Lecture 6

Goals:
- Discuss circular motion

Chapters 5 & 6
- Recognize different types of forces and know how they act on an object in a particle representation
- Identify forces and draw a Free Body Diagram
- Begin to solve 1D and 2D problems with forces in equilibrium and non-equilibrium (i.e., acceleration) using Newton’s 1st and 2nd laws.

Assignment: HW3, (Chapters 4 & 5, due 2/10, Wednesday)
Finish reading Chapter 6
Exam 1 Wed, Feb. 17 from 7:15-8:45 PM Chapters 1-7

Concept Check

Q1. You drop a ball from rest, how much of the acceleration from gravity goes to changing its speed?
   A. All of it
   B. Most of it
   C. Some of it
   D. None of it

Q2. A hockey puck slide off the edge of the table, at the instant it leaves the table, how much of the acceleration from gravity goes to changing its speed?
   A. All of it
   B. Most of it
   C. Some of it
   D. None of it
Uniform Circular Motion (UCM)

- Arc traversed \( s = \theta r \)
- Tangential speed \( |v_t| = \Delta s / \Delta t \) or (in the limit) \( ds/dt = r \, d\theta / dt \)
- Period \( T = 2\pi r / |v_t| \)
- Frequency \( f = 1 / T \)
- Angular position \( \theta \)
- Angular velocity \( \omega = d\theta / dt = |v_t| / r \)

Period \( (T) \): The time required to do one full revolution, 360° or 2\pi radians

Frequency \( (f) \): \( 1/T \), number of cycles per unit time

Angular velocity or speed \( \omega = 2\pi f = 2\pi / T \), number of radians traced out per unit time (in UCM average and instantaneous will be the same)

Example

- A horizontally mounted disk 2 meters in diameter spins at constant angular speed such that it first undergoes
  (1) 10 counter clockwise revolutions in 5 seconds and then, again at constant angular speed,
  (2) 2 counter clockwise revolutions in 5 seconds.
- 1 What is \( T \) the period of the initial rotation?

\[
T = \text{time for 1 revolution} = 5 \text{ sec} / 10 \text{ rev} = 0.5 \text{ s}
\]

also \( T = 2\pi r / |v_t| \)

( just like \( x = x_0 + v \Delta t \) \( \Rightarrow \Delta t = (x- x_0) / v \) )
Example

- A horizontally mounted disk 2 meters in diameter spins at constant angular speed such that it first undergoes 10 counter clockwise revolutions in 5 seconds and then, again at constant angular speed, 2 counter clockwise revolutions in 5 seconds.

1. What is $T$ the period of the initial rotation?
2. What is $\omega$ the initial angular velocity?

$$\omega = \frac{d\theta}{dt} = \frac{\Delta \theta}{\Delta t}$$

$$\omega = 10 \cdot 2\pi \text{ radians} / 5 \text{ seconds}$$

$$= 12.6 \text{ rad} / \text{s} \quad (\text{also} \quad 2\pi f = 2\pi / T)$$

Example

- A horizontally mounted disk 2 meters in diameter spins at constant angular speed such that it first undergoes 10 counter clockwise revolutions in 5 seconds and then, again at constant angular speed, 2 counter clockwise revolutions in 5 seconds.

1. What is $T$ the period of the initial rotation?
2. What is $\omega$ the initial angular velocity?
3. What is the tangential speed of a point on the rim during this initial period?

$$|v_t| = ds/dt = (r d\theta)/dt = r \omega$$

$$|v_t| = r \omega = 1 \text{ m} \cdot 12.6 \text{ rad/s} = 12.6 \text{ m/s}$$
Angular displacement and velocity

- Notice that if $\omega \equiv \frac{d\theta}{dt}$ and, if $\omega$ is constant, then integrating $\omega = \frac{d\theta}{dt}$, we obtain: $\theta = \theta_0 + \omega \Delta t$

( In one dimensional motion if 
$v = \frac{dx}{dt} = \text{constant} \text{ then } x = x_0 + v \Delta t )$

Counter-clockwise is positive, clockwise is negative

$$\theta = \theta_0 + \omega \Delta t$$

Example

- A horizontally mounted disk 2 meters in diameter spins at constant angular speed such that it first undergoes 10 counter clockwise revolutions in 5 seconds and then, again at constant angular speed, 2 counter clockwise revolutions in 5 seconds.
- 1 What is $T$ the period of the initial rotation?
- 2 What is $\omega$ the initial angular velocity?
- 3 What is the tangential speed of a point on the rim during this initial period?
- 4 Sketch the $\theta$ (angular displacement) versus time plot.
Example

- A horizontally mounted disk 2 meters in diameter spins at constant angular speed such that it first undergoes 10 counter clockwise revolutions in 5 seconds and then, again at constant angular speed, 2 counter clockwise revolutions in 5 seconds.

