzed and recorded. To reflect these changes in the introductory laboratory we have endeavored to create a educational tool which utilizes this technology; hopefully while enhancing the learning process and the understanding of physics principles. Thus, when fully deployed, this new edition will be available not only in hard copy but also as a fully integrated web document so that the manual itself has become an interactive tool in the laboratory environment.

As always we are indebted to the hard work and efforts by Joe Sylvester to maintain the labortory equipment in excellent working condition.

M.J. Winokur
M. Thompson

From the original edition

The experiments in this manual evolved from many years of use at the University of Wisconsin. Past manuals have included "cookbooks" with directions so complete and detailed that you can perform an experiment without knowing what you are doing or why, and manuals in which theory is so complete that no reference to text or lecture was necessary.

This manual avoids the"cookbook" approach and assumes that correlation between lecture and lab is sufficiently close that explanations (and theory) can be brief: in many cases merely a list of suggestions and precautions. Generally you will need at least an elementary understanding of the material in order to perform the experiment expeditiously and well. We hope that by the time you have completed an experiment, your understanding will have deepened in a manner not achievable by reading books or by working "paper problems". If the lab should get ahead of the lecture, please read the pertinent material, as recommended by the instructor, before doing the experiment.

The manual does not describe equipment in detail. We find it more efficient to have the apparatus out on a table and take a few minutes at the start to name the pieces and give suggestions for use. Also in this way changes in equipment, (sometimes necessary), need not cause confusion.

Many faculty members have contributed to this manual. Professors Barschall, Blanchard, Camerini, Erwin, Haeberli, Miller, Olsson, Visiting Professor Wickliffe and former Professor Moran have been especially helpful. However, any deficiencies or errors are our responsibility. We welcome suggestions for improvements.

Our lab support staff, Joe Sylvester and Harley Nelson (now retired), have made important contributions not only in maintaining the equipment in good working order, but also in improving the mechanical and aesthetic design of the apparatus.

Likewise our electronic support staff not only maintain the electronic equipment, but also have contributed excellent original circuits and component design for many of the experiments.

R. Rollefson
H. T. Richards

Introduction

General Instructions and Helpful Hints

Goals

Physics 201/202 and 207/208 are introductory calculus-based physics courses which introduce the undergraduate student to a broad spectrum of fundamental physical laws spanning from mechanics to heat and thermodynamics to electricity and magnetism to waves and light. To help develop a meaningful understanding of these physics principles the beginning student is presented with a variety of resources: textbooks, lectures and demonstrations, problem solving, discussion sessions and the laboratory.

Of these, the laboratory component furnishes a unique opportunity for demonstrating physical principles in both a qualitative and quantitative hands-on fashion. An inseparable aspect of this laboratory experience should be the realization that physics is, first or foremost, an experimental science in which the limitations of the instrumentation and the technique of the experimenter can heavily impact the scientific process. Hence this laboratory experience is intended to provide the student with a diverse set of experiences including: a realistic feeling for the origin and limitations of physical concepts; an awareness of experimental errors, of ways to minimize them and how to estimate the reliability of the result in an experiment; an appreciation of the need for keeping clear and accurate records of experimental investigations.

Throughout this laboratory experience there is one crucial step for achieving these stated goals in an enduring way: Simply put, a clearly written laboratory notebook in which each of the aforementioned components is documented and recorded. This lab notebook, at a minimum, should contain the following:

1. *Heading of the Experiment:* Copy from the manual the number and nameof the experiment.
   Include both the current date and the name(s) of your partner(s).

2. *Original data:* Original data must always be recorded directly into your notebook as they are gathered. "Original data" are the actual readings you have taken. For example if you know that each in a series of distance measurements is in error by a constant offset of 0.006 mm, then you should record the actual readings (containing this error) and then, afterwards, either correct each data point or the average. In this way it will always be clear that you have made appropriate corrections. Also, when you take 5 or 6 successive readings of a measurement, record each reading, not just the average. From the scatter of the readings, you can estimate the precision of the measurement. Both partners should record data, so that errors of recording show up. (Complete trackability, say if you were producing a part for the space shuttle, would require that you record serial numbers of equipment. You could then find the same equipment to check results later.) Arrange data in tabular form when appropriate, and properly label each item or table.

3. *Housekeeping deletions:* You may think that a notebook combining all work would soon become quite a mess and have a proliferation of erroneous and superseded material. Indeed it might, but you can improve matters greatly with a little housekeeping work every hour or so. Just draw a box around any erroneous or unnecessary material and hatch three or four parallel diagonal lines across this box. (This way

you can come back and rescue the deleted calculations later if you should discover that the first idea was right after all. It occasionally happens.) Append a note to the margin of box explaining to yourself what was wrong.

We expect you to keep up your notes as you go along. Don't take your notebook home to "write it up" – you probably have more important things to do than making a beautiful notebook. (Instructors may permit occasional exceptions if they are satisfied that you have a good enough reason.)

4. *Remarks and sketches:* Avoid, when possible, "pictorial" sketches of apparatus. On the other hand, a simple diagrammatic sketch is useful and is sometimes the simplest and clearest way to define the various quantities indicated in a table of data; *a phrase or sentence introducing each table or calculation is essential for making sense out of the notebook record.* When a useful result occurs at any stage, describe it with at least a word or phrase.

5. *Graphs:* There are three appropriate methods:

   A. Affix furnished graph paper in your notebook with transparent tape.
   B. Affix a computer generated graph paper in your notebook with transparent tape.
   C. Mark out and plot a simple graph directly in your notebook.

   Show points as dots, circles, or crosses, i.e., $\cdot$, $\circ$, or $\times$. Instead of connecting points by straight lines, draw a smooth curve which may actually miss most of the points but which shows the functional relationship between the plotted quantities. Fasten directly into the notebook any original data in graphic form (such as the spark tapes of Experiment M4).

6. *Units, coordinate labels:* Physical quantities *always* require a number and a dimensional unit to have meaning. Likewise, graphs have *abscissas* and *ordinates* which always need labeling.

7. *Final data, results and conclusions:* At the end of an experiment some written comments and a neat summary of data and results will make your notebook more meaningful to both you and your instructor. Note that perfect results are not essential when making a quantitative measurement. "Good" results occur when your value agrees, within appropriate limits of error, with the expected result. "Bad" results occur if the measured value falls outside the range given by uncertainty. This latter result may be perfectly acceptable if a satisfactory explanations (i.e., a legitimate error) for the failure can be forwarded. In fact, most people seem to learn more from their failures rather than their successes.

Expt. M1 Systematic and Random Errors, Significant Figures,
Density of a Solid

NAME: Jane.Q. Student Date: 2/29/00

Partner: John Q. Student

Purpose: To develop a basic understanding of systematic and random errors in a physical measurement by obtaining the density of metal cylinder.

Equiment: Venier caliper, micrometer, precision gauge block, precision balance

Theory: ρ= mass/($\pi *$ r**2 h)

$$\Delta\rho = \sqrt{(\Delta\ m\ /m)**2.+(\Delta\, h/h)**2.+(\ 2\,\Delta\, r/r)**2.}\quad \rho$$

DATA:

1. Calibration of micrometer

Reading with jaws fully closed:

1. 0.000013 mm $\pm$ 0.000001 mm
2. 0.000014 mm
3. 0.000012 mm
4. 0.000014 mm
5. 0.000015 mm

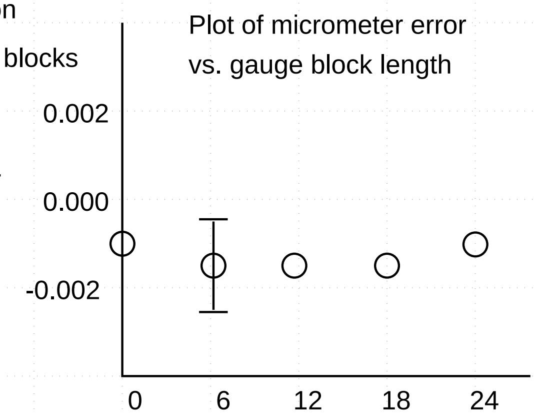
Micrometer exhibits a systematic zero offset

Ave, $\pm$ Standard Deviation

Measure four calibraton gauge blocks

Plot of micrometer error vs. gauge block length

Micrometer error (mm)



Gauge block length (mm)

Measure of cylinder diameter:

Measure of cylinder height:

Measure of cylinder mass

CALCULATIONS:

Density= ??? Uncertainty from propagation of error.

RESULTS and CONCLUSIONS:

PARTNERS

Limitations of space and equipment usually require that one works with a partner. In addition, discussing your work with someone as you go along is often stimulating and of educational value.

Independent calculations; checks: If possible both partners should perform completely independent calculations. Mistakes in calculation are inevitable, and the more complete the independence of the two calculations, the better is the check against these mistakes. Poor results on experiments sometimes arise from computational errors.

CHOICE OF NOTEBOOK

We recommend a large bound or spiral notebook with paper of good enough quality to stand occasional erasures (needed most commonly in improving pencil sketches or graphs). To correct a wrong number always cross it out instead of erasing: thus ~~3.1461~~ 3.1416 since occasionally the correction turns out to be a mistake, and the original number was right. Coarse (1/4 inch) cross-ruled pages are more versatile than blank or line pages. They are useful for tables, crude graphs and sketches while still providing the horizontal lines needed for plain writing. Put everything that you commit to paper right into your notebook. Avoid scribbling notes on loose paper; such scraps often get lost. A good plan is to write initially only on the right-hand pages, leaving the left page for afterthoughts and for the kind of exploratory calculations that you might do on scratch paper.

COMPLETION OF WORK

Plan your work so that you can complete calculations, graphing and miscellaneous discussions before you leave the laboratory. Your instructor will check each completed lab report and will usually write down some comments, suggestions or questions in your notebook.

Your instructor can help deepen your understanding and "feel" for the subject. Feel free to talk over your work with him or her.

Using the Computers: Printing You will want to avoid printing two copies in rapid succession. Wait for your computer to finish "spooling" before sending your next print job, or you risk crashing your computer and thereby loosing all your data.

Errors and Uncertainties

Reliability estimates of measurements greatly enhance their value. Thus, saying that the average diameter of a cylinder is 10.00 ± 0.02 *mm* tells much more than the statement that the cylinder is a centimeter in diameter.

To physicists the term "**error**" is interchangeable with "**uncertainty**" and does **not** have the same meaning as "**mistake**". Mistakes, such as "errors" in calculations, should be corrected before estimating the experimental error. In estimating the reliability of a single quantity (such as the diameter of a cylinder) we recognize several different kinds and sources of error:

FIRST, are **actual variations** of the quantity being measured, e.g. the diameter of a cylinder may actually be different in different places. You must then specify where the measurement was made; or if one wants the diameter in order to calculate the volume, first find the average diameter by means of a number of measurements at carefully selected places. Then the scatter of the measurements will give a first estimate of the reliability of the average diameter.

SECOND, the micrometer *caliper used may itself* be in error. The errors thus introduced will of course not lie equally on both sides of the true value so that averaging a large number of readings is no help. To eliminate (or at least reduce) such errors, we calibrate the measuring instrument: in the case of the micrometer caliper by taking the zero error (the reading when the jaws are closed) and the readings on selected precision gauges of dimensions approximately equal to those of the cylinder to be measured. We call such errors **systematic**, and these cause errors on accuracy.

THIRD, Another type of **systematic** error can occur in the measurement of a cylinder: The micrometer will always measure the largest diameter between its jaws; hence if there are small bumps or depressions on the cylinder, the average of a large number of measurements will not give the true average diameter but a quantity somewhat larger. (This error can of course be reduced by making the jaws of the caliper smaller in cross section.)

FINALLY, if one measures something of definite size with a calibrated instrument, errors of measurement still exist which (one hopes) are as often **positive** as **negative** and hence will **average out** in a large number of trials. Such errors are called random, and result in less precision. For example, the reading of the micrometer caliper may vary because one can't close it with the same force every time. Also the observer's estimate of the fraction of the smallest division varies from trial to trial. Hence the average of a number of these measurements should be closer to the true value than any one measurement. Also the **deviations** of the individual measurements from the **average** give an indication of the **reliability** of that average value. The typical value of this deviation is a measure of the precision. This average deviation has to be calculated from the absolute values of the deviations, since otherwise the fact that there are both positive and negative deviations means that they will cancel. If one finds the average of the absolute values of the deviations, this **"average deviation from the mean"** may serve as a measure of reliability. For example, let column 1 represent 10 readings of the diameter of a cylinder taken at one place so that variations in the cylinder do not come into consideration, then column 2 gives the magnitude (absolute) of each reading's deviation from the mean.

Measurements	Deviation from Ave.	
9.943 mm	0.000	
9.942	0.001	
9.944	0.001	
9.941	0.002	
9.943	0.000	
9.943	0.000	
9.945	0.002	Diameter =
9.943	0.000	
9.941	0.002	9.943±0.001 mm
9.942	0.001	
Ave = 9.943 mm	Ave = 0.0009 mm≈0.001 mm	

Expressed algebraically, the average deviation from the mean is = $(\sum |x_i - \bar{x}|)/n)$, where x_i is the ith measurement of n taken, and $\bar{x}$ is the mean or arithmetic average of the readings.

Standard Deviation:

The average deviation shown above is a measure of the spread in a set of measurements. A more easily calculated version of this is the standard deviation σ (or **root mean square deviation**). You calculate σ by evaluating

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2}$$

where $\overline{x}$ is the mean or arithmetical average of the set of n measurements and x_i is the i^{th} measurement.

Because of the square, the standard deviation σ weights large deviations more heavily than the average deviation and thus gives a less optimistic estimate of the reliability. In fact, for subtle reasons involving degrees of freedom, σ is really

$$\sigma = \sqrt{\frac{1}{(n-1)}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

σ tells you the typical deviation from the mean you will find for an individual measurement. *The mean $\bar{x}$ itself should be more reliable.* That is, if you did several sets of n measurements, the typical means from different sets will be closer to each other than the individual measurements within a set. In other words, the uncertainty in the mean should be less than σ. It turns out to reduce like $1/\sqrt{n}$, and is called **the error in the mean** σ_μ:

$$\sigma_\mu = \mathrm{error\,in\,mean} = \frac{\sigma}{\sqrt{\mathrm{n}}} = \frac{1}{\sqrt{\mathrm{n}}}\sqrt{\frac{1}{\mathrm{n}-1}\sum_{\mathrm{i}=1}^{\mathrm{n}}(\mathrm{x_i} - \bar{\mathrm{x}})^2}$$

For an explanation of the $(n-1)$ factor and a clear discussion of erros, see P.R. Bevington and D.K Robinson, *Data Reduction and Error Analysis for the Physical Sciences*, McGraw Hill 1992, p. 11.

If the error distribution is "normal" (i.e. the errors, ϵ have a Gaussian distribution, $e^{-\epsilon^2}$, about zero), then on average 68% of a large number of measurements will lie closer than σ to the true value. While few measurement sets have precisely a "normal" distribution, the main differences tend to be in the tails of the distributions. If the set of trial measurements are generally bell shaped in the central regions, the "normal" approximation generally suffices.

Relative error and percentage error:
Let ϵ be the error in a measurement whose value is a. Then $(\frac{\epsilon}{a})$ is the relative error of the measurement, and 100 $(\frac{\epsilon}{a})\%$ is the percentage error. These terms are useful in laboratory work.

SYSTEMATIC ERRORS IN THE LABORATORY STANDARDS OF LENGTH, TIME AND MASS

For the experiments in this manual these systematic errors are usually negligible compared to other uncertainties. An exception sometimes occurs for the larger masses especially the 100 gram, the 500 gram, and 1 kg masses. Some contain drilled holes into which lead shot and a plug have been added to adjust the mass to within tolerance (typically 1.000 ± 0.003 kg). Occasionally a plug works loose and the calibration lead shot is lost. You can check the assigned mass values by weighing them on the triple beam balances. Report any deviations greater than 0.4% to the instructor.

UNCERTAINTY ESTIMATE FOR A RESULT INVOLVING MEASUREMENTS OF SEVERAL INDEPENDENT QUANTITIES

A.) If the desired result is the *sum* or *difference* of two measurements, the *ABSOLUTE uncertainties ADD*:
Let Δx and Δy be the errors in x and y respectively. For the *sum* we have $z = x + \Delta x + y + \Delta y = x + y + \Delta x + \Delta y$ and the relative error is $\frac{\Delta x + \Delta y}{x+y}$. Since the signs of Δx and Δy can be opposite, adding the absolute values gives a pessimistic estimate of the uncertainty. If errors have a normal or Gaussian distribution and are independent, they combine in quadrature, i.e. the square root of the sum of the squares, i.e.,

$$\Delta z = \sqrt{\Delta x^2 + \Delta y^2}$$

For the *difference* of two measurements we obtain a relative error of $\frac{\Delta x + \Delta y}{x-y}$. which becomes very large if x is nearly equal to y. Hence avoid, if possible, designing an experiment where one measures two large quantities and takes their difference to obtain the desired quantity.

B.) If the desired result involves **multiplying (or dividing)** measured quantities, then the **RELATIVE uncertainty** of the result is the *SUM of the RELATIVE errors* in each of the measured quantities.

Proof:

$$\text{Let} \quad z = \frac{x_1 \; x_2 \; x_3 .....}{y_1 \; y_2 \; y_3 .....} \quad \text{and hence}$$

$$\ln z = \ln x_1 + \ln x_2 + \ln x_3 + ... - \ln y_1 - \ln y_2 - \ln y_3 - .....$$

Then find the differential, $d(\ln z)$:

$$d(\ln z) = \frac{dz}{z} = \frac{dx_1}{x_1} + \frac{dx_2}{x_2} + \frac{dx_3}{x_3} + ... - \frac{dy_1}{y_1} + \frac{dy_2}{y_2} + \frac{dy_3}{y_3} - .....$$

Consider finite differentials, Δz, etc. and note that the most pessimistic case corresponds to adding the absolute value of each term since Δx_i and Δy_i can be of either sign. Thus

$$\frac{\Delta z}{z} = \sum_i (\frac{\Delta x}{x}) + \sum_i (\frac{\Delta y}{y})$$

Again, if the measurement errors are independent and have a Gaussian distribution, the relative errors will add in quadrature:

$$\frac{\Delta z}{z} = \sqrt{\sum_i (\frac{\Delta x}{x})^2 + \sum_i (\frac{\Delta y}{y})^2}$$

C.) **Corollary:** If the desired result is a *POWER* of the measured quantity, the *RELATIVE ERROR* in the result is the relative error in the measured quantity *MULTIPLIED by the POWER:* Thus $z = x^n$ and

$$\frac{\Delta z}{z} = n\frac{\Delta x}{x}.$$

The above results also follow in more general form: Let $R = f(x, y, z)$ be the functional relationship between three measurements and the desired result. If one differentiates R, then

$$dR = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$$

gives the uncertainty in R when the uncertainties dx, dy and dz are known.

For example, consider the density of a solid (Exp. M1). The relation is

$$\rho \ = \ \frac{m}{\pi r^2 L}$$

where m = mass, r = radius, L = length, are the three measured quantities and ρ = density. Hence

$$\frac{\partial \rho}{\partial m} = \frac{1}{\pi r^2 L} \qquad \frac{\partial \rho}{\partial r} = \frac{-2m}{\pi r^3 L} \qquad \frac{\partial \rho}{\partial L} = \frac{-m}{\pi r^2 L^2}$$

and so

$$d\rho = \frac{1}{\pi r^2 L}\ dm + \ \frac{-2m}{\pi r^3 L}\ dr + \ \frac{-m}{\pi r^2 L^2}\ dL.$$

To get the relative error divide by $\rho = m/\pi r^2 L$. The result, if one drops the negative signs, is

$$\frac{d\rho}{\rho} = \frac{dm}{m} + 2\frac{dr}{r} + \frac{dL}{L}$$

and represents a worst possible combination of errors. For small increments:

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + 2\frac{\Delta r}{r} + \frac{\Delta L}{L}$$

and

$$\Delta\rho = \rho\left[\frac{\Delta m}{m} + 2\frac{\Delta r}{r} + \frac{\Delta L}{L}\right]$$

Again if the errors have normal distribution, then

$$\frac{\Delta\rho}{\rho} = \sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(2\frac{\Delta r}{r}\right)^2 + \left(\frac{\Delta L}{L}\right)^2}$$

SIGNIFICANT FIGURES

Suppose you have measured the diameter of a circular disc and wish to compute its area $A = \pi d^2/4 = \pi r^2$. Let the average value of the diameter be 24.326 ± 0.003 mm ; dividing d by 2 to get r we obtain 12.163 ± 0.0015 mm with a relative error $\frac{\Delta r}{r}$ of $\frac{0.0015}{12} = 0.00012$. Squaring r (using a calculator) we have $r^2 = 147.938569$, with a relative error $2\Delta r/r = 0.00024$, or an absolute error in r^2 of $0.00024 \times 147.93\cdots = 0.036 \approx 0.04$.

Thus we can write $r^2 = 147.94 \pm 0.04$, any additional places in r^2 being unreliable. Hence for this example the first five figures are called *significant*.

Now in computing the area $A = \pi r^2$ how many digits of π must be used? A pocket calculator with $\pi = 3.141592654$ gives

$$A = \pi r^2 = \pi \times (147.94 \pm 0.04) = 464.77 \pm 0.11 \text{ mm}^2$$

Note that $\frac{\Delta A}{A} = 2\frac{\Delta r}{r} = 0.00024$. Note also that the same answer results from $\pi = 3.1416$, but that $\pi = 3.142$ gives $A = 464.83 \pm 0.11$ mm^2 which differs from the correct value by 0.06 mm^2, an amount comparable to the estimated uncertainty.

A good rule is to use one more digit in constants than is available in your measurements, and to save one more digit in computations than the number of significant figures in the data. When you use a calculator you usually get many more digits than you need. Therefore **at the end**, be sure to **round off** the final answer to display the correct number of **significant figures.**

SAMPLE QUESTIONS

1. How many significant figures are there in the following number?
   - (a) 976.45
   - (b) 4.000
   - (c) 10
2. Round off each of the following numbers to three significant figures.
   - (a) 4.455
   - (b) 4.6675
   - (c) 2.045
3. A function has the relationship $Z(A, B) = A + B^3$ where A and B are found to have uncertainties of $\pm\Delta A$ and $\pm\Delta B$ respectively. Find ΔZ in term of A, B and the respective uncertainties assuming the errors are uncorrelated.
4. What happens to σ, the standard deviation, as you make more and more measurements? what happens to $\overline{\sigma}$, the standard deviation of the mean?
   - (a) They both remain same
   - (b) They both decrease
   - (c) σ increases and $\overline{\sigma}$ decreases
   - (d) σ approachs a constant and $\overline{\sigma}$ decreases

Suggestions on Form for Lab Notebooks:

NUMBER AND TITLE (e.g. E1. ELECTROSTATICS)

Date performed:____________ Partner:__________________

Subdivisions: If appropriate, name and number each section as in the manual.

DATA:

Label numbers and give units. In a few words, state what quantities you measured. If appropriate, record the data in tabular form. Label the tables and give units.

CALCULATIONS:

State the equations used and present a sample calculation. (Inclusion of the arithmetic is not necessary.)

CONCLUSIONS:

If any important conclusions follow from the experiments, state them and show by a brief statement how they follow. Compare your results with accepted values if the experiment has involved the measurements of a physical constant.

Errors:

Some of your experiments will be qualitative while others will involve quantitative measurements of physical constants. Where it is appropriate, estimate the uncertainty of each measurement used in a calculation and compute the uncertainty of the result. Does your estimate of uncertainty indicate satisfactory agreement between your result and the accepted one (or between your several values if you have several)? Intelligent discussion is welcomed, but don't make this section a burden on you.

Using the Computers: Printing

You will want to avoid printing two copies in rapid succession. Wait for your computer to finish "spooling" before sending your next print job, or you risk crashing your computer and thereby loosing all your data.

MC-1 Errors & Motion

MC-1a Measurement and Error

OBJECTIVES:

The major objectives of this first, short (est. 1 hr) computer lab are to begin to develop an basic understanding of what it means to make an experimental measurement and provide a methodology for assessing random and systematic errors in this measurement process. In addition this lab will also give you a minimal framework in which to introduce you to the PASCO© (page 109) interface hardware and software.

THEORY:

By now you should have had numerous opportunities to become familiar with time and the concept of a time interval. The increment of one second will be used as an intuitive reference point. In this lab you will test your ability to internalize this one second time interval by making and recording a repetitive flicking motion with your finger. By flicking your finger back and forth you will move it though an infrared beam sensor (i.e. the PASCO photogate) and each full cycle (back and forth, approximately 2 seconds) will be simultaneously recorded, plotted and tabulated by the PASCO interface software.

Your goal in this experiment is to assess the size of systematic and random errors in your data set and learn a simple methodology for distinguishing between the two.
SYSTEMATIC ERRORS: These are errors which affect the *accuracy* of a measurement. Typically they are reproducible so that they always affect the data in the same way. For instance if a clock runs slowly you will make a time measurement which is less than the actual reading.
RANDOM ERRORS: These are errors which affect the *precision* of a measurement. A process itself may have a random component (as in radioactive decay) or the measurement technique may introduce noise that causes the readings to fluctuate. If many measurements are made, a statistical analysis will reduce the uncertainty from random errors by averaging.

APPARATUS:

⇒ Computer with monitor, keyboard and mouse.

⇒ A PASCO photogate and stand: This device emits a narrow infrared beam in the gap and occluding the beam prevents it from reaching a photodetector. When the beam is interrupted the red LED should become lit. (Plugged into DIGITAL CHANNEL #1.)

⇒ A PASCO Signal Interface (CI-700 or CI-750) monitors the photodetector output *vs* time and can be configured to tabulate, plot and analyze this data.

PROCEDURE:

To configure the experiment you should refer to Fig. 1 below. Adjust the photogate so that one member can easily and repetitively flick his/her finger through the gap. The phone-jack cable from the photogate should be plugged into the DIGITAL CHANNEL #1 socket. Ignore the other sensors which may already be plugged into other sockets. It is *important* that the PASCO interface be turned on before the computer. If not the computer will not recognize it and, therefore, it must be rebooted to properly communicate with the PASCO module.

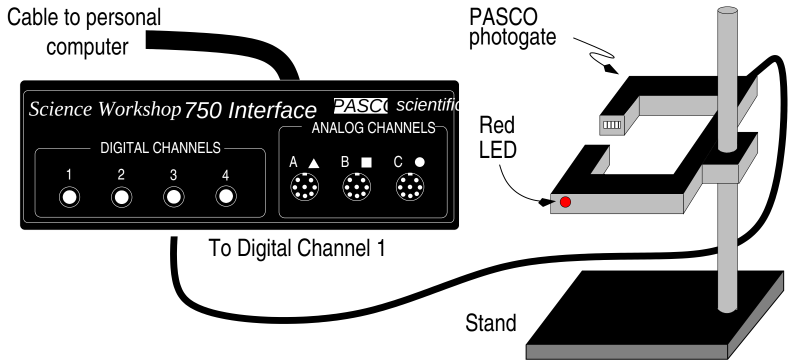


Figure 1: A schematic of the MC1a components and layout.

To initiate the PASCO (page 109) interface software you will need to click the computer mouse on the telescope icon in the "toolkit" area below (web version). Fig. 2 below gives a good idea of how the display should appear. Note that, while you are able to reconfigure the display parameters, the default values that are specified on start-up will allow you to do most of this experiment without necessitating any major changes.

You will note that a "dummy" first data set already exists on start-up showing a typical data run. In the table you can view all 47 data points and the statistical analysis, including mean and standard deviation. In addition there should be a plot of this data and a histogram.

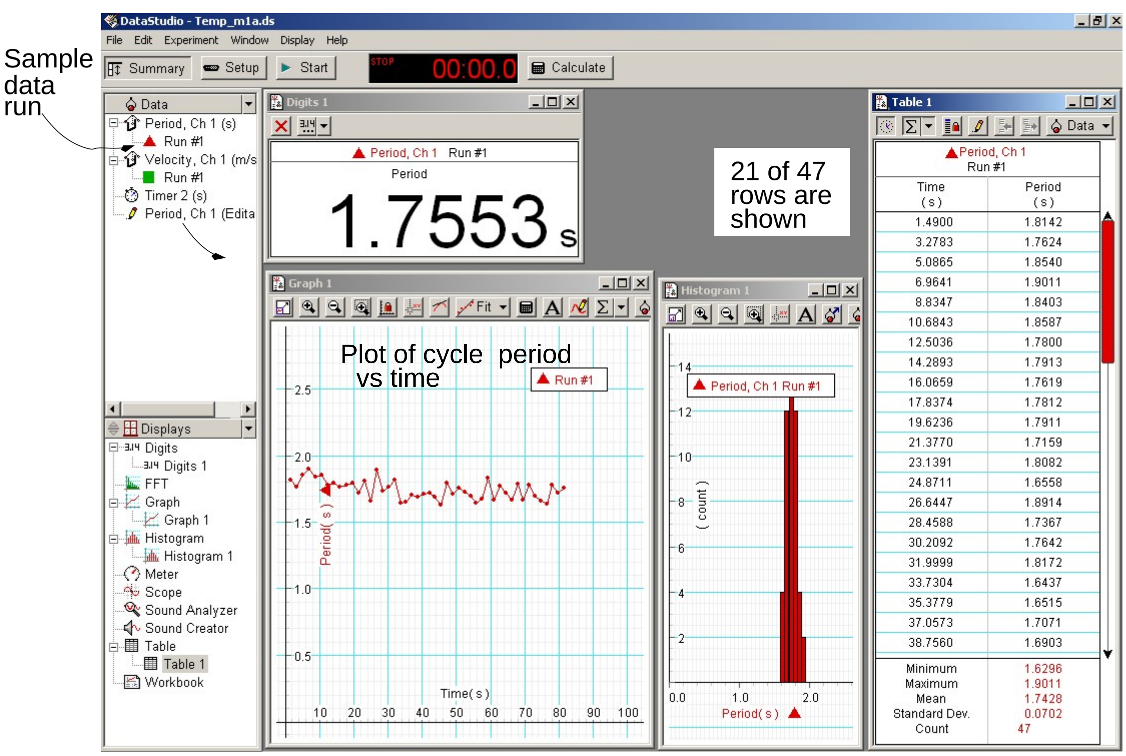


Figure 2: The PASCO Data Studio display window

SUGGESTED PROCEDURE:

1. Start the preliminaries by CLICKing on the  icon and practice "flicking" a finger back and forth so that a two second interval appears in the window. CLICK on the **Stop** icon when done. The same person need not perform both operations. This will produce a second data set. (There is also a "monitor" function which can permit adjustments and trials without storing the results in memory. To access this type ALT-M).

2. Each run gets its own data set in the "Data" display window. (If there are any data sets in existence you will *not* be able to reconfigure the interface parameters or sensor inputs.) A data set can be deleted by moving the mouse cursor to the "Run # 1" position, CLICKing the left mouse button and then striking the "Delete" key.

3. Once you are comfortable with the procedure then click on the  icon and cycle a finger back and forth over fifty times. The click on the "Stop" icon. DO NOT watch the time display while you do this, since you want to find out how accurately and precisely you can reproduce a time interval of 2 s using only your mind.

4. What is the mean time per cycle? What is the standard deviation? The mean $\overline{t}$ and standard deviation σ are given by:

$$\overline{t} = \sum_{i=1}^{N} t_i/N \quad \text{and} \quad \sigma = \sqrt{\sum_{i=1}^{N} (t_i - \overline{t})^2/[N-1]} \quad .$$

   Questions to consider:
   I. Is your mean suggestive of a systematic error?
   II. Does your data qualitatively give the appearance of a normal distribution (i.e. a Gaussian bell curve.)

5. For analyzing and quantifying random errors, you need to asses how a data set is distributed about the mean. The standard deviation σ is one common calculation that does this. In the case of a normal distribution approximately 68% of the data points fall within $\pm 1\sigma$ of the mean (90% within $\pm 2\sigma$). Is your data consistent with this attribute?

6. Assessing the possibility of systematic behavior is somewhat more subtle. In general σ is a measure of how much a *single* measurement fluctuates from the mean. In this run you have made fifty presumed identical measurements. A better estimate of how well you have really determined the mean is to calculate the *standard deviation of the mean* $\overline{\sigma} = \sigma/\sqrt{N}$. After recording $\overline{\sigma}$ in you lab book, can you now observe any evidence that there is a systemic error in your data? Answer this same question with respect to the first "sample" data set.

7. OPTIONAL: Systematic errors can sometimes drift over time. In the best-case scenario they drift up and down so that they hopefully average out to zero. (Clearly it would be better if they could be eliminated entirely.) With respect to $\overline{t}$ and $\overline{\sigma}$ for the first 25 and second 25 cycles do you observe any systematic trends? Use the "Zoom Select" feature on the Graph. Simply by draging the mouse will highlight a subset of the data. The mean, σ and other statistical attributes will appear at the bottom of the table.

MC-1b Errors and the Density of a Solid

OBJECTIVES:

To learn about systematic and random errors; to understand significant figures; to estimate the reliability of one's measurements; and to calculate the reliability of the final result.

NOTE: This experiment illustrates the earlier sections on Errors and Significant Figures. The actual density of the metal is incidental. However, the accuracy of your estimate of reliability will show whether you have mastered the material in the earlier sections.

APPARATUS:

Metal cylinders of varying sizes, micrometer and vernier calipers, precision gauge blocks, precision balance.

Precautions:

Avoid dropping or deforming in any other way the metal cylinders. Avoid damage to the precision screw of the micrometer by turning only the friction head to open or close the caliper jaws. Be sure to disengage the caliper lock before using. (The caliper lock lets you preserve a reading.) Improper weighing procedures may damage the precision balance. Consult your instructor if in doubt. In handling the gauge blocks avoid touching the polished surfaces since body acids are corrosive.

