Lecture 25

Today Review:

- Exam covers Chapters 14-17 plus angular momentum, statics

- Assignment
  - For Thursday, read through all of Chapter 18

Angular Momentum Exercise

- A mass $m=0.10 \text{ kg}$ is attached to a cord passing through a small hole in a frictionless, horizontal surface as in the Figure. The mass is initially orbiting with speed $\omega_i = 5 \text{ rad} / \text{s}$ in a circle of radius $r_i = 0.20 \text{ m}$. The cord is then slowly pulled from below, and the radius decreases to $r = 0.10 \text{ m}$.
- What is the final angular velocity?
- Underlying concept: Conservation of Momentum
Angular Momentum Exercise

- A mass $m=0.10 \text{ kg}$ is attached to a cord passing through a small hole in a frictionless, horizontal surface as in the Figure. The mass is initially orbiting with speed $\omega_i = 5 \text{ rad} / \text{s}$ in a circle of radius $r_i = 0.20 \text{ m}$. The cord is then slowly pulled from below, and the radius decreases to $r = 0.10 \text{ m}$.
- What is the final angular velocity?

No external torque implies

$\Delta L = 0 \text{ or } L_i = L_c$

$I_i \omega_i = I_f \omega_f$

$I$ for a point mass is $mr^2$ where $r$ is the distance to the axis of rotation

$m r_i^2 \omega_i = m r_f^2 \omega_f$

$\omega_f = \frac{r_i^2 \omega_i}{r_f^2} = \left(\frac{0.20}{0.10}\right)^2 \frac{5 \text{ rad/s}}{20 \text{ rad/s}}$

Example: Throwing ball from stool

- A student sits on a stool, initially at rest, but which is free to rotate. The moment of inertia of the student plus the stool is $I$. They throw a heavy ball of mass $M$ with speed $v$ such that its velocity vector has a perpendicular distance $d$ from the axis of rotation.
- What is the angular speed $\omega_f$ of the student-stool system after they throw the ball?
Example: Throwing ball from stool

- What is the angular speed $\omega_f$ of the student-stool system after they throw the ball?
- Process: (1) Define system (2) Identify Conditions

(1) System: student, stool and ball (No Ext. torque, $L$ is constant)
(2) Momentum is conserved (check $|L| = |r| |p| \sin \theta$ for sign)

$$L_{\text{init}} = 0 = L_{\text{final}} = -Mv_d + I \omega_f$$

Walking the plank...

- A uniform rectangular beam of length $L= 5$ m and mass $M=40$ kg is supported, but not attached, to two posts which are length $D=3$ m apart. A child of mass $W=20$ kg starts walking along the beam.

- Assuming infinitely rigid posts, how close can the child get to the right end of the beam without it falling over? [Hint: The upward force exerted by the beam on the left cannot be negative – this is the limiting condition on how far the child can be to the right. Set up the static equilibrium condition with the pivot about the left end of the beam.]
Walking the plank…

- A uniform rectangular beam of length $L = 5$ m and mass $M = 40$ kg is supported, but not attached, to two posts which are length $D = 3$ m apart. A child of mass $W = 20$ kg starts walking along the beam.

- Sum of forces and torques are zero
- As the child walks from left to right $N_L$ goes from positive to “negative”
- If $N_L < 0$ (a pull) then the board will tip.
- Choose blue dot as pivot point.

\[ \sum \tau = 0 = -N_L \times 3 + 400 \times 0.5 + N_R \times 0 - d \times 200 \]

\[ 0 = 0 + 200 + 0 - d \times 200 \]

$d = 1$ meter from pivot point which is 2.0 meter from right side distance to end (2 – 1) meters or 1 meter
Velocity and Acceleration

Position: \( x(t) = A \cos(\omega t + \phi) \)
Velocity: \( v(t) = -\omega A \sin(\omega t + \phi) \)
Acceleration: \( a(t) = -\omega^2 A \cos(\omega t + \phi) \)

\[
\begin{align*}
    x_{\text{max}} &= A \\
    v_{\text{max}} &= \omega A \\
    a_{\text{max}} &= \omega^2 A
\end{align*}
\]

