Wisconsin Summer School for Quantum Science

A summer school introducing quantum science to upper-level undergraduate and early-career graduate students

27-29 May 2026

Lectures: 2241 Chamberlin Hall

Wi-Fi: eduroam

Lunch: Chadbourne dorms

Lab tours:

Wed. 3:45 PM — Sinclair Lab

Thu. 3:45 PM — Kuzmin Lab

Fri. 2:30 PM — Song Lab

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unintuitive $\neq$ vague or imprecise

Precision of QM

g-factor of the electron:

$$\vec{\mu} = g \frac{e}{2m} \vec{S}, \quad (S = \hbar/2)$$

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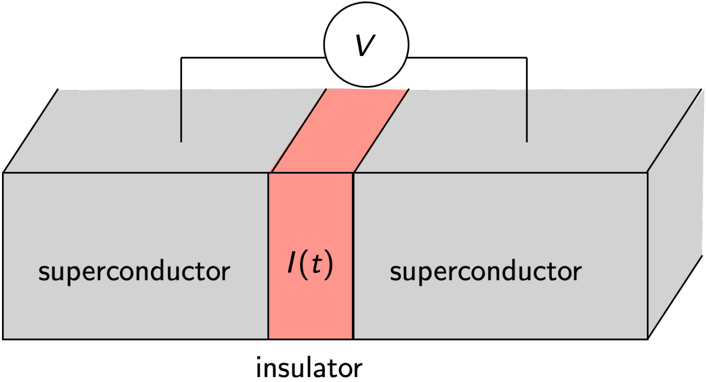
$$g^{\text{theory}} = 2.00231930436\ldots$$

$$g^{\text{expt}} = 2.00231930436118(27)$$

Fan, *et. al.* (2023)

Precision of QM

Metrology: Voltage standard and the Josephson effect



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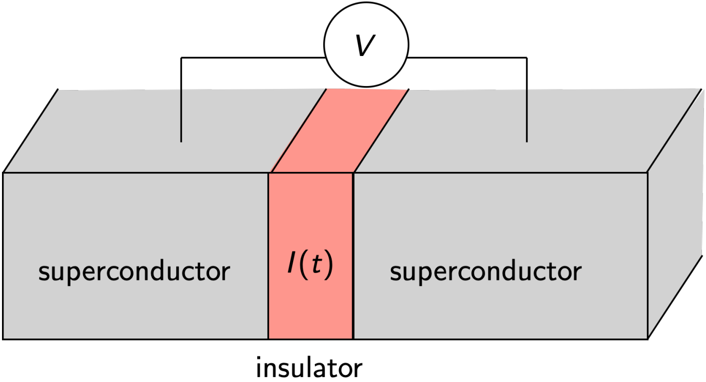
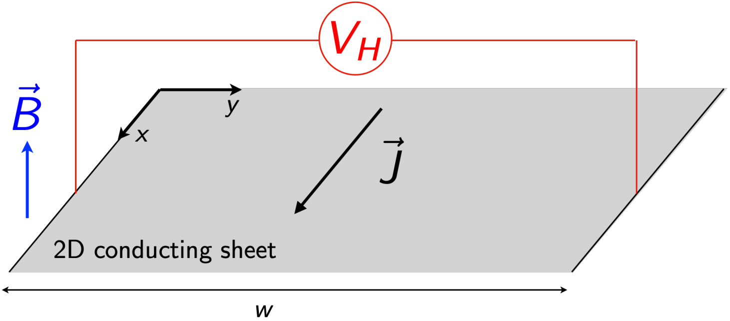


$$I(t) = I_c \sin(\omega t), \qquad \omega = \frac{2e}{\hbar} V$$

(cf. Planck relation $E = \hbar\omega$)

Precision of QM

Metrology: Resistance standard and the quantum Hall effect



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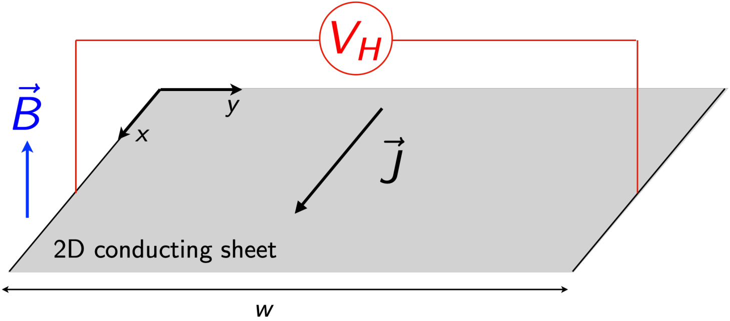

