Lecture 9

• Today:

❖ Review session

Assignment: For Thursday, Read through Chapter 8 (first four sections)
Exam Wed., Feb. 17th from 7:15-8:45 PM Chapters 1-6
One 8½ X 11 hand written note sheet and a calculator (for trig.)
Place: Room 2103 (Sec. 301 & 303+)
                Room 2120 (Sec. 302, TR 2:25-3:15PM, H. Yoo)

Textbook Chapters

• Chapter 1  Concept of Motion
• Chapter 2  1D Kinematics
• Chapter 3  Vector and Coordinate Systems
• Chapter 4  Dynamics I, Two-dimensional motion
• Chapter 5  Forces and Free Body Diagrams
• Chapter 6  Force and Newton’s 1st and 2nd Laws
                (no inclines with friction)

Exam will reflect most key points (but not all)
25-30% of the exam will be more conceptual
70-75% of the exam is problem solving
Example

- The velocity of an object as a function of time is shown in the graph at right. Which graph below best represents the net force vs time relationship for this object?

![Graphs A, B, C, D, E representing force vs time](Physics 207: Lecture 9, Pg 3)

Example

- The velocity of an object as a function of time is shown in the graph at right. Which graph below best represents the net force vs time relationship for this object?

![Graphs A, B, C, D, E representing force vs time](Physics 207: Lecture 9, Pg 4)
Chapter 2

General Principles

Kinematics describes motion in terms of position, velocity, and acceleration. General kinematic relationships are given mathematically by:

\[ \dot{v}_i = \frac{ds}{dt} = \text{slope of position graph} \]
\[ \ddot{a}_i = \frac{dv_i}{dt} = \text{slope of velocity graph} \]

Final position
\[ s_f = s_i + \int_{t_i}^{t_f} v_i \, dt = s_i + \text{area under the velocity curve from } t_i \text{ to } t_f \]

Final velocity
\[ v_f = v_i + \int_{t_i}^{t_f} a_i \, dt = v_i + \text{area under the acceleration curve from } t_i \text{ to } t_f \]

Motion with constant acceleration is uniformly accelerated motion. The kinematic equations are:

\[ v_f = v_i + a_i \Delta t \]
\[ s_f = s_i + v_i \Delta t + \frac{1}{2} a_i (\Delta t)^2 \]
\[ v_f^2 = v_i^2 + 2a_i \Delta s \]

Uniform motion is motion with constant velocity and zero acceleration:
\[ s_f = s_i + v_e \Delta t \]

Important Concepts

Position, velocity, and acceleration are related graphically:

- The slope of the position-versus-time graph is the value on the velocity graph.
- The slope of the velocity graph is the value on the acceleration graph.
- \( v_i \) is a maximum or minimum at a turning point, and \( v_f = 0 \).

Also average speed and average velocity
Chapter 3

Important Concepts

A vector is a quantity described by both a magnitude and a direction.

The vector describes the situation at this point.

The length or magnitude is denoted $A$. Magnitude is a scalar.

Unit Vectors

Unit vectors have magnitude 1 and no units. Unit vectors $i$ and $j$

define the directions of the $x$- and $y$-axes.

Using Vectors

Components

The component vectors are parallel to the $x$- and $y$-axes:

\[ \vec{A} = A_x \hat{i} + A_y \hat{j} \]

In the figure at the right, for example:

\[ A_x = A \cos \theta \quad \theta = \tan^{-1}(A_y/A_x) \]

\[ A_y = A \sin \theta \]

\[ A_x = 0 \quad A_y > 0 \]

\[ A_x > 0 \quad A_y = 0 \]

\[ A_x < 0 \quad A_y > 0 \]

\[ A_x < 0 \quad A_y < 0 \]

\[ A_x < 0 \quad A_y < 0 \]

Minus signs need to be included if the vector points
down or left.

Chapter 3

Working Graphically

Addition

\[ \vec{A} + \vec{B} = \vec{C} \]

Negative

\[ \vec{A} - \vec{B} = \vec{C} \]

Subtraction

\[ \vec{A} - \vec{B} = \vec{C} \]

Multiplication

\[ A \vec{i} \]

Working Algebraically

Vector calculations are done component by component:

\[ \vec{C} = A \vec{i} + B \vec{j} \]

\[ C_x = 2A_x + B_x \]

\[ C_y = 2A_y + B_y \]

The magnitude of $\vec{C}$ is then

\[ C = \sqrt{C_x^2 + C_y^2} \]

and its direction is found using $\tan^{-1}$. 
Chapter 4

**General Principles**

The instantaneous velocity
\[ \vec{v} = \frac{d\vec{x}}{dt} \]
is a vector tangent to the trajectory.

