Lecture 13

Goals:
• Chapter 10
  ❖ Understand the relationship between motion and energy
  ❖ Define Potential Energy in a Hooke’s Law spring
  ❖ Develop and exploit conservation of energy principle in problem solving
• Chapter 11
  ❖ Understand the relationship between force, displacement and work

Assignments:
• HW6, due tomorrow, HW7 due Wednesday, Mar. 10
• For Thursday, Read all of Chapter 11

Newton’s Laws rearranged

From motion in the y-dir, \( F_y = m a_y \) and let accel. be constant

- \( y(t) = y_0 + v_{y0} \Delta t + \frac{1}{2} a_y \Delta t^2 \rightarrow \Delta y = y(t) - y_0 = v_{y0} \Delta t + \frac{1}{2} a_y \Delta t^2 \)

- \( v_y(t) = v_{y0} + a_y \Delta t \rightarrow \Delta t = (v_y - v_{y0}) / a_y \)

and eliminating \( \Delta t \) yields

- \( a_y \Delta y = \frac{1}{2} (v_y^2 - v_{y0}^2) / a_y \)

- \( -mg \Delta y = \frac{1}{2} m (v_y^2 - v_{y0}^2) \)
Energy

\[-mg \Delta y = \frac{1}{2} \, m \left( v_y^2 - v_{y0}^2 \right)\]

\[-mg \left( y_f - y_i \right) = \frac{1}{2} \, m \left( v_{yf}^2 - v_{yi}^2 \right)\]

A relationship between

\textit{y-displacement} and change in the \textit{y-speed} squared

Rearranging to give initial on the left and final on the right

\[\frac{1}{2} \, m \, v_{yi}^2 + mgy_i = \frac{1}{2} \, m \, v_{yi}^2 + mgy_f\]

We now define \(mgy\) as the “gravitational potential energy”

Energy

- Notice that if we only consider gravity as the external force then the \(x\) and \(z\) velocities remain constant
- To \[\frac{1}{2} \, m \, v_{yi}^2 + mgy_i = \frac{1}{2} \, m \, v_{yi}^2 + mgy_f\]
- Add \[\frac{1}{2} \, m \, v_{xi}^2 + \frac{1}{2} \, m \, v_{zi}^2\] and \[\frac{1}{2} \, m \, v_{xf}^2 + \frac{1}{2} \, m \, v_{zf}^2\]

\[\frac{1}{2} \, m \, v_i^2 + mgy_i = \frac{1}{2} \, m \, v_f^2 + mgy_f\]

- where \[v_i^2 \equiv v_{xi}^2 + v_{yi}^2 + v_{zi}^2\]

\[\frac{1}{2} \, m \, v^2\] terms are defined to be kinetic energies

(A scalar quantity of motion)
Inelastic collision in 1-D: Example 1

• A block of mass $M$ is initially at rest on a frictionless horizontal surface. A bullet of mass $m$ is fired at the block with a muzzle velocity (speed) $v$. The bullet lodges in the block, and the block ends up with a speed $V$.

❖ What is the initial energy of the system?
❖ What is the final energy of the system?
❖ Is energy conserved?

Inelastic collision in 1-D: Example 1

What is the momentum of the bullet with speed $v$? $m\vec{v}$

❖ What is the initial energy of the system? $\frac{1}{2}m\vec{v} \cdot \vec{v} = \frac{1}{2}mv^2$
❖ What is the final energy of the system? $\frac{1}{2}(m+M)V^2$
❖ Is momentum conserved (yes)? $mv + M0 = (m+M)V$
❖ Is energy conserved? Examine $E_{\text{before}} - E_{\text{after}} = \frac{1}{2}mv^2 - \frac{1}{2}(m+M)V^2 = \frac{1}{2}mv^2 - \frac{1}{2}(mv) \cdot \frac{m}{m+M} v = \frac{1}{2}mv^2 \left(1 - \frac{m}{m+M}\right)$

Physics 207: Lecture 13, Pg 5
Kinetic & Potential energies

- Kinetic energy, $K = \frac{1}{2}mv^2$, is defined to be the large scale collective motion of one or a set of masses.

- Potential energy, $U$, is defined to be the “hidden” energy in an object which, in principle, can be converted back to kinetic energy.