Measuring \( g \)

- To measure the magnitude of the acceleration due to gravity, \( g \), in an unorthodox manner, a student places a ball bearing on the concave side of a flexible speaker cone. The speaker cone acts as a simple harmonic oscillator whose amplitude is \( A \) and whose frequency \( \omega \) can be varied. The student can measure both \( A \) and \( \omega \). The equations of motion for the speaker are

\[
\begin{align*}
    \Sigma F_y &= m a_y = -mg + N \\
    v_y(t) &= -\omega A \sin(\omega t + \phi) \\
    a_y(t) &= -\omega^2 A \cos(\omega t + \phi)
\end{align*}
\]

- If the ball bearing is in contact with the speaker, then

\[
\begin{align*}
    \Sigma F_y &= mA_y = -mg + N \quad \text{so that} \quad N = mg + ma_y \\
    N &= mg - m\omega^2 A \cos(\omega t + \phi) \\
    \text{Maximum: } N &= mg + m\omega^2 A \quad \text{Minimum: } mg - m\omega^2 A \quad \text{but must be } > 0!
\end{align*}
\]
Bernoulli Equation \(
\bar{p} + \frac{1}{2} \rho v_1^2 + \rho g y_1 = \text{constant}
\)

A 5 cm radius horizontal pipe carries water at 10 m/s into a 10 cm radius. (\(\rho_{\text{water}} = 10^3 \text{ kg/m}^3\))

What is the pressure difference?

\[
P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2
\]

\[
\Delta P = \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2
\]

and \(A_1 v_1 = A_2 v_2\)

\[
\Delta P = \frac{1}{2} \rho v_2^2 (1 - (\frac{A_2}{A_1})^2)
\]

\[
= 0.5 \times 1000 \text{ kg/m} \times 100 \text{ m}^2/\text{s}^2 (1 - (25/100)^2) = 47000 \text{ Pa}
\]

A water fountain

- A fountain, at sea level, consists of a 10 cm radius pipe with a 5 cm radius nozzle. The water sprays up to a height of 20 m.

  - What is the velocity of the water as it leaves the nozzle?
  - What volume of the water per second as it leaves the nozzle?
  - What is the velocity of the water in the pipe?
  - What is the pressure in the pipe?
  - How many watts must the water pump supply?
A water fountain

- A fountain, at sea level, consists of a 10 cm radius pipe with a 5 cm radius nozzle. The water sprays up to a height of 20 m.
- What is the velocity of the water as it leaves the nozzle?
  \[
  \text{Simple Picture: } \frac{1}{2}mv^2 = mgh \\
  v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20} = 20 \text{ m/s}
  \]
- What volume of the water per second as it leaves the nozzle?
  \[
  Q = A_v n = 0.0025 \times 20 \times 3.14 = 0.155 \text{ m}^3/\text{s}
  \]
- What is the velocity of the water in the pipe?
  \[
  A_v n v = A_p v_p \Rightarrow v_p = Q / 4 = 5 \text{ m/s}
  \]
- What is the pressure in the pipe?
  \[
  1 \text{ atm} + \frac{1}{2} \rho v^2_n = 1 \text{ atm} + \Delta P + \frac{1}{2} \rho v^2_p \rightarrow 1.9 \times 10^5 \text{ N/m}^2
  \]
- How many watts must the water pump supply?
  \[
  \text{Power} = Q \rho g h = 0.0155 \text{ m}^3/\text{s} \times 10^3 \text{ kg/m}^3 \times 9.8 \text{ m/s}^2 \times 20 \text{ m} = 3 \times 10^4 \text{ W}
  \]