The instantaneous acceleration is
\[ \vec{a} = \frac{d\vec{v}}{dt} \]
\( \vec{a} \), the component of \( \vec{a} \) parallel to \( \vec{v} \), is responsible for change of speed. \( \vec{a}_r \), the component of \( \vec{a} \) perpendicular to \( \vec{v} \), is responsible for change of direction.

Relative motion
Inertial reference frames move relative to each other with constant velocity \( \vec{V} \). Measurements of position and velocity measured in frame \( S \) are related to measurements in frame \( S' \) by the Galilean transformations:
\[
\begin{align*}
  x' &= x - V_x t \\
  y' &= y - V_y t \\
  z' &= z - V_z t
\end{align*}
\]

**Important Concepts**

**Uniform Circular Motion**

Angular velocity \( \omega = \frac{d\theta}{dt} \). 
\( v_r \) and \( \omega \) are constant:
\[ v_r = \omega r \]
The centripetal acceleration points toward the center of the circle:
\[ a_c = \frac{v_r^2}{r} = \omega^2 r \]
It changes the particle’s direction but not its speed.

**Nonuniform Circular Motion**

Angular acceleration \( \alpha = \frac{da}{dt} \). 
The radial acceleration changes the particle’s direction. The tangential component changes the particle’s speed.

**Applications**

Kinematics in two dimensions
If \( \vec{d} \) is constant, then the \( x \)- and \( y \)-components of motion are independent of each other.
\[
\begin{align*}
  x &= x_0 + v_x \Delta t + \frac{1}{2} a_x \Delta t^2 \\
  y &= y_0 + v_y \Delta t + \frac{1}{2} a_y \Delta t^2 \\
  v_x &= v_{x0} + a_x \Delta t \\
  v_y &= v_{y0} + a_y \Delta t
\end{align*}
\]

Projectile motion occurs if the object moves under the influence of only gravity. The motion is a parabola.

- Uniform motion in the horizontal direction with \( v_{x0} = v_0 \cos \theta \).
- Free-fall motion in the vertical direction with \( v_{y0} = v_0 \sin \theta \).
- The \( x \) and \( y \) kinematic equations have the same value for \( \Delta t \).

**Circular motion kinematics**

Period \( T = \frac{2\pi \omega}{v} \)
Angular position \( \theta = \frac{\tau}{2\pi} \)
\[
\begin{align*}
  \omega &= \omega_0 + \alpha \Delta \tau \\
  \theta &= \theta_0 + \omega_0 \Delta \tau + \frac{1}{2} \alpha (\Delta \tau)^2 \\
  \omega' &= \omega_0' + \alpha (\Delta \tau)
\end{align*}
\]

Angle, angular velocity, and angular acceleration are related graphically.

- The angular velocity is the slope of the angular position graph.
- The angular acceleration is the slope of the angular velocity graph.
Chapter 5

**General Principles**

**Newton’s First Law**
An object at rest will remain at rest, or an object that is moving will continue to move in a straight line with constant velocity, if and only if the net force on the object is zero.

\[ F_{\text{net}} = 0 \]

\[ a = 0 \]

The first law tells us that no “cause” is needed for motion. Uniform motion is the “natural state” of an object.

**Important Concepts**

**Acceleration** is the link to kinematics.

From \( F_{\text{net}} \), find \( a \).

From \( a \), find \( v \) and \( x \).

\( \dot{a} = 0 \) is the condition for equilibrium.

**Static equilibrium** if \( \vec{F} = 0 \).

**Dynamic equilibrium** if \( \vec{F} \) is constant.

Equilibrium occurs if and only if \( F_{\text{net}} = 0 \).

**Newton’s Second Law**
An object with mass \( m \) will undergo acceleration

\[ \vec{a} = \frac{1}{m} \vec{F}_{\text{net}} \]

where \( \vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots \) is the vector sum of all the individual forces acting on the object.

