Physics 201, Lecture 8

Today’s Topics

- Physics 201, Review 1

Important Notes:
- This review is not designed to be complete on its own. It is not meant to replace your own preparation efforts.
- Exercises used in this review do not form a test problem pool.
- I do not endorse any past exams.
- Please practice more with end of chapter problems.
About Exam 1

- When and where
  - Monday Oct 2 5:30-7:00 pm
  - Room allocation: Ag Hall 125

- Format
  - Closed book
  - One 8x11 formula sheet allowed, must be self prepared. (Absolutely no sample problems, examples, class lectures, HW etc. And no photocopying)
  - 20 multiple choice questions.
  - Bring a calculator (but no computer). Only basic calculation functionality can be used (see my earlier email for details)
  - A B2 pencil is required to do with Scantron

- Special needs/ conflicts:
  - Should have been settled by now (except for emergency).
  - All alternative test sessions are in our lab room, only for approved requests.
Chapters Covered

- Chapter 1: Physics and Measurement.
- Chapter 2: Motion in 1-D
  - Sections 2.1-2.7.
- Chapter 3: Vectors
  (will largely be tested indirectly via physics problems)
- Chapter 4: Motion in 2-D
  - All Sections

Super Friday Tomorrow
From 10am-5pm, the lab room will be staffed by TAs to answer questions. You are welcome to drop by at any time, regardless of your sections.
Basic Concepts and Quantities

- Measurements
  - Units and Units Conversion
  - Significant figures
  - Scalar and Vector Quantities

- Kinematics:
  - General quantities: Displacement, Travel distance, Time interval, (average, instantaneous) Velocity/Speed/Acceleration.
  - Circular motion specific: radius, circumference, Centripetal acceleration, period, angular velocity, linear velocity
  - Kinematical equations
  - Relative velocity.

- End of chapter “Concepts and Principles” section is a good helper.
Basic Techniques

- Count number of sig. figures.
- Apply addition/subtraction, multiplication/division rules for sig. figures.
- Basic vector operations
- Read \(x-t, v-t\), graphs (1D only).
- Use kinematical equation to convert among \(x, t, v, a\).
- For uniform circular motion, relate \(a_c\) to \(v, r, \omega\).
- For uniform circular motion, calculate \(T\) from \(r\) and \(v\).
- Decompose a vector quantity \((v, a)\) into \(x, y\) projection. (For now, only 2D projection is required.)
- Calculate relative velocity when switching from one reference frame to another.
- Deal with applications such as:
  - Free Fall and Projectile (1D and 2D)
    - Flight time, maximum height, range...
  - Kinematics of Uniform Circular Motion.
A Trivial Conceptual Exercise: Number of Sig. Figures

What is the number of sig. figures for : 0.0043500?

A: 3
B: 5
C: 8
D: none of above

Answer: 5. Sig. figures do not include leading zeros.
Reminder: Rules On Significant Figures

- **Rule 1:**
  - In direct measurement, keep all “sure” digits but only one **estimated digit**. (estimated digit $\leftrightarrow$ the least significant figure).
  - e.g. 41.2 cm, 23.5s, ...

- **Rule 2:**
  - (Definition). **Number of sig. figures** = number of significant digits not counting leading zeros
  - e.g. 41.2 $\rightarrow$ N of sig. fig. = 3, 0.0032 $\rightarrow$ N of sig. fig. = 2, 3.20 $\rightarrow$ N of sig. fig. = 3.

