CHAPTER 23 | ALTERNATING CURRENT CIRCUITS

CONCEPTUAL QUESTIONS

1. **REASONING AND SOLUTION** A light bulb and a parallel plate capacitor (including a dielectric material between the plates) are connected in series to the 60-Hz ac voltage present at a wall outlet. Since the capacitor and the light bulb are in series, the rms current at any instant is the same through each element, and is given by Equation 23.6: \( I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} \), where \( Z \) is the impedance of the circuit. The impedance of the circuit is given by Equation 23.7 with \( X_L = 0 \) (since there is no inductance in the circuit): \( Z = \sqrt{R^2 + (-X_C)^2} = \sqrt{R^2 + X_C^2} \). According to Equation 19.10, if the dielectric between the plates of the capacitor is removed, the capacitance decreases by a factor of \( k \), where \( k \) is the dielectric constant. From Equation 23.2, \( X_C = \frac{1}{2\pi fC} \), we see that decreasing the capacitance increases the capacitive reactance \( X_C \). Therefore, the impedance of the circuit, \( \sqrt{R^2 + X_C^2} \), increases. The rms current in the circuit is \( I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} \) and will, therefore, be less than it was before the dielectric was removed. As a result, the brightness of the bulb will decrease.

2. **REASONING AND SOLUTION** The ends of a long straight wire are connected to the terminals of an ac generator, and the current is measured. The wire is then disconnected, wound into the shape of a multiple-turn coil, and reconnected to the generator. After the wire is wound into a coil, it has a greater inductance. When the generator is turned on, the coil will develop a voltage that opposes a change in the current according to Faraday’s law. Since the induced voltage opposes the rise in current, the rms current in the circuit will be less than it was before the wire was wound into a coil.

   This can also be seen by considering Equations 23.6 and 23.7. Before the wire was wound into a coil, its primary property was that of resistance, and the current through the wire was given by \( I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} \) with \( Z = R \), or \( I_{\text{rms}} = \frac{V_{\text{rms}}}{R} \). After the wire is wound into a coil, the wire possesses both resistance and inductance. Now the current in the coil is given by \( I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} \), where \( Z = \sqrt{R^2 + X_L^2} \). Since \( Z \) is necessarily greater than \( R \), the rms current is less when the wire is wound into a coil.

3. **REASONING AND SOLUTION** An air-core inductor is connected in series with a light bulb of resistance \( R \). This circuit is plugged into an electrical outlet. The current in the circuit is given by Equation 23.6, \( I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} \), where, from Equation 23.7, \( Z = \sqrt{R^2 + X_L^2} \). When a piece of iron is inserted in the inductor, the magnetic field in the inductor is enhanced relative to that in air, and the inductance increases. Equation 23.4, \( X_L = 2\pi fL \), shows that when the inductance \( L \) increases, the inductive reactance \( X_L \) also increases. The impedance of the circuit, therefore, increases, and the current in the circuit, \( I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} \), decreases. Thus, the brightness of the bulb decreases.
4. **REASONING AND SOLUTION** Consider two ac circuits. The generators in each circuit are identical (same frequency, same rms voltage). In circuit I, only a resistor is connected across the generator. In circuit II, the same resistor is in series with an inductor.

The average power used by an ac circuit is given by Equation 23.9: $\overline{P} = I_{\text{rms}} V_{\text{rms}} \cos \phi$, where the term $\cos \phi$ is the power factor of the circuit and is equal to $R/Z$. Therefore, $\overline{P} = I_{\text{rms}} V_{\text{rms}} R / Z$. But according to Equation 23.6, $I_{\text{rms}} = V_{\text{rms}} / Z$. Therefore,

$$\overline{P} = \left( \frac{V_{\text{rms}}}{Z} \right) \frac{V_{\text{rms}} R}{Z} = \frac{V_{\text{rms}}^2 R}{Z^2}.$$  

For circuit I, the impedance is $Z = R$, so that $\overline{P}_I = \frac{V_{\text{rms}}^2}{R}$. For circuit II, Equation 23.7 indicates that $Z^2 = R^2 + X_L^2$. Substituting this expression for $Z^2$ we find that

$$\overline{P}_\text{II} = \frac{V_{\text{rms}}^2 R}{R^2 + X_L^2} = \frac{\frac{V_{\text{rms}}^2}{R}}{1 + \frac{X_L^2}{R^2}} = \frac{\overline{P}_I}{R^2 + X_L^2 / R^2}.  