INTRODUCTION:

First read the material on Errors and Significant Figures in the Manual Introduction). Since density is the mass per unit volume, you must measure the mass (on a balance) and compute the volume ($h\pi r^2 = h\pi d^2/4$) from measurements of the cylinder's dimensions where d is the diameter and h is the height. Any one of three length measuring devices may be used. These include a micrometer, a vernier caliper and/or a simple metric ruler. The micrometer will permit the highest precision measurements but using one can be cumbersome, especially when reading the vernier scale. All methods will demonstrate the aforementioned objectives. Your instructor will give you guidance in choosing an appropriate measurement device.

THE MICROMETER:

Record the serial number of your micrometer. Then familiarize yourself with the operation of the caliper and the reading of the scales: **work through Appendix** A. on the Micrometer. Note that use of the "friction" head in closing the jaws insures the same pressure on the measured object each time. Always estimate tenths of the smallest division. Some micrometers have verniers to assist the estimation.

THE VERNIER CALIPER:

Work through Appendix A. on the Vernier Caliper. You may also wish to try this java applet vernier at http://webphysics.davidson.edu/. Experiment with one of the large verniers in the lab until you are sure you understand it. Note that verniers need not be decimal: for many inch scales the vernier estimates 1/8's of the 1/16 inch division, i.e. 1/128's of an inch. However vernier calipers divide the inch into 50 divisions and the vernier estimates 1/25 of the 1/50 inch divisions, i.e. 1/1000 inch or 1 mil. The vernier was invented 1631 by Pierre Vernier.

Precautions on use of the calipers:

1. Unclamp **both** top thumbscrews to permit moving caliper jaws.
2. Open caliper to within a few mm of the dimension being measured.
3. Close right thumbscrew to lock position of lower horizontal knurled cylinder which executes fine motions of caliper jaw. **Never over tighten!**

CALIBRATION OF THE MICROMETER (or VERNIER, ETC.):

1. OPTIONAL: Wipe the micrometer caliper jaws with cleaning paper. Then determine the zero error by closing the jaws. Make and record five readings. The variation of these repeated readings gives you an estimate of the reproducibility of the measurements. (For those using the micrometers they have been given a small zero error. Thus a zero error correction is necessary.) In general any measurement device can have a zero error.
2. Measure all four calibration gauge blocks (6, 12, 18 and 24 mm): Set the gauge blocks on end, well-in from the edge of the table, and thus freeing both hands to handle the caliper. Record the actual (uncorrected) reading. A single measurement of each block will suffice.
3. Plot a correction curve for your micrometer, i.e. plot errors as ordinates and nominal blocks sizes (0, 6, 12, 18, and 24 mm) as abscissa. Normally the correction will not vary from block to block by more than 0.003 mm (for the micrometer). If it is larger, consult your instructor.

DENSITY DETERMINATION:

1. Make five measurements (should be in millimeters) of the height and five of the diameter. Since our object is to determine the volume of the cylinder, distribute your measurements so as to get an appropriate average length and average diameter. Avoid any small projections which would result in a misleading measurement. If not possible to avoid, estimate their importance to the result. Record actual readings and indicate, in your lab book, how you distributed them.
2. Calculate the average length, average diameters and the respective standard deviation.
3. Use your correction curve to correct these average readings. If you were to use the uncorrected values, how much relative error would this introduce?
4. Weigh the cylinder twice on the electronic balance; estimate to 0.1 mg.
5. From the average dimensions and the mass, calculate the volume and density. Make a quantitative (refer to the Error and Uncertainties section on page 12) estimate of the uncertainty in the density. The sample worksheet asks for both the maximum and minimum values. You should use, as your starting point:

$$\text{Density}(\rho \pm \Delta\rho) = \rho(h \pm \Delta h, d \pm \Delta d, m \pm \Delta m) = \frac{m}{\pi(d/2)^2\ h}$$

6. Compare the density with the tabulated value. Tabulated values are averages over samples whose densities vary slightly depending upon how the material was cast and worked; also on impurity concentrations.

7. In your notebook or lab form summarize the data and results. Also record your result on the blackboard. Is the distribution of blackboard values reasonable, i.e. "normal" distribution (refer to the section on page 10)?

   To test how accurately you can estimate a fraction of a division, estimate the fractions on the vernier caliper before reading the vernier. Record both your estimate and the vernier reading.

Related facts and URL links:

Question: Why are there are *exactly* 25.4 mm in 1 inch?

Answer(not verified): The Treaty of the Meter (Convention du Metre) in the late 19th century established the first centralized international system of metrology. This defined the meter.

In 1959, the countries of the world that were using Imperial units defined them uniformly based on the metric units. The inch was simply defined that way and agreed to by all. Before 1959, different countries related inches to meters in other ways. Among them was the United States. The Metric Act of 1866 defined the meter in terms of inches (i.e., before the Treaty of the Meter), and that relationship had continued to been used even after the Treaty fixed the length of the meter.

Changing the definition in the U.S. in 1959 caused very little problem, except for the U.S. Geological Survey. When you deal with things 10^5 meters big (like the sizes of the states), even 1 part in 10^5 changes affect the specifications of boundary lines in significant ways. So, to this day the U.S. has two different systems of inches, feet, and yards (in the ratio of 36:3:1, for both). There's the usual inch, foot, and yard; and there's the survey foot (and inch and yard), which is based on the pre-1959 definition. (The ambiguity goes away, of course, when metric specification is used. Newer USGS maps are metric.)

Links to metrology site(s):
National Institute of Science & Technology at http://www.nist.gov/

Experiment 1b Worksheet

1.

CALIBRATION TABLE			
Micrometer Seriel # =	______		
MEASUREMENTS			
Zero gap 1 =	______	±	______
2 =	______		
3 =	______		
4 =	______		
5 =	______		
Mean$\pm\sigma$=	______	±	______
Gauge block 6 mm	______		
12 mm	______		
18 mm	______		
24 mm	______		

2. Make a plot of Gauge block thickness (x-axis) versus Micrometer reading.
   Next fit the data to a line; this generates a correction curve where
   Corrected value = actual × slope + intercept
   slope = ______
   intercept = ______

3. Now for the unknown value

CYLINDER	Height (± ____)	Width (± ____)	Mass (± ____)
1=	______	______	______
2=	______	______	______
3=	______	______	______
4=	______	______	______
5=	______	______	______
Mean$\pm\sigma$ =	____ ± ____	____ ± ____	____ ± ____
Corrected Mean$\pm\sigma$=	____ ± ____	____ ± ____	____ ± ____
Density =	______		
Max. Density =	______		
Min. Density =	______		

4. Final result, density = ____ ± ____

5. Cut and tape this into you lab notebook.

6. Answer the following questions in your notebook.
   A. Identify two sources of systematic error and give their magnitude.
   B. Identify two examples of random error and give their magnitude.

MC-1c Motion, Velocity and Acceleration

OBJECTIVES:

The major objectives of this exploratory computer lab are two-fold. Since you will be using computer based data-acquisition throughout this course, we expect you to become familiar with the PASCO© (page 109) interface hardware and software. Our second objective is for you to develop an intuition for Newtonian mechanics by experimenting with 1-D motion. With the exception of the first part, calibration of the sonic sensor, there is no extensive write-up in this lab, but only a series of recommended experiments and the requirement to write down your observations in your lab book/form.

THEORY:

The motion of an object is described by indicating its distances x_1 and x_2 from a fixed reference point at two different times t_1 and t_2. From the change in position between these two times one calculates the average *velocity* (remember that direction is implied) for the time interval:

$$\text{average velocity} \equiv \overline{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}\ m/s$$

The *acceleration* of an object is found by finding its velocity v_1 and v_2 at two different times t_1 and t_2. From the change in velocity between two different times one calculates the average acceleration for the time interval:

$$\text{average acceleration} \equiv \overline{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}\ m/s^2$$

FUNDAMENTAL CONCEPTS:

1. The equation that describes the motion of an object that moves with *constant velocity* is: $x = A + B \cdot t$.
   If you make a plot of x versus t, you find that it describes a straight line. The letter A indicates the position of the object at time $t = 0$. The letter B is the slope of the line, and is equal to the velocity of the object. So we can rewrite this $x = x_0 + v \cdot t$.

2. The equation that describes the motion of an object that moves with *constant acceleration* is: $x = x_0 + v_0 \cdot t + \frac{1}{2}at^2$ and $v = v_0 + at$.
   So we can rewrite this as: $x = A + Bt + Ct^2$ and $v = B + 2Ct$. The letter A indicates the position of the object at time $t = 0$. The letter B is the the velocity of the object at time $t = 0$, and is the slope of the graph at this time. The letter C is equal to half the acceleration.

PRECAUTIONS:

In order for the position sensor to work properly it must be pointed in such away that it "sees" the vane, and doesn't identify the front of the cart; that means that it must be pointed slightly upwards. The sonic ranger tends to "see" the closest reflecting surface. In addition the minimum range is approximately 40 *cm*.
Make sure you do not drop the carts or allow them to roll off the table, because it

damages the bearings and they begin to suffer too much friction. Try to arrange to keep the cart on the track all the time.

APPARATUS

- ⇒ Computer with monitor, keyboard and mouse.
- ⇒ A PASCO position sensor; this device emits a series of short pulses of sound, and receives the echo of the sound reflected by a nearby object. The length of the time interval between the emission and the reception of the sound pulse depends on the distance to the reflecting object.
  This method of locating an object is the same as the one used by bats to find flying insects or by navy ships to locate submarines.
- ⇒ A PASCO Signal Interface converts the time interval between the emission and reception of the sound pulse to digital form, i.e., numbers that can be then plotted on the monitor.
- ⇒ PASCO dynamic track with magnetic bumpers; cart with reflecting vane; meter stick; one or two steel blocks.

PROCEDURE:

Your instructor will demonstrate how to configure the experiment. To initiate the PASCO(page 109) interface software you will need to click the computer mouse on the telescope icon in the "toolkit" area below. The Fig. 1 below shows how the display should appear. Note that, while you are able to reconfigure the display parameters, the default values that are specified on start-up will allow you to do this experiment without necessitating any changes. All three measured quantities, position, velocity, and acceleration, are displayed simultaneously. Since velocity is determined from the position data and acceleration from the velocity the "scatter" in the data will become progressively more pronounced.

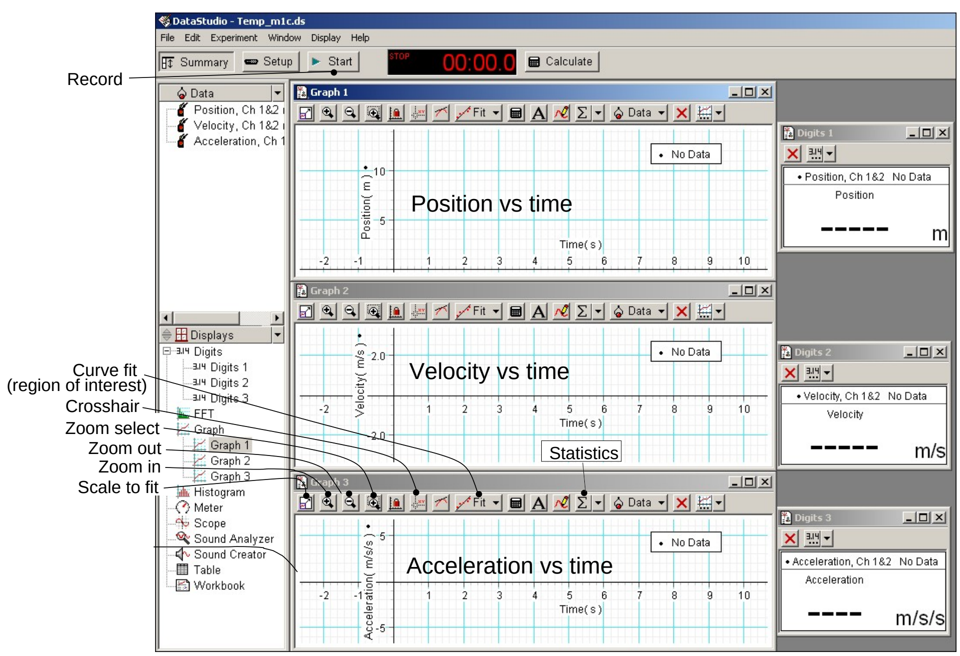


Figure 1: The PASCO Data Studio display format

Experiment I, Basic Operation and Sonic ranger calibration:

1. To start the data acquisition CLICK on the START icon. To stop it CLICK on the same stop which will now become the STOP icon. Each run gets its own data set in the "Data" display window. If there are any data sets in existence you will *not* be able to reconfigure the interface parameters or sensor inputs, unless you clear (delete) all data. The is also a monitor feature that can be accessed on the EXPERIMENT pull down menu (or Alt-M).

2. With the data acquisition started move the cart *to* and *fro* and watch the position, velocity and acceleration displays.

3. STOP the data acquisition.

4. CLICK once (or twice) on the SCALE TO FIT icon. CLICK on the ZOOM SELECT icon and move to one of graph regions and CLICK and DRAG the mouse. CLICK on the CROSSHAIR icon and move about on the various graphs. Change the plot display region be manual adjusting the x-min, x-max, etc. values. To do this double CLICK within a plot window. CLICK on the STATISTICS icon once and then again. *Make* sure that all members of the group have an opportunity to test these components. It will facilitate the rest of the lab course if the basic operations on the software interface are understood by everyone!

5. DELETE the data set by a CLICK on the RUN #1 item in the "Data" window and then striking the "Delete" (<DEL>) key.

6. Start the data acquisition and observe the closest distance to the sonic ranger at which it still functions. This value is supposed to be close to 40 cm. If it is much larger, re-aim the position sensor.

7. Configure the distances so that when the cart nearly touches the near magnetic bumper the sonic ranger still records accurately.

8. Measure the position at two distances approximately 1 m apart and compare the printed centimeter scale with the position sensor readout. By how much do the readings differ?

Experiment II, Inclined Plane and Motion:

1. Raise the side of the track furthest from the position sensor using one of the supplied blocks. Hang the 500 g weight off the other side to weigh the front end down and prevent unwanted slippage of the track (See the demonstration set-up to see how the weight should be positioned.)

2. Find an appropriate release point that allows the cart to roll down the track *without* striking the magnetic bumper.

3. CLICK the START button and release the cart letting it bounce three or four times and the CLICK the STOP button.

4. Qualitatively describe the shape of the three curves: position, velocity and acceleration and discuss how they evolve with time.

5. Obtain a hard copy of this data by simultaneously pressing the **CTRL-p** keys.

6. Label/identify the various key features in the various curves by writing directly on the hard copies. Paste the graphs in your note book.

QUESTIONS: (to be discussed as a group)

1. Does the velocity increase or decrease linearly with time when it is sliding up or down the track?

2. When the velocity is close to zero can you observe any discrepancies in the data? Can you think of a reason for any deviations from linearity?

3. Why does the maximum of the position readout fall with each subsequent bounce?

4. Is the collision with the magnetic bumper or the residual friction in the bearing the most likely source of loss?

5. Can you think of a method using your data to determine which of these two proposed mechanisms is the most likely culprit?

Experiment III, Reproducing Expt. II manually:

1. Remove the block so that the cart is no longer raised.

2. Start a new data set (by clicking the START button) and try to move the cart back and forth at constant velocity using your hand from the side. Alternatively have one team member aim the position sensor at another member holding a book waist-high and walk towards the sensor or away from the sensor.

3. Using the cross hair estimate your velocity in either m/s or cm/s.

4. Repeat step 3. but now try to obtain a region of constant acceleration.

Experiment IV, Acceleration at g (9.8 m/s^2):

1. Have one member of the group stand *carefully* on a chair and hold the position sensor facing downward above another member's head while he or she is holding a notebook on their head.

2. Start a new data set and have the student holding the notebook jump up and down a few times.

3. Stop the recording and determine whether free-fall yields a constant acceleration close to the accepted value.

4. You may wish to try to fit data and measure the acceleration. First use the ZOOM SELECT icon to choose a range of data. The CLICK on the FIT icon to get at the pull down menu. You could use the position data and use a quadratic function or the velocity data and use a linear fit (or even the acceleration curve). Ask you lab instructor for a demonstration if you are unsure.

5. Fini.

M-2 Equilibrium of Forces

OBJECTIVE: To experimentally verify vector addition of forces.

APPARATUS:

Circular horizontal force table with 360° scale, pulleys, weight hangers, slotted masses, protractor, rulers.

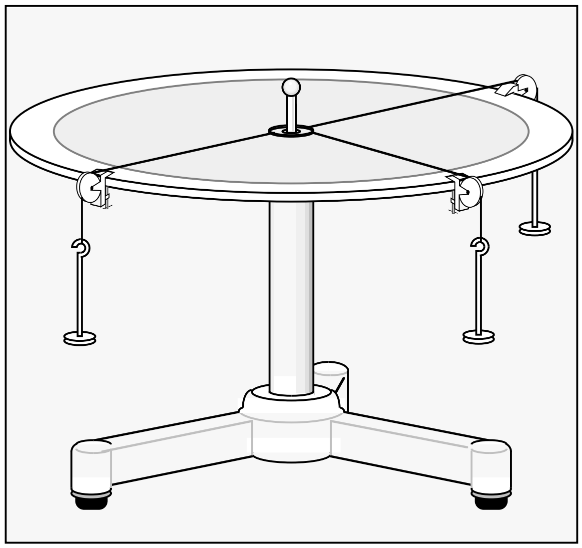


Figure 1: The horizontal force table

PROCEDURE:

Choose two concurrent forces, each between 100 and 500 gram weight and not in the same direction. Add these two vectors to find the resultant force: use both a graphical construction and a numerical computation.

Check your result by setting up the two original forces on the force table, and finding experimentally the single force which will hold them in equilibrium. Random errors are due mainly to friction. You can estimate them by repeating the measurement several times. Are your measured values consistent with the computation and graphical construction?

Repeat the experiment but use three different original forces.

QUESTION: If the weight hangers used all have identical masses, can their weight be neglected?

M-3 Static Forces and Moments

OBJECTIVE: To check experimentally the conditions for equilibrium of a rigid body.

APPARATUS:

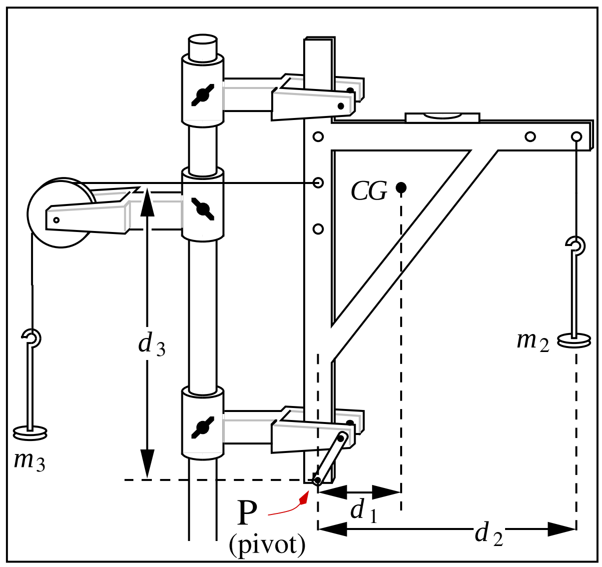
Model of rigid derrick, slotted masses and weight hangers, knife edge (mounted in wall bracket), vernier caliper, single pan balance.

INTRODUCTION:

Equilibrium requires that the net linear acceleration and the net angular acceleration be zero. Hence $\sum \vec{F}_{\text{ext}} = 0$ and $\sum \vec{\tau}_{\text{ext}} = 0$. We treat the rigid derrick as a two dimensional structure so the vector equations become: $\sum \vec{F}_x = 0$, $\sum \vec{F}_y = 0$ and $\sum \vec{\tau} = 0$. The choice of the perpendicular axis about which one computes torques is arbitrary so in part 4 below we choose an axis which simplifies calculations.

SUGGESTED PROCEDURES:

1. Place a load of about 2 kg for m_2.
2. Determine experimentally the force m_3 to hold the derrick in equilibrium with the top member level. Since this is an unstable equilibrium, adjust m_3 so that the derrick will fall either way when displaced equally from equilibrium. To find the uncertainty in m_3 increase or decrease m_3 until you know the smallest force m_3 which is definitely too large and the largest which is definitely too small.



3. Weigh the derrick (use single pan balance) and find the horizontal distance between the center of the stirrup and the vertical line through the center of gravity (c.g.) of the derrick. (Use the knife edge mounted in a wall bracket for locating the c.g.)
4. Calculate how accurately the $\sum \vec{\tau}_{\text{ext}} = 0$ condition is satisfied about the point where the lower stirrup supports the derrick. Note: distance from rotation axis must include stirrup axle radius (use vernier caliper).
5. Calculate the force $\vec{F}$ exerted by the stirrup on the derrick.
6. Choose an axis that is *not* on the line of action of any force and calculate how closely is satisfied about that axis. Is the discrepancy reasonable? Make your answer as quantitative (refer to Section) as you can. (Include the uncertainty in $\vec{F}$ and any other measurements).

M-4 Acceleration in Free Fall

OBJECTIVE: To measure "g", the acceleration of gravity. To observe error propagation.

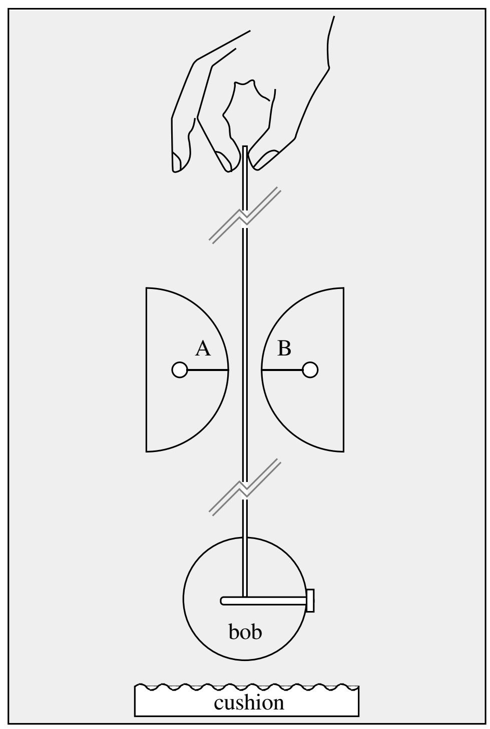
APPARATUS:

Free fall equipment: cylindrical bobs (identical except in mass) which attach to paper tape for recording spark positions; spark timer giving sparks every 1/60 second; cushion; non-streamline bobs to study air resistance (by adding a front plate).

INTRODUCTION:

As shown in figure below the spark timer causes sparks to jump from sharp point A (flush with the convex surface of the plastic insulator) through the falling vertical paper tape to the opposite sharp point B flush with the other insulator surface. As the bob (plus paper tape) undergoes free fall, sparks from A to B mark the paper tape's position every 1/60 second. These data give the bob's acceleration in free fall, g, if air resistance is negligible. Air resistance will be considered later.

EXPERIMENTAL SUGGESTIONS:



1. Position spark timer chassis near table edge and with the convex surfaces (A and B) extended beyond table edge. Put cushion on floor directly under A and B.

2. Select the heaviest bob (no front plate). Insert one end of about a meter of paper tape between the two halves of the cylindrical bob and fasten together with thumb screw.

3. Insert paper tape between A and B. Hold the tape end high enough (vertically) above A and B that the bob just touches below A and B (and is centered). Start the spark and immediately release tape plus bob. Discard any part of tape which fell through the spark gap after bob hit the cushion.

4. Fasten sparked tape to table top with masking tape. Place a meter stick on its side (on top of tape) so that ends of the mm graduations touch the dot track on the tape; this minimizes any parallax error, see Appendix D.

5. **Ignore** the first spark dot; then mark and measure the position of **every other** of the first 24 dots, thus using 1/30 s as the time interval instead of 1/60 s. **Estimate the dot positions to 0.1 mm, and assume this is your uncertainty,** $\delta_r = 0.1\,$**mm. Don't move meter stick between readings!** Tabulate as in sample below.

6. Check the measurement set by remeasuring the tape **after moving the meter stick so that the recorded dot positions will be different**. The differences should be the same. How close are they?

EXAMPLE OF TABULAR FORM FOR DATA:

Spark interval	Real time	Position of every 2nd spark (dot)	Average velocity	Average acceleration
i	$t(i)$	$\overline{r}(i) \pm \Delta r$	$\overline{v}(i) \pm \Delta v$	$\overline{a}(i) \pm \Delta a$
Units	sec	mm or cm	...	...
Uncertainty	$\delta_t =$	$\delta_r =$	$\delta_v =$	$\delta_a =$
0	$t(0)$	a	—	—
1	$t(1)$	b	$b-a$	$(c-b)-(b-a)$
2	$t(2)$	c	$c-b$	$(d-c)-(c-b)$
3	$t(3)$	d	$d-c$	$(e-d)-(d-c)$
4	$t(4)$	e	$e-d$	$(f-e)-(e-d)$
5	$t(5)$	f	$f-e$	$(g-f)-(f-e)$
6	$t(6)$	g	$g-f$	$(h-g)-(g-f)$
7	$t(7)$	h	$h-g$	$(i-h)-(h-g)$
8	$t(8)$	i	$i-h$	$(j-i)-(i-h)$
9	$t(9)$	j	$j-i$	$(k-j)-(j-i)$
10	$t(10)$	k	$k-j$	$(l-k)-(k-j)$
11	$t(11)$	l	$l-k$	$(m-l)-(l-k)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

SUGGESTIONS ON HANDLING DATA:

1. Let $t = 0$ or $1/60$ second represent where your readings start. The actual starting value is arbitrary. Tabulate the actual readings, r, on the meter stick at the end of each time interval (as in column 3 above); calculate the average velocity, $\overline{v}$ in each interval; then calculate the average acceleration, $\overline{a}$ during each interval. If you do the table by hand it may be easier to compute everything in per time interval units and scale your answer when everything is done. OPTION: Alternately you may use the above spreadsheet application link (web version only) to expedite the analysis. Columns B and D through G already have the necessary formulas entered in. Enter in your actual measurements into the correct row of column C and then apply the automated formula option to column E. (Make sure you understand how it works.) Thus use the graph wizard feature to make a graph or use whatever other analysis tools are available.

2. Find the **average** of the **average** $\overline{a}$ in each interval and convert (if necessary). This average of the average $\overline{a}$, $\langle\overline{a}\rangle$" is much more accurate than fluctuations in the $\overline{a}$ column might indicate. Explain why. (Hint: Are the values independent? See last part, Suggestion 4) below.

3. With 1/30 s as the time interval, plot "r" vs time, $\overline{v}$ vs time, and $\overline{a}$ vs time. Note that for $\overline{v}$ the value is the average of v between to adjacent rows, i and $i+1$, and therefore equals the **instantaneous** v at the *middle* of the interval.

4. Use the slope of the velocity curve to find acceleration. Your can do this by hand but graphical analysis tools should be available. This approach is a better way of using all the data than the above numerical one since averaging the $\overline{a}$ column involves summing the $\overline{a}$ column and in such a sum all readings except $(b-a)$ and $(m-l)$ drop out!

ERROR ANALYSIS:

If readings a, b, c, etc., are good to $\approx \frac{1}{2}$ mm because of irregularity in the spark path, the *worst case* would be for $(b-a)$ to be in error by 1 mm. This is in excess of the 0.1 mm measurement uncertainty. Since the two spark position measurements are to first order, uncorrelated, the errors should add in quadrature (i.e., sum of the squares) [see section on Errors, pg. 9]. Using every $v(i)$ for your slope determination is problematic because each $v(i)$ and $v(i+1)$ is correlated. This means that if $v(1) \propto b-a$ then $v(2) \propto c-b$, so that a fluctuation in b will always push the adjacent values of v apart. Thus a linear-fit of slope suffers from the same shortcoming as the average of the average acceleration calculation noted in Suggestion 4! Calculate the slope using only every other $v(i)$ because $v(1)$, $v(3)$, $v(5)$ etc. use independent data. Does your new value for g vary much? Estimate the uncertainty in the slope. Spark timing errors are negligible. Air resistance is a systematic error, small for the streamline bobs at low velocities, but see the optional experiment below.

EQUIVALENCE OF GRAVITATIONAL AND INERTIAL MASS:

Galileo showed (crudely) that the acceleration of falling bodies was independent of the mass. Use the light plastic bob (identical in size and shape to the massive bob) to repeat the free fall experiment and thus check quantitatively this equivalence (when air resistance effects are small enough). If you are planning to do the optional experiment A which follows then just skip this part.

LOCAL VALUE OF g: The UW Geophysics Department determined g accurately for room 4300 Sterling Hall. There you may find a plaque affixed on the northwest window sill that reads $g = 9.803636 + 0.000001$ m/s^2.

QUESTION: Do your value(s) of " g " agree within your assigned errors? (You may simply use the standard deviation of the mean from the four average g values in the right-most table.)

OPTIONAL EXPERIMENTS

A. EFFECT OF AIR RESISTANCE: While air effects are small for streamlined objects at low velocities, they can become large, e.g. on a parachute. To observe and correct for them, first make the bobs **non-streamline** by inserting into the bottom of the bob the banana plug holding a small flat plate. Then repeat the experiment for the **non-streamline bobs (front plate attached), identical except for mass.** There are four possible mass combinations. Since the force of air resistance, $f(v)$, is a function of only velocity if the size, shape and roughness are the same, then $f(v)$ on the two bobs will be almost the same since their velocities are similar. The net force on the falling body is then $F = mg - f(v)$. Hence

$$a = \frac{F}{m} = \frac{mg - f(v)}{m} = g - \frac{f(v)}{m}$$

and

$$\langle a \rangle \approx g - \left(\frac{1}{m}\right) \langle f(v) \rangle \quad .$$

Thus if we measure $\langle a \rangle$ for bobs identical except in mass and plot $\langle a \rangle$ against $1/m$, we should obtain a straight line whose extrapolation to $(1/m) = 0$ should give g. To further test the validity of this hypothesis you should plot the same data as recorded by other lab groups on your plot. If the data permits a simple linear fit you should be able extrapolate to infinite mass (i.e. the y-intercept).

B. MEASUREMENT OF REACTION TIME BY FREE FALL:

(1) With thumb and forefinger grasp a vertical meter stick at the 50 cm mark. Release and grab it again as quickly as possible. From the distance through which the 50 cm mark fell, calculate the time of free fall of the meter stick. This time is your total reaction time involved in releasing and grasping again.

(2) Have your partner hold the vertical meter stick while you place your thumb and forefinger opposite the 50 cm mark but not grasping it. When your partner releases the stick, grab it as soon as possible. Again from the distance through which the 50 cm mark fell, calculate the time of free fall of the meter stick. Compare this time with the other method. Why may these reaction times be different?

OPTIONAL QUESTIONS:

1. Estimate the effect on your g value of the air's buoyant force, F_a for air density, $\rho = 1.2$ kg/m^3 and brass bob density, $\rho_{\mathrm{bob}} = 8700$ kg/m^3.
   Hint: $F_a = \rho_a m_{\mathrm{bob}} g/\rho_{\mathrm{bob}}$ (why?), and $a = g[1 - (\rho_a/\rho_{\mathrm{bob}})]$ (why?).

2. According to universal gravitation, the moon also accelerates the bob with a value a_m of

   $a_m = \frac{GM_m}{r^2} = \frac{6.67\times10^{-11}(7.34\times10^{22})}{(3.84\times10^8)^2} = 0.000033\ m/s^2$

   where G is the constant of universal gravitation, M_m is the mass of the moon, and r is the distance between the moon and the bob. Since this a_m is 33 times the uncertainty quoted for the local g value, why doesn't the plaque also indicate the position of the moon at the time of measurement?

   Hint: Remember the acceleration g in an orbiting earth satellite provides the centripetal acceleration for the circular motion but does not appear as "weight" of an object in the satellite. While to first order the moon and sun effects are negligible, there are detectable tidal effects in the earth ($\approx 10^{-7}\ g$) which one corrects for in the absolute measurements. See Handbuch der Physics, Vol XLVIII, p. 811; also Wollard and Rose, "International Gravity Measurements", UW Geophysical and Polar Research Center, (1963) p. 183. On pages 211 and 236, they also describe the accurate absolute g determination with two quartz pendula in room 70 Science Hall from which the plaque value in Room 4300 Sterling derives.

M-5 Projectile Motion

OBJECTIVE: To find the initial velocity and predict the range of a projectile.
APPARATUS:

Ballistic pendulum with spring gun and plumb bob, projectile, single pan balance, elevation stand.

PART I. BALLISTIC PENDULUM INTRODUCTION:

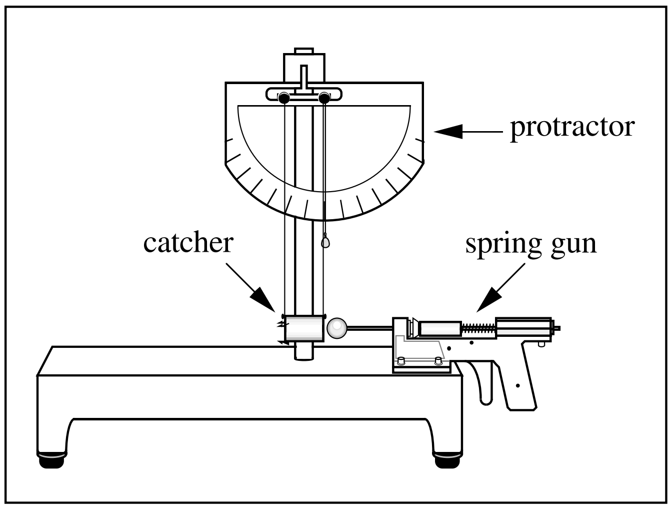


Figure 1: The spring gun.

