Lecture 10

Goals:

- Exploit Newton’s 3rd Law in problems with friction
- Employ Newton’s Laws in 2D problems with circular motion

Assignment: HW5, (Chapter 7, due 2/24, Wednesday)
For Tuesday: Finish reading Chapter 8, First four sections in Chapter 9

Example: Friction and Motion

- A box of mass $m_1 = 1$ kg is being pulled by a horizontal string having tension $T = 40$ N. It slides with friction ($\mu_k = 0.5$) on top of a second box having mass $m_2 = 2$ kg, which in turn slides on a smooth (frictionless) surface.
- What is the acceleration of the bottom box?

Key Question: What is the force on mass 2 from mass 1?

![Diagram of the friction and motion example]
Example Solution

- First draw FBD of the top box:

- Newtons 3\textsuperscript{rd} law says the force box 2 exerts on box 1 is equal and opposite to the force box 1 exerts on box 2.

- As we just saw, this force is due to friction:
Now consider the FBD of box 2:

\[ N_1 = m_1 g \]

\[ f_{2,1} = \mu_k m_1 g \]

\[ m_2 \]

\[ m_2 g \]

Finally, solve \( F_x = ma_x \) in the horizontal direction:

\[ \mu_k m_1 g = m_2 a_x \]

\[ a_x = \frac{m_1 \mu_k g}{m_2} = \frac{5 \text{ N}}{2 \text{ kg}} \]

\[ = 2.5 \text{ m/s}^2 \]
Home Exercise
Friction and Motion, Replay

- A box of mass $m_1 = 1 \text{ kg}$, initially at rest, is now pulled by a horizontal string having tension $T = 10 \text{ N}$. This box (1) is on top of a second box of mass $m_2 = 2 \text{ kg}$. The static and kinetic coefficients of friction between the 2 boxes are $\mu_s = 1.5$ and $\mu_k = 0.5$. The second box can slide freely (frictionless) on an smooth surface.

Question:
Compare the acceleration of box 1 to the acceleration of box 2?

In the case of “no slippage” what is the maximum frictional force between boxes 1 & 2?

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Home Exercise
Friction and Motion, Replay in the static case

- A box of mass $m_1 = 1 \text{ kg}$, initially at rest, is now pulled by a horizontal string having tension $T = 10 \text{ N}$. This box (1) is on top of a second box of mass $m_2 = 2 \text{ kg}$. The static and kinetic coefficients of friction between the 2 boxes are $\mu_s = 1.5$ and $\mu_k = 0.5$. The second box can slide freely on an smooth surface (frictionless).

In the case of “no slippage” what is the maximum frictional force between boxes 1 & 2?
**Home Exercise**
Friction and Motion

\[ f_s \leq \mu_s N = \mu_s m_1 g = 1.5 \times 1 \text{ kg} \times 10 \text{ m/s}^2 \]
which is 15 N (so \( m_2 \) can’t break free)

\[ f_s = 10 \text{ N} \text{ and the acceleration of box 1 is} \]

Acceleration of box 2 equals that of box 1, with \(|a| = |T| / (m_1+m_2)\)
and the frictional force \( f \) is \( m_2 g \)
(Notice that if \( T \) were in excess of 15 N then it would break free)

\[ \mu_s = 1.5 \text{ and } \mu_k = 0.5 \]

slides without friction

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**Exercise** Tension example

Compare the strings below in settings (a) and (b) and their tensions.

A. \( T_a = \frac{1}{2} T_b \)
B. \( T_a = 2 T_b \)
C. \( T_a = T_b \)
D. Correct answer is not given
Problem 7.34 Hint (HW 6)

Suggested Steps

- **Two** independent free body diagrams are necessary
- Draw in the forces on the top and bottom blocks
- **Top Block**
  - Forces: 1. normal to bottom block 2. weight 3. rope tension and 4. friction with bottom block (model with sliding)
- **Bottom Block**
  - Forces: 1. normal to bottom surface 2. normal to top block interface 3. rope tension (to the left) 4. weight (2 kg) 5. friction with top block 6. friction with surface 7. 20 N

Use Newton's 3rd Law to deal with the force pairs (horizontal & vertical) between the top and bottom block.

On to Chapter 8
Reprisal of : Uniform Circular Motion

For an object moving along a curved trajectory with constant speed
\[ \mathbf{a} = \mathbf{a}_r \text{ (radial only)} \]

\[ |\mathbf{a}_r| = \frac{v_t^2}{r} \]
Non-uniform Circular Motion

For an object moving along a curved trajectory, with non-uniform speed
\[ \mathbf{a} = \mathbf{a}_r + \mathbf{a}_t \] (radial and tangential)

\[ |\mathbf{a}_r| = \frac{v_T^2}{r} \]

\[ |\mathbf{a}_t| = \frac{d|\vec{v}|}{dt} \]

Key steps

- Identify forces (i.e., a FBD)
- Identify axis of rotation
- Apply conditions (position, velocity & acceleration)
Example
The pendulum

Consider a person on a swing:

When is the tension on the rope largest?
And at that point is it:
(A) greater than
(B) the same as
(C) less than
the force due to gravity acting on the person?

