CHAPTER 18  ELECTRIC FORCES AND ELECTRIC FIELDS

CONCEPTUAL QUESTIONS

1. **REASONING AND SOLUTION**  In Figure 18.9, the grounding wire is removed first, followed by the rod, and the sphere is left with a positive charge. If the rod were removed first, followed by the grounding wire, the sphere would not be left with a charge. Once the rod is removed, the repulsive force caused by the presence of the rubber rod is no longer present. Since the wire is still attached, free electrons will enter the sphere from the ground until the sphere is once again neutral.

2. **REASONING AND SOLUTION**  A metallic rod is given a positive charge by the process of induction as illustrated in Figure 18.9.

   a. The metallic object becomes positive because, during the induction process, electrons are forced from the object to the earth. The mass of the object will decrease by an amount equal to the mass of the electrons that left the metallic object.

   b. The metallic object becomes negative because, during the induction process, electrons are pulled onto the object from the earth. The mass of the object will increase by an amount that is equal to the mass of the "excess" electrons that are pulled onto the object.

3. **REASONING AND SOLUTION**  When the charged insulating rod is brought near to (but not touching) the sphere, the free electrons in the sphere will move. If the rod is negatively charged, the free electrons will move to the side of the sphere that is opposite to the side where the rod is; if the rod is positively charged, the free electrons will migrate to the side of the sphere where the rod is. In either case, the region of the sphere near the vicinity of the rod will acquire a charge that has the opposite sign as the charge on the rod.

   a. Since oppositely charged objects always attract each other, the rod and sphere will always experience a mutual attraction.

   b. Since the side of the sphere in the vicinity of the rod will always have charge that is opposite in sign to the charge on the rod, the rod and the sphere will always attract each other. They never repel each other.

4. **REASONING AND SOLUTION**  On a dry day, just after washing your hair to remove natural oils and drying it thoroughly, you run a plastic comb through it. As the surface of the comb rubs against your hair, the comb becomes electrically charged. If the comb is brought near small bits of paper, the charge on the comb causes a separation of charge on the bits of paper. Since the paper is neutral, it contains equal amounts of positive and negative charge. The charge on the comb causes the regions
of the bits of paper that are closest to the comb to become oppositely charged; therefore, the bits of paper are attracted to the comb. This situation is analogous to that in Figure 18.10.

5. **REASONING AND SOLUTION** A balloon is blown up and rubbed against a person's shirt a number of times. The balloon is then touched to a ceiling. Upon being released, the balloon remains stuck to the ceiling. The balloon is charged by contact. The ceiling is neutral. The charged balloon will induce a slight surface charge on the ceiling that is opposite in sign to the charge on the balloon. Since the charge on the balloon and the ceiling are opposite in sign, they will attract each other. Since both the balloon and the ceiling are insulators, charge cannot flow from one to the other. The charge on the balloon remains fixed on the balloon, while the charge on the ceiling remains fixed on the ceiling. The electrostatic force that the ceiling exerts on the balloon is sufficient to hold the balloon in place.

6. **REASONING AND SOLUTION** A proton and electron are held in place on the x axis. The proton is at \( x = -d \), while the electron is at \( x = +d \). They are released simultaneously, and the only force that affects their motions is the electrostatic force of attraction that each applies to the other. According to Newton's third law, the force \( F_{pe} \) exerted on the proton by the electron is equal in magnitude and opposite in direction to the force \( F_{ep} \) exerted on the electron by the proton. In other words, \( F_{pe} = -F_{ep} \). According to Newton's second law, this equation can be written

\[
m_p a_p = -m_e a_e
\]

where \( m_p \) and \( m_e \) are the respective masses and \( a_p \) and \( a_e \) are the respective accelerations of the proton and the electron. Since the mass of the electron is considerably smaller than the mass of the proton, the acceleration of the electron at any instant must be considerably greater than the acceleration of the proton at that instant in order for Equation (1) to hold. Since the electron has a much greater acceleration than the proton, it will attain greater velocities than the proton and, therefore, reach the origin first.

7. **REASONING AND SOLUTION** In order for a particle to execute simple harmonic motion, it must obey a force law of the form of Equation 10.6, \( F = -kx \), where \( x \) is the displacement of the object from its equilibrium position, and \( k \) is the spring constant. The force described by Equation 10.6 is a restoring force in the sense that it always pulls the particle toward its equilibrium position.