- **Rule 3:**
  - In additions or subtractions, the least significant figure of the final result can not be more accurate than that of any operands.
  - e.g. 13.8m + 2.05m - 0.062m ($=15.788m$) $=15.8m$

- **Rule 4:**
  - In multiplication or division, the N of sig. figures of the final result equals the lowest num. of sig. figures among all operands.
  - e.g. 13.8 m x 2.05 m x 0.062 m ($=1.75398$) $= 1.8 \text{ m}^3$
Exercise 1: Sig. Figure Rules

- A rectangular wood block is measured 3.20 cm, 75 mm, and 0.1431 m in height (h), width (w), and depth (d), respectively.
  With the sig. fig. rule, what is its side length (s=h+w+d) ?
  a. 2.501 x 10^1 cm
  b. 2.50 x 10^1 cm
  c. 2.5 x 10^1 cm
  d. 3 x 10^1 cm

Solution: see board. Please make sure that the values are converted to the same unit first.
Reminder: Kinematical Quantities to Describe a Motion

**Basic Quantities**

- **Displacement:** change of position from \( t_1 \rightarrow t_2 \)
- **Velocity:** rate of position change.
  - Average: \( \Delta x / \Delta t \)
  - Instantaneous: \( dx/dt \)
- **Acceleration:** rate of velocity change.
  - Average: \( \Delta v / \Delta t \)
  - Instantaneous : \( dv/dt \)
Exercise 2: Average Velocity

- An object moving uniformly around a circle of radius $r$ has a period $T$.
  - what is its average velocity over the period $T$?

  - A: 0
  - B: $2\pi r/T$
  - C: $r/T$
  - D: Not enough information as the mass of the object is not given.
  - E: None of above

- Average velocity is displacement divided by time interval. And displacement depends only on initial and final positions.

- Conceptual questions like this will make up about 1/3 of the exam.
Exercise 3: Acceleration and Speed

A particle moving in 1-D has a negative constant acceleration, which of the following statement is true?

A: The particle’s velocity must be in negative direction.
B: The particle must be speeding up
C: The particle must be slowing down.
D: None of above is necessarily true.

In physics, the acceleration is defined per *velocity* while in daily language, the term acceleration is defined per *speed*. Note the difference.
Reminder: Basic Kinematics

- Useful formulas for motion with constant acceleration:
  - $a(t) = a_0$
  - $v(t) = v_0 + a_0 t$
  - $v_{av} = (v_0 + v)/2$
  - $x(t) = x_0 + v_0 t + \frac{1}{2} a_0 t^2$
  - $v(t)^2 = v_0^2 + 2a_0(x-x_0)$

- If two dimensional, x and y projections are independent of each other
  - X projection:
    - $v_x(t) = v_{x0} + a_{x0} t$
    - $x(t) = x_0 + v_{x0} t + \frac{1}{2} a_{x0} t^2$
    - $v_x(t)^2 = v_{0x}^2 + 2a_{x0}(x-x_0)$
  - Y projection:
    - $v_y(t) = v_{y0} + a_{y0} t$
    - $y(t) = y_0 + v_{y0} t + \frac{1}{2} a_{y0} t^2$
    - $v_y(t)^2 = v_{0y}^2 + 2a_{y0}(y-y_0)$

- Free fall and projectile: $a_x=0$, $a_y=-g$ (if up = positive y)

set $a_0=0$ for constant velocity
Exercise 4: Use Basic Kinematic Equations

A boat is traveling at 4.0 m/s as it passes the starting line of a race. If the boat accelerates at 1.0 m/s$^2$, the distance the boat has traveled after 6.0 seconds is:

- A. 42m
- B. 18m
- C. 26m
- D. 20m
- E. 14m

Questions on basic calculations like this one will make up another 1/3 of the exam.

\[ x(t) = x_0 + v_0 t + \frac{1}{2} a_0 t^2 \]

\[ = 0 + 4 \times 6.0 + \frac{1}{2} \times 1.0 \times 6.0^2 \]

= 42 m
Review: Projectile

horizontal: constant velocity
\[ a_x = 0, \quad v_x = v_{xi} = v_i \cos \theta_i, \quad x = x_0 + v_{xi} t \]

vertical: constant acceleration
\[ a_y = -g, \quad v_y = v_{yi} \sin \theta_i, \quad v_y = v_{yi} - gt \]
\[ y = y_0 + v_{yi} t + \frac{1}{2} gt^2 \]
\[ v_y(t)^2 = v_{yi}^2 - 2g(y - y_0) \]

at top: \( v_y = 0 \)

\[ v = \sqrt{v_x^2 + v_y^2} \]
\[ \tan \theta_y = \frac{v_y}{v_x} \]
Practical Technique: Decompose Kinematic Parameters