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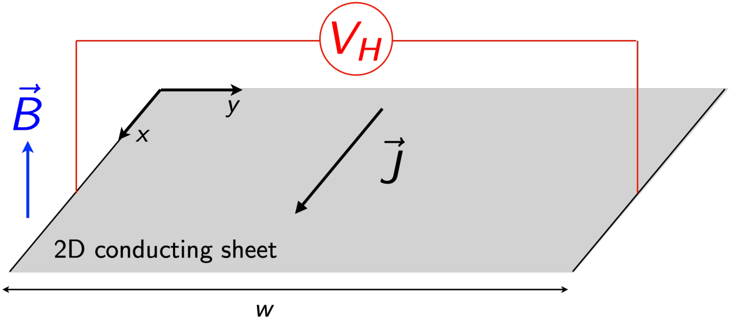

steady state: $F_y = ev_x B - eV_H/w = 0 \quad \Rightarrow V_H = v_x B w$

$$I = (env_x)w \quad \Rightarrow R_H = \frac{V_H}{I} = \frac{B}{ne}$$

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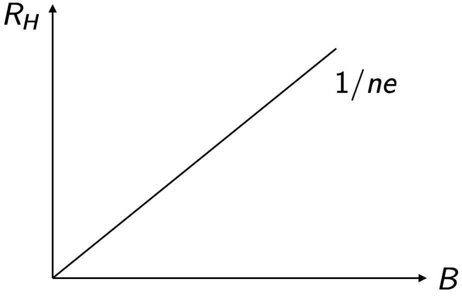

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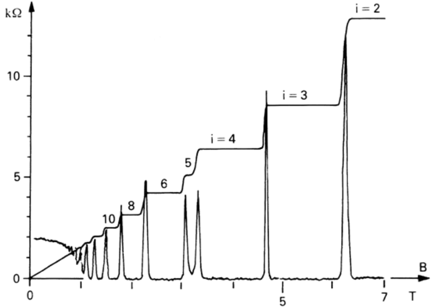

$$R_H = \frac{h}{ie^2}, \quad i = 1, 2, 3, \ldots$$

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von Klitzing, *et. al.* (1980)

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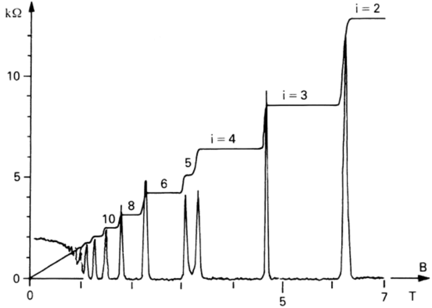

$$R_H = \frac{1}{i} R_K, \qquad R_K = \frac{h}{e^2} = 25\ 812.807\ldots\ \Omega$$

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Quantum phenomena on human scales

Stability of matter

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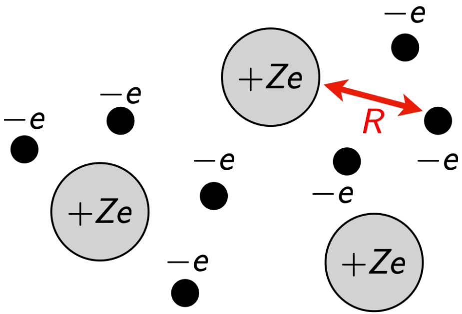
Stability of matter

Matter = collections of protons (or ions) and electrons



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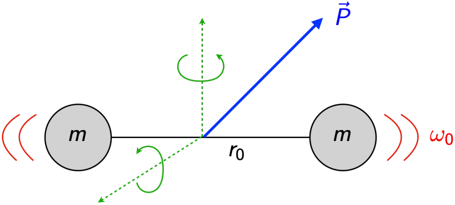
$$\langle T \rangle \sim \langle U \rangle \quad \Rightarrow \text{quantum of density} \sim \frac{m^3 e^6}{\hbar^6} \sim 10^{22}\ \text{cm}^{-3}$$

Quantum phenomena on human scales

Heat capacity of polyatomic gases



$M = 2m$

$\mu = m/2$

$$H_{\text{diatomic}} = \frac{\vec{P}^2}{2M} + \frac{\vec{L}^2}{2\mu r_0^2} + \frac{p^2}{2\mu} + \frac{1}{2}\mu\omega_0^2(r - r_0)^2$$

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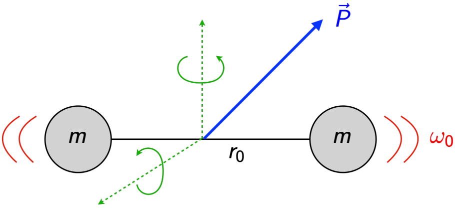

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classical heat capacity (equipartition): $C_V/R = 3 \times \frac{1}{2} + 2 \times \frac{1}{2} + 2 \times \frac{1}{2} = \frac{7}{2}$

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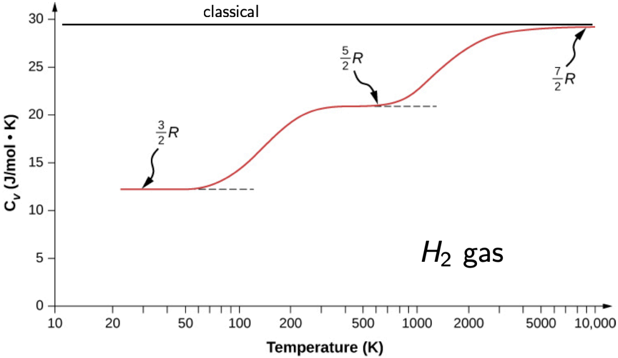