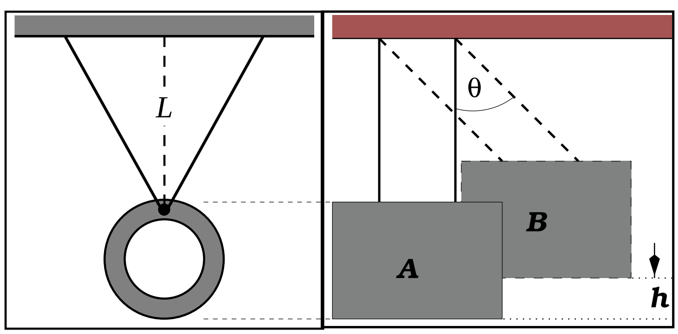


Figure 2: A side view of the catcher

A properly aligned spring gun shoots a ball of mass m into a pendulum catcher of mass M. See Figs. 1 and 2. The pendulum traps the ball; thereafter the two move together in **pure translation**. Since ball and pendulum have no relative motion, the collision is inelastic and thus does not conserve mechanical energy (where does it go?). Of course linear momentum is still conserved, and hence momentum of ball before impact, mu, equals the momentum of ball plus pendulum after impact, $(m + M)V$:

$$mu = (m + M)V \tag{1}$$

where u = ball's velocity before impact and V = initial velocity of combined pendulum plus ball.

To find V, note that although the impact doesn't conserve mechanical energy, the motion **after** impact is almost frictionless and thus conserves mechanical energy. Hence the kinetic energy of the ball plus pendulum at A in Fig. 2, just after impact, equals the potential energy of the two at the top of the swing (at B). Thus

$$\frac{1}{2}(M+m)V^2 = (M+m)gh \ . \qquad \text{Hence,} \qquad V = \sqrt{2gh} \ . \tag{2}$$

(An on-line prelab quiz is available if you wish to test your knowledge.)

ALIGNMENT:

If properly aligned, our bifilar type of suspension for the pendulum (see Figs. 1 and 2) prevents rotation of the bob of mass M. Hence the motion is pure translation. To ensure proper alignment, adjust the three knurled screws on the base so that

A. A plumb bob hangs parallel to the vertical axis of the protractor, and
B. The uncocked gun axis points along the axis of the cylindrical bob. (You may need to adjust the lengths of the supporting strings.)

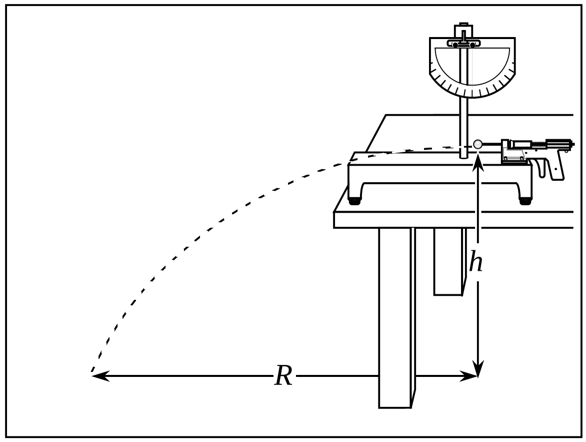
SUGGESTED PROCEDURE:

> *PRECAUTIONS:* Some spring guns have three different cocking stops. For consistent results be sure to recock the gun to the same stop.

1. Find m and M by weighing on the single pan balance.
2. Find the height h by measuring angle θ and the length L of the pendulum. *NOTE:* $h = L - L\cos\theta$ and L is not the length of the string, (see Fig. 2).
   Suggestions on finding θ: Find an approximate θ, and then make a masking tape slit on the back of the protractor at this approximate θ so your eye can locate the proper viewing area to find a more accurate θ on subsequent firings. To avoid parallax (Appendix D.) in reading the protractor choose a line of sight determined by the string in front of the protractor and the string in back of the protractor. Careful observation after a little practice will enable you to get θ to within a degree.
3. Calculate the initial velocity V of the combined pendulum bob and ball.
4. Calculate the initial velocity of the ball, u, as it leaves the gun.
5. Estimate the uncertainty in u. [Hint: Since the largest uncertainty is likely ΔV, then Δh is important. While h is a function of the measured L and θ, the uncertainty in the angle measurement, $\Delta\theta$, will probably dominate. To manually calculate the uncertainty in the resulting trigonometric function, sometimes it is simpler to calculate separately the function for $\theta + \Delta\theta$ and for $\theta - \Delta\theta$. Remember to use absolute, not relative errors when propagating errors through addition (See "Errors and Uncertainties" in the manual.)

PART II. RANGE MEASUREMENTS HORIZONTAL SHOT:

1. After finding u, (the velocity of the ball leaving the gun) predict the impact point on the floor for the ball when shot horizontally from a position on the table.



2. To check your prediction experimentally:
   (i) Use the plumb bob to check that the initial velocity is horizontal.
   (ii) Measure all distances from where the ball starts free fall (not from the cocked position). All measurements of course refer to the bottom of the ball so $x = 0$ corresponds to the radius of the ball beyond the end of the gun rod. Check that the gun's recoil does not change x.
   (iii) Fasten (with masking tape) a piece of computer paper at the calculated point of impact, and just beyond the paper place a box to catch the ball on the first bounce.
   (iv) Record results of several shots. (The ball's impact on the paper leaves a visible mark.) Estimate the uncertainty in the observed range.
   (v) Is the observed range (including uncertainty) within that predicted.

3. Work backwards from the observed range to calculate the initial velocity u. This u is probably more accurate than the value obtained with the ballistic pendulum.

ELEVATED SHOT:

1. Use the stand provided to elevate the gun at an angle above the horizontal. The plumb bob will give the angle of elevation, e.g. 90° - protractor reading. For the elevated gun, be sure to include the additional initial height above the floor of the **uncocked** ball.

2. Before actually trying a shot at an angle, again predict the range but use the value of u which you found from the horizontal shot. (See item 3 above).

3. Make several shots, record the results and compare with predictions.

QUESTION:

From the measured values of u and V in Part I of this experiment, calculate the kinetic energy of the ball before impact, $\frac{1}{2}mu^2$ and the ball and pendulum together after impact, $\frac{1}{2}(m + M)V^2$. What became of the difference?

OPTIONAL:

1. Show that for momentum to be conserved: $\dfrac{KE_{\text{before impact}}}{KE_{\text{after impact}}} = \dfrac{m+M}{m}$

2. Find the spring constant k of the gun from $\frac{1}{2}mu^2 = \frac{1}{2}kx^2$.

M-6 Uniform Circular Motion

OBJECTIVE: To measure the centripetal force, F_c, and compare to

$$F_c = \frac{mv^2}{r} = m\omega^2 r$$

APPARATUS:

Fig. 1 is a schematic of the equipment. The bobs and springs are removable for weighing. Not shown are table clamp and pulley, slotted masses and weight hanger.

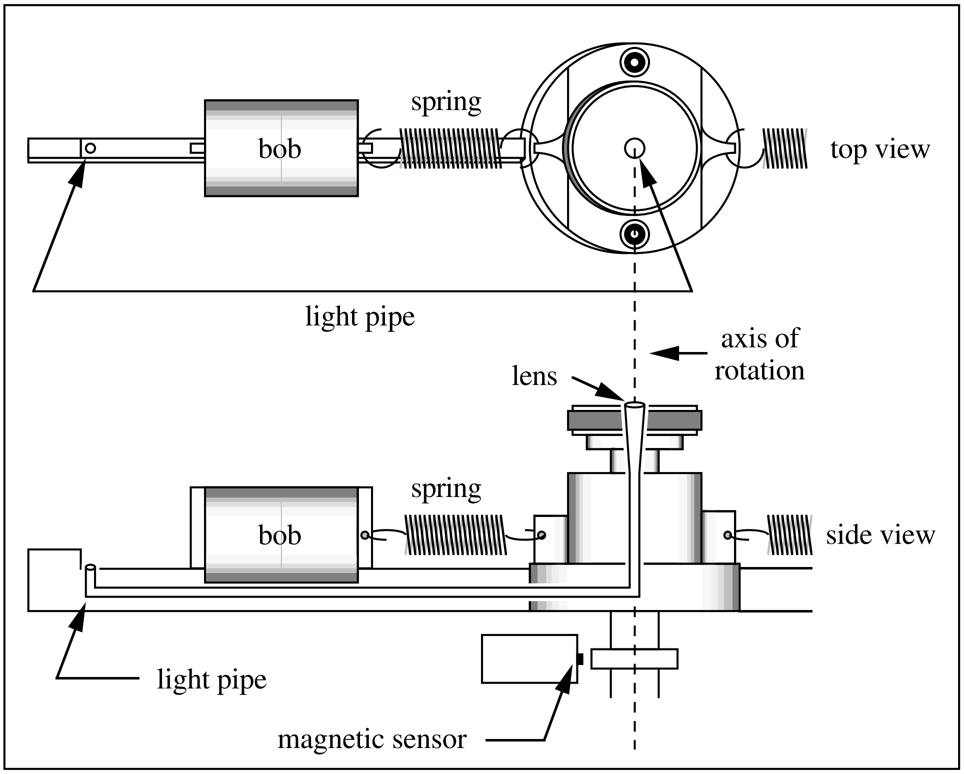


Figure 1: The UCM apparatus.

INTRODUCTION:

A variable speed motor drives the rotating system which has two slotted bobs which slide on a low-friction bar. One adjusts the speed until one bob just covers the optical light pipe and thus reduces the signal seen at the center of the rotating system to zero. A revolutions counter is on the shaft. The counter operates by sensing the rotating magnetic poles and electronically reads out directly the frequency of revolution in rpm. A spring (plus any friction) supplies the centripetal force required to keep the bob traveling in a circle.

If you measure first the frequency of rotation required to make the bob just cover the optical light pipe, and if you then measure the force required to pull the bob out the same distance when the system is not rotating, you can determine F_c and compare your result to

$$F = m\omega^2 r\,,$$

where r is the distance from the axis of rotation to the center of mass of the bob.

SUGGESTIONS:

1. Find the mass of the nickel plated brass bob; also the aluminum bob.

2. **Dynamic measurement of the force**: Attach the brass bob to the spring. Replace the lucite cover, and adjust the motor speed until the light from the light pipe at the center of the rotating system goes to zero. Record the rotation frequency.

   To correct for frictional effects of the bob on the bar, record the frequencies both as the speed is slowly *increased* to the correct value and and as the speed is *decreased* from too high a value. Since the direction of the frictional force reverses for the two cases, the average should eliminate the frictional effect.

   Repeat several times so you can estimate the average and the standard deviation of your values.

3. **Static measurement of the force**: Use the string, pulley and weight holder plus slotted weights to measure the force required to stretch the spring so that the optical light pipe is again just covered. Devise a way to avoid error caused by the friction at the pulley and of the sliding bob on the bar.

4. While the spring is stretched to its proper length (item 3 above), measure the distance r from the axis of rotation to the center of mass of the bob. The center of mass is marked on the bob.

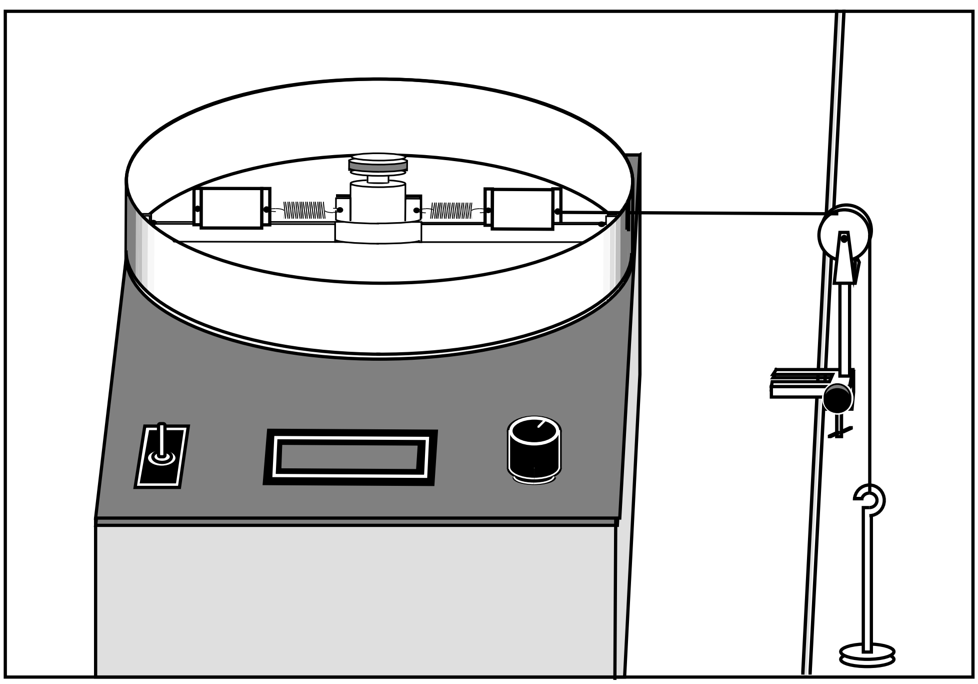


Figure 2: Static measurement of the force using hanging weights

5. Compare the measured centripetal force to the computation $F_c = m\omega^2 r$. In computing the centripetal force, also take into consideration the mass of the spring. One can show (Weinstock, *American Journal of Physics*, 32,p. 370, 1964) that $\approx (1/3)$ of the spring mass should be added to the mass of the bob to obtain the total effective mass.

6. Repeat the above item 1 through item 5 but for the aluminum bob.

QUESTIONS:

1. Estimate the reliability of your measurements. How well do the measured and computed forces agree? Try to account for any discrepancy.

MC-7 Simple Pendulum

OBJECTIVES:

1. To measure how the period of a simple pendulum depends on amplitude.
2. To measure how the pendulum period depends on length if the amplitude is small enough that the variation with amplitude is negligible.
3. To measure the acceleration of gravity.

VIRTUAL PRE-LAB EXPERIMENT:

1. For students wishing to try this experiment on-line, there is a simulation of a pendulum included in the web version of the lab manual...click on the Launch Virtual Pendulum button.
2. Start the pendulum swinging and then let it swing for about 10 periods. Estimate the mean and standard deviation of a single measurement.
3. Perform the required investigations as below except use the virtual pendulum.

APPARATUS:

Basic equipment: Pendulum ball with bifilar support so ball swings in a plane parallel to wall, protractor, infrared photogate and mount, single pan balance.

Computer equipment: Personal computer set to the MC-7 lab manual web-page; PASCO interface module; photogate sensor (plugged into DIGITAL input #1).

NOTE:The period is the time for a complete swing of the pendulum. For the most sensitivity the start and stop of the photogate timer should occur at the bottom of the swing where the velocity is maximum.

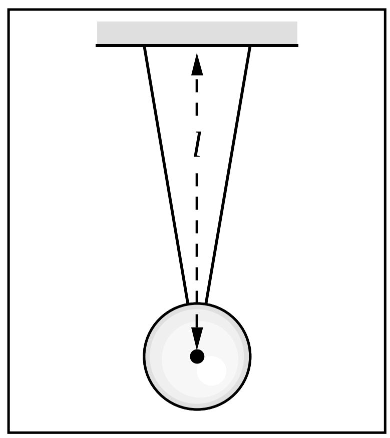


Fig. 1: Side view of pendulum ball and bifilar support. NOTE: L and l are equivalent.

SUGGESTED EXPERIMENTAL TECHNIQUE:

1. Adjust the infrared photogate height so that the bob interrupts the beam at the bottom of the swing. (Make sure the PASCO interface is on and that the phone jack is plugged into the first slot.) There is a red LED on the photogate that will light up when the bob interrupts the beam. Rotating the photogate may help you to intercept the bob at the bottom of the swing (but perfect alignment is not essential).
2. To initiate the PASCO interface software click the computer mouse on the telescope icon in the "toolkit" area. There will be just a single table for recording the measured period.

3. Start the pendulum swinging and then start the data acquisition by clicking the Start icon. Let the bob swing for about 17 periods. Calculate the mean and standard deviation by simply clicking on the table statistics icon (i.e. $\boxed{\Sigma}$). For a single measurement of the period the standard deviation is a reasonable measure of the uncertainty. With 17 measurements the *uncertainty* of the mean is $\sigma/\sqrt{N-1}$ with $N = 17$. (This is the "standard deviation of the mean." Refer to the error section in this manualon page 10).)

REQUIRED INVESTIGATIONS: (Error analysis required only for item 3)

1. Period *vs* Amplitude: For a pendulum of convenient length L (about 0.5 m) determine the dependence of period on angular amplitude. (Do **not** cut string to decrease L; there are clips on the strings to adjust the length.) Use at least amplitudes between 5 and 50 degrees. Measuring the angle is a bit hard; to avoid parallax (see Appendix D. on page 104) effects, position your eye so the two strings are aligned with each other and read the protractor. [Of course, the amplitude of the swing will decrease slowly because of friction. Keep the number of swings that you time small enough that the amplitude changes (because of friction) by well under 5° during the timing. This effect is especially apparent at large amplitudes.] For each group of swings timed, record the average angular amplitude and the standard deviation in a table in your lab book. (Check with your instructor to see if an Excel template document is available. If then the columns will already have a layout including the period versus amplitude formula.)

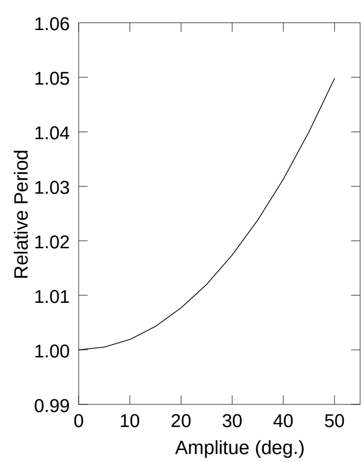
   Plot the measured period as a function of angular amplitude including a few error bars.

   The accurate formula for period as a function of amplitude θ is:

$$T = T_0 \left(1 + \frac{1}{2^2} \sin^2 \frac{\theta}{2} + \frac{1(3^2)}{2^2(4^2)} \sin^4 \frac{\theta}{2} + \cdots \right)$$

   where $T_0 = 2\pi\sqrt{(L/g)}$ and θ is the angular amplitude. The results follow:

θ(deg)	T/T_0	% Change
5	1.0005	0.05
10	1.0019	0.19
15	1.0043	0.43
20	1.0077	0.77
25	1.0120	1.20
30	1.0174	1.74
35	1.0238	2.38
40	1.0313	3.13
45	1.0400	4.00
50	1.0498	4.98



   Compare your plot (above) with values predicted from this table.

2. *Period vs. Length:* For an amplitude small enough that the period is almost independent of amplitude, determine the variation of period with length. (The length

of the pendulum is the vertical distance from the support to the center of the bob.) Try at least five (or more) lengths from 0.10 m to 0.60 m. Remember to correct T to T_0 to account for the offset.

Note: Change lengths by loosening the two spring-loaded clamps above protractor.

I. Plot period (T_0) versus length (L) and extend the curve to $L = 0$. Can you tell at a glance how T_0 depends on L?

II. Plot T_0^2 vs L. What is the shape of this curve? What can you tell from this curve about the dependence of T_0 on L?

3. *Measurement of* **g**: With a pendulum length at its maximum make a measurement of **g**, the acceleration of gravity. (See your text for proof that a simple pendulum swinging through a **small** angle has $T = 2\pi\sqrt{(L/g)}$ where T is the period, L the length and g is the acceleration of gravity.) Take enough measurements to estimate the reliability of your period determination.

4. *Calculate the uncertainty in your determination of* g.

   Note that $\Delta g = g\,\sqrt{(\Delta L/L)^2 + (2\Delta T/T)^2}$. (Take a look at the "Errors and Uncertainties" section of this manual.) Is the accepted value within the limits you have set? If not, can you explain it? (Consider optional items below. The string lineal density is 0.5 g/m, and $M_{\text{bob}} = 62$ g.)

EXERCISE: At right is a sketch of a compound pendulum. There is a bar that you can swing into place which will give half the swing a length $L_1 + L_2$ and the half of the swing a length L_2. The bar should be positioned so the right face just touches the string when the pendulum is at rest and hangs freely. In your lab book first estimate the period of the motion (explain your logic) and then conduct the experiment (stating the steps in your experiment). Is your measured value close to what you expected?

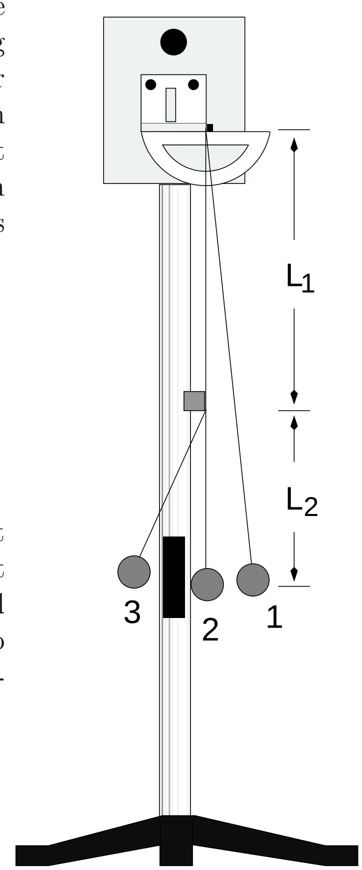


Fig. 2: A compound pendulum.

OPTIONAL CALCULATIONS (these pertain item 4 above):

1. Show that the buoyant force of air increases the period to

$$T = T_0\left(1 + \frac{\rho_{\text{air}}}{2\rho_{\text{bob}}}\right)$$

   where T_0 is the period in vacuum and ρ is the density. Test by swinging simultaneously two pendula of equal length but with bobs of quite different densities: aluminum, lead, and wooden pendulum balls are available. (Air resistance will also increase T a comparable amount. See Birkhoff "Hydrodynamics," p. 155.)

2. The finite mass of the string, m, decreases the period to

$$T = T_0\left(1 - \frac{m}{12M}\right)$$

   where M is the mass of the bob (S.T. Epstein and M.G. Olsson, *American Journal of Physics* 45, 671, 1977). Correct your value of g for the mass of the string.

3. The finite size of a spherical bob with radius r increases the period slightly. When L is the length from support to center of the sphere, then the period becomes (see Tipler "Physics" 2nd ed. p. 346, problem 26):

$$T = T_0\sqrt{1 + \frac{2r^2}{5L^2}} \;\simeq\; T_0\left(1 + \frac{r^2}{5L^2}\right) \ .$$

   What is the resulting percent error in your determination of "g"?

OPTIONAL EXPERIMENT:
In addition to changing the angle or the length of the string there is also a box of differing mass bobs (of approximately the same radius) that can be used to verify the mass portion of the equation and the impact of air resistance/buoyancy.

NOTE: For a comprehensive discussion of pendulum corrections needed for an accurate measurement of g to four significant figures, see R. A. Nelson and M. G. Olsson, *American Journal of Physics* 54, 112, (1986).

MC-8 The Physical Pendulum

Do either PART A or PART B but not both!

PART A

OBJECTIVE:

To measure the rotational inertia of a ring by swinging it as a pendulum from a point of the rim and to compare the value to that computed.

APPARATUS:

Basic equipment: Metal rings, (full, half, and quarter rings); knife edge and 2-point supports.

Computer equipment: Personal computer set to the MC-8 lab manual web-page; PASCO interface module; photogate sensor and extension jack(plugged into DIGITAL input #1).

INTRODUCTION:

The period of a rigid body swinging on an axis as a pendulum is (for small amplitudes $\simeq 5°$):

$$T = 2\pi\sqrt{\frac{I_0}{Mgh}}$$

where

$$\begin{aligned} T &= \text{period} \\ I_0 &= \text{rotational inertia relative to axis about which ring swings} \\ M &= \text{mass of the body} \\ g &= \text{acceleration of gravity} \\ h &= \text{distance from axis to center of mass.} \end{aligned}$$

REQUIRED INVESTIGATIONS:

1. Initiate the PASCO interface software by clicking on the telescope icon below (web version). There will be just a single table for recording the measured period.

2. Measure the period of the ring supported on the knife edge and swinging in its own plane. From the period calculate I_0. Use the parallel axis theorem to also calculate I_c, the rotational inertia about an axis through the c. of m. and perpendicular to plane of the ring. Compare this I_c with the computed value of $I_c = M(r_1^2 + r_2^2)/2$ where the r's are the inner and outer radii of the ring. Do the two I_c's agree within reasonable experimental error? Explain.

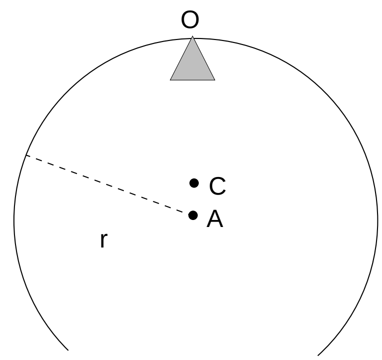
3. Measure the period of the ring when supported on the two sharp points but swinging perpendicular to its plane. From this period calculate the rotational inertia about an axis through those two points. Then use the parallel axis theorem to find the rotational inertia about a diameter of the ring. How does this rotational inertia compare with that about an axis perpendicular to the plane of the ring and through the center of mass?

OPTIONAL PROBLEM: Prove that this relationship should exist by calculating the rotational inertias about the two orthogonal axes.

4. One circular hoop has been cut in sections. Measure the period of a half-hoop when set at its midpoint on a knife edge; also the period of a quarter-hoop. (Be sure hoop sections have the same radius of curvature r).

Proof that any section of a thin hoop has the same period if oscillating in the plane of the hoop:

Let r = radius of the thin hoop
O = axis about which it swings (knife edge)
C = center of mass of the partial hoop
m = mass of the partial hoop
h = OC = distance from c. of m. to axis



Note that $I_A = mr^2$ for any partial hoop.
The parallel axis theorem then gives:

$$I_A = I_C + m(r-h)^2 \qquad \text{or} \qquad I_C = I_A - m(r-h)^2$$

Hence

$$I_C \;=\; mr^2 - mr^2 + 2mrh - mh^2 \;=\; 2mrh - mh^2 \;.$$

When one substitutes this I_C into

$$T = 2\pi\sqrt{I_0/(mgh)} \;=\; 2\pi\sqrt{(I_C + mh^2)/(mgh)}$$

one finds

$$T = 2\pi\sqrt{(2mrh - mh^2 + mh^2)/mgh} \;=\; 2\pi\sqrt{2r/g}$$

or a period independent of what fraction of a hoop is used!

OPTIONAL PROBLEM:

For a thick hoop, the above relationship does not hold exactly. Show that the thickness of the laboratory hoops accounts for the small difference between T for a whole hoop and T for a half hoop. (This is a rather difficult problem. For the older style laboratory hoops the finite thickness increases the period by $\simeq 1.8\%$; the period of the half-hoop will be $\simeq 5.2\%$ larger, and the quarter-hoop over 22% longer!)

PART B of M-8

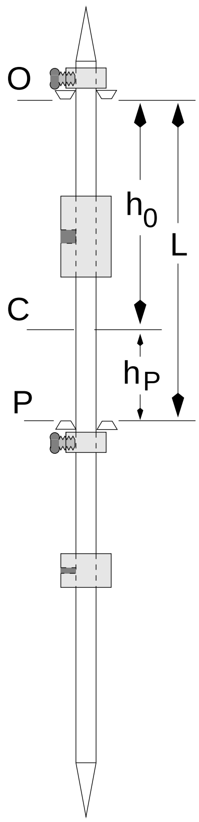
KATER'S REVERSIBLE PENDULUM

OBJECTIVE: To study conjugate centers of oscillation and to measure g accurately.

APPARATUS:

Long rod with movable weights and knife edges; bearing surface (Kater's pendulum mount); infrared photogate & support stand; computer equipment as in PART A.

INTRODUCTION:



When swung from O the period is:

$$T_0 = 2\pi\sqrt{\frac{I_0}{mgh_0}} = 2\pi\sqrt{\frac{I_C + mh_0^2}{mgh_0}}$$

When swung from P: $$T_P = 2\pi\sqrt{\frac{I_P}{mgh_P}} = 2\pi\sqrt{\frac{I_C + mh_P^2}{mgh_P}} .$$

By substitution one easily verifies that $T_0 = T_P$ for two arrangements:

1) the trivial solution $h_0 = h_P$, and

2) when $I_C = mh_0h_P$.

For the second case:

$$T_0 = T_P = 2\pi\sqrt{(h_0 + h_P)/g} = 2\pi\sqrt{L/g} \tag{1}$$

where O and P are called conjugate centers of oscillation. (See note at end of experiment.) A measurement of the period (T) and distance ($L = h_0 + h_P$) between knife edges then gives an accurate value of g.

Empirically finding an L to give $T_0 = T_P$ is tedious. Instead, find an L for which $T_0 \sim T_P$ and then eliminate I_C from the first two equations above to give:

$$\frac{4\pi^2}{g} = \frac{h_0T_0^2 - h_PT_P^2}{h_0^2 - h_P^2} = \frac{T_0^2 + T_P^2}{2(h_0 + h_P)} + \frac{T_0^2 - T_P^2}{2(h_0 - h_P)} . \tag{2}$$

If one chooses an asymmetric geometry for the location of the weights, one can avoid $h_O \sim h_P$. Note then that for $T_0 \sim T_P$ the first term dominates, and an accurate value of g results if one knows *accurately* $L = h_0 + h_P$, (the distance between the knife edges), and to much less accuracy the difference $h_0 - h_P$.

SUGGESTED PROCEDURE:

1. Find the approximate c. of m. of the pendulum plus asymmetrically located weights. Then set one knife edge as far from the c. of m. as feasible. This avoids $h_0 \sim h_P$ which would make the last term of Eqn. 2 large.
   NOTE: Keep this knife edge position fixed throughout the experiment.

2. Determine the period of the pendulum, T_0, when swinging from this knife edge. Keep amplitude small ($< 5°$) and use a photogate timer.

3. Calculate the length, L, of a simple pendulum to give the same period.

4. Set the second knife edge at this distance L from the first knife edge.

5. Determine the new period of the pendulum, T_P, for swinging from the second knife edge. This period T_P will *not quite equal* T_0 since moving the second knife edge has changed slightly the c. of m. and hence I_C, h_0, h_P, and also both T_0 and T_P.

6. Recalculate the length L of the equivalent simple pendulum to give this new period T_P. Then reset the knife edge accordingly.

7. Redetermine the period about the first knife edge.

8. One can continue this iterative process (6 thru 7) until the two periods are arbitrarily close to each other, and hence g is given by Eqn. 1. However this is not necessary if one accurately finds the new c. of m. (e.g. by balancing the pendulum on the knife edge for the hoops of M-8A) and then uses Eqn. 2.

9. Estimate your uncertainty in g and compare with accepted value (see M-4).

NOTE: These two conjugate centers of oscillation (O and P) exist of course for any rigid physical pendulum, e.g. *a baseball bat*:

Let one center of oscillation be where the batter grasps the bat. The conjugate center of oscillation is then called the *center of percussion* because if the ball hits the bat at this point, the blow rotates the bat about the other center of oscillation, (i.e. the batter's hands) and so the bat transmits no "sting" to the hands. However, if the ball hits very far from the *center of percussion*, the hands receive much of the blow and an unpleasant "sting" can result.

MC-9 Angular Acceleration and Moments of Inertia

OBJECTIVES:

- To study rotational motion resulting from constant torque.
- To investigate the rotational inertia of various objects.
- To experimentally observe the parallel axis theorems.
- If time permits, there is an optional experiment that will demonstrate conservation of angular motion in a "collision".

FUNDAMENTAL CONCEPTS OF ROTATIONAL MOTION:

1. The equation that describes the *rotational* motion of an object moving with *constant angular velocity* are completely analogous to those of linear motion with:

   $$\theta(t) = \theta_0 + \omega \cdot t$$

   If you make a plot of θ versus t, you find that it describes a straight line. The Greek letter θ_0 indicates the angular position of the object at time $t = 0$. The letter ω is the slope of the line and this is equal to the angular velocity of the object.

2. The equations that describe the *rotational* motion of an object that moves with *constant angular acceleration* are: $\theta(t) = \theta_0 + \omega_0 \cdot t + \frac{1}{2}\alpha t^2$ and $\omega(t) = \omega_0 + \alpha t$. The Greek letter θ_0 again indicates the angular position of the object at time $t = 0$. The letter ω_0 is the the angular velocity of the object at time $t = 0$, and α (the angular acceleration) is the slope of the graph of ω *vs* t.

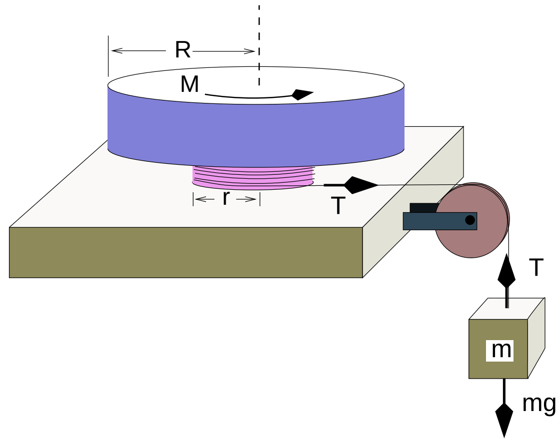
3. It is important to stress that the natural units of angular displacement are in *radians* and NOT degrees. One full revolution of an object corresponds to 2π radians or, equivalently, 360°. Typically units for angular velocity are in rad/sec and for angular acceleration are in rad/sec^2. Notice that the unit dimensions for equivalent dynamical variables in rotational and linear motion do not match. *A linear velocity can never be compared directly with a rotational velocity.* Ask your lab instructor for further clarification if this distinction is unclear.

APPARATUS:

Basic equipment: PASCO rotational assembly; PASCO photogate sensor and support; PASCO super pulley; solid plastic disk; aluminum bar; black metal cylinder; black metal squre; a length of string with metal hooks at both ends; weight hanger and various slotted masses; scale for mass measurements.

