# HW 11

1) see Mathematica

2) $$[A, J_z] = [A, \tfrac{[J_x, J_y]}{i}]$$

$$= -i\left( -[J_x, \cancel{[J_y, A]}] - [J_y, \cancel{[A, J_x]}] \right) \text{ (Jacobi)}$$

$$= 0$$

3) $$[L_j, x_k] = \sum_{lm} \epsilon_{jlm} [x_l p_m, x_k] = -\sum_{lm} [x_k, x_l p_m] \epsilon_{jlm}$$

$$= -\sum_{lm} \epsilon_{jlm} \left( \cancelto{0}{[x_k, x_l]} p_m + x_l \underbrace{[x_k, p_m]}_{i\hbar\delta_{km}} \right)$$

$$= -\sum_{l} \epsilon_{jlk} \, i\hbar x_l = i\hbar \epsilon_{jkl} x_l$$

$$[L_j, p_k] = -\sum_{lm} \epsilon_{jlm} [p_k, x_l p_m]$$

$$= -\sum_{lm} \epsilon_{jlm} \underbrace{[p_k, x_l]}_{-i\hbar\delta_{kl}} p_m = i \epsilon_{jkm} p_m$$

$$[L_i, p^2] = \sum_j [L_i, p_j^2] = \sum_j [L_i, p_j] p_j + p_j [L_i, p_j]$$

$$= \sum_j i \epsilon_{ijk} (p_k p_j + p_j p_k)$$

$$[L_x, p^2] = i \underset{= 1}{\epsilon_{xyz}} (p_z p_y + p_y p_z) + i \underset{= -1}{\epsilon_{xzy}} (p_z p_y + p_y p_z) = 0 \checkmark$$

$$[L_x, r^2] = (\text{same thing only } p \to x)$$
$$= 0$$

4) $$H = -\mu B_z m + \sum_{ij} Q_{ij} J_i J_j$$

only terms with zero expectation value are

$$Q_{33}, \quad Q_{12}, \quad Q_{11}, \quad Q_{22}, \quad Q_{21}$$

$$\langle m | J_x J_y | m \rangle = \langle m | \left(\frac{J_+ + J_-}{2}\right)\left(\frac{J_+ - J_-}{2i}\right) | m \rangle$$
$$= \frac{1}{4i} \langle m | J_- J_+ - J_+ J_- | m \rangle$$

$$[J_-, J_+] = J_- J_+ - J_+ J_- = (J^2 - J_z^2 - J_z) - (J^2 - J_z^2 + J_z)$$
$$= -2J_z$$

$$= \frac{-J_z}{2i}$$

$$\langle m | J_y J_x | m \rangle = \frac{J_z}{2i}$$

$$\langle m | J_x^2 | m \rangle = \langle m | J_y^2 | m \rangle = \frac{1}{2} \langle m | J^2 - J_z^2 | m \rangle = \frac{j(j+1) - m^2}{2}$$

$$\therefore \; E = -\mu B_z m + Q_{33} m^2 + (Q_{22} + Q_{11}) \frac{j(j+1) - m^2}{2}$$
$$- \frac{m}{2i} (Q_{12} - Q_{21})$$

- #1

In[199]:= `$Assumptions = {b > 0, a > 0}`

Out[199]= $\{b > 0, a > 0\}$

In[200]:= $\text{psi} = \left(\frac{\sqrt{\frac{\pi}{2}}}{\sqrt{a}}\right)^{-1/2} \text{Exp}[-a\,x^2]$

Out[200]= $a^{1/4}\, e^{-a x^2} \left(\frac{2}{\pi}\right)^{1/4}$

In[201]:= $\text{Integrate}[\text{psi}^2, \{x, -\infty, \infty\}]$ // PowerExpand

Out[201]= $1$

In[202]:= $\text{Integrate}[\text{psi}\ (\text{Exp}[-x^2/b^2])\ \text{psi}, \{x, -\infty, \infty\}]$ // Simplify

Out[202]= $\sqrt{2}\sqrt{\frac{a}{2 a + \frac{1}{b^2}}}$

In[203]:= `Integrate[psi D[psi, {x, 2}], {x, -∞, ∞}] // Simplify`

Out[203]= $-a$

In[204]:= $\text{energy} = \frac{-\hbar^2}{2 m}(-a) - V0\sqrt{2}\sqrt{\frac{a}{2 a + \frac{1}{b^2}}}$ /. $b \to 10.\ (m\,V0/\hbar^2)^{-1/2}$ // Simplify

Out[204]= $-\sqrt{2}\ V0\sqrt{\frac{a}{2 a + \frac{0.01\, m\, V0}{\hbar^2}}} + \frac{a\,\hbar^2}{2 m}$

In[205]:= `D[energy, a] // Simplify // Together`

Out[205]= $\frac{-0.00707107\, m^2\, V0^2\sqrt{\frac{a}{2 a + \frac{0.01\, m\, V0}{\hbar^2}}} + 0.005\, a\, m\, V0\, \hbar^2 + 1.\, a^2\,\hbar^4}{a\, m\,(0.01\, m\, V0 + 2.\, a\,\hbar^2)}$

In[206]:= `Solve[% == 0, a]`

Out[206]= $\left\{\left\{a \to -\frac{0.0744931\, m\, V0}{\hbar^2}\right\}, \left\{a \to \frac{0.0669946\, m\, V0}{\hbar^2}\right\}\right\}$

In[207]:= energy /. $a \to \frac{0.06699461793568152\text{`}\ m\, V0}{\hbar^2}$

Out[207]= $-0.931153\ V0$