Fluids Buoyancy

- A metal cylinder, 0.5 m in radius and 4.0 m high is lowered, as shown, from a massless rope into a vat of oil and water. The tension, T, in the rope goes to zero when the cylinder is half in the oil and half in the water. The densities of the oil is 0.9 gm/cm$^3$ and the water is 1.0 gm/cm$^3$
- What is the average density of the cylinder?
- What was the tension in the rope when the cylinder was submerged in the oil?
Fluids Buoyancy

- $r = 0.5 \text{ m}$, $h = 4.0 \text{ m}$
- $\rho_{\text{oil}} = 0.9 \text{ gm/cm}^3$, $\rho_{\text{water}} = 1.0 \text{ gm/cm}^3$
- What is the average density of the cylinder?

When $T = 0$

$F_{\text{buoyancy}} = W_{\text{cylinder}}$

$F_{\text{buoyancy}} = \rho_{\text{oil}} g \frac{1}{2} V_{\text{cyl}} + \rho_{\text{water}} g \frac{1}{2} V_{\text{cyl}}$

$W_{\text{cylinder}} = \rho_{\text{cyl}} g V_{\text{cyl}}$

$\rho_{\text{cyl}} g V_{\text{cyl}} = \rho_{\text{oil}} g \frac{1}{2} V_{\text{cyl}} + \rho_{\text{water}} g \frac{1}{2} V_{\text{cyl}}$

$\rho_{\text{cyl}} = \frac{1}{2} \rho_{\text{oil}} + \frac{1}{2} \rho_{\text{water}}$

What was the tension in the rope when the cylinder was submerged in the oil?

Use a Free Body Diagram!

\[ \sum F_z = 0 = T - W_{\text{cylinder}} + F_{\text{buoyancy}} \]

\[ T = W_{\text{cyl}} - F_{\text{buoy}} = g \left( \rho_{\text{cyl}} - \rho_{\text{oil}} \right) V_{\text{cyl}} \]

\[ T = 9.8 \times 0.05 \times 10^3 \times \pi \times 0.5^2 \times 4.0 = 1500 \text{ N} \]
**Fluids Buoyancy**

- \( r = 0.5 \text{ m}, \ h = 4.0 \text{ m} \)
- \( V_{\text{cyl}} = \pi r^2 h \)
- \( \rho_{\text{oil}} = 0.9 \text{ gm/cm}^3 \)
- \( \rho_{\text{water}} = 1.0 \text{ gm/cm}^3 \)
- What is the average density of the cylinder?
  
  When \( T = 0 \)  
  \( F_{\text{buoyancy}} = W_{\text{cylinder}} \)
  
  \[ \rho_{\text{cyl}} = \frac{1}{2} \rho_{\text{oil}} + \frac{1}{2} \rho_{\text{water}} = 0.95 \text{ gm/cm}^3 \]

What was the tension in the rope when the cylinder was submerged in the oil?

Use a Free Body Diagram!

\[ \sum F_z = 0 = T - W_{\text{cylinder}} + F_{\text{buoyancy}} \]

\[ T = W_{\text{cyl}} - F_{\text{buoy}} = g (\rho_{\text{cyl}} - \rho_{\text{oil}}) V_{\text{cyl}} \]

\[ T = 9.8 \times 0.05 \times 10^3 \times \pi \times 0.5^2 \times 4.0 = 1500 \text{ N} \]

---

**A new trick**

- Two trapeze artists, of mass 100 kg and 50 kg respectively are testing a new trick and want to get the timing right. They both start at the same time using ropes of 10 m in length and, at the turnaround point the smaller grabs hold of the larger artist and together they swing back to the starting platform. A model of the stunt is shown at right.

- How long will this stunt require if the angle is small?
A new trick

- How long will this stunt require?

Period of a pendulum is just
\[ \omega = \left(\frac{g}{L}\right)^{\frac{1}{2}} \]
\[ T = 2\pi \left(\frac{L}{g}\right)^{\frac{1}{2}} \]

Time before ½ period
Time after ½ period
So, \( t = T = 2\pi \left(\frac{L}{g}\right)^{\frac{1}{2}} = 2\pi \text{ sec} \)

Key points: Period is one full swing and independent of mass (this is SHM but very different than a spring. SHM requires only a linear restoring force.)