- **Decomposition**

  \[ \mathbf{r} = x \mathbf{i} + y \mathbf{j} \]
  \[ x = r \cos \theta \]
  \[ y = r \sin \theta \]

- **Inversely:**

  \[ r = \sqrt{x^2 + y^2} \]
  \[ \tan \theta = \frac{y}{x} \]
  \[ v = \sqrt{v_x^2 + v_y^2} \]
  \[ \tan \theta_v = \frac{v_y}{v_x} \]
  \[ a = \sqrt{a_x^2 + a_y^2} \]
  \[ \tan \theta_a = \frac{a_y}{a_x} \]
A projectile is shot at an initial speed $v_i$ at an angle $\theta$. After which, it is in motion only under gravitational force. Find position at any time, air time, the range, maximum height.

**Step 1: decompose $v_i \rightarrow v_i = (v_{ix}, v_{iy}) = (v_i \cos \theta, v_i \sin \theta)$, $a_x=0$, $a_y=-g$**

- Position at any $t$ (Treat $x$, and $y$ separately):
  
  $x(t) = 0 + v_{ix} t = v_i \cos \theta \ t$
  
  $y(t) = 0 + v_{iy} t + \frac{1}{2} a_y t^2 = v_i \sin \theta \ t - \frac{1}{2} gt^2$

- Air time (think vertically $\Delta y=0$):
  
  at $t_B$, $\Delta y=0 \rightarrow t_B = \frac{2 v_i \sin \theta}{g} = T_{air}$

- Range (Think horizontally)
  
  $R = v_i \cos \theta \ T_{air} = 2 v_i^2 \sin \theta \cos \theta / g$

- Maximum height (think vertically $v_{y_A}=0$):
  
  at $A$: $v_y=0 \rightarrow t_A = \frac{v_i \sin \theta}{g} (= \frac{1}{2} t_B !)$,  
  
  $h = y_A = v_{iy} t_A - \frac{1}{2} gt_A^2 = \frac{1}{2} v_i^2 \sin^2 \theta / g$

  (or use $0^2 = v_{iy}^2 - 2hg$)
Conceptual Exercise: Projectile

- Two balls have trajectories A and B, as shown below. (ignore air friction). Without further information, what can we say about the motions?

  a. The launch speed of ball B must be greater than that of ball A.
  b. The launch speed of ball A must be greater than that of ball B.
  c. Ball A is in the air for a longer time than ball B.
  d. Ball B is in the air for a longer time than ball A.
  e. None of above can be concluded with given information.
Exercise 5: Projectile

A projectile is projected on the ground with a velocity of 45.0 m/s at an angle of 60.0 degrees above the horizontal. On its way down, it lands on a rooftop of 4m high. What is the flight time?

- A. 0.1 s
- B. 7.9 s
- C. 2.5 s
- D. 15.0 s

Solution 1:
\[ v_{yi} = v \sin 60^\circ = 39.0 \text{ m/s} \]
\[ y = y_0 + v_{yi}t - \frac{1}{2} gt^2 \]
\[ \frac{1}{2} gt^2 + 39t + 4 = 0 \]
Solving quadratic eq. gives \( t = 7.9 \) s

Solution 2:
\[ v_{yi} = v \sin 60^\circ = 39.0 \text{ m/s} \]
\[ v_y(t)^2 = v_{yi}^2 - 2g(y-y_0) \]
\[ v_y = -38.0 \text{ m/s} \]
\[ t = \frac{(v_y(t) - v_{yi})}{(-g)} = 7.9 \text{ s} \]
Exercise 6: Same Projectile

A projectile is projected on the ground with a velocity of 45.0 m/s at an angle of 60.0 degrees above the horizontal. On its way down, it lands on a rooftop of 4m high. What is the horizontal distance between the launching and landing points?