Thus, since the term $X_L^2 / R^2$ in the denominator on the right is positive, we find that $\overline{P}_I$ is greater than $\overline{P}_\text{II}$. Therefore, circuit I consumes more average power.

5. **REASONING AND SOLUTION** Consider the series RCL circuit shown in Figure 23.9. The impedance of the circuit is given by Equation 23.7: $Z = \sqrt{R^2 + (X_L - X_C)^2}$, where the capacitive reactance is given by Equation 23.2 [$X_C = 1/(2\pi f C)$] and the inductive reactance is given by Equation 23.4 [$X_L = 2\pi f L$]. One example of this expression for $Z$ is plotted in Figure 23.10 (see the red curve). The vertical axis in this figure gives the impedance, while the horizontal axis gives the frequency. A horizontal line drawn to intersect the vertical axis above the minimum in the curve will intersect the curve at two places. These places correspond to two different frequencies on the horizontal axis. Thus, the circuit in Figure 23.9 can have the same impedance at two different frequencies.

6. **REASONING AND SOLUTION** An inductor and a capacitor are connected in parallel across the terminals of a generator.

a. *The frequency of the generator becomes very large.* From Equation 23.2 (or Figure 23.2) we see that, in the high frequency limit, the capacitive reactance approaches zero. Therefore, in the high frequency limit, the capacitor behaves as if it were replaced by a wire with zero resistance, and the generator delivers a very large current to the capacitor. From Equation 23.4 (or Figure 23.6) we see that, in the high frequency limit, the inductive reactance becomes very large. In this limit, the inductor behaves as if it has been cut out of the circuit, leaving a gap in the wire, and the generator delivers no current to the inductor. Therefore, when the frequency of the generator becomes very large, the current through the capacitor becomes large, and no current flows through the inductor. The total current delivered by the generator is large.
b. The frequency of the generator becomes very small. From Equation 23.2 we see that, in the low frequency limit, the capacitive reactance approaches infinity. The capacitor acts as if it has been cut from the circuit, leaving a gap in the wire. As a result, the generator delivers no current to the capacitor. From Equation 23.4 we see that, in the low frequency limit, the inductive reactance approaches zero. In this limit, the inductor acts as if it has been replaced by a wire with zero resistance, and the generator delivers a very large current to the inductor. Therefore, when the frequency of the generator becomes very small, the current through the inductor becomes very large and the current through the capacitor is zero. The total current delivered by the generator is again large.

7. REASONING AND SOLUTION The rms current is given by Equation 23.6: \( I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} \), where \( V_{\text{rms}} \) is the rms voltage of the generator and \( Z \) is the impedance of the circuit. Since the rms voltage of the generator is the same in both cases, the greater current is delivered to the circuit with the smaller impedance. We see from Equation 23.2 \( X_C = \frac{1}{2\pi fC} \) and Equation 23.4 \( X_L = 2\pi fL \) that in the high frequency limit, the capacitive reactance is nearly zero and the inductive reactance is essentially infinite. In other words, in the high frequency limit, the capacitors behave as if they have been replaced by wires of zero resistance and the inductors behave as if they have been cut from the circuit leaving gaps in the wires. The figure below shows the two circuits in this limit.
Circuit I behaves as if it consists only of two identical resistors in parallel; therefore, the impedance of circuit I is \( Z = 1 / (R + 1 / R) \), or \( Z = R / 2 \). Circuit II behaves as if it consists only of two identical resistors in series; therefore, the impedance of circuit II is \( Z = 2R \). At a very high frequency, then, circuit I has the smaller impedance, and, therefore, its generator supplies the greater rms current.