The second law tells us that a net force causes an object to accelerate. This is the connection between force and motion that we are seeking.

**Key Skills**

**Identifying Forces**

Forces are identified by locating the points where other objects touch the object of interest. These are points where contact forces are exerted. In addition, objects with mass feel a long-range gravitational force.

**Free-Body Diagrams**

A free-body diagram represents the object as a particle at the origin of a coordinate system. Force vectors are drawn with their tails on the particle. The net force vector is drawn beside the diagram.

**General Strategy**

All examples in this chapter follow a four-part strategy. You’ll become a better problem solver if you adhere to it as you do the homework problems. The Dynamics Worksheet in the Student Workbook will help you structure your work in this way.

**Equilibrium Problems**

Object at rest or moving with constant velocity:

**MODEL** Make simplifying assumptions.

**VISUALIZE**

- Translate words into symbols.
- Identify forces.
- Draw a free-body diagram.

**SOLVE**

Use Newton’s first law:

\[ F_{\text{net}} = \sum \vec{F} = 0 \]

“Read” the vectors from the free-body diagram.

\[ \text{Is the result reasonable?} \]

**Dynamics Problems**

Object accelerating:

**MODEL** Make simplifying assumptions.

**VISUALIZE**

- Translate words into symbols.
- Draw a sketch to define the situation.
- Draw a motion diagram.
- Identify forces.
- Draw a free-body diagram.

**SOLVE**

Use Newton’s second law:

\[ F_{\text{net}} = \sum \vec{F} = ma \]

“Read” the vectors from the free-body diagram.

Use kinematics to find velocities and positions.

\[ \text{Is the result reasonable?} \]
Important Concepts

<table>
<thead>
<tr>
<th>Specific information about three important forces:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gravity</strong></td>
</tr>
<tr>
<td><strong>Friction</strong></td>
</tr>
<tr>
<td>( F_f = (\mu_w \text{, direction opposite the motion}) )</td>
</tr>
</tbody>
</table>

Newton’s laws are vector expressions. You must write them out by components:
\[
(F_{\text{net}})_x = \sum F_x = ma_x \text{ or } 0
\]
\[
(F_{\text{net}})_y = \sum F_y = ma_y \text{ or } 0
\]

Including static and kinetic friction models

Short word problems

- After breakfast, I weighed myself and the scale read 588 N. On my way out, I decide to take my bathroom scale in the elevator with me. What does the scale read as the elevator accelerates downwards with an acceleration of 1.5 m/s\(^2\) ?

- A bear starts out and walks 1\(^{st}\) with a velocity of 0.60 \(\hat{j}\) m/s for 10 seconds and then walks at 0.40 \(\hat{i}\) m/s for 20 seconds. What was the bear’s average velocity on the walk? What was the bear’s average speed on the walk (with respect to the total distance travelled) ?
Conceptual Problem

The pictures below depict cannonballs of identical mass which are launched upwards and forward. The cannonballs are launched at various angles above the horizontal, and with various velocities, but all have the same vertical component of velocity.

\[
\begin{align*}
\text{v}_y &= 20 \text{ m/s} \\
v &= 40 \text{ m/s} & 30^\circ & \text{ (III)} \\
v &= 28.3 \text{ m/s} & 45^\circ & \text{ (II)} \\
v &= 22.01 \text{ m/s} & 60^\circ & \text{ (I)}
\end{align*}
\]

Which of the following correctly ranks the time the balls are in the air, from shortest to longest?

a) I, II, III
b) III, II, I
c) I, III, II
d) All the balls are in the air for the same amount of time.

Conceptual Problem

A bird sits in a birdfeeder suspended from a tree by a wire, as shown in the diagram at left.

Let \( W_B \) and \( W_F \) be the weight of the bird and the feeder respectively. Let \( T \) be the tension in the wire and \( N \) be the normal force of the feeder on the bird. Which of the following free-body diagrams best represents the birdfeeder? (The force vectors are not drawn to scale and are only meant to show the direction, not the magnitude, of each force.)

\[\text{Diagram with force vectors labeled:}\]

\( W_B \), \( W_F \), \( T \), \( N \)
Graphing problem

The figure shows a plot of velocity vs. time for an object moving along the $x$-axis. Which of the following statements is true?