Example
Gravity, Normal Forces etc.

\[ F_r = m \frac{0^2}{r} = 0 = T - mg \cos \theta \]
\[ T = mg \cos \theta \]
\[ T < mg \]

\[ F_r = m a_c = m \frac{v_r^2}{r} = T - mg \]
\[ T = mg + m \frac{v_r^2}{r} \]
\[ T > mg \]
Conical Pendulum (very different)

- Swinging a ball on a string of length L around your head
  \( r = L \sin \theta \)

\[ \sum F_r = ma_r = T \sin \theta \]

\[ \sum F_z = 0 = T \cos \theta - mg \]

so

\[ T = mg / \cos \theta \ (> mg) \]

\[ ma_r = (mg / \cos \theta) (\sin \theta) \]

\[ a_r = g \tan \theta = v_T^2 / r \]

\[ v_T = (gr \tan \theta)^{1/2} \]

Period:

\[ t = 2\pi \frac{r}{v_T} = 2\pi \left( r \cot \theta / g \right)^{1/2} \]

\[ = 2\pi \left( L \cos \theta / g \right)^{1/2} \]

= 2\pi (5 \cos 5 / 9.8)^{1/2} = 4.38 \text{s}

= 2\pi (5 \cos 10 / 9.8)^{1/2} = 4.36 \text{s}

= 2\pi (5 \cos 15 / 9.8)^{1/2} = 4.32 \text{s}
Another example of circular motion
Loop-the-loop 1

A match box car is going to do a loop-the-loop of radius $r$.
What must be its minimum speed $v_t$ at the top so that it can manage the loop successfully?

To navigate the top of the circle its tangential velocity $v_T$ must be such that its centripetal acceleration at least equals the force due to gravity. At this point N, the normal force, goes to zero (just touching).

$$F_r = ma_r = mg = \frac{mv_T^2}{r}$$

$$v_T = (gr)^{1/2}$$
The match box car is going to do a loop-the-loop. If the speed at the bottom is $v_B$, what is the normal force, $N$, at that point?

Hint: The car is constrained to the track.

\[ F_r = ma_r = mv_B^2/r = N - mg \]

\[ N = mv_B^2/r + mg \]

Once again the car is going to execute a loop-the-loop. What must be its minimum speed at the bottom so that it can make the loop successfully?

This is a difficult problem to solve using just forces. We will skip it now and revisit it using energy considerations later on...
Home Exercise

Swinging around a ball on a rope in a “nearly” horizontal circle over your head. Eventually the rope breaks. If the rope breaks at 64 N, the ball’s mass is 0.10 kg and the rope is 0.10 m
How fast is the ball going when the rope breaks?
(neglect mg contribution, 1 N << 40 N)

\[ T = 40 \text{ N} \]
\[ F_r = m \frac{v_T^2}{r} \approx T \]
\[ v_T = (r F_r / m)^{1/2} \]
\[ v_T = (0.10 \times 64 / 0.10)^{1/2} \text{ m/s} \]
\[ v_T = 8 \text{ m/s} \]

Example, Circular Motion Forces with Friction
(recall \( m a_r = m |v_T|^2 / r \quad F_f \leq \mu_s N \))

• How fast can the race car go?
(How fast can it round a corner with this radius of curvature?)

\[ m_{car} = 1600 \text{ kg} \]
\[ \mu_S = 0.5 \text{ for tire/road} \]
\[ r = 80 \text{ m} \]
\[ g = 10 \text{ m/s}^2 \]
Example

- Only one force is in the horizontal direction: **static friction**

  \[ x \text{-dir: } F_r = ma = -m |v_T|^2 / r = F_s = -\mu_s N \text{ (at maximum)} \]

  \[ y \text{-dir: } ma = 0 = N - mg \quad N = mg \]

  \[ v_T = (\mu_s \ m \ g \ r / \ m)^{1/2} \]

  \[ v_T = (\mu_s \ g \ r)^{1/2} = (0.5 \times 10 \times 80)^{1/2} \]

  \[ v_T = 20 \text{ m/s} \]

  \[ m_{\text{car}} = 1600 \text{ kg} \]

  \[ \mu_S = 0.5 \text{ for tire/road} \]

  \[ r = 80 \text{ m} \]

  \[ g = 10 \text{ m/s}^2 \]

Another Example

- A horizontal disk is initially at rest and very slowly undergoes constant angular acceleration. A 2 kg puck is located a point 0.5 m away from the axis. At what angular velocity does it slip (assuming \( a_t \ll a_r \) at that time) if \( \mu_s = 0.8 \)?

- Only one force is in the horizontal direction: **static friction**

  \[ x \text{-dir: } F_r = ma = -m |v_T|^2 / r = F_s = -\mu_s N \text{ (at } \omega) \]

  \[ y \text{-dir: } ma = 0 = N - mg \quad N = mg \]

  \[ v_T = (\mu_s \ m \ g \ r / \ m)^{1/2} \]

  \[ v_T = (\mu_s \ g \ r)^{1/2} = (0.8 \times 10 \times 0.5)^{1/2} \]

  \[ v_T = 2 \text{ m/s} \rightarrow \omega = v_T / r = 4 \text{ rad/s} \]

  \[ m_{\text{puck}} = 2 \text{ kg} \]

  \[ \mu_S = 0.8 \]

  \[ r = 0.5 \text{ m} \]

  \[ g = 10 \text{ m/s}^2 \]
Banked Curves

In the previous car scenario, we drew the following free body diagram for a race car going around a curve on a flat track.

What differs on a banked curve?

Lecture 10

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For Tuesday: Finish reading Chapter 8 and start Chapter 9