Computer equipment: Personal computer set to the M9 lab manual web-page; PASCO CI 750 or 700 interface module; photogate sensor/smart pulley (this should be plugged into DIGITAL CHANNEL input #3); also a 2nd photogate sensor directly sensing the wheel's rotation (this should be plugged into DIGITAL CHANNEL input #4).
NOTE: If a flywheel needs much more than ten grams on the end of string to maintain constant rotational velocity, notify the instructor.

DERIVATIONS: As a prelab exercise the web version of this lab contains a quiz that covers the basic derivations for the angular acceleration. Here you are expected to conduct the following calculations (next page):



PROBLEM: The figure at right sketch depicts a solid disk of mass M and radius R that is free to rotate about the axis as shown. The inner hub is massless but has radius r. Around this inner hub is wrapped a string (as shown) that goes over a frictionless pulley and this is attached to a hanging mass m. The mass m is acted upon by gravity. The normal force of the table cancel the weight of the disk. The disk starts at rest and, thereafter, the string slowly unwinds without slipping on the pulley or the hub.

You need to obtain expressions that relate 1: the linear acceleration of the mass m to the tension T in the string, 2: the angular acceleration α to the tension T in the string in terms of the masses (M and m) and the radii (R and r) and 3: the linear acceleration of the mass m to the angular acceleration α.

EXPERIMENT I: MEASURING CONSTANT ANGULAR ACCELERATION (suggested set-up procedures):

1. Make sure the PASCO interface has been turned on. If not you may have to reboot the computer. Next make sure that the phone jack connector from the photogate (super pulley) has been plugged DIGITAL channel # 3 position of the PASCO interface module. The computer itself is configured to measure the on/off timing between two adjacent slots in the super pulley wheel and is also calibrated for the pulley's diameter. Thus the read out is in linear velocity of the moving string if it makes non-slipping contact with the pulley.
2. Before (as shown in Figure 2, left panel) mounting the solid grey disc hook the wire through one of the holes in the center pulley hub assembly. It will work best if the string is wrapped about the channel formed by the top pulley hub. There are three hubs that one could use but alignment will be easiest with the top one. Unless there is a second hole in the pulley support rod the other two hubs may not line up correctly. Your instructor may wish to demonstrate "good" winding technique.
3. Mount the solid grey disc horizontally onto the rotation shaft as shown in Figure 2.
4. Make sure the pulley/infrared photogate assembly is properly aligned. As the pulley rotates the little red LED sensor turns off when each slot move past the photogate. *More important* is make sure that the string runs parallel to the super pulley's channel and the string is tangential to the hub. It may be necessary to rotate the super pulley/photogate assembly.
5. To initiate the PASCO interface software you will need to click on the telescope icon in the "toolkit." The image below gives a good idea of how the display appears in the web-based version. Note that, while you are able to reconfigure the display parameters, the default values that are specified on start-up should allow you to do this experiment without necessitating any major changes. The only value that can be displayed is the linear velocity of the string as it moves and rotates the super pulley. This calibration is fixed and cannot be altered. There is a bogus data set in the starting file. You should delete this before beginning.

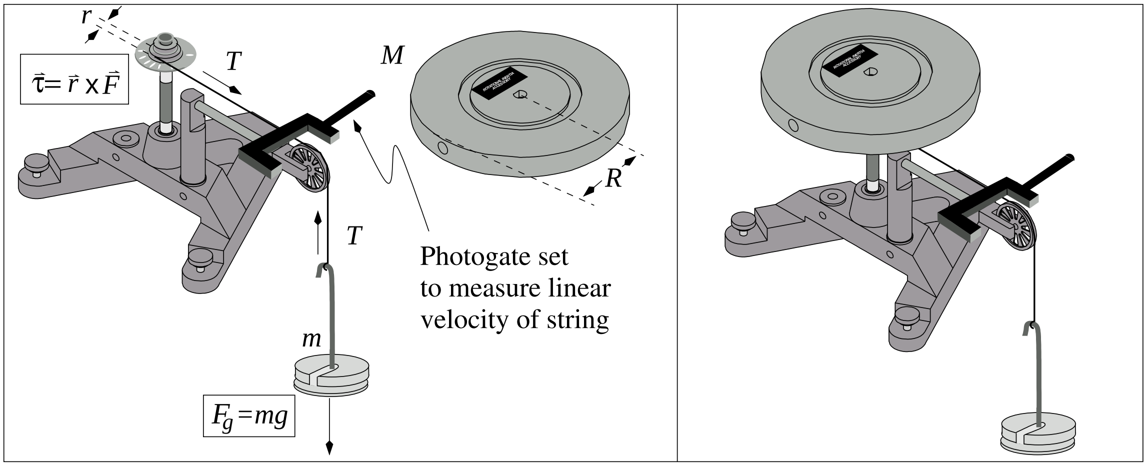


Figure 2: The PASCO rotational motion assembly. At left with the solid disc removed and, at right with the disc mounted.

6. Start the PASCO data acquisition by CLICKing on the START icon. To stop the data acquisition CLICK on the STOP icon. (If there are any preexisting data sets you cannot reconfigure either the interface parameters or sensor inputs.)

7. Using the 50 gram hanger plus another 100 or 150 grams release the disc while a lab partner starts the data acquisition. Stop the data acquisition and the rotating disk before the hanger strikes the floor. Since the torque is constant (see the on-line prelab M9 quiz or paper a copy....this quiz is in lieu of explicitly showing the full calculation) the velocity on screen should increase linearly. Conduct a number of trials to make sure of your technique.

8. The friction may be small but it is *not* zero. To extract the *net* torque you need to subtract off the hanging weight required to overcome friction. Take off the 50 gram weight hanger and suspend two or three grams at a time at the end of the string. When the mass descends with nearly constant velocity that will adequately identify the frictional forces. The mass must be subtracted from the full value. Call this mass m_0 and so the effective mass that is available for providing a net accelerating torque is $m - m_0$ or m'.

9. The plot velocity is proportional to the angular velocity of the wheel and the average acceleration a (i.e., the average slope) is proportional to the angular acceleration α. By using the PASCO plot ”zoom select” function (4th icon from left on the graph toolbar), you can magnify and rescale features of interest in the plot display. For the graphical analysis you need to CLICK on the plot window “Fit” icon and choose the linear function. Your lab instructor can provide assistance if necessary.

To obtain the angular acceleration you will need to divide the linear accelations by the hub’s radius r. In principle you could simply measure the radius of the hub with a verier caliper but this would neglect both the string diameter and tracking. A direct measure is a better approach.

DETERMINING THE EFFECTIVE HUB RADIUS:

(i) Align a meter stick vertically from the floor so that you can track the net displacement of the string.

(i) Identify a calibration point on the rim of the grey disc. (Masking tape should work well enough.) Align this near the edge of the meter stick and use the meter stick edge to locate a reproducible starting point.

(ii) Use the weight hanger to identify a height reading. Then lower the string and hanger ten or eleven (or, if using the midde hub, seven or eight) full turns of the wheel and take a second height reading,

(iii) The relationship between the change in height and angle is given by, $\Delta h = r\ \Delta\theta$ where θ is in *radians*. Notice

$$\begin{aligned}
\frac{\Delta h}{\Delta t} &= r\ \frac{\Delta\theta}{\Delta t} \\
v &= r\ \omega \\
dv/dt &= r\ d\omega/dt \\
a &= r\ \alpha
\end{aligned}$$

COMPARING THEORY WITH EXPERIMENT:

Once the effective hub radius is obtained you can directly compare your measured accelerations with those calculated. You will need to measure the mass, M, of the disc (you may neglect the mass and rotational inertia of the center support assembly).

1. If the mass of the suspended weight is small then you can approximate (see the DERIVATION section and/or on-line quiz) the calculated acceleration by

$$a = \frac{m'r^2 g}{I + m'r^2} \approx \frac{2m'r^2 g}{MR^2}$$

where

$$\begin{aligned}
\text{Let } I &= \text{grey disk moment of inertia (calculated from } \tfrac{1}{2}MR^2\text{)} \\
a &= \text{linear acceleration of the mass} \\
m' &= \text{effective mass as hung on string } (m' = m - m_0) \\
M &= \text{mass of grey disc} \\
r, R &= \text{radii of the inner hub and grey disc, respectively.}
\end{aligned}$$

2. For two additional values of masses suspended on the mass hanger obtain their respective average accelerations. Compare the measured accelerations with those calculated. Clearly mark them in your lab manual. How well do they agree?

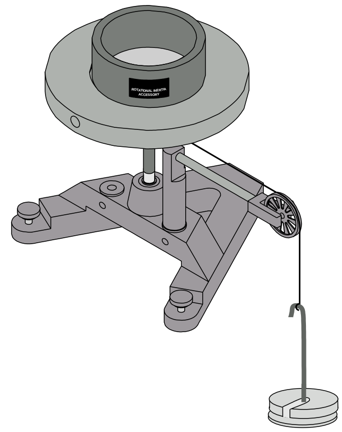
EXPERIMENT II (MOMENT OF INERTIA and the PARALLEL AXIS THEOREM)
Suggested Procedures:

1. Measuring other moments of inertia are no more difficult. In this case it is easy to retain the $m'r^2$ term and rewrite the above equation to give

$$I = m'r^2\frac{g}{a} - m'r^2 = m'r^2(g/a - 1)$$

(Once again correcting for friction.)

2. The rotational inertia expression for a hollow cylinder (rotating about its axis) is $\frac{1}{2}M'(R_1^2 + R_2^2)$ where R_1 and R_2 are the inner and outer radii, respectively, and M' is the mass. Place the black metal ring as shown in the adjacent figure and measure the rotational inertia of the two objects. This is simply $I_{\text{grey disc}} + I_{\text{black disk}}$. Compare your calculated and measured values. How well do they agree?



3. Now remove both discs and mount just the solid grey disk vertically using one of the holes drilled in its side. Experimentally measure the rotational moment of inertia. How does it compare with that calculated for horizontal mounting case?
4. Now remove the disc and replace it with the aluminum bar and measure the rotational inertia of the bar. In this case the aluminum bar approximates a thin long rod. Thus $I = \frac{1}{12}ML^2$ where M is the mass of the aluminum and L is the overall length. Choose hanging masses that give good reproducibility. Compare your calculated and measure values.
5. The aluminum bar is designed with a channel that allows you to mount a second object with variable displacements from the rotation axis. Mount a single black rectangular bar (approximately 4.5 cm on a side) in three position. The first should be centered on the axis of rotation and the latter two approximately 15 cm and 20 cm from the axis of rotation. By the parallel axis theorem, $I_m = I_c + mR'^2$ where I_c is the rotational inertia of the added square black mass m about its own center of mass. So the full express for I is $I_{bar} + I_c + mR'^2$. Plotting I against R'^2 should give a straight line. Do your three points line up?

EXPERIMENT III: CONSERVATION OF ANGULAR MOMENTUM (optional):

1. This experiment is designed to mimic the layout of the fourth quiz problem. The problem is: Two uniform density cylinders, one solid (the larger) and one hollow (the smaller) and differing radii are mounted so that the smaller cylinder is held centered and directly above the larger radius mass. Initially the bottom mass is spinning at a uniform angular velocity while the top mass is at rest. You will need to remove the string and hanging mass and use the second rotational sensor directly sensing the angular velocity of the horizontal wheel (see below or on the next page). To conduct this experiment a second PASCO setup file is accessible by clicking on the telescope icon in the web version of the experiment.
2. Use the solid grey disk and hollow black disk on the PASCO rotational assembly to recreate this problem. The initial values are:

$$L_i = I_{\text{grey disk}}\,\omega_i \qquad \text{and} \qquad E_i = \frac{1}{2} I_{\text{grey disk}}\,\omega_i^2$$

and the final values are:

$$L_f = (I_{\text{grey disk}} + I_{\text{black disk}})\,\omega_f \qquad \text{and} \qquad E_f = \frac{1}{2}(I_{\text{grey disk}} + I_{\text{black disk}})\,\omega_f^2$$

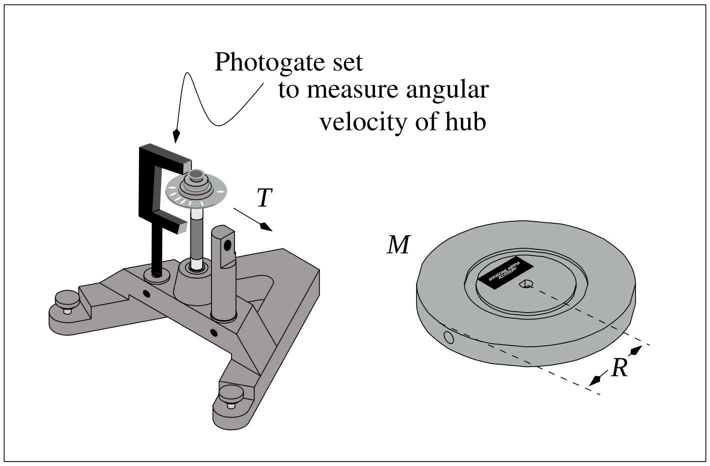


Figure 5: The PASCO rotational motion assembly with the photogate reset to measure the angular velocity of the hub directly. Although the super pulley is not shown it does not need to be removed.

3. Move the photogate to the new position and move the plug from Digital Channel #3 to number #4.
4. Slowly spin the grey disc and CLICK on the START icon.
5. While the grey disc it spinning hold the top mass (i.e., the black metal ring) just above the spin gray disc. The top mass should be very carefully dropped onto the bottom so that it remains centered and, afterwards, they will "stick" such that the two masses rotate at the same angular velocity.
6. Stop the data acquistion. Repeat the experiment a few times if necessary.
7. Compare the initial and final rotational momenta and energies. How much rotational energy was lost? Were you able to verify conservation of angular momentum? Why or why not?
8. NOTE: Before leaving please return the super pulley and photogate to the original position and switch the plug back to Digital Channel #3.

M-10 Young's Modulus of Elasticity and Hooke's Law

OBJECTIVE:

To study the elastic properties of piano wire under tension (Hooke's law and Young's modulus).

APPARATUS:

Frame for holding the steel wire, optical lever, telescope & support stand for measuring elongation (alternatively, dial gauges), 1 kg slotted masses, micrometer, tape measure.

INTRODUCTION:

Young's modulus M_Y is the ratio of longitudinal stress to the resultant longitudinal strain:

$$M_Y = \frac{\text{stress}}{\text{strain}} = \frac{\Delta F/A}{\Delta L/L}$$

where

$$\begin{aligned} \Delta F &= \text{longitudinal force in newtons} \\ A &= \text{area in square meters} \\ \Delta L &= \text{elongation} \\ L &= \text{length of wire undergoing the elongation (not the total length!)} \end{aligned}$$

Note that

$$\Delta F = \left\{ \frac{M_Y A}{L} \right\} \Delta L = k(\Delta L)$$

is Hooke's law where k is a constant if the elastic limit is not exceeded.

SUGGESTIONS:

1. First read about the optical lever (Appendix C) and about parallax and focusing a telescope (Appendix D). Also be sure that the frame holding the wire, and the stand holding the telescope are on solid bases. If you use a table, avoid leaning on it. With the telescope check the table sag resulting from leaning on it (least sag when units are near the table legs).

2. Adjust the height of the platform holding the optical lever so that the long arm of the lever is approximately horizontal.

PROCEDURE:

1. Put a load of three kilograms on the wire to straighten it.

2. Measure the successive deflections as you increase the load one kilogram at a time up to a total of 10 kg (i.e., 7 kg additional).

3. Repeat (2) but reducing the load 1 kg at a time.

4. Convert differences in scale readings to elongations (see Appendix C).

5. Plot total elongation as abscissa against load in Newtons as ordinate. Find the average slope (i.e. $\Delta F/\Delta L = k$).

6. Measure A and L; compute Young's modulus. The value for piano wire is $\simeq 20 \times 10^{10}\ N/m^2$.

QUESTIONS:

1. Piano wire has a tensile strength (breaking stress) of 19 to 23 $\times$ 10^8 N/m^2. (The elastic limit may be about 0.7 the breaking stress). Calculate the maximum load your wire could stand. At what load would you pass the elastic limit?

   Note that the wire fails at a stress which is 100 x less than Young's modulus. Can you understand why the two values are not inconsistent? (Hint: If the stress equaled M_Y, what would be the strain?).

2. Discuss the sources of error in this experiment and estimate the reliability of your result. Is the accepted value for piano wire steel within the limits you have estimated?

3. How could you detect slipping of the wire in the chuck during the experiment?

4. How could you modify the experiment so as to detect and correct for any sagging of the support frame?

5. Poisson's ratio, $\left(\frac{-\Delta r/r}{\Delta L/L}\right)$, for steel is about 0.3.

   Could you notice the decrease in diameter of the wire in this experiment by use of a micrometer caliper? If so, try it.

   Suggested additional experiment: Determine the dependence of elongation on stretching force for a rubber band over a wide range of elongations (up to 3 or more times the unstretched length). Does it obey Hooke's law? Can you suggest reasons for its behavior?

MC-11–Elastic and Inelastic Collisions

MC-11a Collisions Between Rolling Carts

NOTE TO INSTRUCTORS:

This lab uses PASCO carts and avoids many of the problems of the air track. The older version of this lab that uses gliders on the air track can be found in MC-14b. The three procedures outlined below will certainly take more than the three hours allotted. Please do procedure I, then as many of the others as time allows.

OBJECTIVE:

In this experiment you will observe elastic and inelastic collisions between two PASCO carts; the carts are provided with Velcro bumpers on one end, and magnetic bumpers on the other.

THEORY:

Collisions are a common experience in our lives–the collisions of billiard balls, the collisions of two football players, the collisions of cars. Various 'natural philosophers' from Galileo onwards discussed the laws that govern such collisions. As early as the XVIIth century it was understood that collisions between hard bodies, like billiard balls, behaved differently from collisions like car wrecks, where the two masses stick together after the collision. The concept of Momentum was first introduced in mechanics by Descartes as $momentum = mass \cdot velocity$; he did not get it quite right however, because he thought of momentum as a **scalar** quantity. It was Leibnitz that defined momentum as a **vector** quantity:

$$\vec{p} = m \cdot \vec{v}.$$

Newton's 2nd Law $\vec{F} = m\vec{a}$ can be rewritten

$$\vec{F} = m\frac{\Delta\vec{v}}{\Delta t};$$

now $m\Delta\vec{v}$ is the change in the quantity $\vec{p} = m\vec{v}$, so one can finally rewrite Newton's IInd law as

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

or

$$\vec{F}\Delta t = \Delta\vec{p}.$$

The quantity $\vec{F}\Delta t$ is called the **impulse** and is equal to the change in momentum.

FUNDAMENTAL CONCEPTS:

A physical quantity is said to be conserved in a process if its value does not change even though other quantities are changing. An example of a conserved quantity is energy. When a body falls freely its total energy (potential energy plus kinetic energy) remains unchanged while the position and velocity of the body change with time.

Linear momentum is conserved if the net force acting on an object is zero. This follows from the equation which relates the change in momentum to the impulse given to the object. Clearly if the force F is zero, the impulse is zero and the change in momentum is zero, hence the momentum remains constant.

This same principle becomes more useful when it is applied to an *isolated system* of objects. An isolated system is one where the only forces acting on an object *in* the system are due to the interaction with another object *within* the same system: there are no *external* forces.

A simple such system may consist of two particles, let us call them A and B, interacting with each other. The force $\vec{F}_A$ on particle A is equal and opposite to the force $\vec{F}_B$ acting on particle B: $\vec{F}_A = -\vec{F}_B$. Because the times during which the forces act are the same, it follows that the changes in momentum of the two particles are also equal and opposite, so that the total change in momentum is zero. The conservation of momentum is therefore a consequence of Newton's IIIrd Law. In collisions, provided there is no net external force on any of the bodies, the sum of the initial momenta equals the sum of the final momenta:

$$\vec{p_1} + \vec{p_2} = \vec{p_1}' + \vec{p_2}'$$

where the unprimed quantities refer to the velocities before the collision, and the primed quantities refer to the velocities after the collision.

For one dimensional processes the physical quantity that is conserved is linear momentum.

A collision is called totally inelastic if the two bodies stick together after colliding. The conservation of momentum for a *one dimensional* totally inelastic collision is then:

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) \cdot v'.$$

Energy is not conserved in inelastic collisions.

A collision of two bodies is called totally elastic if energy is conserved in the process. In this case the result of the collision in one dimension can be calculated by

$$(v_1 - v_2) = -(v_1' - v_2')'.$$

It is interesting to note that $(v_1 - v_2)$ and $(v_1' - v_2')$ are the relative velocities of body # 1 relative to body #‘2 before and after the collision.

APPARATUS:

1. Personal Computer with monitor, keyboard, and mouse
2. PASCO dynamic track with magnetic bumpers
3. PASCO signal interface
4. Two PASCO carts with a “picket fence”
5. A set of 500 g masses
6. One meter long ruler
7. Two photogates (plugged into DIGITAL channels #1 and #2): these devices consist of
   (a) a source which produces a narrow beam of infrared radiation
   (b) an infrared detector at the other side of the gate that senses the radiation.

When the beam between the source and detector is blocked, a red Light Emitting Diode (LED) on the top of the gate lights up; concurrently an electrical signal is sent to the Signal Interface which converts the time during which the beam was blocked into the velocity of the fence. NOTE: for the photogates to work properly, the picket fence must be on the opposite side of the cart closest to the LED.

SUGGESTIONS:

1. Measure the mass of the empty carts using the pan balance; MAKE SURE YOU PUT THE CART ON ITS SIDE ON THE PAN. Record these masses in your lab notebook. Note that the cart with the magnet bumper is somewhat heavier than the cart without magnet, you may wish to tape some masses on the lighter cart to equalize the weights.

2. Initiate the PASCO interface software in the usual way. The monitor should now look as shown in Figure 1.

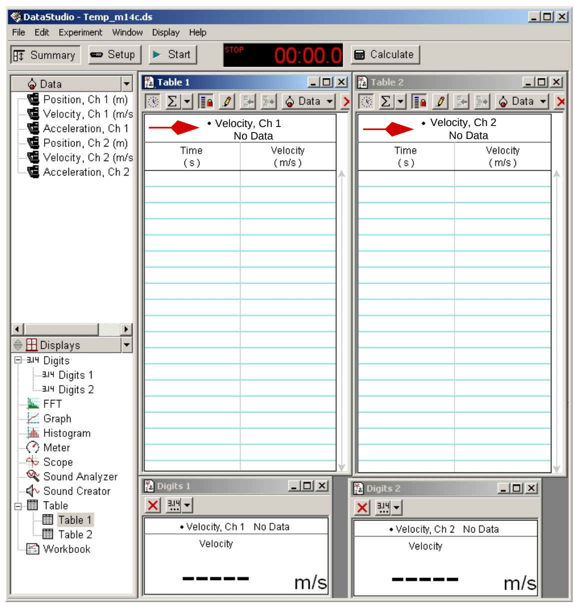


Figure 1: The Pasco DataStudio display for M14.

On the computer monitor you see a table of velocities for each of the two photogates. As seen by a person looking at the computer monitor the photogate on the left is # 1 and the one on the right is # 2.

3. Level the track so the carts do not move to the right or the left.

4. Place the photogates 15 cm apart.

PROCEDURE I - INELASTIC COLLISION - EQUAL MASSES

1. Prepare a table like the one below:

Run #	Velocity # 1 (m/s)	Velocity #1 (m/s)
1		
2		
3		

Table 1: Sample layout for inelastic collison data.

2. Install a picket fence on one cart (on the side opposite the LED). Make sure that it is the longest black stripe on the fence that intercepts the LED beam. Put the cart at the left end of the track. Place the othercart just to the right of photogate #2 as shown below. Check that the carts do not repel each other; the Velcro hooks and loops must be able to stick together easily.

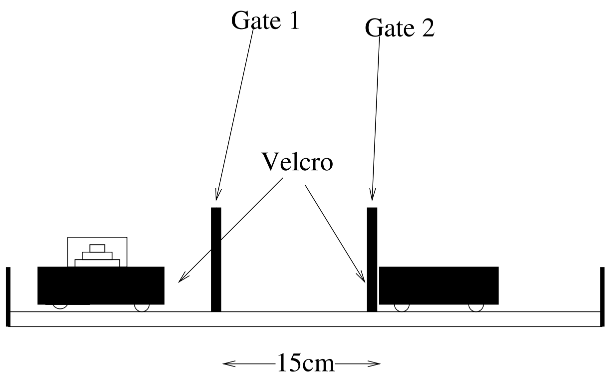


Figure 2:

3. CLK on REC. Gently push the projectile cart to the right toward the first photogate and the target cart. Catch the carts before they hit the end of the track. CLK on STOP. Record the velocity of the projectile cart before and after the collision in Table 1.

4. Take two more runs.

5. ANALYSIS OF THE DATA.

   Prepare tables as shown below in Table 2 and 3.

Run #	Car 1 p_{ini} (kg m/s)	Cars 1+2 p_{final} (kg m/s)	$100 \times \Delta p / p_{ini}$
1			
2			
3			
Avg			

Table 2: Momentum

(a) Calculate the initial and final momenta and enter them in Table 2.

(b) Calculate the initial and final kinetic energies and enter them in Table 3

[(c)] Calculate and record in Table 2 the percent difference in the momenta: $100 \times (p_{ini} - p_{fin})/p_{ini}$. Calculate the average percent difference in the momentum.

[(d)] Calculate and record in Table 3 the percent difference in energy: $100 \times (E_{ini} - E_{fin})/E_{ini}$. Calculate the average percent difference in the energy.

	Car 1	Cars 1+2	
Run #	E_{ini} (J)	E_{final} (J)	$100 \times \Delta E/E_ini$
1			
2			
3			
Avg			

Table 3: Energy

1. QUESTIONS:

   A. Give a likely physical cause for the observed difference between initial and final momentum.

   B. What fraction of the energy was dissipated?

   C. If you could remove the physical cause you listed, what fraction of energy would you expect to find dissipated?

PROCEDURE II - ELASTIC COLLISION - EQUAL MASSES
In this experiment you will be measuring the velocities of two bodies with equal masses, before and after a totally elastic collision.

1. Place the photogates about 35 cm apart with one cart just to the left of photogate # 2. *Make sure the carts will repel each other.* (One cart has one end without any magnets-that end will not repel the other cart.) Place the other cart on the left end of the track. as shown in Figure 3 below. Cart # 2 should be close to gate # 2, (not as shown in the figure). Make sure that there is enough space, so that cart # 2 starts moving **after** cart # 1 has passed completely through gate # 1.

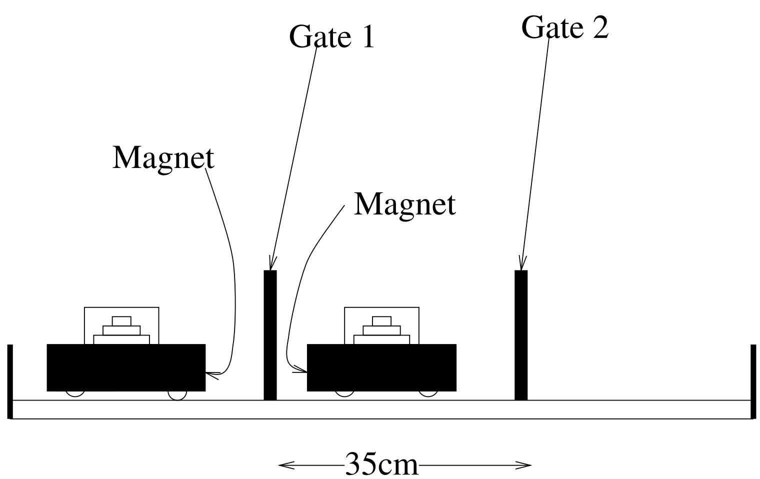


Figure 3:

2. Prepare a table like Table 4.

Run #	Velocity # 1 (m/s)	Velocity #1 (m/s)
1		
2		
3		

Table 4:

3. CLICK on the START icons. Gently push the projectile cart (the one on the left) to the right toward the first photogate and the target cart. The projectile cart should stop between the gates and the target will move through photogate # 2. Catch it before it has time to hit the end of the track; CLK on STOP. Record the velocities in Table 4. Take two more runs.

4. ANALYSIS OF THE DATA

   Prepare tables like Tables 5 and 6.

	Car 1	Cars 1+2	
Run #	p_1 (kg m/s)	p_2 (kg m/s)	$100 \times \Delta p/p_1$
1			
2			
3			
Avg			

Table 5: Momentum

	Car 1	Cars 1+2	
Run #	E_1 (J)	E_2 (J)	$100 \times \Delta E/E_1$
1			
2			
3			
Avg			

Table 6: Energy

A. Calculate the initial momentum of Car # 1 and final momentum of Car # 2 and enter them in Table 5.

B. Calculate and record the percent difference of the momenta: $100\times(p_1-p_2)/p_1$.

C. Calculate the average percent difference in the momentum.

D. Calculate the initial and final kinetic energies and enter them in Table 6.

E. Calculate and record the percent difference of the energies: $100 \times (E_1 - E_2)/E_1$.

5. QUESTIONS

   What is a reason for the observed difference between p_1 and p_2?

   What fraction of the energy was dissipated?

   What fraction of the energy should have been dissipated in the absence of friction?

PROCEDURE III - ELASTIC COLLISION - UNEQUAL MASSES

1. The photogates should be about 35 cm apart. Find and record the masses of two 500g masses using the triple beam balance; place them on the target cart, place this cart just to the left of photogate # 2 as shown in Figure 3 (not as shown in the figure) . The carts should be placed so they will repel each other. The lighter cart is 'the projectile' and should be placed on the left end of the track.

2. Prepare a table like that of Table 7.

Run #	V_{1ini} (m/s)	V_{1fin} (m/s)	V_{2fin} (m/s)
1			
2			
3			

Table 7:

3. CLICK on the START icon. Gently push the projectile cart (the lighter one on the left) to the right toward the first photogate and the target cart. Try to predict what will happen. Catch the carts before they hit the end of the track. CLICK on STOP. Record the velocities in Table 7

4. Take two more runs.

5. ANALYSIS OF THE DATA

   Prepare tables like Tables 8 and 9.

	Car 1 ini	Car 1 fin	Car 2	
Run #	p_{1ini} (kg m/s)	p_{1fin} (kg m/s)	p_{2fin}(kg m/s)	$100 \times \Delta p/p_{1ini}$
1				
2				
3				
Avg				

Table 8: Momentum

Calculate the initial and final momentum of each cart and enter them in Table 8. Calculate the total momentum of the system p_{fin} after the collision and record it in Table 8. Remember that the final momentum is the sum of the momenta of carts # 1 and # 2 after the collision.

Run #	Car 1 ini E_{1ini} (J)	Car 1 fin E_{1fin} (J)	Car 2 E_{fin} (J)	$100 \times \Delta E/E_{1ini}$
1				
2				
3				
Avg				

Table 9: Energy

Calculate the initial and final kinetic energies and enter them in Table 9. Calculate the total energy of the system E_{fin} after the collision and record it in Table 9. Remember that the final energy is the sum of the energies of carts # 1 and # 2 after the collision.

Calculate and record the percent difference $100 \times (p_{1ini} - p_{fin})/p_{1ini}$.

Calculate and record the percent difference $100 \times (E_{1ini} - E_{fin})/E_{1ini}$.

6. QUESTIONS:

   What is a reason for the observed difference?

   Was momentum approximately conserved?

   What fraction of the energy was dissipated?

   What fraction of the energy should have been dissipated in the absence of friction?

JAVA APPLET:

If time permits and you are interested the web version of the lab has a link to an applet which animates elastic collisions of two point masses.

MC-11b Air Track Collisions

OBJECTIVE: To study conservation of energy and momentum in collision.
APPARATUS:

Air track, assorted slotted masses, air supply, hose, gliders; photogates & support stands and PASCO interface and computer.

> **PRECAUTIONS:** *The soft aluminum gliders and track surfaces damage easily: Don't drop!* With the air pressure on, use a glider to *check that the track is level and free of high friction areas* (from scratches or plugged air holes). Getting good results can, at times, be surprisingly difficult in this lab. All collisions must be free from any glider contact with the rail. In general speeds which are too slow are overly influenced by residual friction and air track leveling errors. On the other hand, speeds which too high invariably cause pitch and yaw motions of the gliders which increase the likelihood of physical contact with the track. Good alignment of the needle assembly is also a necessity. You should perform a number of preliminary trials to discern which speeds work best.

SUGGESTIONS:

1. Use the large gliders whenever possible. Small ones often tilt upon impact and hence give excessive friction.
2. Turn on the air supply and experiment with the gliders. Adjust the leveling screw so that the nominal cart acceleration is minimized. Adjust the air flow so the gliders move freely without rocking side to side.
3. Make sure that the photogates are plugged into the first two phone jack inputs in the PASCO interface module. Also set the two photogates approximately 40 to 50 cm apart and so that they track the 10 cm long plate atop the glider.
4. Click on the telescope icon below (web version) to launch the PASCO software. It should already be configured to display a two column table which will display the *speed* of the glider as it passes through the infrared photogate. (It already assumes that the plate is exactly 10 cm long and actually measures the time.) The monitor should now look as shown in Figure 1 at right.
5. To start the data acquisition CLICK on the START icon. To stop it, CLICK on the STOP icon. Each time a glider passes the photogate an entry will appear in the appropriate table column. NOTE: The photogates measure the speed but do NOT sense the direction of motion. You are responsible for the latter.