The force of repulsion that a charge \( q_1 \) feels when it is pushed toward another charge \( q_2 \) of the same polarity is given by Coulomb's law, Equation 18.1, \( F = kq_1 q_2 / r^2 \). Clearly, Coulomb's law is an inverse square law. It does not have the same mathematical form as Equation 10.6. Therefore, a charged particle that is pushed toward another charged particle of the same polarity that is fixed in position, will not exhibit simple harmonic motion when it is released. Coulomb's law does not describe a restoring force. When \( q_1 \) is released, it simply "flies away" from \( q_2 \) and never returns.
8. **REASONING AND SOLUTION** Identical point charges are fixed to opposite corners of a square, as shown in the figure at the right.

There exists an electric field at each point in space in the vicinity of this configuration. The electric field at a given point is the resultant of the electric field at that point due to the charge at corner 1 and the charge at corner 3.

The magnitude of the electric field due to a single point charge of strength \( q \) is given by Equation 18.3: \( E = \frac{kq}{r^2} \), where \( r \) is the distance from the charge to the point in question.

Since the point charges at corners 1 and 3 are identical, they each have the same value of \( q \). Furthermore, they are equidistant from the center point \( C \). Therefore, at the center of the square, each charge gives rise to an electric field that is equal in magnitude and opposite in direction. The resultant electric field at the center of the square is, therefore, zero. Note that this result is independent of the polarity of the charges.

The distance from either empty corner, 2 or 4, to either of the charges is the same. Since the charges are equidistant from either empty corner, each charge gives rise to an electric field that is equal in magnitude. Since the direction of the electric field due to a point charge is radial (radially inward for negative charges and radially outward for positive charges), we see that the electric fields due to each of the two point charges will be mutually perpendicular. Their resultant can be found by using the Pythagorean theorem.

The magnitude of the force experienced by a third point charge placed in this system is, from Equation 18.2: \( F = q_0 E \), where \( q_0 \) is the magnitude of the charge and \( E \) is the magnitude of the electric field at the location of the charge. Since the electric field is zero at the center of the square, a third point charge will experience no force there. Thus, a third point charge will experience the greater force at one of the empty corners of the square.

9. **REASONING AND SOLUTION** The figure shows a thin, nonconducting rod, on which positive charges are spread evenly, so that there is the same amount of charge per unit length at every point. We wish to deduce the direction of the electric field at the point \( P \), which is directly above the midpoint of the rod.

We first imagine dividing the rod up into a large number of small length elements. Since the charge per unit length is the same along the rod, each element will contain an equal amount of charge \( \Delta q \). If the elements are chosen small enough, then the amount of charge on each may be treated as a point charge. That is, the charge on each element gives rise to an electric field at the point \( P \) given by \( \Delta E = \frac{k\Delta q}{r^2} \), where \( r \) is the distance between that particular element and the point \( P \). The electric field at \( P \) is found by adding, vectorially, the effects of all the elements. Since the rod is positively charged, the direction of \( \Delta E \) is away from the rod and along the line that connects the point \( P \) to the particular element of charge. In general, each element \( \Delta q \) is a different distance \( r \) from the point \( P \),
and, therefore, gives rise to a different electric field \( \Delta E \) at the point \( P \). However, every element on the left half of the rod can be paired with an element on the right half of the rod that is the same distance \( r \) from the point \( P \). Thus, each element on the left can be paired with an element on the right that gives rise to an electric field at \( P \) that has the same magnitude \( \Delta E \).

We notice that each \( \Delta E \) has both \( x \) and \( y \) components; however, for charge elements on the right, the \( x \) component points in the negative \( x \) direction, while for elements on the left, the \( x \) component points in the positive \( x \) direction, as shown below.

Thus, the \( x \) field components from the charges on the left cancel pair-wise with the \( x \) field components from the charges on the right. The \( y \) field components from all of the charge elements point in the positive \( y \) direction. Therefore, the net electric field at the point \( P \) will point only in the positive \( y \) direction, that is, perpendicularly away from the rod.

On another identical rod, positive charges are spread evenly over only the left half of the rod, and the same amount of negative charges are spread evenly over the right half. Again, we wish to deduce the direction of the electric field at the point \( P \), which is located above the midpoint of the rod.

We proceed as we did previously. Since the left half of the rod is positively charged, the direction of \( \Delta E \) is away from the rod and along the line that connects the point \( P \) to the particular element of charge on the left half. Conversely, the right half of the rod is negatively charged. Therefore, the direction of \( \Delta E \) is toward the rod and along the line that connects the point \( P \) to the particular element of charge on the right half. As with the first rod, each element of charge \( \Delta q \) gives rise to a different electric field \( \Delta E \) at the point \( P \). But once again, each element on the left half of the rod can be paired with an element on the right half of the rod that is the same distance \( r \) from the point \( P \) and, therefore, gives rise to an electric field at \( P \) of the same magnitude \( \Delta E \).