- Mechanical energy, $E_{\text{Mech}}$, is defined to be the sum of $U$ and $K$.

Energy

- If only “conservative” forces are present, the total mechanical energy (sum of potential, $U$, and kinetic energies, $K$) of a system is conserved.

For an object in a gravitational “field”

$$\frac{1}{2}m v_{yi}^2 + mgy_i = \frac{1}{2}m v_{yi}^2 + mgy_f$$

$$K \equiv \frac{1}{2}mv^2 \quad U \equiv mgy$$

$$E_{\text{mech}} = K + U$$

$$E_{\text{mech}} = K + U = \text{constant}$$

- $K$ and $U$ may change, but $E_{\text{mech}} = K + U$ remains a fixed value.

$E_{\text{mech}}$ is called “mechanical energy”
Example of a conservative system:
The simple pendulum.

- Suppose we release a mass \(m\) from rest a distance \(h_f\) above its lowest possible point.
  - What is the maximum speed of the mass and where does this happen?
  - To what height \(h_2\) does it rise on the other side?

![Diagram of simple pendulum](Physics_207_Lecture_13_Pg_9.png)

Example: The simple pendulum.

- What is the maximum speed of the mass and where does this happen?
  - \(E = K + U = \text{constant}\) and so \(K\) is maximum when \(U\) is a minimum.
Example: The simple pendulum.

- What is the maximum speed of the mass and where does this happen?
  
  \[ E = K + U = \text{constant} \]  
  
  and so \( K \) is maximum when \( U \) is a minimum

  \[ E = mgh_1 \text{ at top} \]

  \[ E = mgh_1 = \frac{1}{2} mv^2 \text{ at bottom of the swing} \]

Example: The simple pendulum.

To what height \( h_2 \) does it rise on the other side?

\[ E = K + U = \text{constant} \]  

and so when \( U \) is maximum again (when \( K = 0 \)) it will be at its highest point.

\[ E = mgh_1 = mgh_2 \quad \text{or} \quad h_1 = h_2 \]
Example
The Loop-the-Loop ... again

- To complete the loop the loop, how high do we have to let the release the car?
- Condition for completing the loop the loop: Circular motion at the top of the loop \( a_c = \frac{v^2}{R} \)
- Exploit the fact that \( E = U + K = \text{constant} \) (frictionless)

Recall that “g” is the source of the centripetal acceleration and \( N \) just goes to zero is the limiting case.
Also recall the minimum speed at the top is \( v = \sqrt{gR} \)

\[ U_b = mgh \]
\[ U = mg2R \]
\[ h = \frac{5}{2} R \]

Example
The Loop-the-Loop ... again

- Use \( E = K + U = \text{constant} \)
- \( mgh + 0 = mg 2R + \frac{1}{2} mv^2 \)
  - \( mgh = mg 2R + \frac{1}{2} mgR = \frac{5}{2} mgR \)

\[ h = \frac{5}{2} R \]
Example
Skateboard

- What speed will the skateboarder reach halfway down the hill if there is no friction and the skateboarder starts at rest?
- Assume we can treat the skateboarder as a “point”
- Assume zero of gravitational U is at bottom of the hill

\[ m = 25 \text{ kg} \]

\[ R = 10 \text{ m} \]

\[ y = 5 \text{ m} \]

\[ \theta = 30^\circ \]

\[ E = K + U = \text{constant} \]

\[ E_{\text{before}} = E_{\text{after}} \]

\[ 0 + mgR = \frac{1}{2}mv^2 + \frac{1}{2}mgR \]

\[ mgR/2 = \frac{1}{2}mv^2 \]

\[ gR = v^2 \rightarrow v = (gR)^{\frac{1}{2}} \]

\[ v = (10 \times 10)^{\frac{1}{2}} = 10 \text{ m/s} \]
Potential Energy, Energy Transfer and Path

- A ball of mass $m$, initially at rest, is released and follows three different paths. All surfaces are frictionless
1. The ball is dropped
2. The ball slides down a straight incline
3. The ball slides down a curved incline

After traveling a vertical distance $h$, how do the three speeds compare?

(A) $1 > 2 > 3$  (B) $3 > 2 > 1$  (C) $3 = 2 = 1$  (D) Can't tell

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Example Skateboard

- What is the normal force on the skateboarder?