Example

- A Hooke’s Law spring, \( k = 200 \text{ N/m} \), is on a horizontal frictionless surface is stretched 2.0 m from its equilibrium position. A 1.0 kg mass is initially attached to the spring however, at a displacement of 1.0 m a 2.0 kg lump of clay is dropped onto the mass. The clay sticks.

What is the new amplitude?
Example

- A Hooke’s Law spring, $k=200$ N/m, is on a horizontal frictionless surface is stretched 2.0 m from its equilibrium position. A 1.0 kg mass is initially attached to the spring however, at a displacement of 1.0 m a 2.0 kg lump of clay is dropped onto the mass.

What is the new amplitude?

Sequence: SHM, collision, SHM

\[ \frac{1}{2} k A_0^2 = \text{const.} \]

\[ \frac{1}{2} k A_0^2 = \frac{1}{2} mv^2 + \frac{1}{2} k (A_0/2)^2 \]

\[ \frac{3}{4} k A_0^2 = m v^2 \Rightarrow v = \left( \frac{3}{4} k A_0^2 / m \right)^{1/2} \]

\[ v = \left( \frac{0.75 \times 200 \times 4}{1} \right)^{1/2} = 24.5 \text{ m/s} \]

Conservation of x-momentum:

\[ mv = (m+M) V \Rightarrow V = \frac{mv}{(m+M)} \]

\[ V = \frac{24.5}{3} \text{ m/s} = 8.2 \text{ m/s} \]

Key point: $K+U$ is constant in SHM

Example

- A Hooke’s Law spring, $k=200$ N/m, is on a horizontal frictionless surface is stretched 2.0 m from its equilibrium position. A 1.0 kg mass is initially attached to the spring however, at a displacement of 1.0 m a 2.0 kg lump of clay is dropped onto the mass. The clay sticks.

What is the new amplitude?

Sequence: SHM, collision, SHM

\[ V = \frac{24.5}{3} \text{ m/s} = 8.2 \text{ m/s} \]

\[ \frac{1}{2} k A_f^2 = \text{const.} \]

\[ \frac{1}{2} k A_f^2 = \frac{1}{2} (m+M)V^2 + \frac{1}{2} k (A_f)^2 \]

\[ A_f^2 = [(m+M)V^2 / k + (A_i)^2]^{1/2} \]

\[ A_f^2 = \left[ 3 \times 8.2^2 / 200 + (1)^2 \right]^{1/2} \]

\[ A_f^2 = \left[ 1 + 1 \right]^{1/2} \Rightarrow A_f^2 = 1.4 \text{ m} \]

Key point: $K+U$ is constant in SHM
Fluids Buoyancy & SHM

- A metal cylinder, 0.5 m in radius and 4.0 m high is lowered, as shown, from a rope into a vat of oil and water. The tension, T, in the rope goes to zero when the cylinder is half in the oil and half in the water. The densities of the oil is 0.9 gm/cm$^3$ and the water is 1.0 gm/cm$^3$
- Refer to earlier example
- Now the metal cylinder is lifted slightly from its equilibrium position. What is the relationship between the displacement and the rope’s tension?
- If the rope is cut and the drum undergoes SHM, what is the period of the oscillation if undamped?

\[0 = T + F_{\text{buoyancy}} - W_{\text{cylinder}}\]
\[T = -F_{\text{buoyancy}} + W_{\text{cylinder}}\]
\[T = -[\rho_o g (h/2 + \Delta y) A_c + \rho_w g A_c (h/2 - \Delta y)] + W_{\text{cyl}}\]
\[T = -[g A_c (\rho_o + \rho_w)/2 + \Delta y g A_c (\rho_o - \rho_w)] + W_{\text{cyl}}\]
\[T = -[W_{\text{cyl}} + \Delta y g A_c (\rho_o - \rho_w)] + W_{\text{cyl}}\]
\[T = \frac{g A_c (\rho_w - \rho_o)}{\Delta y}\]