(A) The average acceleration over the 11.0 second interval is $-0.36 \text{ m/s}^2$  
(B) The instantaneous acceleration at $t = 5.0$ s is $-4.0 \text{ m/s}^2$  
(C) Both A and B are correct.  
(D) Neither A nor B are correct.

Conceptual Problem

A cart on a roller-coaster rolls down the track shown below. As the cart rolls beyond the point shown, what happens to its speed and acceleration in the direction of motion?

A. Both decrease.  
B. The speed decreases, but the acceleration increases.  
C. Both remain constant.  
D. The speed increases, but acceleration decreases.  
E. Both increase.  
F. Other
Conceptual Problem

- A person initially at point $P$ in the illustration stays there a moment and then moves along the axis to $Q$ and stays there a moment. She then runs quickly to $R$, stays there a moment, and then strolls slowly back to $P$. Which of the position vs. time graphs below correctly represents this motion?

Sample Problem

- You have been hired to measure the coefficients of friction for the newly discovered substance jelloium. Today you will measure the coefficient of kinetic friction for jelloium sliding on steel. To do so, you pull a 200 g chunk of jelloium across a horizontal steel table with a constant string tension of 1.00 N. A motion detector records the motion and displays the graph shown.

- What is the value of $\mu_k$ for jelloium on steel?
Sample Problem

\[ \sum F_x = ma = F - f = F - \mu_k N = F - \mu_k mg \]
\[ \sum F_y = 0 = N - mg \]

\[ \mu_k = (F - ma) / mg \quad \& \quad x = \frac{1}{2} a t^2 \rightarrow 0.80 \text{ m} = \frac{1}{2} a \cdot 4 \text{ s}^2 \]
\[ a = 0.40 \text{ m/s}^2 \]
\[ \mu_k = (1.00 - 0.20 \cdot 0.40 ) / (0.20 \cdot10.) = 0.46 \]

Exercise: Newton’s 2nd Law

A force of 2 Newtons acts on a cart that is initially at rest on an air track with no air and pushed for 1 second. Because there is friction (no air), the cart stops immediately after I finish pushing. It has traveled a distance, \( D \).

Next, the force of 2 Newtons acts again but is applied for 2 seconds.

The new distance the cart moves relative to \( D \) is:

A. 8 x as far
B. 4 x as far
C. 2 x as far
D. 1/4 x as far
Exercise: Solution

We know that under constant acceleration,
\[
\Delta x = \frac{a (\Delta t)^2}{2} \quad \text{(when } v_0 = 0)\]

Here \(\Delta t_2 = 2 \Delta t_1\), \(F_2 = F_1 \Rightarrow a_2 = a_1\)

\[
\frac{\Delta x_2}{\Delta x_1} = \frac{\frac{1}{2} a \Delta t_2^2}{\frac{1}{2} a \Delta t_1^2} = \frac{(2 \Delta t_1)^2}{\Delta t_1^2} = 4
\]

(B) 4 x as long

Mass-based separation with a centrifuge

A centrifuge with a radius of 0.10 m spins at a frequency of \(10^4\) rpm. How many \(g\)'s?
Mass-based separation with a centrifuge

How many g’s?

\[ a_r = \frac{v_t^2}{r} \quad \text{and} \quad v_t = \omega r = 2\pi f r \]

\[ v_t = (2\pi \times 10^4 / 60) \times 0.10 \text{ m/s} = 100 \text{ m/s} \]

\[ a_r = 1 \times 10^4 / 0.10 \text{ m/s}^2 = 10000 \text{ g’s} \]

Another question to ponder

How high will it go?

- One day you are sitting somewhat pensively in an airplane seat and notice, looking out the window, one of the jet engines running at full throttle. From the pitch of the engine you estimate that the turbine is rotating at 3000 rpm and, give or take, the turbine blade has a radius of 1.00 m. If the tip of the blade were to suddenly break off (it occasionally does happen with negative consequences) and fly directly upwards, then how high would it go (assuming no air resistance and ignoring the fact that it would have to penetrate the metal cowling of the engine.)
Another question to ponder

How high will it go?