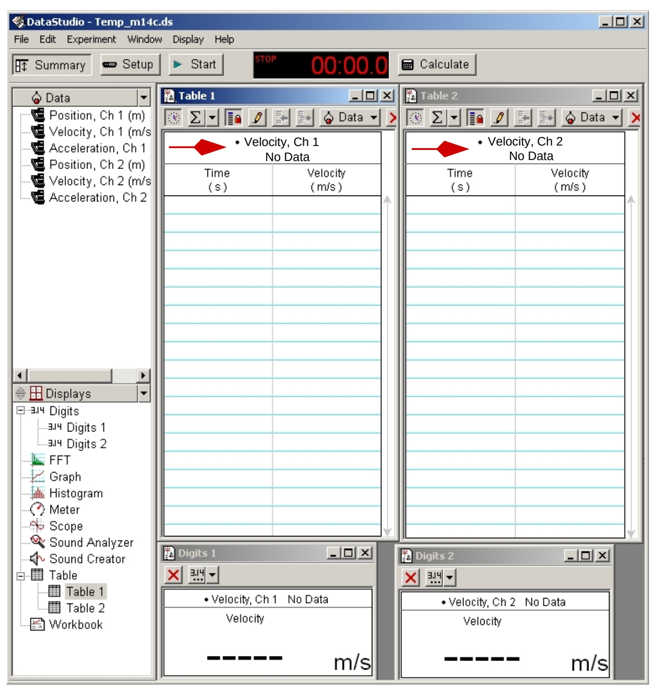


Figure 1: PASCO DataStudio display for M14.

6. The results of this experiment are very technique sensitive. Take a single glider and practice sending it through both photogates until you are able to get reasonably equivalent speed readings from both photogates. Check the behavior by launching the glider from both ends and record, in your lab book, the speed measured in four or five satisfactory trials. Estimate the precision associated with a pair of velocity measurements and show how this will impact your momentum and kinetic energy measurements.

EXPERIMENT I:

1. Choose gliders of equal mass (or make them approximately the same by fastening weights on one). Qualitatively predict the expected outcome of the next step (i.e., item 2) before attempting the experiment.
2. With glider #1 at the end of track and glider #2 at rest near the center, give #1 a push toward glider #2.
   To help prevent confusion, stop glider #2 before it bounces back.
   As before you should perform multiple trials until you achieve consistent results and record a few of them.

   Check conservation of momentum and energy in the impact. In equation,

   $$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2,$$

   call velocities to the right positive, those to the left negative.
3. Comment on how well these two quantities are conserved and, if your results seem poorer than expected, suggest possible sources of error.
4. Devise a method for determining how elastic is a rubber band collision and record your results.

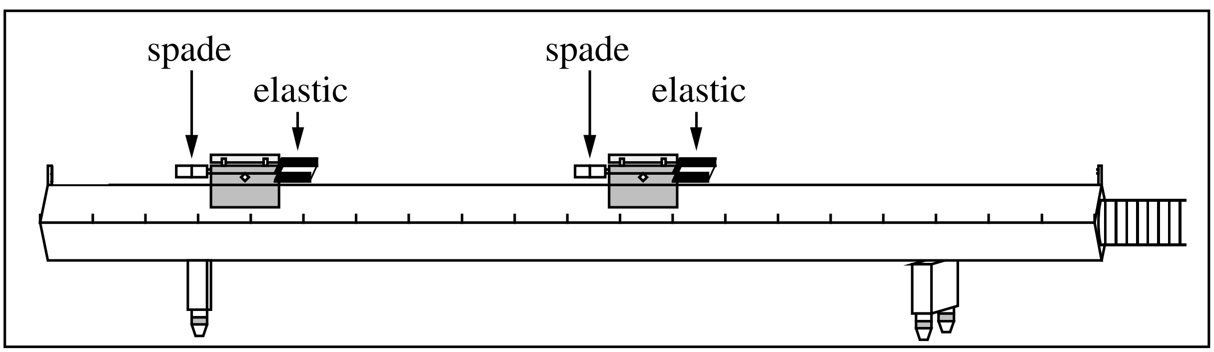


Figure 2: Sketch of the air track configuration for elastic collisions.

Suggested tabulations:

Glider:	#1	#2	Velocity Readings:	#1	#2
mass			Before impact (u)		
length			After impact (v)		

Before Impact		After Impact	
$u_1 =$ ______	$u_2 =$ ______	$v_1 =$ ______	$v_2 =$ ______
$m_1u_1 =$ ______	$m_2u_2 =$ ______	$m_1v_1 =$ ______	$m_2v_2 =$ ______
$\frac{1}{2}m_1u_1^2 =$ ______	$\frac{1}{2}m_2u_2^2 =$ ______	$\frac{1}{2}m_1v_1^2 =$ ______	$\frac{1}{2}m_2v_2^2 =$ ______

change in momentum = ______; % change in momentum = ________.

change in energy = ______; % change in energy = ________.

EXPERIMENT II:

Perform the same procedure as Exp. I (steps 1, 2 and 3) except start both gliders from opposite ends of the air track and with considerably different velocities.

EXPERIMENT III:

Repeat Exp. II but for inelastic collisions by attaching cylinders with needle and wax inserts. Note that the needle must lines up exactly with the insert or there will be significant sideways motion when the two gliders strike.

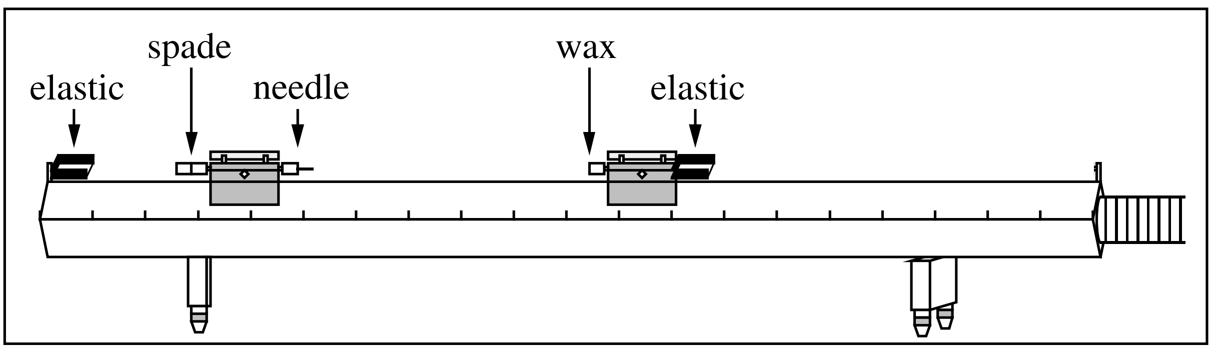


Figure 3: Sketch of the air track configuration for inelastic collisions.

EXPERIMENT IV:

Increase m_1 by adding masses a nd repeat EXP. I.

MEASUREMENT OF FRICTION:

Estimate the frictional force between the glider and track by using the velocity data recorded before Exp. I. From any net decrease in velocity you should be able to obtain the frictional force. Does this information help you understand the experimental data?

JAVA APPLET:

If time permits and you are interested the web version of the lab has a link to an applet which animates elastic collisions of two point masses.

MC-12–Simple Harmonic Motion and Resonance

MC-12a Simple Harmonic Motion and Resonance (Rolling Carts)

NOTE TO INSTRUCTORS: This lab uses PASCO rolling carts instead of the more problematic gliders on the air track. The older airtrack version of this lab is MC-15b.
OBJECTIVES:

- To study the period of Simple Harmonic Motion (SHM) as a function of oscillation amplitude.
- To demonstrate Hooke's Law, $F = -kx$, where k is the spring constant, and x is the elongation.
- To study the period T of SHM as a function of the mass m which is oscillating, investigating $\sqrt{m/k} \propto T(= 2\pi/\omega = 1/f)$.
- To observe the relationships between the potential and kinetic energy.

THEORY:

The restoring force (F) on an object attached to a "simple" one-dimensional spring is proportional to the displacement from equilibrium and has the form, $F = -k(x - x_0)$, where k is the spring constant (or stiffness in N/m), x_0 is the equilibrium position (i.e., no net force) and x is the position of the object. This is Hooke's Law. Remember that the simple harmonic oscillator is a good approximation to physical systems in the real world, so we want to understand it well. That's the purpose of this lab!

The expression $F = ma = -k(x - x_0)$ is a 2nd order differential equation with

$$a = \frac{d^2x'}{dt^2} = -\frac{k}{m}(x')$$

where $x' \equiv x - x_0$. The most general solution for this expression is often given as

$$x'(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$$

where $\omega_0 \equiv \sqrt{k/m}$. ω_0 is defined as the *natural frequency* for the *undamped* harmonic oscillator. A and B are arbitrary initial displacement parameters. Alternatively the solution is often specified as

$$x'(t) = C\sin(\omega_0 t + \phi) \qquad \text{or} \qquad x'(t) = C\cos(\omega_0 t + \phi)$$

where ϕ is the starting phase and C is the displacement. It is also possible to write down these solutions using complex numbers as in

$$x'(t) = A'e^{i\omega_0 t} + B'e^{-i\omega_0 t} \quad .$$

The relative merit of these equivalent expressions will become clearer in EXPTS. V and VI.

FUNDAMENTAL CONCEPTS:

1. The solution to the undamped (i.e., no frictional or drag forces) harmonic oscillator is time dependent and periodic. When the $\omega_0 t$ term varies by 2π (or one period $T = 1/f = 2\pi/\omega_0$) both the position, $x'(t+T) = x'(t)$, and velocity, $v'(t+T) = v'(t)$ return to their previous values.

2. Total energy (TE) is conserved in SHM motion. As time evolves kinetic energy (KE) is transferred to (and from) potential energy (PE). Thus at any time t:

$$TE = \text{constant} = KE(t) + PE(t) = \frac{1}{2}m[v'(t)]^2 + \frac{1}{2}k[x'(t)]^2 \quad .$$

APPARATUS:

Basic equipment: PASCO dynamic track, PASCO cart with "picket fence", PASCO cart with aluminum plate, adjustable stop, assorted masses, springs: *this experiment works best with a pair of short (10 mm unstretched length) springs*, timer, photogate & support stand, knife edge assembly.

Computer equipment: Personal computer set to the MC15a lab manual web-page; PASCO interface module; photogate sensor and extension jack, PASCO sonic position sensor, speaker with driver stem, power amplifier module.

PRECAUTIONS:

1. See MC-14a
2. **When setting up the springs, use the adjustable end stop on the track so the springs are stretched properly: don't stretch them beyond their elastic limit and don't let them sag and drag on the track**. Stretching the springs to reach the ends of the track will damage them.
3. *Keep the amplitudes small enough that a slack spring doesn't touch the track nor a stretched spring exceed its elastic limit.*

SUGGESTIONS: To measure the period of the oscillating cart:

1. Install a "picket fence" on the cart.
2. Locate the photogate so the black stripe on the fence just cuts off the LED beam (see MC-14a) when the cart is in the equilibrium position ($x - x_0 = 0$). The photogate phone jack should be in the first PASCO interface position. Choose two springs having a similar length and refer to the above precaution.
3. To initiate the PASCO interface software click the mouse on the telescope icon in the "toolkit" area below. There will be just a single table for recording the measured period.
4. Start the cart by displacing it from equilibrium and then releasing it. Then start the data acquisition by clicking the START icon. Let the cart oscillate for about 10 periods. Calculate the mean and standard deviation by simply clicking on the statistics icon (i.e. $\boxed{\Sigma}$) on the data table.

UNDAMPED SIMPLE HARMONIC MOTION:

1. Using sketches in your lab book, both at equilibrium and after a displacement from equilibrium, show that the effective force constant for 2 identical springs of force constant k on either side of an oscillating mass is $2k$.

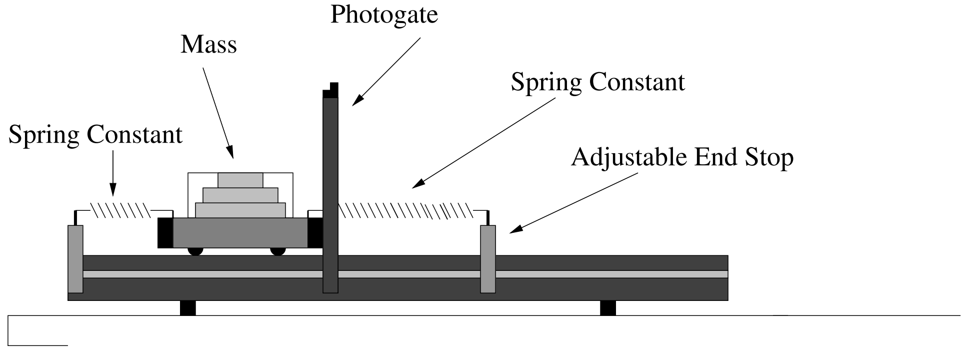


Figure 1: Sketch of the dynamic track configuration for SHM.

2. **EXPERIMENT I:** Show experimentally that the period is independent of the amplitude. Try amplitudes of *approximately* 10 cm, 20 cm, and 30 cm. (Friction may be a problem at very small amplitudes).

3. **EXPERIMENT II:** By adding mass to the cart, study the period versus total oscillating mass. The latter must include a correction for the oscillating springs whose effective oscillating mass (see the note below) is approximately $m_s/3$ where m_s is the mass of the *two* springs. Explain, in words, how this correction may be qualitatively justified (Why not m_s or $m_s/2$?) By inspection of your T vs $[M + (m_s/3)]$ curve, what function of T might yield a straight line when plotted versus $[M + (m_s/3)]$? Prepare this plot. Calculate the effective force constant, k', (equal to $k_1 + k_2$, the two springs will actually differ slightly) of the system from the slope of this straight line graph. (Remember $T = 2\pi\sqrt{M_{\text{eff}}/k'}$ where $M_{\text{eff}} \simeq \mathrm{M} + \mathrm{m}_s/3$.) Estimate the uncertainty in k' by solving for k' at each cart mass and calculating the standard deviation of the mean.

   NOTE: Theoretically $m_{\text{eff}} = m/3$ only for $M/m_s = \infty$ For $M = 0$, the effective mass is $4m/\pi^2 = 0.405\,\mathrm{m}$. However for $M/m_s = 5$, the effective mass is already $\simeq 0.336\ m_s$. See Fig. 1 of J.G. Fox and J. Makanty, *American Journal of Physics*, **38**, 98 (1970).

4. **EXPERIMENT III :** Also measure k directly by hanging the two springs vertically on a "knife edge" assembly in the laboratory. Record the stretch produced by a series of weights, *but do not exceed the elastic limit of either spring!* Graph F vs y for each spring and obtain a best-fit straight line. The slope should be the spring constant. Compare this value of k' with that obtained in part #3.

5. QUESTION: Assuming the spring constant doubles, how would T vary?

FURTHER INVESTIGATIONS OF "UNDAMPED" SIMPLE HARMONIC MOTION:

Up to this point you have characterized the SHM of a spring-mass assembly in terms of only a single parameter T (the period). One of the possible time-dependent expressions for describing the motion was

$$x'(t) = A\cos(\omega_0 t + \phi_0) \quad \text{and} \quad v'(t) = -A\omega_0 \sin(\omega_0 t + \phi_0)$$

or

$$v'(t) = \ A\omega_0 \cos(\omega_0 t + \phi_0 - \frac{\pi}{2})$$

where A is the amplitude (displacement from equilibrium), ω_0 is the *natural* angular frequency and ϕ_0 is the starting phase. Notice that the velocity has a *phase* shift of $\frac{\pi}{2}$ relative to the displacement.

Equally characteristic of SHM is the process of energy transference: kinetic energy of motion is transferred into potential energy (stored in the spring) and back again. Friction is an ever present energy loss process so that the total energy always diminishes with time.

To capture this rather rapid cyclic process we will again use the PASCO interface while replacing the photogate sensor with the sonic position sensor.

EXPERIMENT IV: Measuring the x vs t behavior:

1. Replace the cart with the picket fence by a cart with an aluminum vane attached to it. You will want to measure the mass of this new cart and predict the new natural frequency.

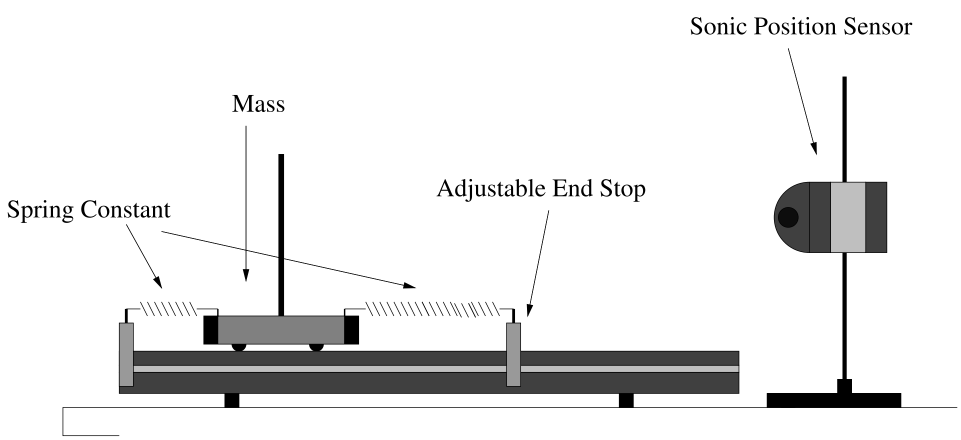


Figure 2: Sketch of the dynamic track with the sonic position sensor.

2. Place the position sensor approximately 60 cm from the vane in the direction of oscillatory motion. Make sure the yellow phone jack is in the third slot and the black phone jack is in the fourth slot. Alignment is very important so that the sensor senses only the vane and not the cart. A slight upward tilt may help (or raising the vane up slightly as well).
3. CLICK on the telescope icon below to initiate the PASCO© interface software.
4. Displace the cart approximately 20 cm from equilibrium to initiate the oscillatory motion. CLICK on the the START icon to start your data acquisition. The graph will simultaneously display both absolute position and velocity versus time.
5. Practice a few times to make sure you can obtain smoothly varying sinusoidal curves. Then run the data acquisition for just over ten cycles and use the cross-hair feature to read out the time increment for ten full cycles. How does your prediction check out? Determine the initial phase (i.e., at $t = 0$). Record the equilibrium position (x_0) as well.

6. Use the "Zoom select" feature of the PASCO graph toolbar to better view a single full cycle by clicking on the magnifying glass icon (4th from left) and then select two points in the graph using a CLICK and DRAG motion of the mouse.

7. Print out (click in graph region and then type ALT, CTRL-P) or, alternatively, sketch the position and velocity curves in your lab book identifying key features in the time dependent curves. In particular identify the characteristic(s) which demonstrate the $\pi/2$ phase difference between the velocity and displacement curves.

8. Using the PASCO cross-hair option to read out the relevant time, position and velocity, make a table as below and fill in the missing entries (identify units).

time	phase($\omega_0 t + \phi_0$)	$x'(t)$	$v'(t)$	KE	PE	TE
	0					
	$\pi/4$ (45 deg)					
	$\pi/2$ (90 deg)					
	$3\pi/4$ (135 deg)					
	π (180 deg)					

9. Is the total energy a constant of the motion?

10. How much displacement amplitude and energy are lost after five full cycles? What is the approximate friction coefficient?

OPTIONAL INVESTIGATIONS OF "UNDER-DAMPED" SIMPLE HARMONIC MOTION:

In the real world friction is an ever present process. In the case of SHM friction can have a profound effect. Damping of unwanted vibrations is important in a myriad of situations. (Imagine what driving a car would be like if there were no shock absorbers!)

Introducing friction can be done by simply adding one more term in the force expression, $F_{\text{drag}} \equiv -Rv$, a drag force which is proportional to the velocity where R is the drag coefficient [units of kg m/s or N/(m/s)] . This is appropriate for motion thru a viscous fluid but it is really only a rough approximation for the frictional forces in the track. The modified force expression now becomes

$$F = ma = -kx' - Rv' \quad \text{or} \quad 0 = \frac{d^2x'}{dt^2} + \frac{R}{m}\frac{dx'}{dt} + \omega^2 x'$$

Since energy is continuously lost, solutions of this expression will be *time dependent* but NOT periodic. Adding this "simple" term dramatically complicates the process of finding appropriate solutions. The most general form of the solution is

$$x' = e^{-(R/2m)t} \left[Ae^{+\sqrt{(\frac{R^2}{4m^2} - \omega_0^2)}\; t} + Be^{-\sqrt{(\frac{R^2}{4m^2} - \omega_0^2)}\; t} \right]$$

which is quite formidable. Since the track drag is low (i.e., R is relatively small) the solution is said to be *underdamped* and oscillatory when $\omega_0^2 > R^2/4m^2$. The solution in this case becomes:

$$x' = Ce^{-(R/2m)t} \cos(\omega_1 t + D) \quad \text{where} \quad \omega_1^2 \equiv \omega_0^2 - \frac{R^2}{4m^2}$$

where ω_1 is the *natural frequency* of the *underdamped* system and C and D are the initial displacement and phase. Because the frequency is lowered, the period lengthens. This is consistent with one's intuition; drag works against oscillatory motion.

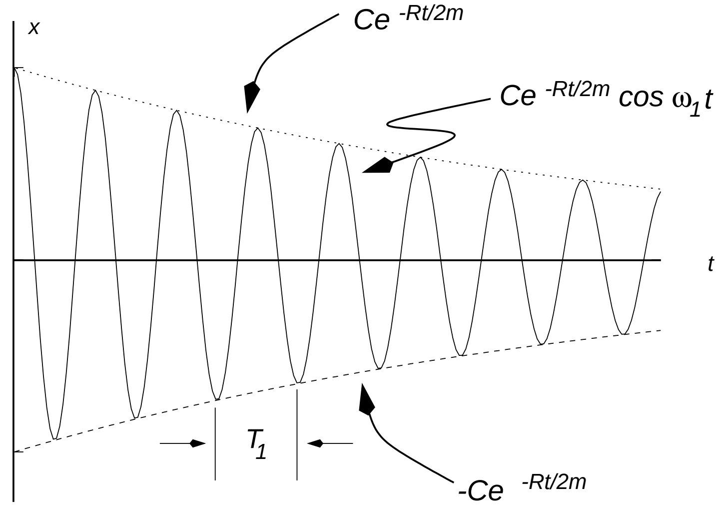


Figure 3: Sketch of underdamped harmonic motion with $\omega_0^2 > R^2/4m^2$.

EXPERIMENT V:

1. CLICK on the telescope icon below (web-version) to initiate the next PASCO© interface application. In addition to the graph there will be a "two" column table displaying the time and position then the time and velocity.
2. Displace the cart approximately 20 cm from equilibrium to initiate the oscillatory motion. CLICK on the the START icon to start your data acquisition and record enough cycles to see the amplitude diminish by two thirds.
3. As the amplitude of oscillation decreases, does the period T change? Base the answer on measurements of T using the cursor in the PASCO display.
4. Select six representative times using the table (identifying where the velocity changes sign) and make a table of t vs maximum displacement. Use these points in the graphical analysis package and fit these points to the expression $C \exp -(R/2m)t$ or, in terms of the explicit analysis formula, $y = C * \exp[-B * (x - x_0)]$.
5. How good or poor is the assumption that the drag force is proportional to the velocity?
6. Sketch out an approximate curve of x vs t if R were significantly larger. Which R would be more appropriate for absorbing and dissipating a physical "shock" (and why)?

OPTIONAL INVESTIGATIONS OF RESONANCE:

One of the most important situations of the harmonic oscillator is that of FORCED, damped harmonic motion. In one-dimension the applied force is typically sinusoid and when ω approaches the natural frequency of the system (nominally ω_1 and, if $R^2/4m^2 \ll 1$, also ω_0) the energy of the driver is additively coupled to the moving mass (e.g., a glider) and resonance occurs. Resonance is a *very* important aspect of the world around us and many mechanical and electronic devices employ resonant behavior as a fundamental aspect of their operation (e.g., musical instruments, radios, televisions).

Along with the frictional drag (Rv' where R is the drag coefficient) one more force term must be added, that of the mechanical driver, with $F_{\text{driver}} = F_d \ \cos(\omega t)$. The new force expression is conventionally written as:

$$F = ma = -kx' - Rv' + F_d \cos \omega t \quad \text{ or } \quad \frac{F_d}{m} \cos \omega t = \frac{d^2x'}{dt^2} + \frac{R}{m}\frac{dx'}{dt} + \omega^2 x' \ .$$

Since the system energy is lost through friction and may be gained through the driver action, solutions of this expression will be *time dependent* but with both *transient* and *steady-state* attributes. In many instances resonant systems respond so quickly that one only views the steady-state behavior. In this lab you will be able to observe BOTH the *transient* and *steady-state* processes.

As you may expect the most complete solution of this new differential equation has a rather complicated form and so is not reproduced here. Since we are interested only in resonance we can simplify the expression by assuming solutions that apply to the *underdamped case* (those with oscillatory behavior). Thus the analytic solution reduces to:

$$x' = Ce^{-(R/2m)t} \ \cos(\omega_1 t + D) + \frac{F_d}{[m^2(\omega_0^2 - \omega^2)^2 + \omega^2 R^2]^{1/2}} \cos(\omega t - \phi)$$

where the first term is the *transient* behavior, identical to that of the simple damped harmonic oscillator (described in the last section) and the second term is the *steady-state* solution. At large times t the first terms dies out exponentially so that x' is approximated by only

$$x'((R/2m)t \gg 1) = \frac{F_d}{[m^2(\omega_0^2 - \omega^2)^2 + \omega^2 R^2]^{1/2}} \cos(\omega t - \phi)$$

where ω_0 is the natural frequency of the undamped harmonic oscillator, ω is the mechanical driver frequency, R is the drag coefficient and ϕ is a measure of the phase difference between the driver motion and the cart motion.

In this lab we will only investigate the nature of the cart displacement with driver frequency (ω) in the vicinity of the resonant frequency. Thus the only relationship of interest becomes amplitude $\propto [m^2(\omega_0^2 - \omega^2)^2 + \omega^2]^{-1/2} \equiv Z^{-1/2}$. Z is a minimum when the driver frequency is set to $\omega_0^2 - R^2/(2m^2)$ or $\omega_1^2 - R^2/(4m^2)$ which is defined to be ω_2. Since $R^2/2m^2$ is small (the PASCO track is a low friction experiment) ω_0 is nearly the *same* and, in addition, x' will be sharply peaked about ω_0.

EXPERIMENT VI: (SUGGESTED PROCEDURE:)

1. Use the rolling cart with the aluminum plate. Weigh the cart and predict the natural frequency for this new arrangement.
2. Place the position sensor approximately 60 cm from the vane in the direction of oscillatory motion. Make sure the yellow phone jack is in the third slot and the black phone jack is in the fourth slot. Alignment is very important so that the sensor senses only the vane and not the cart. A slight upward tilt may help.
3. Detach the fixed spring end stop and place the speaker as shown in the figure above with the spring looped through the small slot in the speaker driver stem using the same considerations for the spring extension as in the previous experiments.

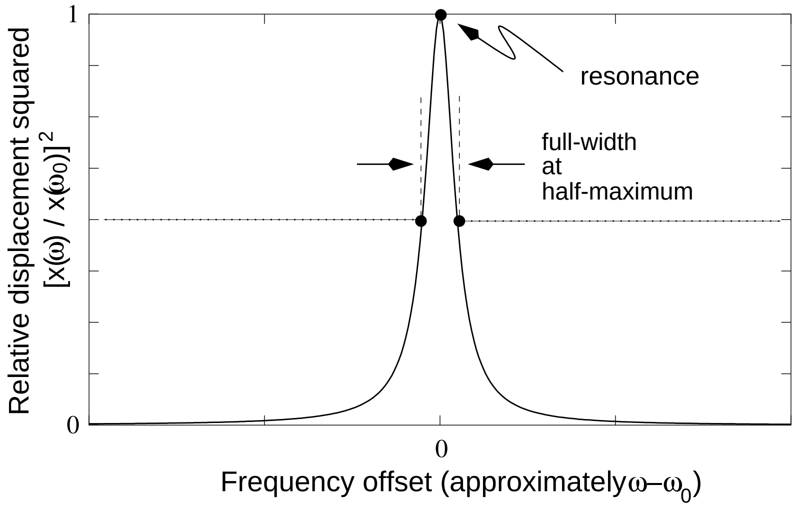


Figure 4: Sketch of the amplitude squared vs driver frequency.

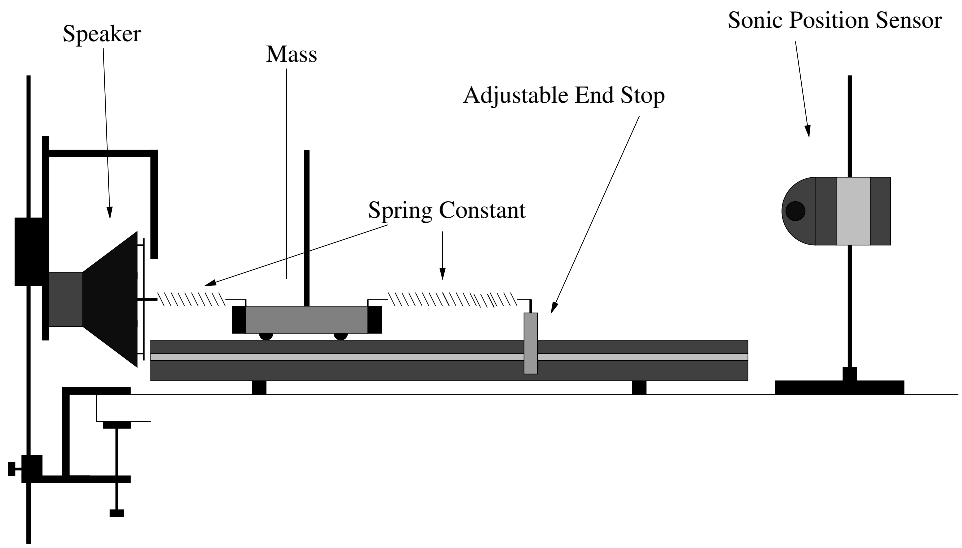


Figure 5: Sketch of the dynamic track with the position sensor and speaker.

4. Make sure the speaker power leads are plugged into the amplifier module and that its power is turned on. Also verify that the DIN-9 pin connector (from the amplifier module) is plugged into the A position in the PASCO interface module.
5. CLICK on the telescope icon (if web version) below to initiate the PASCO© interface software.
6. Make sure the speaker driver window has been switched to the **Off** position. Displace the cart approximately 20 cm from equilibrium to initiate the oscillatory motion. CLICK on the the START icon to start your data acquisition. The graph will simultaneously display glider position and velocity versus time and the amplifier current (which should be zero). Practice a few times to make sure you can obtain smoothly varying sinusoidal curves.
7. Displace the cart approximately 20 cm from equilibrium to initiate the oscillatory motion. CLICK on the the START icon to start your data acquisition and record

enough cycles to see the amplitude diminish by 90%. Compare your data to Fig. 3 and verify that your cart has a similar transient behavior. Determine the *natural frequency* of the system and compare to your prediction. (This frequency is actually ω_1 but it is, for this low friction set-up, nearly the same as either ω_2 or ω_0.) To observe the steady-state properties of resonance you will have to wait for times *longer* than those required for this step.

8. Moving to the PASCO Signal Generator window (shown in Fig. 6), set the amplifier frequency (which is in Hertz or cycles per second) to the closest to resonance (0.01 Hz steps) and engage the speaker but CLICKing the **Auto** icon. The driver output should use the sinusoid AC waveform and, if the overload light on the Power Amplifier flashes on, reduce the voltage setting slightly. Start the data acquisition (and speaker motion) by CLICKing on the START icon and record data until you achieve steady-state behavior. You have to wait a couple of minutes before you reach steady state.

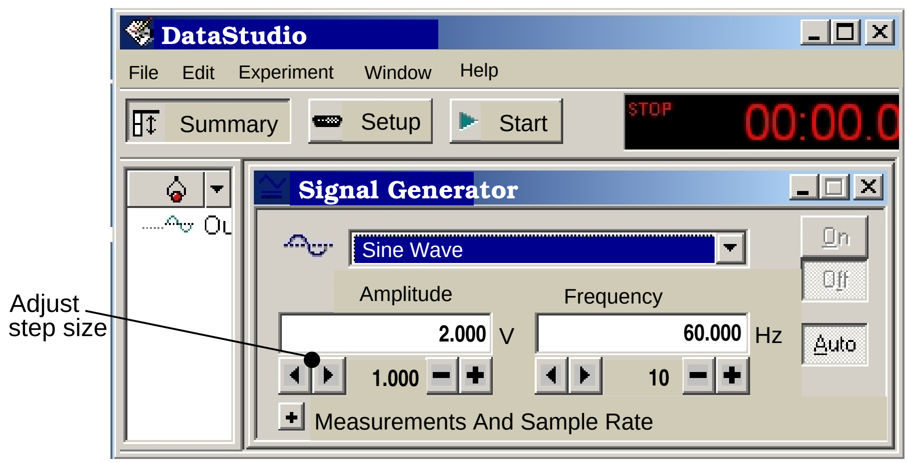


Figure 6: Image of the PASCO software Signal Generator window.

NOTE: The nominal step sizes for adjusting the amplifier frequency and voltage are *very* large. To alter the step size use the ◀ or ▶ buttons. To alter the current or voltage (which of these depends on configuration) use the + or − buttons.

9. Set the amplifier frequency 0.01 Hz steps above and below resonance and record data until you achieve steady-state behavior. Repeat for 0.10 Hz and (if time permits) 0.40 Hz steps. Plot out a few representative sets of data.

10. Determine the maximum steady-state displacement of the cart for each of the measured frequencies and plot the relative amplitude squared $[(x'(\omega)/x'(\omega_0))^2]$ vs frequency offset $(\omega - \omega_0)$. Estimate the full width at half maximum for this curve. This value should be equal to R/m.

11. Discuss the nature of this resonance curve. If you adjust the R/m ratio to further sharpen the resonance curve can you identify a compensating complication if you are interested in achieving steady-state behavior? If the speaker were attached to an amplifier playing audible music (nominally 20-20,000 Hz), what do you think this mass will do?

