Chapter 15, Fluids & Elasticity
This is an actual photo of an iceberg, taken by a rig manager for Global Marine Drilling in St. Johns, Newfoundland. The water was calm and the sun was almost directly overhead so that the diver

Lecture 21
Goals:
• Chapter 15
  ❖ Understand pressure in liquids and gases
  ❖ Use Archimedes’ principle to understand buoyancy
  ❖ Understand the equation of continuity
  ❖ Use an ideal-fluid model to study fluid flow.
  ❖ Investigate the elastic deformation of solids and liquids
• Assignment
  ❖ HW10, Due Wednesday, Apr. 14th
  ❖ Thursday: Read all of Chapter 16
Fluids

- At ordinary temperature, matter exists in one of three states
  - Solid - has a shape and forms a surface
  - Liquid - has no shape but forms a surface
  - Gas - has no shape and forms no surface
- What do we mean by “fluids”?  
  - Fluids are “substances that flow” .... “substances that take the shape of the container”
  - Atoms and molecules are free to move.
  - No long range correlation between positions.

Fluids

- An intrinsic parameter of a fluid
  - Density

\[ \rho = \frac{m}{V} \]

**units:**

\[ \text{kg/m}^3 = 10^{-3} \text{ g/cm}^3 \]

\[
\begin{align*}
\rho(\text{water}) &= 1.000 \times 10^3 \text{ kg/m}^3 = 1.000 \text{ g/cm}^3 \\
\rho(\text{ice}) &= 0.917 \times 10^3 \text{ kg/m}^3 = 0.917 \text{ g/cm}^3 \\
\rho(\text{air}) &= 1.29 \text{ kg/m}^3 = 1.29 \times 10^{-3} \text{ g/cm}^3 \\
\rho(\text{Hg}) &= 13.6 \times 10^3 \text{ kg/m}^3 = 13.6 \text{ g/cm}^3 \\
\rho(\text{W or Au}) &= 19.3 \times 10^3 \text{ kg/m}^3 = 19.3 \text{ g/cm}^3
\end{align*}
\]
Fluids

- Another parameter: Pressure $p = \frac{F}{A}$

- Any force exerted by a fluid is perpendicular to a surface of contact, and is proportional to the area of that surface.

  - Force (a vector) in a fluid can be expressed in terms of pressure (a scalar) as:

    \[
    \vec{F} = pA\hat{n}
    \]

What is the SI unit of pressure?

A. Pascal  
B. Atmosphere  
C. Bernoulli  
D. Young  
E. p.s.i.

Units:

- $1 \text{ N/m}^2 = 1 \text{ Pa (Pascal)}$  
- $1 \text{ atm} = 1.013 \times 10^5 \text{ Pa} = 1013 \text{ mbar} = 760 \text{ Torr} = 14.7 \text{ lb/ in}^2 (=\text{PSI})$
When the pressure is small, relative to the bulk modulus of the fluid, we can treat the density as constant independent of pressure:

incompressible fluid

For an incompressible fluid, the density is the same everywhere, but the pressure is NOT!

\[ p(y) = p_0 - y \, g \, \rho \]

Gauge pressure (subtract \( p_0 \))

\[ p_{\text{Gauge}} = p(y) - p_0 \]

\[ F_2 = F_1 + m \, g \]
\[ = F_1 + \rho \, V \, g \]
\[ F_2 / A = F_1 / A + \rho \, V \, g / A \]
\[ p_2 = p_1 - \rho \, g \, y \]

Pressure vs. Depth

For a uniform fluid in an open container pressure same at a given depth independent of the container

Fluid level is the same everywhere in a connected container, assuming no surface forces
Pressure Measurements: Barometer

- Invented by Torricelli
- A long closed tube is filled with mercury and inverted in a dish of mercury
  - The closed end is nearly a vacuum
- Measures atmospheric pressure as
  1 atm = 0.760 m (of Hg)

Exercise
Pressure

- What happens with two fluids??
- Consider a U tube containing liquids of density \( \rho_1 \) and \( \rho_2 \) as shown:

  Compare the densities of the liquids:

  \[(A) \, \rho_1 < \rho_2 \quad (B) \, \rho_1 = \rho_2 \quad (C) \, \rho_1 > \rho_2 \]
Exercise
Pressure

• What happens with two fluids??

