Lecture 5

• Goals:
  - Address systems with multiple accelerations in 2-dimensions (including linear, projectile and circular motion)
  - Discern different reference frames and understand how they relate to particle motion in stationary and moving frames
  - Begin to recognize different types of forces and know how they act on an object in a particle representation

Assignment: HW2, (Chapters 2 & 3, due Wednesday)
Read through Chapter 6, Sections 1-4

Acceleration

• The average acceleration of particle motion reflects changes in the instantaneous velocity vector

\[ \overrightarrow{a} = \frac{\overrightarrow{v}_f - \overrightarrow{v}_i}{t_f - t_i} = \frac{\Delta \overrightarrow{v}}{\Delta t} \]

• The average acceleration need NOT be along the path
**Instantaneous Acceleration**

- The instantaneous acceleration vector:
  \[ \mathbf{a} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt} \]

- The instantaneous acceleration is a vector with components parallel (tangential) and/or perpendicular (radial) to the tangent of the path.

- **Changes in a particle’s path and speed reflect acceleration**
  - If acceleration is tangential, only the magnitude of the velocity vector changes.
  - If acceleration is perpendicular, only the direction of the velocity vector changes.

**Motion with non-zero acceleration:**

\[ \vec{a} \neq 0 \text{ with } |\vec{a}| = \sqrt{a^2_t + a^2_r} \]

need both path & time

\[ \vec{a} = \vec{a}_t + \vec{a}_r \]

Two possible options:
- Change in the magnitude of \( \vec{v} \) \( \vec{a}_t \neq 0 \)
- Change in the direction of \( \vec{v} \) \( \vec{a}_r \neq 0 \)

Animation
Kinematics in 2 D

- The position, velocity, and acceleration of a particle moving in 2-dimensions can be expressed as:

\[ r = x \mathbf{i} + y \mathbf{j} \]
\[ \mathbf{v} = v_x \mathbf{i} + v_y \mathbf{j} \]
\[ \mathbf{a} = a_x \mathbf{i} + a_y \mathbf{j} \]

\[ x = x(\Delta t) \quad y = y(\Delta t) \]
\[ v_x = \frac{dx}{dt} \quad v_y = \frac{dy}{dt} \]
\[ a_x = \frac{d^2x}{dt^2} \quad a_y = \frac{d^2y}{dt^2} \]

Special Cases:
1. \( a_x = 0 \), \( a_y = -g \)
2. Uniform Circular Motion

Special Case 1: Freefall

\[ x(\Delta t) = x_0 + v_x \Delta t \quad v_x = \text{const.} \]
\[ y(\Delta t) = y_0 + v_{y0}t - \frac{1}{2} g \Delta t^2 \]

\[ v_y(\Delta t) = v_{y0} - g \Delta t \]

\( x \) and \( y \) motion are separate and \( \Delta t \) is common to both.

Now: Let \( g \) act in the \(-y\) direction, \( v_{0x} = v_0 \) and \( v_{0y} = 0 \)
Trajectory with constant acceleration along the vertical

What do the velocity and acceleration vectors look like?

Velocity vector is always tangent to the curve!
Acceleration may or may not be!

Example Problem
Given $\vec{r}_0$ & $\vec{v}_0$
How far does the knife travel (if no air resistance)?

Another trajectory

Can you identify the dynamics in this picture?
How many distinct regimes are there?
Are $v_x$ or $v_y = 0$? Is $v_x >, <$ or $= v_y$?
**Another trajectory**

Can you identify the dynamics in this picture?

How many distinct regimes are there?

\[ 0 < t < 3 \quad 3 < t < 7 \quad 7 < t < 10 \]

- **I.** \( v_x = \text{constant} = v_0 ; \quad v_y = 0 \)
- **II.** \( v_x = -v_y = v_0 \)
- **III.** \( v_x = 0 ; \quad v_y = \text{constant} < v_0 \)

What can you say about the acceleration?

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**Exercise 1 & 2**

**Trajectories with acceleration**

- A rocket is drifting sideways (from left to right) in deep space, with its engine off, from A to B. It is not near any stars or planets or other outside forces.
- Its “constant thrust” engine (i.e., acceleration is constant) is fired at point B and left on for 2 seconds in which time the rocket travels from point B to some point C
  - Sketch the shape of the path from B to C.
- At point C the engine is turned off.
  - Sketch the shape of the path after point C
Exercise 1
Trajectories with acceleration

From B to C?

A. A
B. B
C. C
D. D
E. None of these

Exercise 2
Trajectories with acceleration

After C?

A. A
B. B
C. C
D. D
E. None of these
Exercise 3
Relative Trajectories: Monkey and Hunter

All free objects, if acted on by gravity, accelerate similarly.

A hunter sees a monkey in a tree, aims his gun at the monkey and fires. At the same instant the monkey lets go.

Does the bullet ...