- A. 10 m
- B. 40 m
- C. 121 m
- D. 178 m
- E. 236 m

solution:

\[ v_{xi} = v \cos 60^\circ = 22.5 \text{ m/s} \]
\[ x - x_0 = v_{yxi}t = 22.5 \times 7.9 = 178 \text{ m} \]

Additional possible questions: Maximum height? landing velocity? ...
Uniform Circular Motion: Useful Formulas

- $\omega = \frac{2\pi}{T}$ (or $T = \frac{2\pi}{\omega}$)

- Linear speed $v = r\omega = \frac{2\pi r}{T}$

- Centripetal Acceleration ($a_c$)
  
  $$a_c = r\omega^2 = \frac{v^2}{r}$$

- Be familiar with the directional relationship of $r$, $v$, $a_c$

(Lecture 6) Exercise: Spin of Earth

- The radius of Earth is $6.37 \times 10^6$ m. To a good approximation, the spin of the Earth is uniform with a period $T$.
  
  Quick Quiz: How much is $T$?
  
  Answer: $T = 24$ hr $= 24 \times 3600 = 86400$ s

- What is angular speed of a person standing on Earth?
  
  ($\omega = \frac{2\pi}{T} = 7.27 \times 10^{-5}$ rad/s)

- What is the linear speed of that person?
  
  ($v = r\omega = 463.1$ m/s)

- How much is his acceleration?
  
  ($a_c = r\omega^2 = 0.034$ m/s²)
Exercise 7:
Do not forget basic definitions

Figure below shows a particle in uniform circular motion. The radius of the circle is \( R = 3.0 \text{ m} \) and the period of the motion is \( T = 4.0 \text{ s} \). The particle goes counter-clockwise, at time \( t_A \), the particle passes point A, and at a later time \( t_B = t_A + T/4 \), it passes B.

What is the magnitude of the particle’s average velocity between \( t_A \) and \( t_B \)?

a. 4.7 m/s  
b. 4.2 m/s  
c. 3.5 m/s  
d. 6.4 m/s  
e. none of above is within 5% from the correct answer
Review: Relative Motion

- Conversion between reference frames (Galilean Transformation)

\[ \vec{v}_{\text{obj\_wrt\_FrameB}} = \vec{v}_{\text{obj\_wrt\_FrameA}} + \vec{v}_{\text{FrameA\_wrt\_FrameB}} \]

One example

<table>
<thead>
<tr>
<th>$v_{o_A}$</th>
<th>$v_{o_B}$</th>
<th>$v_{A_B}$</th>
</tr>
</thead>
</table>

Same principle but a different configuration

visualization example : A=bus, B=earth, o=water drops
Exercise 8: Relative Velocity

- If there is no wind, rain drops will fall to earth vertically down. To a passenger in a moving bus, which picture represents the correct vision of rain traces he observes? (Trivial)

- (not so trivial but still doable) If the bus speed is 16 m/s and the speed of rain drops is 9 m/s, To the passengers in the bus, what is the inclination angle the rain drops make with vertical line? (answer 60.6°, Practice after class).

Hint: use relative velocity relationship

- Diagram showing relative velocities of bus and raindrops.
Quick Quiz: Relatively Velocity

Two stones, A and B, are released from rest at a certain height, one after the other. As they are falling down, will the difference in their velocities increase, decrease, stay the same?

Solution:

\[ v_{A \text{ wrt } B} = v_{A \text{ wrt Earth}} - v_{B \text{ wrt Earth}} \]

\[ \frac{\Delta v_{A \text{ wrt } B}}{\Delta t} = \frac{\Delta v_{A \text{ wrt Earth}}}{\Delta t} - \frac{\Delta v_{B \text{ wrt Earth}}}{\Delta t} = g - g = 0 \]
Special Consulting Hours

- As announced, there will be a “super Friday” review session tomorrow in our lab room.

  TAs will be there to answer your questions from 10:00am to 5pm

- In addition, there will still be regular office hours in our consulting room. Check course web for schedule.