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1. Control over single and few-body quantum systems (few-body coherence protected by isolation).
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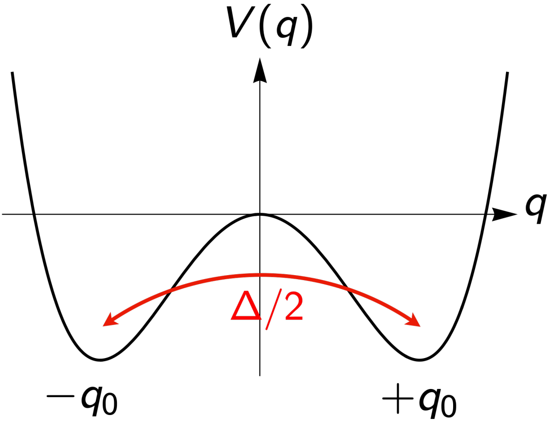
2. Collective behavior of macroscopic collection ($N \rightarrow \infty$) of quantum particles (many-body coherence protected by collective behavior and associated rigidity).

Quantum superposition and coherence

Double-well in the "two-state limit":



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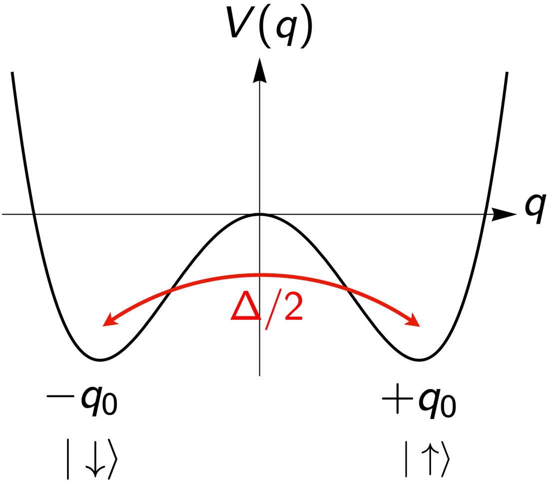

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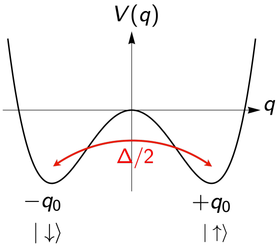

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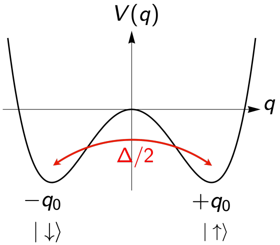

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Ground state: $|0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$

$=$ coherent superposition of localized states

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oscillations are purely quantum: due to coherence of QM superposition

Decoherence

Coupling to environment, modeled as collection of harmonic oscillators:

$$H = -\frac{\Delta}{2}\sigma_x + \sum_\alpha \left(\frac{p_\alpha^2}{2m} + \frac{1}{2} m\omega_\alpha^2 x_\alpha^2 \right) + \sigma_x \sum_\alpha C_\alpha x_\alpha$$

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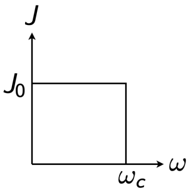
$$\langle \sigma_z(t) \rangle = e^{-\Phi(t)} \cos(\Delta t/\hbar)$$

$$\Phi(t) = \sum_\alpha \frac{C_\alpha^2}{m_\alpha \omega_\alpha^2} [1 - \cos(\omega_\alpha t)] = \int_{-\infty}^{\infty} \mathrm{d}\omega \; J(\omega) \frac{1 - \cos(\omega t)}{\omega^2}$$

$$J(\omega) = \sum_\alpha \frac{C_\alpha^2}{m_\alpha \omega_\alpha^2} \delta(\omega - \omega_\alpha) = \text{bath density of states}$$

Decoherence

$$J(\omega) = J_0\theta(\omega_c - \omega)$$



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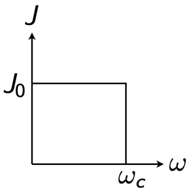

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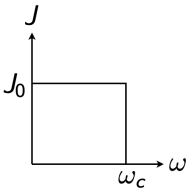

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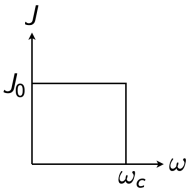

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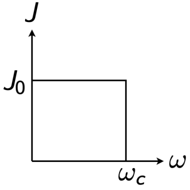

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Strategy to minimize decoherence: bath coupling $C_\alpha \to 0$

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Quantum materials: engineered QM systems to yield phases of matter with desirable coherent phenomena