• Consider a U tube containing liquids of density \( \rho_1 \) and \( \rho_2 \) as shown:

• At the red arrow the pressure must be the same on either side. \( \rho_1 x = \rho_2 (d_1 + y) \)
  ▶ Compare the densities of the liquids:

(A) \( \rho_1 < \rho_2 \)  (B) \( \rho_1 = \rho_2 \)  (C) \( \rho_1 > \rho_2 \)

Archimedes’ Principle: A Eureka Moment

• Suppose we weigh an object in air (1) and in water (2).

How do these weights compare?

\( W_1 < W_2 \)  \( W_1 = W_2 \)  \( W_1 > W_2 \)

• Buoyant force is equal to the weight of the fluid displaced
The Golden Crown

- In the first century BC the Roman architect Vitruvius related a story of how Archimedes uncovered a fraud in the manufacture of a golden crown commissioned by Hiero II, the king of Syracuse. The crown (corona in Vitruvius's Latin) would have been in the form of a wreath, such as one of the three pictured from grave sites in Macedonia and the Dardanelles. Hiero would have placed such a wreath on the statue of a god or goddess. Suspecting that the goldsmith might have replaced some of the gold given to him by an equal weight of silver, Hiero asked Archimedes to determine whether the wreath was pure gold. And because the wreath was a holy object dedicated to the gods, he could not disturb the wreath in any way. (In modern terms, he was to perform nondestructive testing). Archimedes' solution to the problem, as described by Vitruvius, is neatly summarized in the following excerpt from an advertisement:

- The solution which occurred when he stepped into his bath and caused it to overflow was to put a weight of gold equal to the crown, and known to be pure, into a bowl which was filled with water to the brim. Then the gold would be removed and the king's crown put in, in its place. An alloy of lighter silver would increase the bulk of the crown and cause the bowl to overflow.


Archimedes' Principle

- Suppose we weigh an object in air (1) and in water (2).

  - How do these weights compare?

    \[
    W_1 < W_2 \\
    W_1 = W_2 \\
    W_1 > W_2
    \]

  - Why?

    Since the pressure at the bottom of the object is greater than that at the top of the object, the water exerts a net upward force, the buoyant force, on the object.
Sink or Float?

- The buoyant force is **equal** to the weight of the liquid that is displaced.
- If the buoyant force is larger than the weight of the object, it will float; otherwise it will sink.

- We can calculate how much of a floating object will be submerged in the liquid:
  - Object is in equilibrium $\Rightarrow F_B = mg$
  - $\rho_{\text{liquid}} \cdot g \cdot V_{\text{liquid}} = \rho_{\text{object}} \cdot g \cdot V_{\text{object}}$
  - $\frac{V_{\text{liquid}}}{V_{\text{object}}} = \frac{\rho_{\text{object}}}{\rho_{\text{liquid}}}$

**Bar Trick**

What happens to the water level when the ice melts?

Expt. 1: A. It rises  
Expt. 2: B. It stays the same  
C. It drops
Exercise

\[ V_1 = V_2 = V_3 = V_4 = V_5 \]
\[ m_1 < m_2 < m_3 < m_4 < m_5 \]
What is the final position of each block?

---

Exercise

\[ V_1 = V_2 = V_3 = V_4 = V_5 \]
\[ m_1 < m_2 < m_3 < m_4 < m_5 \]
What is the final position of each block?

Not this

But this
Home Exercise: Buoyancy

- A small lead weight is fastened to a large styrofoam block and the combination floats on water with the water level with the top of the styrofoam block as shown.

- If you turn the styrofoam + Pb upside-down, What happens?