Each \( \Delta E \) has both \( x \) and \( y \) components; however, for charge elements on the right, the \( y \) component points in the negative \( y \) direction, while for charge elements on the left, the \( y \) component points in the positive \( y \) direction, as shown in the following drawings.
Thus, the $y$ field components that arise from the charges on the left cancel pair-wise with the $y$ field components that arise from the charges on the right. The $x$ field components from all of the charge elements point in the positive $x$ direction. Therefore, the net electric field at the point $P$ will point only in the positive $x$ direction, that is, parallel to the rod, pointing from the positive toward the negative side.

10. **REASONING AND SOLUTION** From Equation 18.2, we see that the electric field $E$ that exists at a point in space is the force per unit charge, $F/q_0$, experienced by a small test charge $q_0$ placed at that point. When a positive or negative charge is placed at a point where there is an electric field, the charge will experience a force given by $F = q_0 E$. If the charge $q_0$ is positive, then $F$ and $E$ are in the same direction; that is, a positive charge experiences a force in the direction of the electric field. Similarly, if the charge $q_0$ is negative, then $F$ and $E$ are in the opposite direction; a negative charge experiences a force that is opposite to the direction of the electric field.

If two different charges experience forces that differ in both magnitude and direction when they are separately placed at the same point in space, we can conclude that the two charges differ in both magnitude and polarity.

11. **REASONING AND SOLUTION** Three point charges are fixed to the corners of a square (side length $L$ and diagonal length $d$), one to a corner, in such a way that the net electric field at the empty corner is zero.

a. Each charge produces an electric field that is radially toward (positive charges) or radially away from (negative charges) the charge. The net electric field at corner 1 is the vector sum of the electric fields due to each of the charges at corners 2, 3 and 4. Since the electric field at corner 1 is zero, then the charges in corners 2 and 4 must have the same sign and have the opposite sign to the charge in corner 3. Suppose, for example, that the charge in corner 3 is positive. Then the electric field at corner 1 due to the charge in corner 3 must point away from corner 3 along the diagonal. In order for the vector sum of the fields due to all three charges to vanish at corner 1, the electric field due to the charge in corner 2 must cancel the $x$ component of the electric field due to the charge in corner 3. Likewise, the electric field due to the charge in corner 4 must cancel the $y$ component of the electric field due to the charge in corner 3. Thus, at corner 1, the electric field due to the charge at corner 2 must point to the right, while the electric field due to the charge in corner 4 must point...
toward the bottom of the page. The charges in corners 2 and 4, therefore, must be negative. Similar arguments apply if the charge in corner 3 is negative; we can then deduce that the charges in corners 2 and 4 must be positive. We conclude, therefore, that the three charges cannot all have the same sign.

b. Let all three charges have magnitude $Q$. Then the magnitude of the electric field at corner 1 due to the charge in corner 3 is given by Equation 18.3:

$$E_{13} = \frac{kQ}{d^2} = \frac{kQ}{2L^2}$$

As discussed in part (a), this electric field vector points along the diagonal. Therefore, its $x$ and $y$ components have the following magnitudes

$$\left( E_{13} \right)_x = \frac{kQ}{2L^2} \cos 45^\circ = \frac{kQ}{2L^2} \left( \frac{\sqrt{2}}{2} \right) = \frac{kQ}{(2\sqrt{2})L^2}$$

$$\left( E_{13} \right)_y = \frac{kQ}{2L^2} \sin 45^\circ = \frac{kQ}{2L^2} \left( \frac{\sqrt{2}}{2} \right) = \frac{kQ}{(2\sqrt{2})L^2}$$

In order for the net electric field at corner 1 to be zero, the charges in corners 2 and 4 must have the opposite sign to the charge in corner 3 [as discussed in part (a)], and the electric field at corner 1 due to the charge in corner 2 must be equal to $-\left( E_{13} \right)_x$. Similarly, the electric field at corner 1 due to the charge in corner 4 must be equal to $-\left( E_{13} \right)_y$. If the charges in all three corners have the same magnitude, however, the electric field at corner 1 due to the charge at corner 2 and the electric field at corner 1 due to the charge at corner 4 have magnitudes given by Equation 18.3 as

$$E_{12} = E_{14} = \frac{kQ}{L^2}$$

Clearly, $E_{12} \neq \left( E_{13} \right)_x$ and $E_{14} \neq \left( E_{13} \right)_y$; therefore, all three charges cannot have the same magnitude.