- $\omega = 3000 \text{ rpm} = (3000 \times 2\pi / 60) \text{ rad/s} = 314 \text{ rad/s}$
- $r = 1.00 \text{ m}$
- $v_o = \omega r = 314 \text{ m/s} \ (\sim 650 \text{ mph})$
- $h = h_0 + v_0 t - \frac{1}{2} g t^2$
- $v_h = 0 = v_o - g t \rightarrow t = \frac{v_o}{g}$

So
- $h = v_0 t - \frac{1}{2} g t^2 = \frac{1}{2} v_o^2 / g = 0.5 \times 314^2 / 9.8 = 5 \text{ km}$
  or $\sim 3 \text{ miles}$

***Sample exam problem

- You have a 2.0 kg block that moves on a linear path on a horizontal surface. The coefficient of kinetic friction between the block and the path is $\mu_k$. Attached to the block is a horizontally mounted massless string as shown in the figure below. The block includes an accelerometer which records acceleration vs. time. As you increase the tension in the rope the block experiences an increasingly positive acceleration. At some point in time the rope snaps and then the block slides to a stop (at a time of 10 seconds). Gravity, with $g = 10 \text{ m/s}^2$, acts downward.
Sample exam problem

A. At what time does the string break and, in one sentence, explain your reasoning?

B. What speed did the block have when the string broke?

C. What is the value of $\mu_k$?

D. Using $\mu_k$ above (or a value of 0.25 if you don’t have one), what was the tension in the string at $t = 2$ seconds?

Sample exam problem

B. What speed did the block have when the string broke?

Don’t know initial $v$ (t=0) so can’t integrate area at $t < 4$ sec.

$v_f = 0 \text{ m/s}$ and from $t = 4$ to $10$ sec (6 second) $a = -2 \text{ m/s}^2$

$0 = v_i + a t = v_i - 2 \times 6 \text{ m/s} \Rightarrow v_i = 12 \text{ m/s}$
Sample exam problem

C. What is the value of $\mu_k$? Use a FBD!

\[ \sum F_x = ma_x = -f_k = -\mu_k N \]
\[ \sum F_y = 0 = mg - N \rightarrow N = mg \]
So
\[ ma_x = -f_k = -\mu_k mg \rightarrow \mu_k = -\frac{a_x}{g} = -\frac{-2}{10} = 0.20 \]

Sample exam problem

D. What was the tension in the string at $t = 2$ seconds?
Again a FBD!

\[ \sum F_x = ma_x = -f_k + T \]
\[ \sum F_y = 0 = mg - N \rightarrow N = mg \]
\[ T = f_k + ma_x = (0.20 \times 2 \times 10 + 2 \times 3) N = 10 N \]
Sample exam problem

An object is at first travelling due north, turns and finally heads due west while increasing its speed. The average acceleration for this maneuver is pointed

A directly west.
B somewhere between west and northwest.
C somewhere between west and southwest.
D somewhere between northwest and north.
E somewhere between southwest and south.
F None of these are correct

Sample exam problem

An object is at first travelling due north, turns and finally heads due west while increasing its speed. The average acceleration for this maneuver is pointed

\[ a = \frac{v_f - v_i}{\Delta t} \]

A directly west.
B somewhere between west and northwest.
C somewhere between west and southwest.
D somewhere between northwest and north.
E somewhere between southwest and south.
F None of these are correct
*** Sample exam problem

A small block moves along a frictionless incline which is 45° from horizontal. Gravity acts down at 10 m/s². There is a massless cord pulling on the block. The cord runs parallel to the incline over a pulley and then straight down. There is tension, $T_1$, in the cord which accelerates the block at 2.0 m/s² up the incline. The pulley is suspended with a second cord with tension, $T_2$.

A. What is the tension magnitude, $T_1$, in the 1st cord?
B. What is the tension magnitude, $T_2$, in the 2nd cord?

(Assume $T_1 = 50. N$ if you don't have an answer to part A.)

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Sample exam problem

$a = 2.0 \text{ m/s}^2$ up the incline.

What is the tension magnitude, $T_1$, in the 1st cord?

Use a FBD!

Along the block surface

$$\sum F_x = ma_x = -mg \sin \theta + T$$

$$T = 5 \times 2 \text{ N} + 5 \times 10 \times 0.7071 \text{ N}$$

$$= (10 + 35) \text{ N} = 45 \text{ N}$$
Sample exam problem

\[ a = 0.0 \text{ m/s}^2 \] at the pulley.

What is the tension magnitude, \( T_2 \), in the 2\(^{nd} \) cord?

Use a FBD!

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Lecture 9

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