This Lecture:

From previous lecture:
• More on Biot and Savart’s law
• Magnetic dipole moment and Torque on a loop
• Ampere’s law

Magnetism

Ampere’s law: some simplifications

\[ \oint B \cdot ds = \mu_0 I \]

Integral around closed path proportional to current passing through any surface bounded by the path.

Right-hand ‘rule’:
• Thumb in direction of positive current
• Curled fingers show direction integration

Example: field produced by an infinitely long wire

\[ \oint B \cdot ds = B2 \pi a = \mu_0 I \]

Examples: straight wire, solenoid

\[ \oint B \cdot ds = \oint B \cdot ds = B \ell \]

\[ \oint B \cdot ds = B \ell = \mu_0 NI \]

\[ B = \mu_0 \frac{N}{\ell} I = \mu_0 n I \]

\( n \) = number of turns per unit length

Induction and Induced EMF

• Magnet moving toward or away a coil: an emf is induced as shown by galvanometer deflection
• Faraday noticed there is a current in the coil when B-field through it is changing

Magnetic flux

Magnetic flux is defined exactly as electric flux

\[ \Phi_B = \oint B \cdot dA \]

SI unit of magnetic flux is the Weber ( = 1 T-m² )
Michael Faraday & Joseph Henry's Law
- 1831: Faraday in England and Henry independently in the United States: an EMF can be induced in a circuit by changing the magnetic flux and current can be produced despite no battery!

\[ \mathcal{E} = \oint E \cdot ds = - \frac{d}{dt} \Phi_B = - \frac{d}{dt} \int B \cdot dA \]

Integral \( E \cdot ds \) over closed path
Magnetic flux through surface bounded by path
E is not conservative!!!

If \( E \) is not conservative a potential cannot be defined!!

EMF and \( E \cdot ds \)

Remember, work done by E-field = \( W_{\text{ext}} = \int F_{\text{Coulomb}} \cdot ds = -q_{\text{test}} (V_B - V_A) \)

\[ \mathcal{E} = \int E \cdot ds = -(V_B - V_A) \]

“EMF”
\[ \oint E \cdot ds = \int E \cdot ds = 0 \]

- Integral of E-field around closed loop is zero if E is conservative.
- If \( \oint E \cdot ds \neq 0 \) field is not like a Coulomb field produced by a charge since it is a ‘circular’ E-field (it does not start and stop on charges)

Ways of Inducing an emf
1) Magnetic flux change: the magnitude of \( B \) changes in time
\[ \mathcal{E} = \frac{d}{dt} (BA \cos \theta) \]
- The ammeter deflects when the magnet is moving toward or away from the loop or vice versa
- We relate this detection to a change in the magnetic field
- The induced current is produced by an induced emf (the driving energy per unit charge in a circuit!)

Ways of Inducing an emf
2) the area crossed by B lines changes with \( t \) (motional emf)

Ways of Inducing an emf
3) The angle \( \theta \) between \( B \) and normal to loop changes with \( t \)

| Loop oriented parallel to field lines \( \theta = 90^\circ \Rightarrow \Phi_B = 0 \) |
| Loop oriented at an angle \( \theta \) \Rightarrow the projected area is \( A \cos \theta \) |
| Loop perpendicular to B-lines \( \theta = 0^\circ \Rightarrow \Phi_B = AB \) |

Lenz's Law
- The induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop

\[ \mathcal{E} = - \frac{d\Phi_B}{dt} \]

From conservation of energy: if current were opposite there would be a magnetic force on the magnet that increases \( v \) and this would increase the current in the loop and the force would increase...this is a diverging effect that violates energy conservation
### Quick Quiz

A square loop of wire is being pulled at a constant velocity through a region of uniform magnetic field pointing into the screen.

In the loop of wire:

a) there will be a current induced,

b) charge separation will occur,

c) a and b, or

d) neither a nor b.

No Flux change! No current! No induced field!

---

### Quick Quiz: motional emf

A conducting bar moves with velocity \( v \). Which statement is true?

A) + accumulates at the top, and - at the bottom

B) no complete circuit, therefore, no charge accumulation

C) - accumulates at the top, and + at the bottom

---

### Pickup coil in electric guitar

A permanent magnet is placed near the string of the guitar and causes a portion of the string to become magnetized.

- When the string vibrates at some frequency, the magnetized segment produces a changing flux through the coil.
- The induced emf is fed to an amplifier.

Pickups are composed of small electromagnets, which are small magnets with a coil of wire wrapped around it.

---

### Induced current in a moving rod

**Equilibrium between electric and magnetic forces:**

- \( qE = qvB \) or \( E = vB \)
- Potential difference = \( vB \)
- If direction of motion reversed \( \Delta V \) polarity changes too

Equation:

\[
\varepsilon = \frac{\Delta \Phi}{\Delta t} = \frac{d(BL)}{dt} = -\frac{d\Phi}{dt} = -\frac{dB}{dt}L = -BL \frac{dv}{dt}
\]

\[
I = \frac{\varepsilon}{R} = \frac{BvL}{R}
\]

**Power input of applied force**

\[
\Phi = F_{app} v = (I \times B) v = \frac{\varepsilon^2}{R}
\]
Quick Quiz
Consider the case on the left. Then suppose the magnetic field in the image below is reversed so that it now points out of the page instead of inside as shown in the figure. The motion of the rod is as shown in the figure.
To keep the bar moving at the same speed, the force supplied
A) increases
B) remains the same
C) decreases

Rotating loops
- $N$ turns of same area rotating in a uniform $B$
- $\Phi_B = BA \cos \theta = BA \cos \omega t$
- Resulting EMF $\varepsilon = -N \frac{d\Phi_B}{dt} = NAB\omega \sin \omega t$

AC Generators
The AC generator consists of a loop of wire rotated by some external means in a magnetic field

In USA & Canada: $f = \frac{60}{(2\pi)} = 60$ Hz
In Europe $f = 50$ Hz

DC-generator
- DC generator has same components as AC generator
- Main difference: contacts to rotating loop are made using a split ring called a commutator
- In this way the output voltage has always the same polarity
- To obtain a steady DC current, generators use many coils and commutators distributed so pulses are out of phase

Eddy Currents
- Circulating currents (eddy currents) induced in flat copper or Al plate swinging through $B$
- The changing magnetic flux induces an emf in plate hence electron currents that produce a magnetic retarding $F_B$ when plate enters or leaves the field
- Eddy currents are often undesirable because they are a transformation of mechanical energy into internal energy
- Used in subway braking systems: electromagnet = solenoid with Fe core where current induced by relative motion of train on rails ⇒ drag force

Meissner effect
- When a material becomes superconductor, the resistance is zero and super-Eddy-currents are generated by a variation of the B-field when a small permanent magnet is moving towards the superconductor. The levitation results from the repulsion between the permanent magnet producing a field and the field produced by the supercurrents (2 magnets with like poles closer to unlike ones)