12. **REASONING AND SOLUTION** We can think of dividing the line into three distinct regions. Region I to the left of the positive charge, region II between the charges, and region III to the right of the negative charge. The electric field $\mathbf{E}_+$ from the positive charge always points away from the positive charge, while the electric field $\mathbf{E}_-$ always points toward the negative charge.
In order for the electric field to be zero, the resultant of the electric field due to the positive charge $E_+$ and the electric field due to the negative charge $E_-$ must be zero. This can only occur in regions I or III since those are the only two regions where the vectors $E_+$ and $E_-$ point in opposite directions.

The magnitude of the electric field at a distance $r$ from either charge is given by Equation 18.3: $E = \frac{kq}{r^2}$. When the two charges have equal magnitudes, the point on the line where the magnitude of $E_+$ is equal to the magnitude of $E_-$ is midway between the charges, since that point is equidistant from either charge ($q$ and $r$ are the same for both $E_+$ and $E_-$). The field will not be zero, however, because both $E_+$ and $E_-$ point to the right. If the magnitude of the negative charge is greater than the magnitude of the positive charge, then, the position where the magnitude of $E_+$ is equal to the magnitude of $E_-$ will occur closer to the positive charge. There will be a point in region I where the magnitudes of $E_+$ and $E_-$ are the same. Therefore, a point of zero electric field can occur on the line in region I.

13. **REASONING AND SOLUTION** In general, electric field lines are always directed away from positive charges and toward negative charges. At a point in space, the direction of the electric field is tangent to the electric field line that passes through that point. The magnitude of the electric field is strongest in regions where the field lines are closest together. Furthermore, the text states that electric field lines created by a positive point charge are directed radially outward, while those created by a negative point charge are directed radially inward.

(a) *In both regions I and II the electric field is the same everywhere.* This statement is false. By inspection, we see that the field lines in II get closer together as we proceed from left to right in the drawing; hence, the magnitude of the electric field is getting stronger. The direction of the electric field at any point along a field line can be found by constructing a line tangent to the field line. Clearly, the direction of the electric field in II changes as we proceed from left to right in the drawing.

(b) *As you move from left to right in each case, the electric field becomes stronger.* This statement is false. While this is true for the field in II by reasons that are discussed in part (a) above, we can see by inspection that the field lines in I remain equally spaced and point in the same direction as we go from left to right. Thus, the field in I remains constant in both magnitude and direction.

(c) *The electric field in I is the same everywhere but becomes stronger in II as you move from left to right.* This statement is true for the reasons discussed in parts (a) and (b) above.

(d) *The electric fields in both I and II could be created by negative charges located somewhere on the left and positive charges located somewhere on the right.* Since electric field lines are always directed away from positive charges and toward negative charges, this statement is false.

(e) *Both I and II arise from a single point charge located somewhere on the left.* Since the electric field lines are radial in the vicinity of a single point charge, this statement is false.

14. **REASONING AND SOLUTION** The effects of air resistance and gravity are ignored throughout this discussion. As shown in Figure 18.26, the electric field lines between the plates of a parallel plate capacitor are parallel and equally spaced, except near the edges. Ignoring edge effects, then, we
can treat the electric field between the plates as a constant electric field. Since the force on the charge is given by Equation 18.2 as $F = qE$, we can deduce that the force on the positive charge, and therefore its acceleration, is constant in magnitude and is directed toward the negative plate. Since the charge enters the region with a horizontal velocity and the acceleration is downward, we can conclude that the horizontal component of the velocity will remain constant while the vertical component of the velocity will increase in the downward direction as the particle moves from left to right. This situation is analogous to that of a projectile fired horizontally near the surface of the earth. We can deduce, therefore, that the charged particle will follow a parabolic trajectory.

a. Two possible trajectories for the particle are shown below. For case 1, the particle hits the bottom plate and never emerges from the capacitor. The point at which the particle hits the bottom plate (when indeed it does) depends on the initial speed $v$ of the particle when it enters the capacitor and the magnitude $E$ of the electric field between the plates. For certain values of $v$ and $E$, the particle may exit from the right side of the capacitor without striking the bottom plate. This is shown in case 2. Note that after the particle leaves the capacitor, it will move in a straight line at constant speed according to Newton's first law (assuming that no other forces are present).

**Case 1**

![Parabolic path](image1)

**Case 2**

![Parabolic and straight line path](image2)

b. The direction of the particle's instantaneous displacement vector and velocity vector are in the instantaneous direction of motion of the particle. Furthermore, the direction of the linear momentum vector is always in the same direction as the particle's velocity, and, therefore, in the instantaneous
direction of motion. Since the particle travels in a parabolic path as discussed in part (a), we know that the directions of the particle's displacement vector, velocity vector, and linear momentum vector are always tangent to the path shown in part (a). They are not, therefore, parallel to the electric field inside the capacitor. From Newton's second law, we know that the acceleration vector of the particle is in the direction of the net force that acts on the particle. Since the particle is positively charged, we know that the net force on the particle at any point is in the same direction as the direction of the electric field at that point. Therefore, the acceleration vector is parallel to the electric field in the capacitor.