MC-12b Simple Harmonic Motion and Resonance (Air Track)

OBJECTIVES:

1. To study the period of Simple Harmonic Motion (SHM) as a function of oscillation amplitude.
2. To study the period of SHM as a function of oscillating mass.
   Expected result: Period($\equiv\ T\ =\ 2\pi/\omega\ =\ 1/f$)
   is proportional to $\sqrt{\text{Mass/Spring Constant}(k)}$
3. To demonstrate Hooke's Law, $F = -kx$
4. To observe the relationships between the potential and kinetic energy

THEORY:

The restoring force (F) on an object attached to a "simple" one-dimensional spring is proportional to the displacement from equilibrium and has the form, $F = -k(x - x_0)$, where k is the spring constant (or stiffness in N/m), x_0 is the equilibrium position (i.e., no net force) and x is the position of the object. This is Hooke's Law. Remember that the simple harmonic oscillator is a good approximation to physical systems in the real world, so we want to understand it well. That's the purpose of this lab!

The expression $F = ma = -k(x - x_0)$ is a 2nd order differential equation with

$$a = \frac{d^2x'}{dt^2} = -\frac{k}{m}(x')$$

where $x' \equiv x - x_0$. The most general solution for this expression is often given as

$$x'(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$$

where $\omega_0 \equiv \sqrt{k/m}$. ω_0 is defined as the *natural frequency* for the *undamped* harmonic oscillator. A and B are arbitrary initial displacement parameters. Alternatively the solution is often specified as

$$x'(t) = C\sin(\omega_0 t + \phi) \qquad \text{or} \qquad x'(t) = C\cos(\omega_0 t + \phi)$$

where ϕ is the starting phase and C is the displacement. It is also possible to write down these solutions using complex numbers as in

$$x'(t) = A'e^{i\omega_0 t} + B'e^{-i\omega_0 t} \quad .$$

The relative merit of these equivalent expressions will become clearer in EXPTS. V and VI.

FUNDAMENTAL CONCEPTS:

1. The solution to the undamped (i.e., no frictional or drag forces) harmonic oscillator is time dependent and periodic. When the $\omega_0 t$ term varies by 2π (or one period $T = 1/f = 2\pi/\omega_0$) both the position, $x'(t+T) = x'(t)$, and velocity, $v'(t+T) = v'(t)$ return to their previous values.

2. Total energy (TE) is conserved in SHM motion. As time evolves kinetic energy (KE) is transferred to (and from) potential energy (PE). Thus at any time t:

$$TE = \text{constant} = KE(t) + PE(t) = \frac{1}{2}m[v'(t)]^2 + \frac{1}{2}k[x'(t)]^2 \quad .$$

APPARATUS:

Basic equipment: Air track, assorted slotted masses, air supply, hose, adjustable stop, glider, springs, timer, photogate & support stand, knife edge assembly.

Computer equipment: Personal computer set to the MC15A lab manual web-page; PASCO interface module; photogate sensor and extension jack, PASCO sonic position sensor, speaker with driver stem, power amplifier module.

PRECAUTIONS:

1. See MC-14b
2. **When setting up the springs, use the adjustable end stop on the air track so the springs are stretched properly: don't stretch them beyond their elastic limit and don't let them sag and drag on the track**.
3. *Keep the amplitudes small enough that a slack spring doesn't touch the track nor a stretched spring exceed its elastic limit.*

SUGGESTIONS: To measure the period of the oscillating glider:

1. Locate the photogate so the glider just cuts off beam when glider is in the equilibrium position ($x - x_0 = 0$). The photogate phone jack should be in the first PASCO interface position. Choose two springs having a similar length and refer to the above precaution. Turn on the air supply adjust the blower speed to minimize frictional forces.

2. To initiate the PASCO interface software click the computer mouse when centered on the telescope icon in the "toolkit" area below. There will be a just a single table for recording the measured period.

3. Start the glider by displacing it from equilibrium and then releasing it. Then start the data acquisition by clicking the START icon. Let the glider oscillate for about 10 periods. Calculated the mean and standard deviation by simply clicking on the statistics icon (i.e. $\boxed{\Sigma}$) on the data table.

UNDAMPED SIMPLE HARMONIC MOTION:

1. Using sketches in your lab book, both at equilibrium and after a displacement from equilibrium, show that the effective force constant for 2 identical springs of force constant k on either side of an oscillating mass is $2k$.

2. **EXPERIMENT I:** Show experimentally that the period is independent of the amplitude. Try amplitudes of *approximately* 10 cm, 20 cm, and 30 cm. (Friction may be a problem at very small amplitudes).

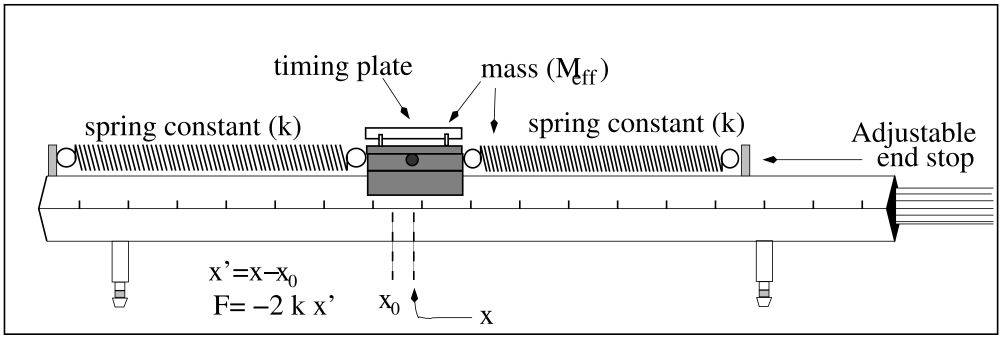


Figure 1: Sketch of the air track configuration for SHM.

3. **EXPERIMENT II:** By adding mass to the glider, study the period versus total oscillating mass. The latter must include a correction for the oscillating springs whose effective oscillating mass (see the note below) is approximately $m_s/3$ where m_s is the mass of the *two* springs. Explain, in words, how this correction may be qualitatively justified (Why not m_s or $m_s/2$?) By inspection of your T vs $[M + (m_s/3)]$ curve, what function of T might yield a straight line when plotted versus $[M + (m_s/3)]$? Prepare this plot. Calculate the effective force constant, k', (equal to $k_1 + k_2$, the two springs will actually differ slightly) of the system from the slope of this straight line graph. (Remember $T = 2\pi\sqrt{M_{\text{eff}}/k'}$ where $M_{\text{eff}} \simeq$ M + m_s/3.) Estimate the uncertainty in k' by solving for k' at each glider mass and calculating the standard deviation of the mean.

   NOTE: Theoretically $m_{\text{eff}} = m/3$ only for $M/m_s = \infty$ For $M = 0$, the effective mass is $4m/\pi^2 = 0.405$ m. However for $M/m_s = 5$, the effective mass is already $\simeq 0.336\ m_s$. See Fig. 1 of J.G. Fox and J. Makanty, *American Journal of Physics*, **38**, 98 (1970).

4. **EXPERIMENT III :** Also measure k directly by hanging the two springs vertically on a "knife edge" assembly in the laboratory. Record the stretch produced by a series of weights, *but do not exceed the elastic limit of either spring!* Graph F vs y for each spring and obtain a best-fit straight line. The slope should be the spring constant. Compare this value of k' with that obtained in part #3.

5. QUESTION: Assuming the spring constant doubles, how would T vary?

FURTHER INVESTIGATIONS OF "UNDAMPED" SIMPLE HARMONIC MOTION:

Up to this point you have characterized the SHM of a spring-mass assembly in terms of only a single parameter T (the period). One of the possible time-dependent expressions for describing the motion was

$$x'(t) = A\cos(\omega_0 t + \phi_0) \quad \text{and} \quad v'(t) = -A\omega_0 \sin(\omega_0 t + \phi_0)$$

or

$$v'(t) = \ A\omega_0 \cos(\omega_0 t + \phi_0 - \frac{\pi}{2})$$

where A is the amplitude (displacement from equilibrium), ω_0 is the *natural* angular frequency and ϕ_0 is the starting phase. Notice that the velocity has a *phase* shift of $\frac{\pi}{2}$ relative to the displacement.

Equally characteristic of SHM is the process of energy transference: kinetic energy of motion is transferred into potential energy (stored in the spring) and back again. Friction is an ever present energy loss process so that the total energy always diminishes with time.

To capture this rather rapid cyclic process we will again use the PASCO interface while replacing the photogate sensor with the sonic position sensor.

EXPERIMENT IV: Measuring the x vs t behavior:

1. Remove the 10 cm timing plate and plug the aluminum vane into the central banana plug position with the vane perpendicular to the long axis of the glider. The mass of this arrangement will now have changed slightly; predict the new natural frequency.

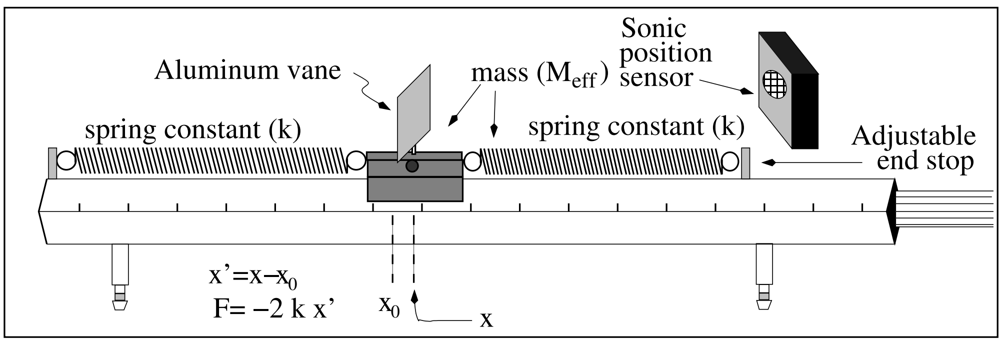


Figure 2: Sketch of the air track with the position sensor.

2. Place the position sensor approximately 60 cm from the vane in the direction of oscillatory motion. Make sure the yellow phone jack is in the third slot and the black phone jack is in the fourth slot. Alignment is very important so that the sensor senses only the vane and not the cart. A slight upward tilt may help (or raising the vane up slightly as well).

3. CLICK on the telescope icon below to initiate the PASCO© interface software.

4. Displace the cart approximately 20 cm from equilibrium to initiate the oscillatory motion. CLICK on the the REC button to start your data acquisition. The graph will simultaneously display both absolute position and velocity versus time.

5. Practice a few times to make sure you can obtain smoothly varying sinusoidal curves. Then run the data acquisition for just over ten cycles and use the cross-hair feature to read out the time increment for ten full cycles. How does your prediction check out? Determine the initial phase (i.e., at $t = 0$). Record the equilibrium position (x_0) as well.

6. Use the magnification option of the PASCO software to better view a single full cycle by clicking on the magnifying glass icon (in the graph window) and then select two points in the graph using a CLICK and DRAG motion of the mouse.

7. Print out (click in graph region and then type ALT, CTRL-P) or, alternatively, sketch the position and velocity curves in your lab book identifying key features

in the time dependent curves. In particular identify the characteristic(s) which demonstrate the $\pi/2$ phase difference between the velocity and displacement curves.

8. Using the PASCO cross-hair option to read out the relevant time, position and velocity, make a table as below and fill in the missing entries (identify units).

time	phase($\omega_0 t + \phi_0$)	$x'(t)$	$v'(t)$	KE	PE	TE
	0					
	$\pi/4$ (45 deg)					
	$\pi/2$ (90 deg)					
	$3\pi/4$ (135 deg)					
	π (180 deg)					

9. Is the total energy a constant of the motion?

10. How much displacement amplitude and energy are lost after five full cycles? What is the approximate friction coefficient?

OPTIONAL INVESTIGATIONS OF "UNDER-DAMPED" SIMPLE HARMONIC MOTION:

In the real world friction is an ever present process. In the case of SHM friction friction can have a profound effect. Damping of unwanted vibrations is important in a myriad of situations. (Imagine what driving a car would be like if there were no shock absorbers!)

Introducing friction can be done by simply adding one more term in the force expression, $F_{\text{drag}} \equiv -Rv$, a drag force which is proportional to the velocity where R is the drag coefficient [units of kg m/s or N/(m/s)] . This is appropriate for motion thru a viscous fluid but it is really only a rough approximation for the frictional forces in the air track. The modified force expression now becomes

$$F = ma = -kx' - Rv' \quad \text{or} \quad 0 = \frac{d^2x'}{dt^2} + \frac{R}{m}\frac{dx'}{dt} + \omega^2 x'$$

Since energy is continually lost solutions of this expression will be *time dependent* but NOT periodic. Adding this "simple" term dramatically complicates the process of finding appropriate solutions. The most general form of the solution is

$$x' = e^{-(R/2m)t} \left[Ae^{+\sqrt{(\frac{R^2}{4m^2}-\omega_0^2)}\ t} + Be^{-\sqrt{(\frac{R^2}{4m^2}-\omega_0^2)}\ t} \right]$$

which is quite formidable. Since the air track drag is low (i.e., R is relatively small) the solution is said to be *underdamped* and oscillatory when $\omega_0^2 > R^2/4m^2$. The solution in this case becomes:

$$x' = Ce^{-(R/2m)t}\ \cos(\omega_1 t + D) \quad \text{where} \quad \omega_1 \equiv \omega_0^2 - \frac{R^2}{4m^2}$$

where ω_1 is the *natural frequency* of the *underdamped* system and C and D are the initial displacement and phase. Because the frequency is lowered, the period lengthens. This is consistent with one's intuition; drag works against oscillatory motion.

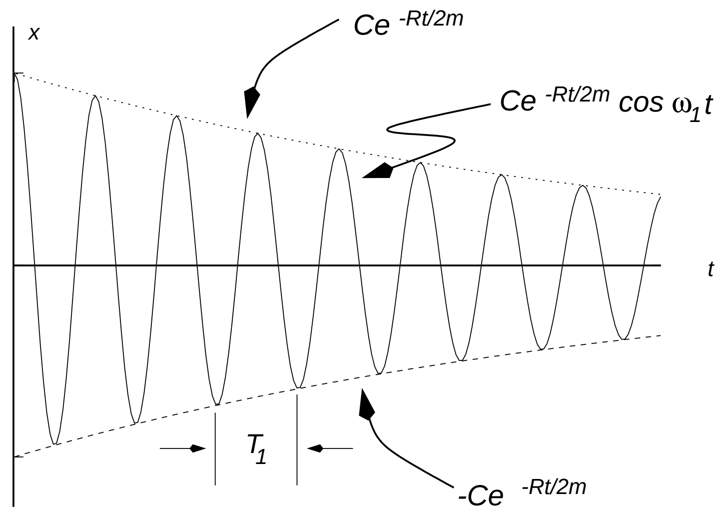


Figure 3: Sketch of underdamped harmonic motion with $\omega_0^2 > R^2/4m^2$.

EXPERIMENT V:

1. Reduce the air flow to the air track so that the amplitude of the glider motion diminishes by 50% (or so) in a few minutes. Typically the lowest blower setting will work well enough. (Make sure the other group is also ready.)

2. CLICK on the telescope icon below to initiate the next PASCO© interface application. In addition to the graph there will be a "two" column table displaying the time and position then the time and velocity.

3. Displace the cart approximately 20 cm from equilibrium to initiate the oscillatory motion. CLICK on the the REC button to start your data acquisition and record enough cycles to see the amplitude diminish by two-thirds.

4. Repeat the experiment at full blower speed for approximately the same period of time. Can you distinguish the difference between "ω_0" and ω_1 (T_0 and T_1)?
HINT: The graph display for the position is configured to overlay the data sets. Use the magnifying glass icon feature to examine the relative phase difference at early time and late time.

5. If so, from the formula $\omega_1^2 = \omega_0^2 - R^2/4m^2$ determine R.

6. For the reduced air flow data, select six representative times using the table (identifying where the velocity changes sign) and make a table of t vs maximum displacement. Use these points in the graphical analysis package and fit these points to the expression $C \exp -(R/2m)t$ or, in terms of the explicit analysis formula, $y = C * \exp[-B * (x - x_0)]$.

7. Does this value compare favorably with the results of item 4? How good or poor is the assumption that the drag force is proportional to the velocity?

8. Sketch out an approximate curve of x vs t if R were significantly larger. Which R would be more appropriate for absorbing and dissipating a physical "shock" (and why)?

OPTIONAL INVESTIGATIONS OF RESONANCE:

One of the most important situations of the harmonic oscillator is that of FORCED, damped harmonic motion. In one-dimension the applied force is typically sinusoid and when ω approaches the natural frequency of the system (nominally ω_1 and, if $R^2/4m^2 \ll 1$, also ω_0) the energy of the driver is additively coupled to the moving mass (e.g., a glider) and resonance occurs. Resonance is a *very* important aspect of the world around us and many mechanical and electronic devices employ resonant behavior as a fundamental aspect of their operation (e.g., musical instruments, radios, televisions).

Along with the frictional drag (Rv' where R is the drag coefficient) one more force term must be added, that of the mechanical driver, with $F_{\text{driver}} = F_d \cos(\omega t)$. The new force expression is conventionally written as:

$$F = ma = -kx' - Rv' + F_d \cos\omega t \quad \text{or} \quad \frac{F_d}{m}\cos\omega t = \frac{d^2x'}{dt^2} + \frac{R}{m}\frac{dx'}{dt} + \omega^2 x' \quad .$$

Since the system energy is lost through friction and may be gained through the driver action, solutions of this expression will be *time dependent* but with both *transient* and *steady-state* attributes. In many instances resonant systems respond so quickly that one only views the steady-state behavior. In this lab you will be able to observe BOTH the *transient* and *steady-state* processes.

As you may expect the most complete solution of this new differential equation has a rather complicated form and so is not reproduced here. Since we are interested only in resonance we can simplify the expression by assuming solutions that apply to the *underdamped case* (those with oscillatory behavior). Thus the analytic solution reduces to:

$$x' = Ce^{-(R/2m)t}\ \cos(\omega_1 t + D) + \frac{F_d}{[m^2(\omega_0^2 - \omega^2)^2 + \omega^2 R^2]^{1/2}}\cos(\omega t - \phi)$$

where the first term is the *transient* behavior, identical to that of the simple damped harmonic oscillator (described in the last section) and the second term is the *steady-state* solution. At large times t the first terms dies out exponentially so that x' is approximated by only

$$x'((R/2m)t \gg 1) = \frac{F_d}{[m^2(\omega_0^2 - \omega^2)^2 + \omega^2 R^2]^{1/2}}\cos(\omega t - \phi)$$

where ω_0 is the natural frequency of the undamped harmonic oscillator, ω is the mechanical driver frequency, R is the drag coefficient and ϕ is a measure of the phase difference between the driver motion and the glider motion.

In this lab we will only investigate the nature of the glider displacement with driver frequency (ω) in the vicinity of the resonant frequency. Thus the only relationship of interest becomes $x' \propto [m^2(\omega_0^2 - \omega^2)^2 + \omega^2]^{-1/2} \equiv Z^{-1/2}$. Z is a minimum when the driver frequency is set to $\omega_0^2 - R^2/(2m^2)$ or $\omega_1^2 - R^2/(4m^2)$ which is defined to be ω_2. Since $R^2/2m^2$ is small (the air-track is a low friction experiment) ω_0 is nearly the *same* and, in addition, x' will be sharply peaked about ω_0.

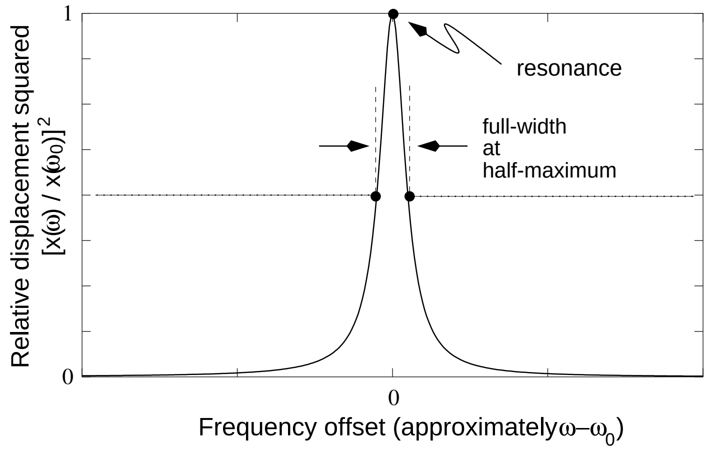


Figure 4: Sketch of the relative maximum displacement squared vs driver frequency.

SUGGESTED PROCEDURE:

1. **EXPERIMENT VI:** Make sure the 10 cm timing plate is replaced by the aluminum vane (in the center banana plug position) with the vane perpendicular to the long axis of the glider. Predict the natural frequency for this new arrangement.

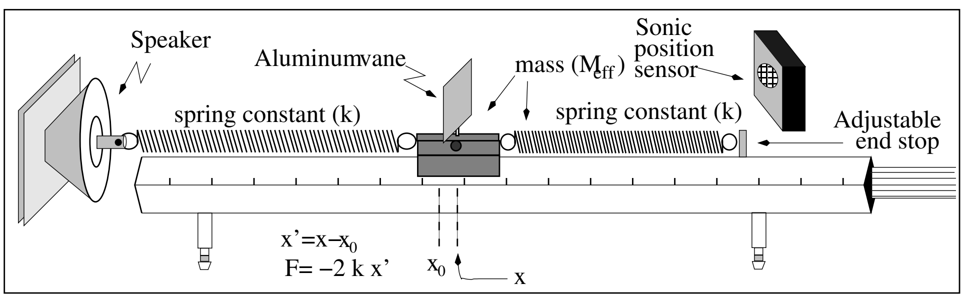


Figure 5: Sketch of the air track with the position sensor and speaker.

2. Place the position sensor approximately 60 cm from the vane in the direction of oscillatory motion. Make sure the yellow phone jack is in the third slot and the black phone jack is in the fourth slot. Alignment is very important so that the sensor senses only the vane and not the cart. A slight upward tilt may help (or raising the vane up slightly as well).
3. Detach the fixed spring end stop and place the speaker as shown in the figure above with the spring looped through the small slot in the speaker driver stem using the same considerations for the spring extension as in the previous experiments.
4. Make sure the speaker power leads are plugged into the amplifier module and that its power is turned on. Also verify that the DIN-9 pin connector (from the amplifier module) is plugged into the A position in the PASCO interface module.
5. CLICK on the telescope icon below (web version only) to initiate the PASCO© interface software.
6. Make sure the speaker driver window has been switched to the **Off** position. With maximum blower airflow, displace the cart approximately 20 cm from equilibrium to initiate the oscillatory motion. CLICK on the the START icon to start your data acquisition. The graph will simultaneously display glider position and velocity

versus time and the amplifier current (which should be zero). Practice a few times to make sure you can obtain smoothly varying sinusoidal curves.

7. Reduce the air flow to the air track so that the amplitude of the glider motion diminishes by 50% (or so) in a few minutes. Typically the lowest blower setting will work well enough. (Make sure the other group is also ready.)
8. Displace the cart approximately 20 cm from equilibrium to initiate the oscillatory motion. CLICK on the the START icon to start your data acquisition and record enough cycles to see the amplitude diminish by 90%. Compare your data to Fig. 3 and verify that your glider has a similar transient behavior. Determine the *natural frequency* of the system and compare to your prediction. (This frequency is actually ω_1 but it is, for this low friction set-up, nearly the same as either ω_2 or ω_0.) To observe the steady-state properties of resonance you will have to wait for times *longer* than those required for this step.
9. Moving to the PASCO Signal Generator window (shown in Fig. 6), set the amplifier frequency (which is in Hertz or cycles per second) to the closest to resonance (0.01 Hz steps) and engage the speaker but CLICKing the **Auto** icon. The driver output should use the sinusoid AC waveform and, if the overload light on the Power Amplifier flashes on, reduce the voltage setting slightly. Start the data acquisition (and speaker motion) by CLICKing on the START icon and record data until you achieve steady-state behavior.
   NOTE: The nominal step sizes for adjusting the amplifier frequency and voltage are *very* large. To alter the step size use the ◀ or ▶ buttons. To alter the current or voltage (which of these depends on configuration) use the + or − buttons.

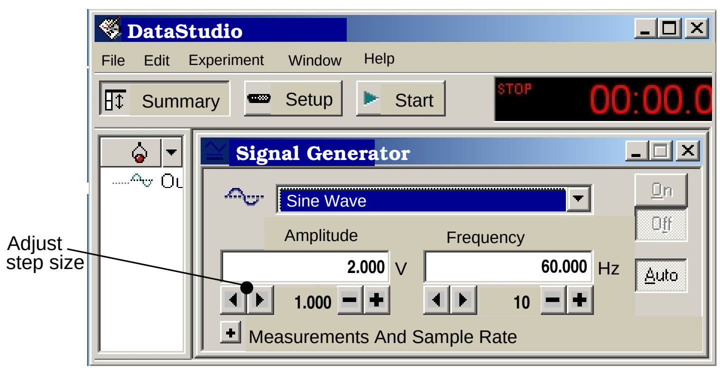


Figure 6: Sketch of the PASCO software Signal Generator window.

10. Set the amplifier frequency 0.01 Hz steps above and below resonance and record data until you achieve steady-state behavior. Repeat for 0.10 Hz and (if time permits) 0.40 Hz steps. Plot out a few representative sets of data.
11. Determine the maximum steady-state displacement of the glider for each of the measured frequencies and plot the relative amplitude squared $[(x'(\omega)/x'(\omega_0))^2]$ vs frequency offset $(\omega - \omega_0)$. Estimate the full width at half maximum for this curve. This value should be equal to R/m.
12. Discuss the nature of this resonance curve. If you adjust the R/m ratio to further sharpen the resonance curve can you identify a compensating complication if you are interested in achieving steady-state behavior? If the speaker were attached to an amplifier playing audible music (nominally 20-20,000 Hz), what do you think this the mass will do?

HC-1 The Ideal Gas Law and Absolute Zero

OBJECTIVES:

To observe the behavior of an "ideal" gas (air) and to determine absolute zero (in Centigrade).

PRECAUTIONS:

Liquid N_2 (go to the large Dewar in room 4329 Chamberlin) is fascinating to work with. However, please keep in mind the following simple safety precautions.

(1) **Never stopper a flask of liquid N_2 with an unperforated stopper. This can result in a dangerous explosion with risk of major injury. Students found attempting this will be referred to the Dean of Students for disciplinary action.**
(2) Have a perforated stopper on the Dewar throughout the experiment to prevent condensation of moisture from the air on the inside of the flask.
(3) **Avoid contact of liquid N_2 with your skin.** The insulating vapor may disappear and severe frost-bite may result.

FUNDAMENTAL CONCEPTS:

The behavior of an ideal gas under varying conditions of pressure and temperature is described by the "Ideal Gas Law":

$$PV = nRT \tag{1}$$

Where P is the pressure (in Pascals, Pa), V is the volume (in m^3), T is the temperature in Kelvins (K= C+273), n is the number of moles present (1 mole $\equiv 6.023 \times 10^{23}$), and R is the gas constant (8.31 $J/mol \cdot K$).
This equation is a combination of two laws which were discovered previously:

(1) *Boyle*'s Law relates volume and pressure of a fixed quantity of gas at a constant temperature: $PV = constant$
(2) *Charles'* Law relates volume and temperature of a fixed quantity of gas at a constant pressure $V/T = constant$

Gay-Lussac finally combined the two laws into the ideal gas law shown in Eqn. 1.
In EXPT. I Boyle's law will be verified by varying V and P (assuming fixed T).
In EXPT. II You will vary T and V to determine the value of absolute zero.

APPARATUS:

Basic equipment: EXPT. I - Lab stand; 60 cm^3 plastic syringe attached to a plastic "quick-connect" coupling.
EXPT. II - PASCO Steam generator; water jacket container; steel Dewar flask; small stainless steel can with a volume of 98 cm^3 with a "quick release" connector; FLUKE digital thermometer and temperature probe

Computer equipment: Personal computer set to the HC1 lab manual web-page; PASCO interface module; PASCO pressure sensor (mounted on lab stand).

EXPERIMENT I: BOYLE'S LAW

1. CLICK on the telescope icon below (web version only) to initiate the PASCO© interface software. The computer, monitor and PASCO interface must already be on and pressure sensor plugged into the **A** DIN connector position. (If not see your instructor.)

2. The computer screen should look something like the figure at top on the next page . In this window there is a graph configured to display pressure *vs* volume

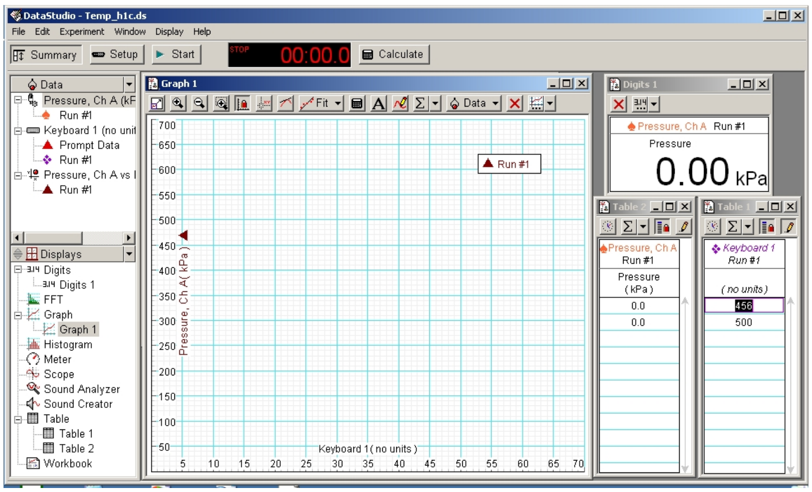


Figure 1: Image of PASCO DataStudio display.

and a table for these three values, a panel meter for the instantaneous pressure reading. Since there is no automated volume measurement you will have to enter these data points manually. After initiating the ▶ Start button it will then change to the ✔ Keep. CLICKing on this button will generate a pop-up window for data entry. To terminate the run you must CLICK on the red square immediately to the right of the ✔ Keep icon.
NOTE: The volume is recorded in *milliliters* (1 mL = 1 cm^3) and the pressure in *kilopascals* (kPa). In this case the ideal gas law ($PV = nRT$, T in Kelvin) uses $R = 8.31 \times 10^3 (\text{kPa}\cdot\text{cm}^3)/(\text{mol}\cdot\text{K})$.

3. If the syringe is attached to the pressure sensor, disconnect it at the twist-lock quick connect coupling. Set the syringe plunger to the 60 cm^3 mark and then reconnect it to the pressure sensor. Make sure that the equipment looks as sketched in Fig. 2.

4. Initiate the data acquisition by CLICKing on the ▶ Start icon. (Data entry is enabled each time you CLICK the ✔ Keep icon.)
Slowly push the plunger in to the 50 cm^3 position (about 10 seconds) and back to the original setting while watching the panel pressure meter and record in your lab-book the readout precision of the pressure sensor.

5. Now begin the actual data logging by again CLICKing the mouse on the KEEP icon. Data logging should be confirmed by observing a new data point on the plots and new values in appropriate row of the table.

6. Reduce the volume slowly in discrete 5 cm^3 increments while logging the data at each step (see the previous item) until you reach 20 cm^3. Then as a check, slowly

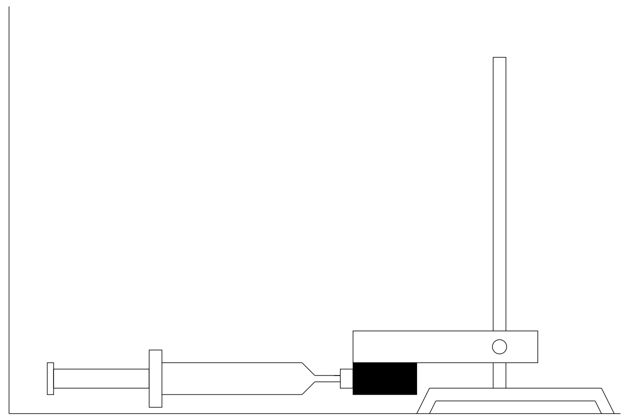


Figure 2: Sketch of pressure sensor and syringe/plunger assembly.

increase the volume in 5 or 10 cm^3 steps (remember to log the data) until you reach 60 cm^3.

7. Stop the data acquisition and transfer your data to a table in your lab write-up and comment on the reproducibility of your measurements. You should make two plots, one with respect to volume and one with respect to *inverse* volume. Do your plots have the correct functional behavior? Your can refrain from printing out the graphs until your have performed a linear regression using either the PASCO graphical analysis capabilities or Excel (or by whatever additional graphical analysis packages are available).

8. Which of the two graphs do you expect to be a straight line and why? For this "curve" what is the expected y intercept? Now fit this data to a line (using linear regression) and record the slope and intercepts with the appropriate units. Print out the respective plots and include them in your lab write-up.

9. OPTIONAL: Repeat the procedure of item 4 except move from 60 to 20 cm^3 as rapidly as possible and record the pressure data at the start and stop points. Are the pressure readings consistent with those of the "slow" moving experiment? If not can you suggest a reason?

QUESTIONS:

I. Does the line go through the origin as expected? By how much should you change the pressure readings so that the line goes through the origin?

II. What are the possible sources of error in this experiment? For each source, try to decide what effect it might have on the experimental results.

III. Read the barometer that is located in your laboratory room, ask your instructor for help if you are having trouble. Convert the pressure from cm of Hg to kPa and compare this value with the one obtained at 60 cm^3 in step 5. Is the difference between the two readings important? Does it affect your conclusions?

IV. OPTIONAL: Assuming that there are 22.4 liters per mole of an ideal gas, how well does your observed slope agree with your expected slope?