8. **REASONING AND SOLUTION**  The phase angle between the current in and the voltage across a series RCL combination is the angle \( \phi \) between the current phasor \( I_0 \) and the voltage phasor \( V_0 \) in Figure 23.12. According to Equation 23.8, the tangent of this angle is related to the resistance \( R \), the inductive reactance \( X_L \), and the capacitive reactance \( X_C \), according to the relation

\[
\tan \phi = \frac{X_L - X_C}{R}
\]

At resonance, \( X_L = X_C \), and so \( \tan \phi = 0 \). Therefore, \( \phi = \tan^{-1}(0) = 0 \); in other words, the phase angle between the current phasor and the voltage phasor is zero. Since the phase angle between the current phasor and the voltage phasor is zero, we can conclude that the current is in phase with the voltage.

9. **REASONING AND SOLUTION**  The resonant frequency of a series RCL circuit is given by Equation 23.10:

\[
f_0 = \frac{1}{2\pi\sqrt{LC}}
\]

a. Since the resonant frequency of a RCL circuit does not depend on the value of \( R \), it is possible for two series RCL circuits to have the same resonant frequencies and yet have different \( R \) values.

b. The resonant frequency of a series RCL circuit is inversely proportional to \( \sqrt{LC} \). It is possible for two series RCL circuits to have the same resonant frequencies and yet have different values of \( C \) and \( L \), provided that the product \( LC \) is the same in both circuits.

10. **REASONING AND SOLUTION**  The generator connected to a series RCL circuit has a frequency that is greater than the resonant frequency of the circuit. Suppose that it is necessary to match the resonant frequency of the circuit to the frequency of the generator. To accomplish this, a second capacitor will be added to the circuit. In order to match the resonant frequency of the circuit to the frequency of the generator, the resonant frequency of the circuit must be increased. The resonant frequency of a series RCL circuit is given by Equation 23.10: \( f_0 = 1/(2\pi\sqrt{LC}) \). Since the resonant frequency is inversely proportional to \( \sqrt{LC} \), the capacitor must be added to the circuit so that it decreases the capacitance. This can be accomplished by placing the second capacitor in series with the first capacitor (see Equation 20.19). With the proper choice of the value of \( C \) for the second capacitor, the capacitance of the series combination will be smaller than the capacitance of the first capacitor alone, and the resonant frequency of the circuit will increase to match the generator frequency.
11. **REASONING AND SOLUTION**  The drawing at the right shows a full-wave rectifier circuit, in which the direction of the current through the load resistor $R$ is the same for both positive and negative halves of the generator's voltage cycle. The points $A$ and $B$ are connected directly to the top and bottom of the generator, respectively. The diodes are labeled $a$, $b$, $c$, and $d$. The direction of the current through the circuit can be found by recalling the fact that charge flows through a diode only when the diode is in a forward bias condition. When the diode is in a forward bias condition, the side of the diode symbol that contains the arrowhead has a positive potential relative to the other side.

a. When the top of the generator is positive and the bottom is negative, point $A$ in the figure has the positive potential and point $B$ the negative potential. Since point $A$ has the positive potential and is connected directly to the arrowhead for diode $a$, diode $a$ is in a forward bias condition. In contrast, we see that for diode $b$, the side opposite the arrowhead is connected to point $A$, so diode $b$ is in a reverse bias condition. Similarly, since point $B$ is negative, and is connected directly to the side of diode $c$ opposite the arrowhead, the arrowhead must be positive, and diode $c$ is in a forward bias condition. In contrast, the arrowhead of diode $d$ is negative, so diode $d$ is in a reverse bias condition. Thus, when the top of the generator is positive and the bottom is negative, only diodes $a$ and $c$ are in the forward bias condition and allow charge to flow through the resistance $R$. The figure below shows the path taken by the current under these circumstances.

b. When the top of the generator is negative and the bottom is positive, reasoning similar to that given above in part (a) indicates that diodes $b$ and $d$ are now forward biased, while diodes $a$ and $c$ are in reverse bias. Furthermore, the forward bias diodes allow charge to flow from left to right through the resistance $R$. The figure below shows the path taken by the current under these circumstances.