15. **REASONING AND SOLUTION**  Refer to Figure 18.28 which shows the electric field lines for two identical positive point charges. Imagine a plane that is perpendicular to the line that passes through the charges and is half into and half out of the paper. The plane is midway between the charges. The electric flux through this plane is given by Equation 18.6: \( \Phi_E = \sum (E \cos \phi) \Delta A \), where \( E \) is the magnitude of the electric field at a point on the plane, \( \phi \) is the angle that the electric field makes with the normal to the plane at that point, \( \Delta A \) is a tiny area that just surrounds the point, and the summation is carried out over the entire plane. Midway between the charges, the electric field is zero (see Figure 18.28). Therefore, the electric field is zero for all points on the plane. The electric flux through the plane is, therefore, also zero.

16. **REASONING AND SOLUTION**  Two charges + \( q \) and − \( q \) are inside a Gaussian surface. Since the net charge inside the Gaussian surface is zero, Gauss' law states that the electric flux \( \Phi_E \) through the surface is also zero. However, the fact that \( \Phi_E \) is zero does not imply that \( E \) is zero at any point on the Gaussian surface.

The flux through a Gaussian surface of any shape that surrounds the two point charges is given by text Equation 18.6: \( \Phi_E = \sum (E \cos \phi) \Delta A \), where \( E \) is the magnitude of the electric field at a point on the surface, \( \phi \) is the angle that the electric field makes with the normal to the surface at that point, \( \Delta A \) is a tiny area that surrounds the point, and the summation is carried out over the entire Gaussian surface. The individual terms \( (E \cos \phi) \Delta A \) can be positive or negative, depending on the sign of the factor \( \cos \phi \).

We can see from Figure 18.27 that the electric field in the vicinity of the two point charges changes from point to point in both magnitude and direction. Furthermore, as Figure 18.27 indicates, the electric field lines are directed away from the positive charge and toward the negative charge. In general, the value of \( E \) is non-zero at every point on any arbitrary Gaussian surface that surrounds the charges. The magnitude and direction of \( E \) at each point is such that when the summation in Equation 18.6 is carried out over the entire Gaussian surface, the electric flux through the entire surface is zero.
17. **REASONING AND SOLUTION**

Gauss' law states that the electric flux \( \Phi_E \) through a Gaussian surface is equal to \( Q/e_0 \), where \( Q \) is the net charge enclosed by the surface and \( e_0 \) is the permittivity of free space.

The drawing shows three charges, labeled \( q_1, q_2, \) and \( q_3 \).

a. Since the charges labeled \( q_1 \) and \( q_2 \) lie within the Gaussian surface, they determine the electric flux through the surface. From Gauss' law, the flux through the surface is

\[
\Phi_E = (q_1 + q_2) e_0
\]

b. Each of the three charges produces its own electric field. The magnitude of the field at the point \( P \) due to any one of the charges is given by Equation 18.3: \( E = kq/r^2 \), where \( q \) is the magnitude of the charge and \( r \) is the distance between the charge and the point \( P \). The net electric field at the point \( P \) is the resultant of the electric fields that are produced by each charge individually. Thus, all three charges contribute to the electric field at the point \( P \).

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18. **REASONING AND SOLUTION**

Gauss' law states that the electric flux \( \Phi_E \) through a Gaussian surface is equal to \( Q/e_0 \), where \( Q \) is the net charge enclosed by the surface and \( e_0 \) is the permittivity of free space. Gauss' law make no reference to position or location other than to specify that \( Q \) refers to the net charged enclosed by the Gaussian surface.

Suppose that a charge \( +q \) is placed inside a spherical Gaussian surface. The charge is positioned so that it is not at the center of the sphere.

a. In order for Gauss' law to tell us anything about the charge within the Gaussian surface, the electric flux through the surface must be known. Since Gauss' law makes no reference to the position of the net charge \( Q \), Gauss' law cannot tell us where the charge is located. If the flux through the surface is known, Gauss' law can only tell us that a net amount of charge \( Q = +q \) is located somewhere within the surface.

b. If we know that the net charge enclosed by the sphere is \( Q = +q \), then, by the statement of Gauss' law, we know that the flux through the surface is \( \Phi_E = +q/e_0 \) (regardless of where the enclosed charges are positioned relative to the center of the sphere).