  (A) It sinks  (B) styrofoam Pb  (C) styrofoam Pb  (D) styrofoam Pb

Exercise: Buoyancy

- A small lead weight is fastened to a large styrofoam block and the combination floats on water with the water level with the top of the styrofoam block as shown.

- If you turn the styrofoam + Pb upside-down, What happens (assuming density of Pb > water)?

  (A) It sinks  (B) styrofoam Pb  (C) styrofoam Pb  (D) styrofoam Pb
Home Exercise
More Buoyancy

Two identical cups are filled to the same level with water. One of the two cups has plastic balls floating in it.

- Which cup weighs more?

(A) Cup I  (B) Cup II  (C) the same  (D) can’t tell

Exercise
More Buoyancy

Two identical cups are filled to the same level with water. One of the two cups has plastic balls floating in it.

- Which cup weighs more?

(A) Cup I  (B) Cup II  (C) the same  (D) can’t tell
Home Exercise
Even More Buoyancy

A plastic ball floats in a cup of water with half of its volume submerged. Next some oil ($\rho_{\text{oil}} < \rho_{\text{ball}} < \rho_{\text{water}}$) is slowly added to the container until it just covers the ball.

- Relative to the water level, the ball will:

   Hint 1: What is the buoyant force of the part in the oil as compared to the air?

(A) move up  (B) move down  (C) stay in same place

Pascal’s Principle

- So far we have discovered (using Newton’s Laws):
  
  - Pressure depends on depth: $\Delta p = - \rho g \Delta y$
  
- Pascal’s Principle addresses how a change in pressure is transmitted through a fluid.

Any change in the pressure applied to an enclosed fluid is transmitted to every portion of the fluid and to the walls of the containing vessel.
Pascal’s Principle in action: Hydraulics, a force amplifier

- Consider the system shown:
  - A downward force $F_1$ is applied to the piston of area $A_1$.
  - This force is transmitted through the liquid to create an upward force $F_2$.
  - Pascal’s Principle says that increased pressure from $F_1$ ($F_1/A_1$) is transmitted throughout the liquid.

- $F_2 > F_1$ with conservation of energy

Continuity Exercise

- Now consider the set up shown on right.
  - In each case, a block of mass $M$ is placed on the piston of the larger cylinder and this results in a difference $d_i$ in the liquid levels on the right.
  - In this case equilibrium is NOT yet established so that the pressure of the fluid in the left column is does not yet support the mass. The mass $M$ simply displaces an equal distance downward.
  - If $A_2 = 2A_1$, how do $d_A$ and $d_B$ compare?
  - Here the fluid volume displaced will be the identical or

$$V_{10} = V_1 = V_2$$
$$d_A A_1 = d_B A_2$$
$$d_A A_1 = d_B 2A_1$$
$$d_A = d_B 2 \text{ with } p_A > p_B$$
Fluids in Motion

- To describe fluid motion, we need something that describes flow:
  - Velocity $\mathbf{v}$

- There are different kinds of fluid flow of varying complexity
  - non-steady / steady
  - compressible / incompressible
  - rotational / irrotational
  - viscous / ideal

Types of Fluid Flow

- Laminar flow
  - Each particle of the fluid follows a smooth path
  - The paths of the different particles never cross each other
  - The path taken by the particles is called a streamline

- Turbulent flow
  - An irregular flow characterized by small whirlpool like regions
  - Turbulent flow occurs when the particles go above some critical speed
Types of Fluid Flow

- Laminar flow
  - Each particle of the fluid follows a smooth path
  - The paths of the different particles never cross each other
  - The path taken by the particles is called a streamline

- Turbulent flow
  - An irregular flow characterized by small whirlpool like regions
  - Turbulent flow occurs when the particles go above some critical speed

Lecture 21

- Question to ponder: Does heavy water (D₂O) ice sink or float?

- Assignment
  - HW10, due Wednesday, Apr. 14th
  - Thursday: Read all of Chapter 16