EXPERIMENT II: IDEAL GAS LAW

In this part the Ideal Gas Law is verified by observing the pressure of a fixed volume of gas at four different temperatures: Room temperature, 100 °C, 0 °C and -197 °C. Since $PV = nRT$ and $T = T_{\text{Celsius}} + T_0$ we can rewrite this as

$$P(T_c) = \frac{nR}{V} T_c + \frac{nR}{V} T_0$$

and thereby determine the value of absolute zero (T_0) from the intercept.

SUGGESTED PROCEDURE:

1. CLICK on the telescope icon below (web version) to initiate the PASCO© interface software. The computer, monitor and PASCO interface must already be on and pressure sensor plugged into the **A** DIN connector position. (If not see your instructor.)

2. Attach the small stainless steel can (volume of 98 cm^3) as shown below to the PASCO pressure sensor using the plastic "quick release" connector. Raise the assembly up if necessary. Turn on the Fluke thermometer. Unless the stainless steel can has been recently used it should now be at room temperature.

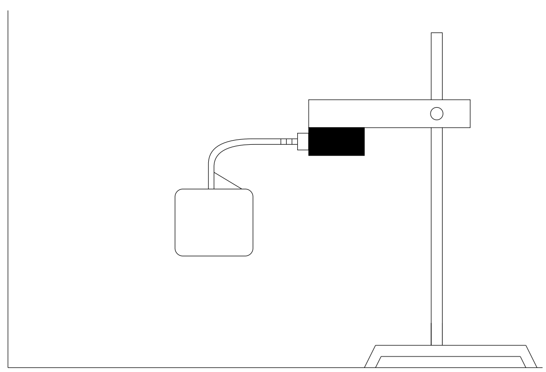


Figure 3: Sketch of pressure sensor and stainless-steel can assembly.

3. Once again initiate the experiment by CLICKing on the 

icon. There should be a single plot (pressure vs. keyboard entry or, equivalently, temperature). Record the Fluke meter reading by CLICKing on the 

icon and then entering the temperature in the keyboard entry window.

4. Fill the PASCO steam generator with water and turn it on. The level of the water should be ~ 3 *cm* below the top, or about 1/2 – 3/4 full (Fig. 3). Turn the knob to 9 until the water boils and then turn it down somewhat.
5. As soon as the water in the steamer is boiling, raise the pressure sensor stand and then lower it so as to immerse the stainless steel can in the boiling water.

6. Wait until the water is boiling again and then put the Fluke thermometer in the boiling water. Resume the data acquisition by clicking on the REC button. Watch the pressure readout and after it is stable (with time) record the Fluke temperature in the data entry window. Turn the PASCO steam generator off and remove the small stainless steel can from the bath. Pause the data acquisition.

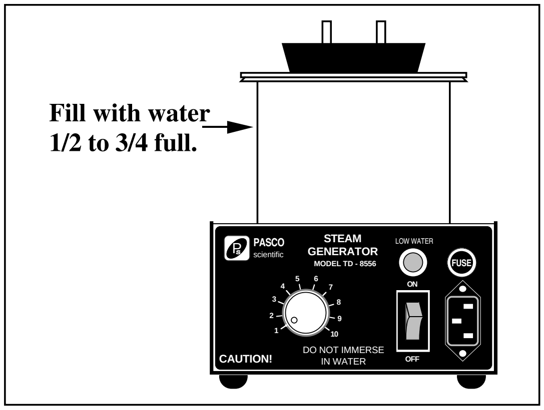


Figure 4: PASCO steam generator.

7. Fill the water jacket container with water and ice. The level of the water should be ~ 5 *cm* below the top. Repeat the last two steps for obtaining the 0 °C pressure reading. Empty and dry the stainless steel dewar.
8. Take the stainless steel dewar and ask the instructor to fill your dewar with liquid Nitrogen and repeat step 3 for the container at liquid Nitrogen temperature.
9. Stop the data acquisition and transfer your data to a table in your write-up. Does your plot have the correct functional behavior? Your can refrain from printing out the data until your have performed a linear regression using either the PASCO graphical analysis capabilities or any other analysis package.
10. For your curve what is the expected y intercept? Now fit this data to a line (using linear regression) and record the slope and intercepts with the appropriate units. Print out the plot and include it in your lab write-up.

QUESTIONS

I. What is the percentage difference between the value you found and the accepted value for absolute zero?

II. Assuming that ice water is exactly at 0 °C and the boiling water is at 100 °C, estimate the systematic error introduced into your absolute zero measurement? Does this improve your results?

III. OPTIONAL: Qualitivity what error results from the gas in the small tube not being always at the can temperature?

IV. OPTIONAL: Does thermal expansion of the can affect your results? In what way?

H-2 Latent heat of fusion of ice

OBJECTIVE: To measure the latent heat of fusion, L_f, of ice.
APPARATUS:

Chrome plated brass calorimeter (cup), brass stirrer, water jacket for thermal ballast, digital thermocouple thermometer, ice bucket; ice (inside room 4329 Chamberlin Hall), double pan balance; 400 ml glass beaker; coffee pot for hot water, selection of slotted masses.

SUGGESTED PROCEDURE:

1) Find the mass of the calorimeter plus stirrer.

2) Add ~300 g of water at temperature T as far above jacket T as one expects the final T will be below it, (thus minimizing heat exchange with the environment). For this estimate assume that one will add 60 g of ice. **Record the mass of the water**.

3) Record the water temperature in the calorimeter each minute for several minutes while gently stirring.

4) Gently add (without splashing!) the ~60 g of ice in one or a few pieces after carefully drying each piece with a paper towel. Continue recording the temperature each minute until five minutes after it begins a slow rise.

5) Record the final mass of calorimeter plus contents. **Deduce the mass of ice added**.

6) Plot the temperature vs time as recorded in 3) and 4).

7) From the data calculate L_f of ice. (You may neglect the heat supplied by the thermocouple type digital thermometer.)

8) Estimate quantitatively the error in L_f. [Recall that **absolute** (not relative) errors add when you add or subtract, whereas **relative** errors add when you multiply or divide.

SPECIFIC HEAT CAPACITIES
(in kcal/kg/K or cal/g/K)

water	1.00	glass	0.199
brass	0.090	Hg	0.033

H-3 Latent heat of vaporization of liquid-N_2

OBJECTIVE:

To measure the heat of vaporization of liquid nitrogen, L_v, at its boiling point (T_b = 77 K at standard atmospheric pressure).

APPARATUS:

Dewar flask; liquid nitrogen (ask for help in getting it from a large storage Dewar in room 4329 Chamberlin Hall); aluminum cylinder on a long thread; double pan balance; calorimeter plus water jacket for thermal ballast (as in H-2a); timer; thermocouple type digital thermometer; selection of slotted masses; coffee pot for hot water.

PRECAUTIONS:

Liquid N_2 is fascinating to work with. However, keep in mind the following simple safety precautions.

1. **Never stopper a flask of liquid N_2 with an unperforated stopper. This can result in a dangerous explosion with risk of major injury. Students found attempting this will be referred to the Dean of Students for disciplinary action.**
2. Have a perforated stopper on the Dewar throughout the experiment to prevent condensation of moisture from the air on the inside of the flask.
3. **Avoid prolonged contact of liquid N_2 with your skin.** The insulating vapor layer may disappear and severe frost-bite may result.

INTRODUCTION:

When one lowers an aluminum cylinder of mass, m_{Al}, and at room temperature, T_r, into liquid N_2 at its boiling temperature, T_b, the cylinder cools to T_b. The heat given off during this cooling, Q_{Al}, will vaporize a mass m_N of liquid N_2.

You might expect to find L_v, of the nitrogen by setting

$$m_N L_v \ = \ Q_{Al} \ = \ m_{Al} c_{Al} (T_r - T_b) \ \ .$$

This method fails because c_{Al} is not constant over the $\sim 220°$C temperature range between T_r and T_b. See figure 1.

We can avoid this difficulty by noting that Q_{Al} is also the heat needed to warm the same aluminum cylinder to from T_b to T_r. You can measure this heat by placing the cold aluminum cylinder (at temperature T_b) in a "calorimeter" that contains water and observing the change in temperature of the water, $-\Delta T$, –provided that the final temperature of the water, T_f, is room temperature, T_r. It is hard to arrange for T_f to end up exactly at room temperature, but if T_f is *close* to T_r, one can accurately correct the calorimeter data for the small additional heat term, namely $m_{Al} c_{Al} (T_r \ - \ T_f)$, since over the small $T_r \ - T_f$ interval, $c_{Al} = 0.212$ cal/g/K is constant.

SUGGESTIONS ON PROCEDURE:

1. To maximize sensitivity (i.e. to get a large temperature change in the water) use only enough water (~125-150 grams) in the calorimeter to cover the metal cylinder.

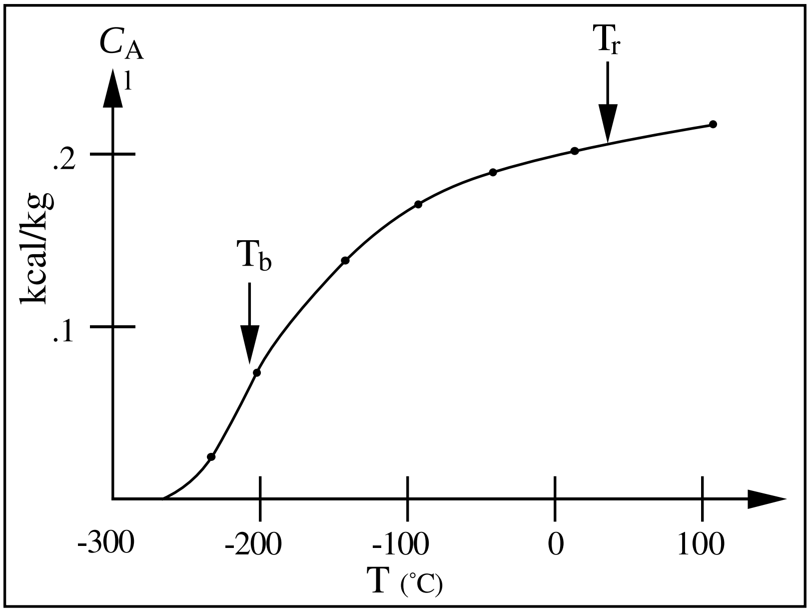


Figure 1: Specific heat capacity of Al vs. temperature.

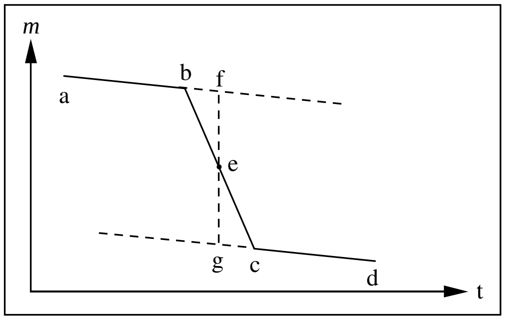
2. The calorimeter is designed to thermally isolate the water from the surroundings. The inner vessel of the calorimeter is mounted within, but thermally isolated from, a surrounding water jacket, which is close to room temperature. The isolation isn't perfect, so here will be some small amount of heat flow between the calorimeter and the jacket. To minimize the *net* heat exchange with the water jacket, you will want to make the initial water temperature as far *above* the jacket temperature as you expect it to end up *below* the jacket temperature after the water in the inner vessel has been cooled by your Al cylinder. (The jacket temperature will remain fairly constant during the experiment.) Estimate roughly the proper initial water and calorimeter temperature. [Use the measured mass of the aluminum cylinder, m_{Al}, its specific heat (0.212 kcal/kg°C) and the b.p. of liquid N_2, $T_b = 77$ K. For this rough calculation, assume that the specific heat of Al *is* constant with temperature.] Determine the water mass, m_w, and appropriate starting temperature.

3. Place the flask containing liquid nitrogen (plus perforated stopper) on one pan of a balance and record the mass each minute for 10 minutes. (Why is the mass decreasing?)

4. Record the temperature of the metal cylinder, T_R, and then lower it (by a thread) gently to the bottom of the flask. Replace the stopper (perforated) on the top of the flask and continue recording the total mass each minute until it shows a slow steady decrease.

5. Record the initial temperature, T_i, of the calorimeter, which you have chosen so cleverly in section 2. Transfer the cold metal cylinder into the calorimeter, and note the calorimeter temperature every two minutes (while gently stirring). Record the mass of the flask of liquid N_2 on alternate minutes. When the calorimeter temperature has reached a slow steady rate of change and the mass of the flask of liquid N_2 is falling at a slow steady rate, discontinue the readings.

6. Plot the mass of the flask plus nitrogen as a function of time. For the minutes that the cylinder of metal was in the flask, subtract the mass of the cylinder. See figure for a typical plot.



How much of the N_2 mass change was caused by the heat from the cylinder? If a-b and c-d were parallel, it would be the vertical distance between these lines. But c-d ordinarily has a smaller slope than a-b, possibly because the evaporation of liquid nitrogen between b and c has cooled the upper part of the flask.

Hence we use the average of the two rates of fall by drawing a vertical line through e, (the midpoint of line b-c). Then f-g estimates the mass, m_N, evaporated by the heat from the cylinder.

7. The final temperature, T_f, of the Al cylinder in the calorimeter usually will not be quite the same as the initial temperature ($= T_r$) of the cylinder before it was lowered into the liquid nitrogen.

Hence Q_{Al} will be the heat to warm the Al cylinder in the calorimeter to T_f *plus* the mass of the cylinder × (specific heat of aluminum) × ($T_r - T_f$).

Specifically if:

L_v = latent heat of vaporization of nitrogen
m_N = mass of nitrogen evaporated by heat from the cylinder
m_{Al}, c_{Al} = mass and specific heat of Al cylinder
T_i = initial temperature of water and calorimeter
m_w, c_w = mass and specific heat of water
m_c, c_c = mass and specific heat of calorimeter and stirrer
h_t = heat capacity of immersed part of thermometer,

then, if we neglect h_t:

$$Q_{Al} = m_N L_v = (m_w c_w + m_c c_c)(T_i - T_f) + m_{Al} c_{Al}(T_r - T_f) \ .$$

Calculate L_v from the above relation. The accepted value is 47.8 kcal/kg.

OPTIONAL:

1. Calculate the **apparent** specific heat of the Al block by use of

$$c_{Al} = \frac{Q}{m_{Al}(T_r - T_b)}$$

and your data. How does your result compare with the accepted value of $c_{Al} = 0.212$ kcal/kg? Explain. (See Introduction).

2. Observe (but do not touch) the following items after immersion in liquid N_2: rubber (get a piece from the instructor), pencil eraser.

3. Pour a little liquid N_2 onto the floor. Explain the behavior of the small spheres of liquid N_2.

SC-1 Transverse Standing Waves on a String

OBJECTIVE: To study propagation of transverse waves in a stretched string.

INTRODUCTION:

A standing wave in a string stretched between two points is equivalent to superposing two traveling waves on the string of equal frequency and amplitude, but opposite directions. The distance between nodes (points of minimum motion) is one half wavelength, $(\lambda/2)$.

The wave velocity, v, for a stretched string is $v = \sqrt{F/\mu}$ where **F = tension in the string** and **μ = mass per unit length**. But v = fλ and hence

$$f = \frac{\sqrt{F/\mu}}{\lambda} \quad . \tag{1}$$

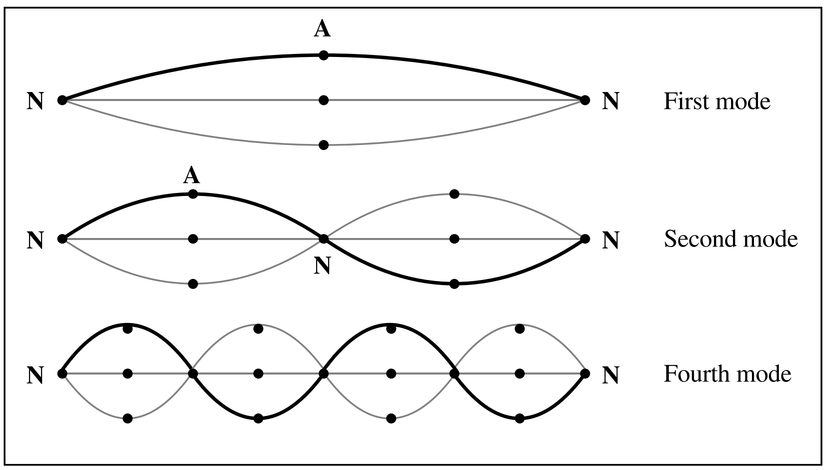


Figure 1: The Modes of a String

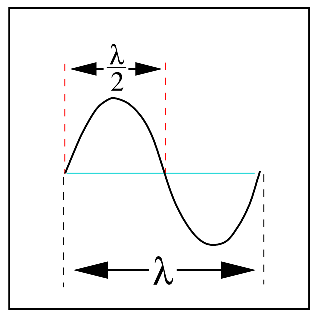


Figure 2: A close-up

PART A: Waves from a mechanical driver (i.e. a speaker)

APPARATUS:

Basic equipment: Electrically driven speaker; pulley & table clamp assembly; weight holder & selection of slotted masses; black Dacron string; electronic balance; stroboscope.

Computer equipment: Personal computer; PASCO© interface module; power amplifier module; various electrical connectors.

The set-up consists of an electrically driven speaker which sets up a standing wave in a string stretched between the speaker driver stem and a pulley. Hanging weights on the end of the string past the pulley provides the tension.

The computer is configured to generate a digitally synthesized sine wave (in volts versus time) with adjustable frequency and amplitude (max: $\sim$10 V).
PASCO interface: This transforms the digital signal into a smooth analog signal for input into the power amplifier.

Power amplifier: The amplifier transforms the voltage sine wave single into a current suitable to drive the loudspeaker. (A few exotic speakers, often referred to as electrostatic speakers, actually utilize high voltages directly to produce sound.)

Precautions: Decrease the amplitude of the signal if the speaker makes a rattling sound, or if the red pilot light on the amplifier is lit. The generator is set to produce sine waves; **do not change the waveform**.

Note: Although the speaker is intended to excite string vibrations only in a plane, the resultant motion often includes a rotation of this plane. This arises from nonlinear effects since the string tension cannot remain constant under the finite amplitude of displacement. [See Elliot, **Am. J Phys.** 50, 1148, (1982)]. Other oscillatory effects arise from coupling to resonant vibrations of the string between pulley and the weight holder; hence keep this length short.

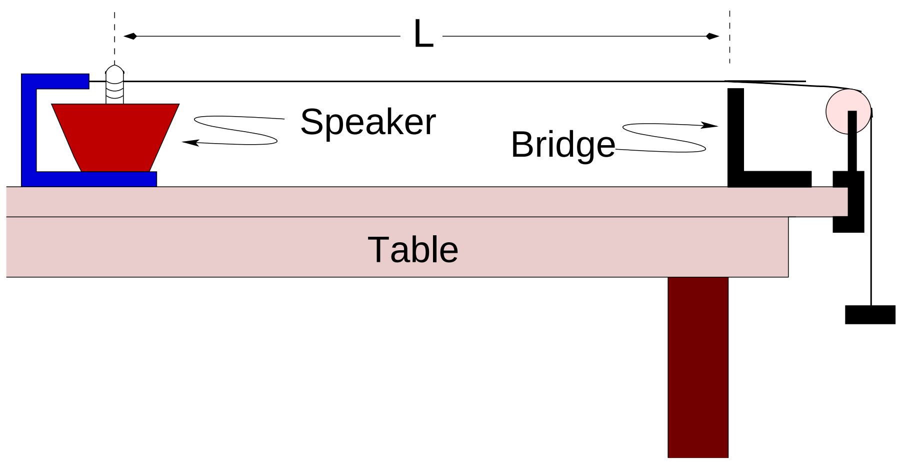


Figure 3: The apparatus

SUGGESTED EXPERIMENTS:

PROCEDURE I: Checking Equation (1)

1. Place the sheet of paper provided on the table; this will make it easier to see the vibration of the string. Measure accurately the distance, L, between the bridge and the pin of the speaker using the two meter ruler; record this in your lab notebook. Click on the LAUNCH EXPERIMENT icon (i.e., the telescope), from the on-line lab manual. The computer monitor will appear as shown in Fig. 4.

2. You will see that the computer is set to produce a 60 Hz sine wave with an amplitude of 2 V. To start the string vibrating **CLICK** the “ON” button.

3. **CLICK** on the up/down arrow in order to change the amplitude or the frequency of the signal although this produces rather large steps. NOTE: The nominal step sizes for adjusting the amplifier frequency and voltage may be much too large. To alter the step size use the ◀ or ▶ buttons. To alter the current or voltage (which of these depends on configuration) use the + or - buttons. You can also change the value directly by CLICKing the mouse cursor in the numeric window and entering a new value with keyboard number entry.

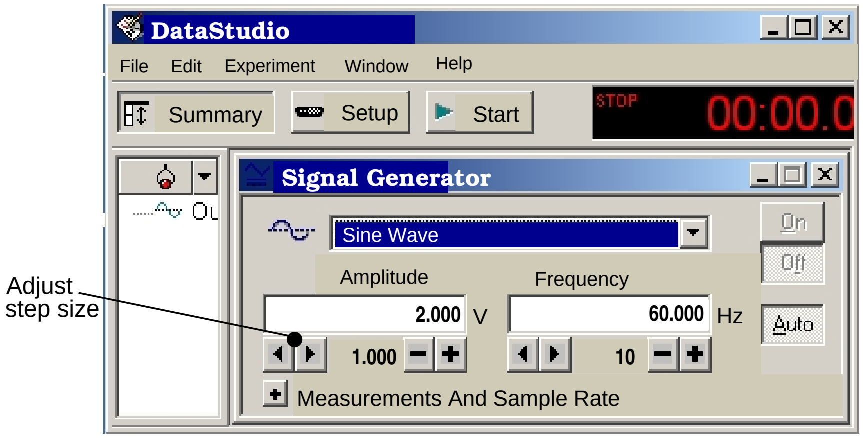


Figure 4: The PASCO DataStudio display.

4. At 60 Hz check eqn. 1 by first calculating the necessary string tension to produce a standing wave in the third or fourth mode. Weigh the string to get μ. Your instructor will provide you with a one meter length of string. (Dacron 30# has $\sim$ 0.283 g/m.) Note that the hanger itself has a 50 g mass so it may not be easy to access the forth mode (depending on L).

   Check your results by adjusting the string tension by increasing/decreasing the weight to find the tension which results in the largest amplitude vibrations. How do the two values (calculated and measured) compare?

5. Now put a 200 g mass on the mass hanger and restart the signal generator. Record the total mass and tension in your lab book.

6. Adjust the frequency so that the amplitude of the oscillation is at its maximum by changing the frequency in 1 Hz steps. This is best done as follows: First decrease the frequency until the amplitude of the string is very small.
   Then increase the frequency in 1 Hz steps, observe that the amplitude first increases and then decreases. Record the best frequency f_2 in your table.

7. Change the frequency to observe the third mode. Find and record the best frequency (using 10 Hz steps at first may be faster).

8. Find and record the frequency of the higher modes.

9. OPTIONAL: Check the frequency f of the string in its 2nd mode with the stroboscope. Note that the stroboscope is calibrated in **RPM** or cycles per minute, NOT Hz (cycles per second). You should find a value close to 70 Hz.

ANALYSIS:

1. Divide the various frequencies f_n by n and enter the values in a table. Calculate the average value of f_n/n; this is the expected value of the frequency of the first mode.

2. Calculate the velocity of propagation on the string using the appropriate equation.

3. Calculate the mass per unit length of the string. How do the two values for the string mass per unit length compare?

PROCEDURE II: f_n vs string tension

In this section you will investigate the dependence of the resonant frequency of a string as a function of the applied tension.

1. Choose six masses between 100 gm and 1 kg and enter the values in the data table.
2. Determine the resonant frequency of the second mode of the string under these different tensions and record your results. (Hint: increasing the mass by a factor of two increases f_n by nominally a factor of $\sqrt{2}$.)
3. Plot a graph of frequency versus mass, m, and include the zero value.
4. Plot a graph of frequency versus $\sqrt{m}$ and again include the zero value.

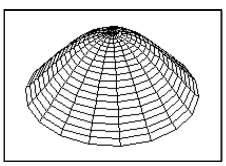
QUESTIONS:

1. Which of the two graphs can be fitted with a straight line? A parabola? Why?
2. From the slope of the graph having the linear relationship obtain the mass per unit length of the string and compare to your previous result.

PART B: "Virtual" waves on a drum head

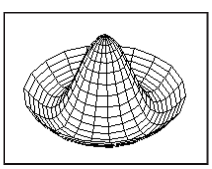
PROCEDURE III: (If time permits)

Vibrations of a circular drum head. In this section you will examine, via a virtual demonstration, the vibrational modes of a two dimensional drum head.



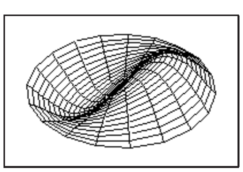
The [0,1] mode.

1. Click on the icon at left to down-load and initiate the MPEG movie viewer to observe the "first" mode.
2. Use the replay and step frame functions to view the motion.
3. Where is the displacement at a maximum? Always at a minimum?



The [0,2] mode.

1. Click on the icon at left to down-load and initiate the MPEG movie viewer to observe the first of the two "second" modes.
2. Use the replay and step frame functions to view the motion.
3. Where is the displacement at a maximum? Always at a minimum?



The [1,1] mode.

1. Click on the icon at left to down-load and initiate the MPEG movie viewer to observe the second of the two "second" modes.
2. Use the replay and step frame functions to view the motion.
3. Where is the displacement at a maximum? Always at a minimum?

JAVA APPLET:

If time permits and you are interested the web version of the lab has a link to an applet ../java/ph14e/stwaverefl.htm which animates transverve 1D motion for a propagating wave incident on a fixed or or a free boundary.

S-2 Velocity of Sound in Air

OBJECTIVE:

To calculate the velocity of sound from measurement of the wavelength in air for sound of a certain frequency.

APPARATUS:

Resonance tube with arrangement for varying water level (use only distilled water); rubber tipped hammer; tuning fork; Hg thermometer.

INTRODUCTION:

For a closed tube, resonance occurs at tube lengths of an odd multiple of one-fourth wavelength, i.e. at $\lambda/4$, $3\lambda/4$, $5\lambda/4$ etc.

SUGGESTIONS:

1. Find the positions of the water level in the tube for the first three of these resonances. Use these readings to calculate the speed of sound, $v = |\vec{v}|$. Initially have enough water that you can raise the level above the first resonance position. The tuning fork frequency is on the fork.

   Since the effective end of the resonance tube is not at the tube's end, **do not use the position of the tube's top** in your calculations, but rather **take differences between the other readings**.

2. Sound waves in gases have a speed $v = \sqrt{\gamma RT/M}$. (Recall the formula for the speed of sound on a string, $v = \sqrt{T/\mu}$ (e.g., Lab SC-1)). Correct your value of v to 0°C (T = 273.16 K) and compare with that accepted for **dry** air at 0°C: 331.29 ± .07 m/s, [Wong, *J. Acoust. Soc. Am.*, **79**, 1559, (1986)]. For humid air see 3. below.

3. We quantify proportions in gas mixtures by the pressure each gas contributes to the total pressure. This is called the "partial" or "vapor" pressure. Think of the speed as resulting from an average $<\gamma/M>$,

   $$<\gamma/M> \;=\; \left[(\gamma_a/M_a)P_a \;+\; (\gamma_w/M_w)P_w\right]/(P_a + P_w)\,,$$

   so that

   $$v_{dry} \;\cong\; v_{humid}\sqrt{(\gamma_a/M_a)/<\gamma/M>}\,,$$

   where $\gamma_{air} = 1.40, \gamma_w = 1.33, P_a$ is the partial pressure of air, P_w is the vapor pressure of water, $M_a \sim 29$ kg and $M_w = 18$ kg.

   How should the v.p. of water, P_w, in the tube affect the speed?

   OPTIONAL: Humidity changes will affect tuning of what musical instruments?

4. What effect does atmospheric pressure have on the velocity of sound in dry air? (Assume air at these pressures is an ideal gas.)

5. Viscosity and heat conduction in the tube may reduce **v** by ~0.1%. See N. Feather, "The Physics of Vibrations and Waves", Edinburgh Univ. Press, (1961), p. 110-120; this reference also has a delightful historical account (including Newton's famous goof).

Appendices

A. Precision Measurement Devices

Vernier Calipers:

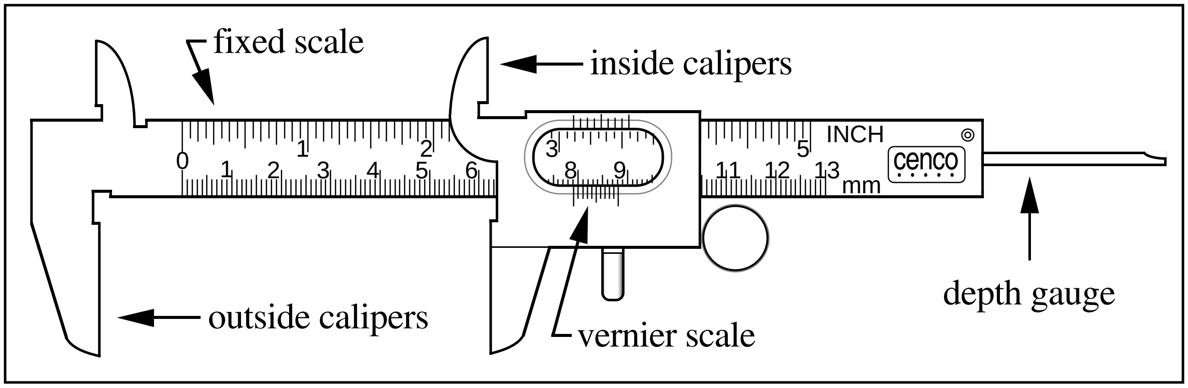


Figure 1: The vernier caliper

A Vernier consists of a fixed scale and a moving *vernier* scale. In a metric vernier the fixed scale is marked in centimeters and millimeters, the vernier scale is nine millimeters long, and is divided into ten parts each 0.9 millimeters long. The distances of each line from the first are therefore 0.9, 1.8, 2.7, . . . ,mm or generally: $d_i = 0.9 \times i$, where d_i is the distance between the zero line and the i^{th} line of the vernier scale. If the vernier caliper is closed, so that the two jaws touch each other, the zero of the fixed scale should coincide with the zero of the vernier scale. Opening the jaws 0.03 $cm = 0.3$ mm will cause the fourth line (the *three* line which is a distance of 2.7 mm from the zero line of the of the vernier scale) to coincide with the 3 mm line of the fixed scale as shown below.

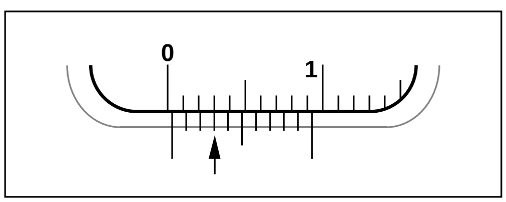


Figure 2: The vernier reads 0.03 cm

Below is another example of vernier reading; the arrow shows which mark on the vernier scale is being used.

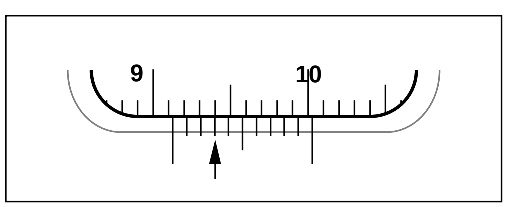


Figure 3: The vernier reads 9.13 cm

EXERCISES:

A. Close the vernier and observe that the first vernier mark coincides with the zero of the centimeter scale.

B. Open the jaws of the vernier very slowly and observe how the different vernier marks coincide successively with the millimeter marks on the fixed scale: the first mark coincides with the 1 *mm* mark on the fixed scale; then the second mark coincides with the 2 *mm* mark on the fixed scale; then the third mark coincides with the 3 *mm* mark on the fixed scale and so on.

C. Estimate the dimension of an object using a meter stick and then Use the vernier caliper to measure the dimension precisely.

D. In the four examples of Fig. 4 determine the actual reading.

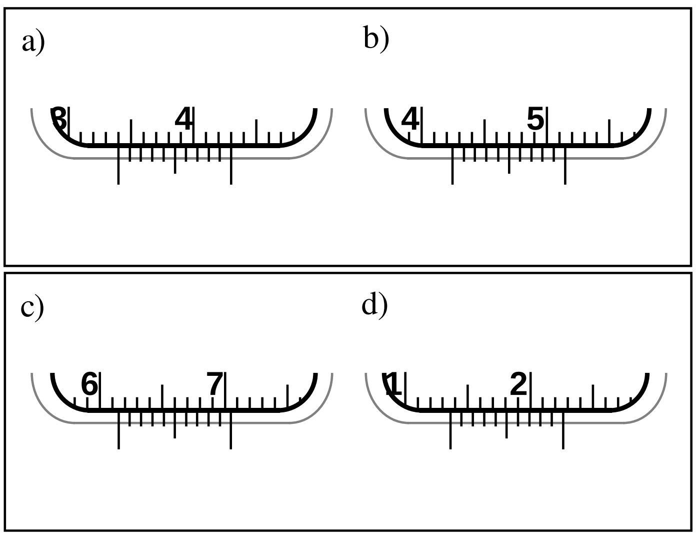


Figure 4: Test cases

Micrometer:

A micrometer can measure distances with more precision than a vernier caliper. The micrometer has a 0.5 *mm* pitch screw, this means that you read millimeters and half millimeters along the barrel. The sleeve is divided into 50 divisions corresponding to one hundredth of a millimeter (0.01 *mm*) or 10 μ each. The vernier scale on the micrometer barrel has ten divisions, marked from 2 to 10 in steps of two. The "zero" line is not marked '0', but is longer than the others. The vernier allows you to read to the nearest thousandth of a millimeter, *i.e.*, to the nearest micron (0.001 $mm = 1\ \mu$).