$m = 25$ kg
**Example Skateboard**

- Now what is the normal force on the skate boarder?

\[
\begin{align*}
\sum F_r &= ma_r = \frac{mv^2}{R} \\
&= N - mg \cos 60^\circ \\
N &= \frac{mv^2}{R} + mg \cos 60^\circ \\
N &= \frac{25 \times 100}{10} + 25 \times 10 \times 0.87 \\
N &= 250 + 220 = 470 \text{ Newtons}
\end{align*}
\]

**Elastic vs. Inelastic Collisions**

- A collision is said to be *elastic* when both energy & momentum are conserved before and after the collision.

\[
K_{\text{before}} = K_{\text{after}}
\]

- Carts colliding with a perfect spring, billiard balls, etc.
Elastic vs. Inelastic Collisions

- A collision is said to be inelastic when energy is not conserved before and after the collision, but momentum is conserved.

\[ K_{\text{before}} \neq K_{\text{after}} \]

- Car crashes, collisions where objects stick together, etc.

Example – Fully Elastic Collision

- Suppose I have 2 identical bumper cars.
- One is motionless and the other is approaching it with velocity \( v_i \). If they collide elastically, what is the final velocity of each car?

Identical means \( m_1 = m_2 = m \)

Initially \( v_{\text{Green}} = v_1 \) and \( v_{\text{Red}} = 0 \)

\[ \text{COM} \rightarrow \quad mv_1 + 0 = mv_{1f} + mv_{2f} \rightarrow v_1 = v_{1f} + v_{2f} \]

\[ \text{COE} \rightarrow \frac{1}{2} mv_1^2 = \frac{1}{2} mv_{1f}^2 + \frac{1}{2} mv_{2f}^2 \rightarrow v_1^2 = v_{1f}^2 + v_{2f}^2 \]

\[ v_1^2 = (v_{1f} + v_{2f})^2 = v_{1f}^2 + 2v_{1f}v_{2f} + v_{2f}^2 \rightarrow 2v_{1f}v_{2f} = 0 \]

- Soln 1: \( v_{1f} = 0 \) and \( v_{2f} = v_1 \)
- Soln 2: \( v_{2f} = 0 \) and \( v_{1f} = v_1 \)
Variable force devices: Hooke’s Law Springs

- Springs are everywhere, (probe microscopes, DNA, an effective interaction between atoms)

- In this spring, the magnitude of the force increases as the spring is further compressed (a displacement).
- Hooke’s Law,
  \[ F_s = -k \Delta s \]

\( \Delta s \) is the amount the spring is stretched or compressed from it resting position.

Hooke’s Law Spring

- For a spring we know that \( F_x = -k s \).
Exercise 2
Hooke’s Law

What is the spring constant “k”?

(A) 50 N/m  (B) 100 N/m  (C) 400 N/m  (D) 500 N/m

ΣF = 0 = F_s - mg = k Δs - mg

mg

Use k = mg/Δs = 500 N / 1.0 m

(A) 50 N/m  (B) 100 N/m  (C) 400 N/m  (D) 500 N/m
**Force vs. Energy for a Hooke’s Law spring**

- \( F = -k(x - x_{\text{equilibrium}}) \)
- \( F = ma = m \frac{dv}{dt} \)
  \[ = m \left( \frac{dv}{dx} \frac{dx}{dt} \right) \]
  \[ = m \frac{dv}{dx} v \]
  \[ = mv \frac{dv}{dx} \]

- So \( k(x - x_{\text{equilibrium}}) \) \( dx = mv \) \( dv \)
- Let \( u = x - x_{\text{eq.}} \) & \( du = dx \)

\[
\int_{u_i}^{u_f} -ku \, du = \int_{v_i}^{v_f} mv \, dv
\]

\[ -\frac{1}{2}ku^2 \bigg|_{u_i}^{u_f} = \frac{1}{2}mv^2 \bigg|_{v_i}^{v_f} \]

\[ -\frac{1}{2}ku_f^2 + \frac{1}{2}ku_i^2 = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \]

\[
\frac{1}{2}ku_i^2 + \frac{1}{2}mv_i^2 = \frac{1}{2}ku_f^2 + \frac{1}{2}mv_f^2
\]

**Lecture 13**

**Assignment:**

- HW6 due Wednesday
- For Thursday: Read all of chapter 11