A. go over the monkey.
B. hit the monkey.
C. go under the monkey.

Schematic of the problem

- $x_B(\Delta t) = d = v_0 \cos \theta \Delta t$
- $y_B(\Delta t) = h_f = v_0 \sin \theta \Delta t - \frac{1}{2} g \Delta t^2$
- $x_M(\Delta t) = d$
- $y_M(\Delta t) = h - \frac{1}{2} g \Delta t^2$

Does $y_M(\Delta t) = y_B(\Delta t) = h_f$?

Does anyone want to change their answer?

What happens if $g=0$?

How does introducing $g$ change things?
Relative motion and frames of reference

- Reference frame $S$ is stationary
- Reference frame $S'$ is moving at $v_o$
  This also means that $S$ moves at $-v_o$ relative to $S'$
- Define time $t = 0$ as that time when the origins coincide

Relative Velocity

- The positions, $r$ and $r'$, as seen from the two reference frames are related through the velocity, $v_o$, where $v_o$ is velocity of the $r'$ reference frame relative to $r$
  \[ r' = r - v_o \Delta t \]
- The derivative of the position equation will give the velocity equation
  \[ v' = v - v_o \]
- These are called the Galilean transformation equations
- Reference frames that move with “constant velocity” (i.e., at constant speed in a straight line) are defined to be inertial reference frames (IRF); anyone in an IRF sees the same acceleration of a particle moving along a trajectory.
  \[ a' = a \quad (dv_o / dt = 0) \]
Central concept for problem solving: “x” and “y” components of motion treated independently.

- Example: Man on cart tosses a ball **straight** up in the air.
- You can view the trajectory from two reference frames:

Reference frame on the moving cart.  

\[ y(t) \text{ motion governed by} \]
\[ 1) \quad \mathbf{a} = -g \mathbf{y} \]
\[ 2) \quad v_y = v_{0y} - g \Delta t \]
\[ 3) \quad y = y_0 + v_{0y} - g \Delta t^2/2 \]

Reference frame on the ground.

x motion: \( x = v_x t \)

Net motion: \[ \mathbf{R} = x(t) \mathbf{i} + y(t) \mathbf{j} \text{ (vector)} \]

Example (with frames of reference)

An experimental aircraft can fly at full throttle in still air at 200 m/s. The pilot has the nose of the plane pointed west (at full throttle) but, unknown to the pilot, the plane is actually flying through a strong wind blowing from the northwest at 140 m/s. Just then the engine fails and the plane starts to fall at 5 m/s\(^2\).

What is the magnitude and directions of the resulting velocity (relative to the ground) the instant the engine fails?

Calculate: \( \mathbf{A} + \mathbf{B} \)

\[ A_x + B_x = -200 + 140 \times 0.71 \quad \text{and} \]
\[ A_y + B_y = 0 - 140 \times 0.71 \]
Exercise,
Relative Motion

- You are swimming across a 50 m wide river in which the current moves at 1 m/s with respect to the shore. Your swimming speed is 2 m/s with respect to the water.

You swim across in such a way that your path is a straight perpendicular line across the river.
- How many seconds does it take you to get across?

\[
\begin{align*}
\text{a)} & \quad 50/2 = 25 \text{ s} \\
\text{b)} & \quad 50/1 = 50 \text{ s} \\
\text{c)} & \quad 50/\sqrt{3} = 29 \text{ s} \\
\text{d)} & \quad 50/\sqrt{2} = 35 \text{ s}
\end{align*}
\]

Exercise

Choose x axis along riverbank and y axis across river.
- The time taken to swim straight across is \((\text{distance across}) / (v_y)\)
- Since you swim straight across, you must be tilted in the water so that your x component of velocity with respect to the water exactly cancels the velocity of the water in the x direction.
Generalized motion with only radial acceleration

Uniform Circular Motion

\[ \vec{a} = \vec{a}_r + \vec{a}_\perp \]

Changes only in the direction of \( \vec{v} \)  \( \vec{a}_\perp \neq 0 \)

A particle doesn’t speed up or slow down!

Uniform Circular Motion (UCM) is common so we have specialized terms

- Arc traversed \( s = \theta r \)
- Tangential velocity \( v_t \)
- Period, \( T \), and frequency, \( f \)
- Angular position, \( \theta \)
- Angular velocity, \( \omega \)

Period (\( T \)): The time required to do one full revolution, 360° or 2\( \pi \) radians

Frequency (\( f \)): 1/\( T \), number of cycles per unit time

Angular velocity or speed \( \omega = 2\pi f = 2\pi/T \), number of radians traced out per unit time (in UCM average and instantaneous will be the same)
**Exercise**

A Ladybug sits at the outer edge of a merry-go-round, and a June bug sits halfway between the outer one and the axis of rotation. The merry-go-round makes a complete revolution once each second. What is the June bug’s angular velocity?

A. half the Ladybug’s.
B. the same as the Ladybug’s.
C. twice the Ladybug’s.
D. impossible to determine.

**Circular Motion**

- UCM enables high accelerations (g's) in a small space

- Comment: In automobile accidents involving rotation severe injury or death can occur even at modest speeds. [In physics speed doesn’t kill….acceleration does (i.e., the sudden change in velocity).]
Recap

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