Precaution: Great care must be taken in using the micrometer caliper; A ratchet knob is provided for closing the caliper on the object being measured without exerting too much force. Treat the micrometer with care, **ALWAYS** close the calipers using the ratchet knob, this prevents tightening the screw too strongly. Closing the calipers too hard damages the precision screw.

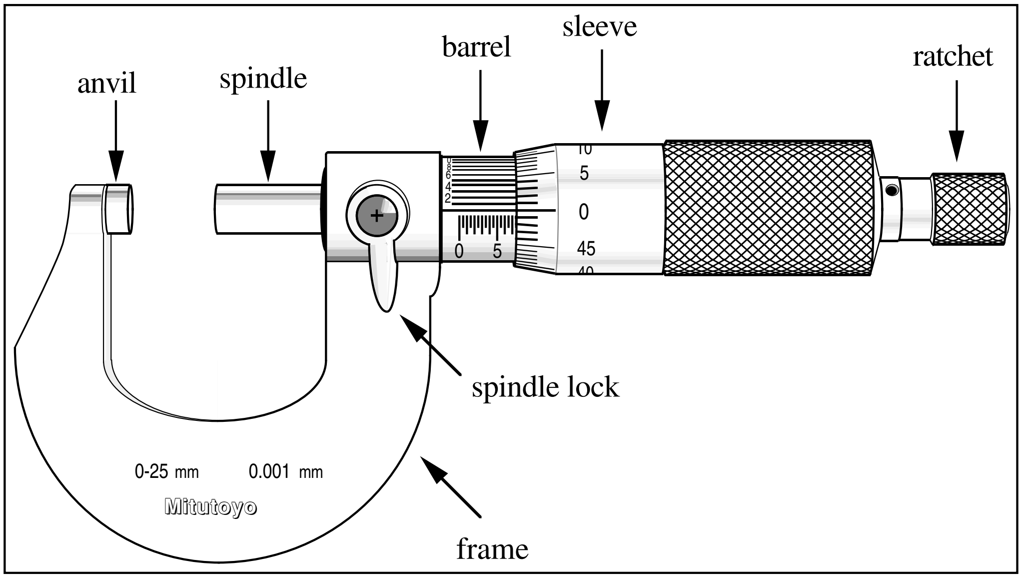


Figure 5: The micrometer calipers

Below are two examples of micrometer reading; the arrow shows which mark on the vernier scale is being used.

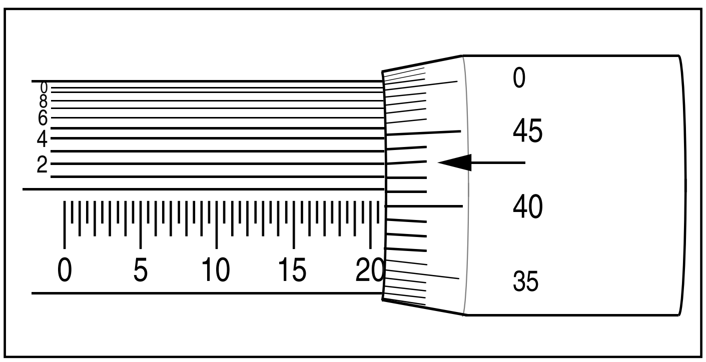


Figure 6: The micrometer reads 20.912 *mm*

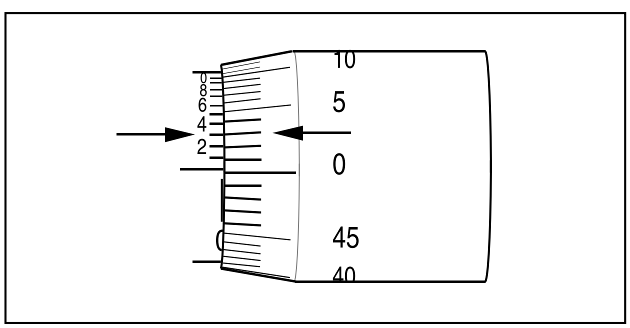


Figure 7: The micrometer reads 3 μ

In Fig. 7 the zero line on the barrel is barely visible, and the vernier reads 0.003 *mm* = 3 μ; the zero error is $\epsilon_0 = 3\mu$.

A negative zero error, as shown below requires a moment of thought.

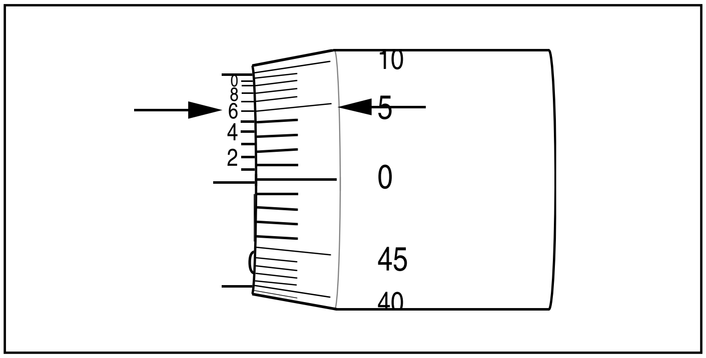


Figure 8: The micrometer reads -4 μ

In Fig. 8 the zero line on the barrel of the micrometer is obscured by the sleeve, (the "zero" line on the sleeve is above the "zero" line on the barrel) this corresponds to a reading of -0.5 *mm*; the vernier reads 0.496 *mm* the zero error is then $\epsilon_0 = -0.5 + 0.496 = -0.004mm \; = -4 \; \mu$.

B. The Travelling Microscope

The sliding carriage of the traveling microscope rides on carefully machined ways, pushed by a nut under the carriage which rides on the micrometer screw. The nut must not fit tightly on the screw or it will bind; hence there is always some slack built into the mechanism.

When the nut is being pulled to the right (dial being turned toward larger numbers), the screw threads will press against the threads in the nut as shown in Fig. 1, with the screw threads in contact with the back side of the threads on the nut. When the direction of turning reverses, the screw threads then push on the front side of the nut threads.

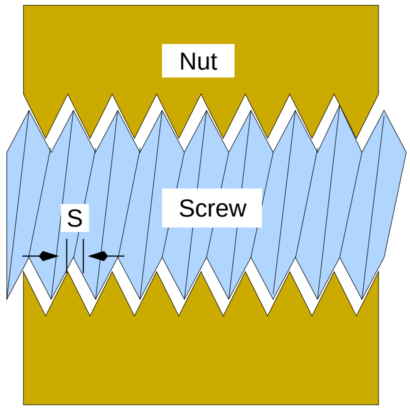


Figure 1: Backlash in screw mechanism.

For the microscope set initially on the same line for both directions of motion, the readings will differ by distance S, the backlash (slop) in the mechanism.

One way to avoid trouble with this slack is always to make settings after turning the screw more than the slack in one direction, say the direction of increasing readings. If one overshoots on a reading, go back by more than the slack and then turn forward again. The screw will then always press on the same side of the nut and no error arises.

A much better experimental technique is to take readings both ways. Suppose one wants to measure the distance between two lines, 1 and 2. Call the reading turned toward larger readings on line 1, D_1 and when turning in the reverse direction, R_1; similarly for D_2 and R_2. Then the distance between the lines will be $D_2 - D_1$ and also $R_2 - R_1$ so that one has immediately two independent readings to compare. More important, $D_1 - R_1$ is the slack in the mechanism; it should equal $D_2 - R_2$ and should be the same for all pairs of readings. If $D - R$ changes by more than the experimental error in setting, you know immediately you have made a blunder in either setting or reading and can immediately repeat the measurement. The constancy of D - R is actually an excellent measure of the uncertainty in the measurements you are taking.

C. The Optical Lever

An optical lever is a convenient device to magnify a small displacement and thus to make possible an accurate measurement of the displacement. Experiment M-11, Young's modulus, uses an optical lever to magnify the extension of a wire produced by a series of different loads.

The plate P carries a mirror M. The mirror mount has two points resting in a fixed groove, F, and at the other end has a single point resting on the object whose displacement one is measuring. Raising the object through a distance ΔL will tilt the mirror through an angle θ or $\Delta L/d$ radians (approximately) but will turn the light beam through an angle 2θ.

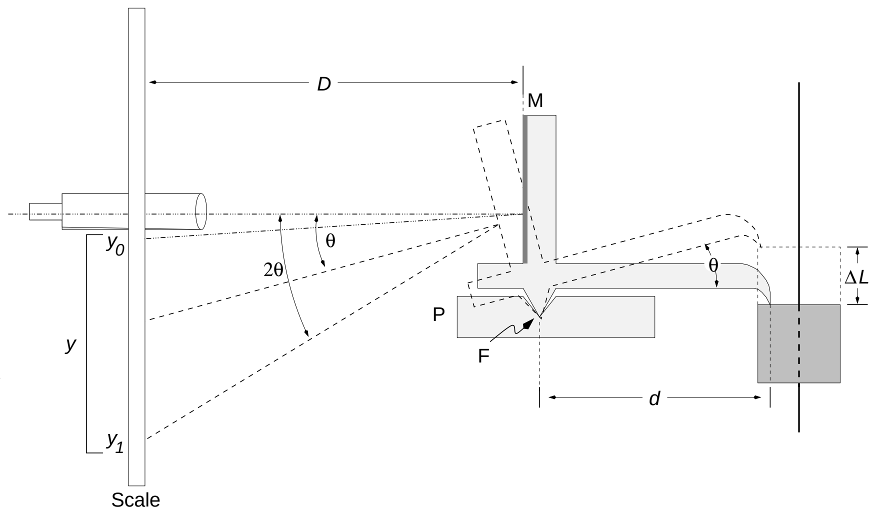


Figure 1: Schematic of the optical lever.

Hence

$$\theta \;=\; \frac{\Delta L}{d} \sim \frac{\frac{1}{2}(y_1 - y_0)}{D}$$

if θ is small so that $\theta \sim \tan\theta$. Therefore

$$2\theta = \frac{2\Delta L}{d} = \frac{y_1 - y_0}{D}, \text{ and } \Delta l = (y_1 - y_0)\left[\frac{d}{2D}\right] \quad .$$

Note that with the telescope nearly perpendicular to the scale at the beginning then y_0 is close to the telescope, and the difference between two elongations $(\Delta L_2 - \Delta L_1)$ is very accurately given by

$$\Delta L_2 \; - \; \Delta L_1 = \frac{y_2 \; d}{2D} - \frac{y_1 \; d}{2D} = \frac{(y_2 - y_1)d}{2D}$$

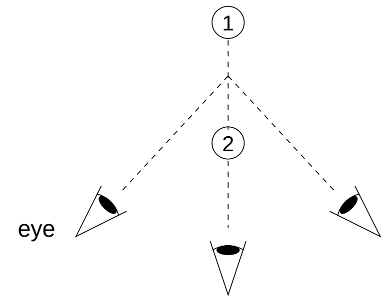
where y_i is the scale reading. This relation holds so long as 2θ is small enough that $\tan 2\theta \;\sim\; 2\theta$.

D. PARALLAX and Notes on using a Telescope

1. *PARALLAX*:

To do quantitative work in optics one must understand *parallax* and how it may be eliminated. PARALLAX is defined as *apparent motion of an object caused by actual motion of the observer*.

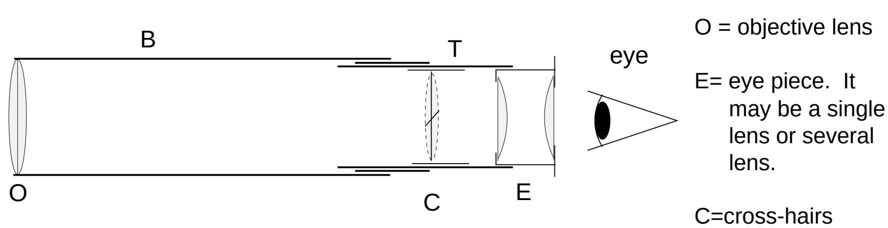
When the observer's eye is in position 1, objects 1 and 2 are in line and may appear to coincide. If the eye is moved to the left to position two, object 1 (i.e. O_1) appears to move to the left with respect to object 2, (i.e. O_2). If the eye moves to the right to position 3, object 1 (O_1) appears to move to the right with respect to object 2 (O_2).



As object 1 moves toward position 2 along the dotted line, its apparent displacement with respect to object 2 caused by motion of the eye from 2 to 3 gets smaller until it vanishes when O_1 and O_2 coincide. When O_1 gets closer to the eye than O_2, the direction of its apparent displacement reverses for the same eye motion. In short, the object farthest from the eye apparently moves in the same direction as the eye. Try this with two fingers.

Note that if O_1 is an image and O_2 a cross hair, the absence of parallax shows that the cross hairs are in the plane of the image.

2. *Focusing a Telescope for Parallel Rays*:



The eyepiece E slides back and forth in the tube T and one should first adjust the eyepiece to give a clear image of the cross hairs. Then move the tube T back and forth in the barrel B until the image of a distant object, formed by the objective O, falls on the plane of the cross hairs. The test for this is the absence of parallax between the cross hairs and image.

The rays from a distant object are nearly parallel. For viewing a distant object, use an open window if the window glass is not accurately plane. Otherwise poor image formation may result. You can check by trying it both ways. At night use a distant object in the hallway.

The telescope, now focused for parallel rays, will stay so as long as the distance between O and C is unchanged. One may still adjust the eyepiece position to suit the observer.

3. *Finding an Image in a Telescope*:

If you have trouble finding an image in a telescope, locate the image first with your unaided eye, and then pull the telescope in front of your eye, aligning it with your line of sight. With high magnification it is difficult to find the image in the telescope because the alignment must be nearly perfect before the image appears in the field of view. The eye has a rather large field of view so that the image will be visible over a range of positions.

E. Notes on Radiation Dosage, Dosimetry, and the Radon Problem

INTRODUCTION:

The biological effects of radiation arise from the absorption of the radiant energy to produce heat, electronic excitation and/or ionization.

Radio, T.V., microwave, visible light, u.v. light, x-rays, γ-rays are all electromagnetic radiation and differ only in wavelength. Electromagnetic radiation may have both beneficial and harmful effects: e.g. u.v. light absorbed by the skin can supply needed vitamin D, but excess u.v. radiation accounts for much skin cancer. X-rays and γ-rays are more penetrating and so can affect tissue below the skin.

Besides electromagnetic radiation one has high velocity charged (and neutral) particles. From *naturally* radioactive materials, the charged particles are either high speed electrons (β's, beta rays) or alpha particles (α's, the nuclei of helium atoms). Both types are rather easily stopped by a small thickness of matter e.g. ~ 1.8 mm Al stops 1.17 MeV β's from RaE, and ~ 0.06 mm Al stops 5.3 Mev α's from Po. Hence natural radioactivities are normally of little concern unless the parent nucleus has been inhaled or ingested in the body. The biological effects of neutral particles (e.g. neutrons and neutrinos) from naturally radioactive materials are normally negligible.

Radioactivity unit: 1 Becquerel (Bq) = 1 disintegration/s $\cong$ 27 pCi (picoCurie)

Radiation dose is the radiant energy absorbed per unit mass.

Dose UNITS: 0.01 J of radiation absorbed/kg of mass = 1 rad
1 J of radiation absorbed/kg of mass = 1 gray (abbreviated **Gy**)

Dose Equivalent: includes the long term *relative biological effects of different types of radiation.* The original unit was the **rem** (**r**ad **e**quivalent **m**an), but the now recommended S.I. unit is the **sievert** (abbreviated Sv) with:

$$1 \text{ Sv} = 100 \text{ rem} = 10^5 \text{ mrem}$$

and so: $1 \text{ mSv} = 100 \text{ mrem}$

Federal laws on permissible doses are:

1. For workers, < 50 mSv/year for a whole body dose, but employer must follow the ALARA (**A**s **L**ow **A**s **R**esonably **A**chievable) principle. For the hands alone, 750 mSv/yr are allowed.

2. For the general population < 5 mSv/yr whole body dose.

RADIATION SOURCES:

Besides the sun's u.v. radiation, the natural environment contributes an unavoidable dose equivalent to ~ 1.3 mSv/yr plus a variable dose from inhaled radon which often is several times larger: see **The Radon Problem** (below). **Hence the average natural background radiation dose is $\sim$ 3 mSv/yr**. About .25 mSv/yr of this dose comes from internal radioactivities in the body (chiefly ^{40}K which constitutes 0.0119% of natural K and has $T_{1/2} \sim 10^9$ years). The rest comes from external natural radioactivities in the earth (chiefly decay chains of uranium and thorium, $T_{1/2} \sim 4 \times 10^9$ years and $T_{1/2} \sim 10^{10}$ years) and from cosmic rays. The cosmic ray contribution increases with altitude and is $\sim$ 30 mSv/yr at 40,000 feet elevation (jet airplane altitudes).

Brick and stone houses often have larger backgrounds. Living in Denver (elevation 5200 ft.) contributes an additional $\sim$ 0.7 mSv/yr. In the Kerala region of India and the Espirito Santo region of Brazil, natural sources give $\sim$ 30 mSv/yr with no obvious abnormality resulting to the indigenous population so the Federal regulations seem very conservative for the general population.

Man-made radiation exposure averages $\sim$ .7 mSv/yr and comes almost exclusively from medical and dental x-rays. A single dental x-ray may involve 7 mSv to the skin. Exposures from nuclear power generating stations are nearly zero. In fact, per KWH of electricity generated, the radioactivity released from coal fired plants is often high compared to that permitted from nuclear plants since many coals contain appreciable uranium and/or thorium plus the equilibrium decay products from these long-lived radioactive nuclei.

For perspective on radiation exposure, Prof. Cameron formerly of the UW Medical Physics Dept. suggests translating doses into a natural unit, the **BERT** defined as the **B**ackground **E**quivalent **R**adiation **T**ime. Thus a **BERT equal to 1 yr would correspond to 3 mSv** (see above discussion on average background dose).

THE RADON PROBLEM:

Radon (an inert gas) from decay of naturally occurring U and Th in the earth continually diffuses into the atmosphere and may cause $\sim$ 10,000 lung cancer cases per year in the U.S. The radon content of outdoor air 1 meter above ground typically gives 4 to 15 becquerels/m^3 The health effects come mainly from inhalation of ^{222}Rn (from U)since this radon isotope has a $T_{1/2}$ of 3.82 days whereas the thorium radon isotope (^{220}Rn) has $T_{1/2}$ of only 56 seconds. The indoor air concentration of radon ($\sim$ 50 Bq/m^3) varies perhaps a factor of a thousand from location to location, depending upon the U content and physical characteristics of the soil, moisture content, building construction, winds, etc. (A house in Maine had a record $\sim$ 160,000 Bq/m^3!)

A radon concentration of 50 Bq/m^3 may result in an annual dose equivalent to bronchial epithelium (site of most radiation induced lung cancer) of $\sim$ 2.5 mSv/yr. Perhaps 25% of Wisconsin houses have concentrations $>$ 150 Bq/m^3 which is the EPA guideline where action should be taken in a few years since the lung cancer risk may be comparable to smoking 3 to 10 cigarettes/day. (M.S. Blumenthal, **Wisconsin Medical Journal**, Vol. 87, May 1988, p.17) In fact 2% of U.S. homes have radon concentrations $>$ 300 Bq/m^3 and occupants should take action to reduce the concentration since they may be receiving an effective dose of $>\sim$ 16 mSv/yr. (By contrast the EPA limit for off-site exposure from nuclear reactors or from nuclear waste depositories is only .25 mSv/yr!) (Bodansky, **Physics and Society** 16, No. 4, p. 6, 1987).

If a house has high radon levels, then the radon ingress usually is from air infiltration from soil beneath the house. Natural convection in the house (chimney effect) tends to pull in the radon contaminated air. A solution is to drive pipes through the basement floor and connect them to fans exhausting to the outside air. See A. V. Nero Jr. "The Indoor Radon Story", **Technology Review** Vol.89, No. 1, p. 28, (1986); also A. Nero, "Earth, Air, Radon and Home", **Physics Today**, Vol 42, No. 4, p.32, (1989).

A good general reference on the subject is **"Radon and its Decay Products in Indoor Air"** edited by William Nazaroff and A.V. Nero, Jr., John Wiley & Sons, 1988. Ground water supplies often contain high concentrations of radon from uranium decay in the aquifers. The radon concentration in public ground water supplies averages $\sim$ 5000 Bq/m^3, and is much higher in some of the New England states. The health hazard is apparently not from drinking the water, but from the water's contribution to the indoor radon air problem: perhaps $\sim$ 5 Bq/m^3. Private wells often have high concentrations. Storage or aeration of the water provides effective control of the hazard.

OTHER ISSUES:

Is a Small Amount of Radiation Healthy - The Hormesis Effect

The following is from RADIATION DOSIMETRY by former Prof. John R. Cameron, Department of Medical Physics, UW, Madison:

> "Studies on nuclear workers often show that they have less cancer than other member of the population and even of other workers with similar jobs. This is usually explained as the 'healthy worker' effect. That is, for reasons not understood, radiation work attract healthy workers. An alternate explanation which is rarely mentioned is the possibility that a small amount of radiation is good for you. This is referred to as the 'hormesis' effect. Since humans and all of our ancestors evolved in a sea of natural radiation, it is possible that mutations have occurred that produce the hormesis effect. Animal experiments have demonstrated the hormesis effect. Rats exposed to increased radiation have a longer survival than their controls."

F. PASCO© Interface and Computer Primer

INTRODUCTION:

The Physics 201/207 207/208 laboratories utilize a Web-browser based display format in combination, when necessary, computer controlled data acquisition interface (typically the PASCO CI-700 or 750). Various sensors are plugged into either digital I/O (phone jack style inputs 1 to 4) or analog I/O ports (DIN-9 style inputs A, B and C). To aid in the data acquisition and analysis PASCO module also requires use of a special purpose software package which can be easily reconfigured for the particular need of a experiment. In general all experiment starting configuration will be preset and launched through a Web-browser button at the appropriate place in the lab.

THE MOUSE

CLICKING: Most of the operations of your computer are controlled by locating the cursor on the appropriate symbol (icon) and by clicking (CLICK) or double clicking (DCLICK) the *left* button of the mouse.

If the operation you have to perform requires clicking the right button this will be shown by CLICK-R or DCLICK-R. Double clicking means pressing the mouse button twice in rapid sequence *without moving the mouse*. The image of an hour glass appears momentarily indicating that the computer is loading the program, that is, getting ready to do what you requested. It will not do this if you moved the mouse while double clicking.

WINDOWS

The monitor usually displays various “windows” with a title bar. If you CLICK anywhere inside the window, the title bar turns blue, and the window is “active” (i.e. the computer will respond to any clicks on the “icons” on the border of the window).

BASIC OPTIONS:

I. CLICK on the head bar to “drag” the window to a different position.
II. Enlarge the window by placing the cursor on the corner, a diagonal arrow will appear, then CLICK and drag to change the size of the window.

Depending on which experiment you are performing you will see various windows. These will be discussed separately.

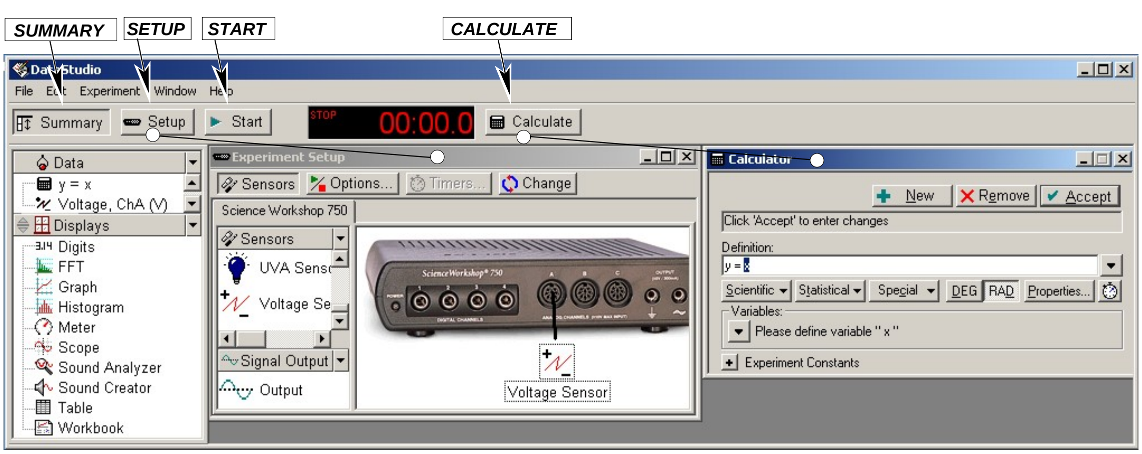


Figure 1: The main PASCO Data Studio window

ICONS:

- SUMMARY: CLICKing on this alternately opens and closes the summary area on left (i.e., frame with Data and Displays).
- SETUP: CLICKing on this open the "Experiment Setup" window.
- START: CLICKing on this begins the data acquisition and the icons changes to "STOP". CLICKing on the STOP ends the data acquisition.
- CALCULATE: CLICKing on this open a calculator window as shown.

THE GRAPH WINDOW:
Across the top of the graph window you will find a litany of icons: The icons that appear at the top right of all windows are (see Fig. 2):

(1) EXIT: The window is removed permanently.

(2) RESIZE: The size is changed from large to small, or vice-versa.

(3) MINIMIZE: The window is shrunk and should appear as an icon in the Data Studio workspace.

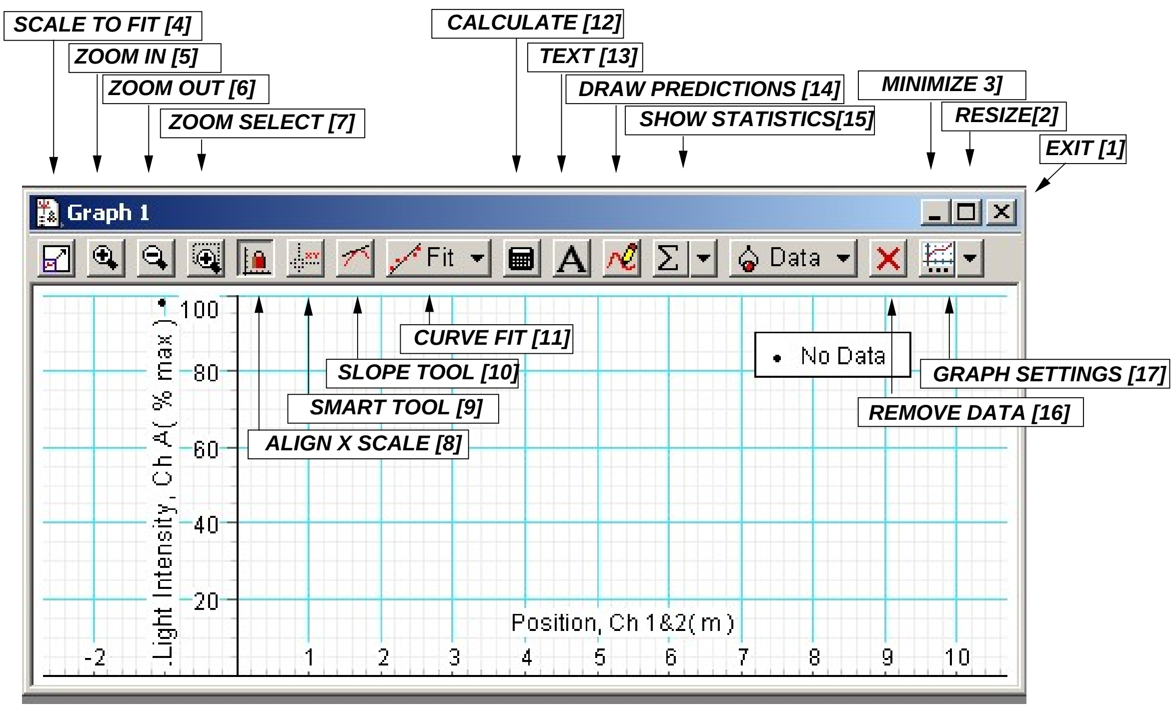


Figure 2: The Graph Window

(4) SCALE TO FIT will rescale the x and y axes to fit the current data set.
(5) ZOOM IN will enhance the size of the graph features.
(6) ZOOM OUT will reduce the size of the graph features.
(7) ZOOM SELECT: After CLICKing on this icon move the cursor into the plot and CLICK then DRAG to select a region of interest. All calculations will refer to this region of interest.
(8) ALIGN X SCALE: If there are multiple graphs this will align all the X axes.
(9) SMART TOOL turns on cross hairs so that graph x,y positions are read out directly.
(10) SLOPE TOOL determines the slope at a point

(11) CURVE FIT
(12) CALCULATE launches the calculator applications
(13) TEXT
(14) DRAW PREDICTIONS
(15) SHOW STATISTICS shows/hides statistics for a selected region of interest. You must first select the area of the graph you want to analyze by CLICKing on the ZOOM SELECT icon and the moving to the upper left corner of the ROI. The drag the cursor (CLICK and hold) diagonally across the graph to generate a rectangle that encloses the area chosen.
(16) REMOVE DATA:
(17) GRAPH SETTINGS: This icon allows for complete customization of the plot.

THE EXPERIMENT SETUP WINDOW:

Usually you will find this window in its "minimized" form but this window control the physical instrumentation connected to the PASCO computer interface

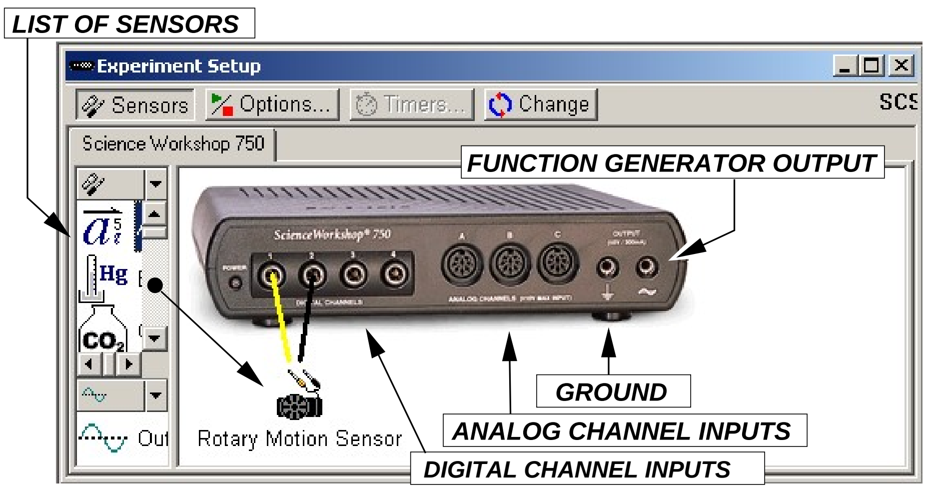


Figure 3: The Experimental Setup Window.

(1) **Sensors** Icon: CLICKing here alternately open and closes the sensor list on left. A sensor must be "grabbed" from the list and then "dropped" onto the appropriate PASCO channel.
(2) **Options** Icon: CLICKing here open a window for various custom data acquisition options (Manual sampling, Delayed acquisition, Automatic start)
(3) **Timers** Icon: CLICKing, if active (by using e.g. the "Time of Flight" sensor), will allow for a customized time sequence.
(4) **Change** Icon: CLICKing here will allow you to change the type of Pasco computer interface (e.g., CI-750, CI-700, etc.)
(5) FUNCTION GENERATOR: Output from a built-in signal generator (e.g. sine or square waves) and allows control of both frequency and amplitude.
(6) DIGITAL CHANNELS: These components produce or require signals (i.e., input/output) that switch between two levels, typically 0 and 5 volts. NOTE: Exceeding 10 volts may damage the port.
(7) ANALOG CHANNELS: These components produce or require signals that have a large range of values. If voltage is specified then the range is typically between -5 and 5 volts. NOTE: Exceeding 10 volts may damage the port.
(8) GROUND: Electrical access for *signal ground*. Note that this does not necessarily mean the ground of the outlet.

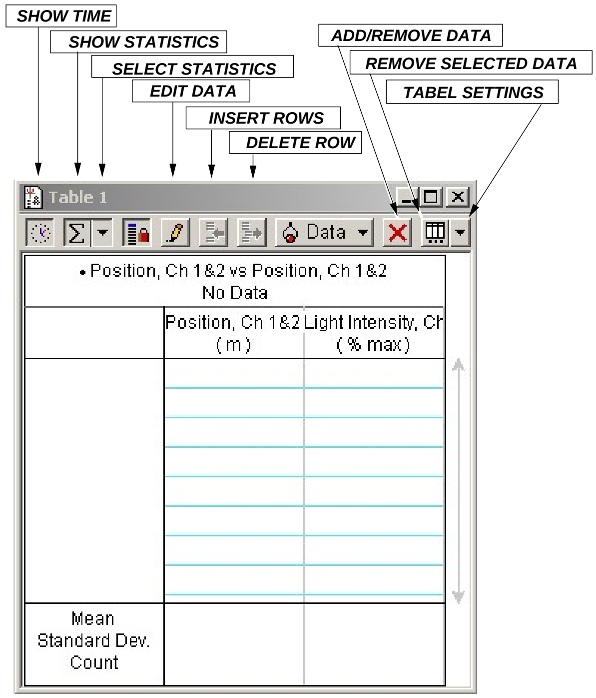


Figure 4: The Table Window

THE TABLE WINDOW:

(1) SHOW TIME: Alter the display to include time at which data was recorded.
(2) SHOW STATISTICS: Toggles off and on a display for various selected values including: minimum, maximum, mean, standard deviation and the count. NOTE: Subsets of the full data set can be analyzed by using the mouse and highlighting (through a CLICK and drag motion) the rows of interest.
(3) Almost all of the headings are self explanatory.