  1. What is T the period of the initial rotation?
  2. What is $\omega$ the initial angular velocity?
  3. What is the tangential speed of a point on the rim during this initial period?
  4. Sketch the $\theta$ (angular displacement) versus time plot.

  5. What is the average angular velocity over the 1st 10 seconds?
Example

- A horizontally mounted disk 2 meters in diameter spins at constant angular speed such that it first undergoes 10 counter clockwise revolutions in 5 seconds and then, again at constant angular speed, 2 counter clockwise revolutions in 5 seconds.

- If now the turntable starts from rest and uniformly accelerates throughout and reaches the same angular displacement in the same time, what must be the angular acceleration?
What if $\omega$ is linearly increasing ...

- Then angular velocity is no longer constant so $d\omega/dt \neq 0$
- Define tangential acceleration as $a_t = d\nu/dt = r \, d\omega/dt$
- So $s = s_0 + (ds/dt)_0 \, \Delta t + \frac{1}{2} a_t \, \Delta t^2$ and $s = \theta \, r$
- We can relate $a_t$ to $d\omega/dt$

\[
\left\{
\begin{align*}
\theta &= \theta_0 + \omega_0 \, \Delta t + \frac{1}{2} \frac{a_t}{r} \, \Delta t^2 \\
\omega &= \omega_0 + \frac{a_t}{r} \, \Delta t
\end{align*}
\right.
\]

- Many analogies to linear motion but it isn’t one-to-one
- Remember: Even if $\omega$ is constant, there is always a radial acceleration.

Circular motion also has a radial (perpendicular) component

Uniform circular motion involves only changes in the direction of the velocity vector, thus acceleration is perpendicular to the trajectory at any point, acceleration is only in the radial direction. Quantitatively (see text)

Centripetal Acceleration

\[a_r = \frac{v_t^2}{r}\]

Circular motion involves continuous radial acceleration
Tangential acceleration?

6 If now the turntable starts from rest and uniformly accelerates throughout and reaches the same angular displacement in the same time, what must the "tangential acceleration" be?

\[ \theta = \theta_0 + \omega_0 \Delta t + \frac{1}{2} \frac{a_t}{r} \Delta t^2 \]

(from plot, after 10 seconds)

\[ 24 \pi \text{ rad} = 0 \text{ rad} + 0 \text{ rad/s} \Delta t + \frac{1}{2} \frac{(a_t/r)}{r} \Delta t^2 \]

\[ 48 \pi \text{ rad} \quad 1 \text{ m} / 100 \text{ s}^2 = a_t \]

7 What is the magnitude and direction of the acceleration after 10 seconds?

Tangential acceleration is too small to plot!

\[ a_t = 0.48 \pi \text{ m} / \text{s}^2 \]

and \[ \omega r = \omega_0 r + r \frac{a_t}{r} \Delta t = 4.8 \pi \text{ m/s} = v_t \]

\[ a_r = \frac{v_t^2}{r} = 23 \pi^2 \text{ m/s}^2 \]

Tangential acceleration is too small to plot!
Angular motion, sign convention

- If angular displacement, velocity, accelerations are counter-clockwise then sign is positive.
- If clockwise then negative.

What causes motion? *(Actually changes in motion)*

What are forces?

What kinds of forces are there?

How are forces and changes in motion related?
Newton’s First Law and IRFs

An object subject to no external forces moves with constant velocity if viewed from an *inertial reference frame (IRF)*.

If **no net force** acting on an object, there is **no acceleration**.

- The above statement can be used to define inertial reference frames.

IRFs

- An IRF is a reference frame that is not accelerating (or rotating) with respect to the “fixed stars”.

- If one IRF exists, infinitely many exist since they are related by any arbitrary constant velocity vector!

- In many cases (i.e., Chapters 5, 6 & 7) the surface of the Earth may be viewed as an IRF
Newton’s Second Law

The acceleration of an object is directly proportional to the net force acting upon it.

The constant of proportionality is the mass.

\[ \sum \vec{F} = \vec{F}_{\text{NET}} = m\vec{a} \]

- This expression is vector expression: \( F_x, F_y, F_z \)
- Units
  - The metric unit of force is \( \text{kg m/s}^2 = \text{Newtons (N)} \)
  - The English unit of force is \( \text{Pounds (lb)} \)

Lecture 6

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Read rest of chapter 6