If the is rope cut, net force is towards equilibrium position with a proportionality constant
\[g A_c (\rho_w - \rho_o) \quad [\& \text{ with } g = 10 \text{ m/s}^2]\]
If \(F = -k \Delta y\) then \(k = g A_c (\rho_o - \rho_w) = \pi/4 \times 10^3 \text{ N/m}\)
Fluids Buoyancy & SHM

- A metal cylinder, 0.5 m in radius and 4.0 m high is lowered, as shown, from a rope into a vat of oil and water. The tension, T, in the rope goes to zero when the cylinder is half in the oil and half in the water. The densities of the oil is 0.9 gm/cm³ and the water is 1.0 gm/cm³.

- If the rope is cut and the drum undergoes SHM, what is the period of the oscillation if undamped?

\[ F = ma = -k \Delta y \] and with SHM \[ \omega = \frac{(k/m)^{1/2}}{\text{where } k \text{ is a "spring" constant and } m \text{ is the inertial mass (resistance to motion), the cylinder}} \]

So \[ \omega = \left(\frac{1000\pi}{4 m_{\text{cyl}}}\right)^{1/2} \]
\[ = \left(\frac{1000\pi}{4 \rho_{\text{cyl}}V_{\text{cyl}}}\right)^{1/2} = \left(\frac{0.25}{0.95}\right)^{1/2} \]
\[ = 0.51 \text{ rad/sec} \]
\[ T = 3.2 \text{ sec} \]

Underdamped SHM

\[ x(t) = A \exp\left(-\frac{bt}{2m}\right) \cos(\omega t + \phi) \text{ if } \omega > b/2m \]

If the period is 2.0 sec and, after four cycles, the amplitude drops by 75%, what is the time constant \( \tau \) if \( \tau = 2m / b \)?

Four cycles implies 8 sec

So

\[ 0.25 A_0 = A_0 \exp\left(-8b / 2m\right) \]
\[ \ln(1/4) = -8 \frac{1}{\tau} \]
\[ \tau = -8 / \ln(1/4) = 5.8 \text{ sec} \]
Anti-global warming or the nuclear winter scenario

- Assume $P/A = \mathcal{P} = 1340 \text{ W/m}^2$ from the sun is incident on a thick dust cloud above the Earth and this energy is absorbed, equilibrated and then reradiated towards space where the Earth’s surface is in thermal equilibrium with cloud. Let $e$ (the emissivity) be unity for all wavelengths of light.

- What is the Earth’s temperature?
  - $P = \sigma A T^4 = \sigma (4\pi r^2) T^4 = \mathcal{P} \pi r^2 \Rightarrow T = \left[ \mathcal{P} (4 \times \sigma) \right]^{1/4}$
  - $\sigma = 5.7 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$
  - $T = 277 \text{ K} \ (A \ little \ on \ the \ chilly \ side.)$

Time for a swim

- At solar equinox, how much could you reasonably expect for the top 1 meter of water in a lake at 45 deg. latitude to warm on a calm but bright sunny day assuming that the sun’s flux ($P/A$) is 1340 W/m$^2$ if all the sun’s energy were absorbed in that layer?
  - At 45 deg., $\sin 45 \deg \times 0.7071 \times 1340$ or 950 W/m$^2$
  - But only 6 hours of full sun
  - So Q available is 6 x 60 x 60 seconds with 950 J/s/m$^2$
  - $Q = 2 \times 10^7 \text{ J} \ \text{in one square meter}$
  - $Q = m c \Delta T \ \text{with} \ m \ \rightarrow 10^3 \text{ kg/m}^3 \ \text{and} \ c = 4.2 \times 10^3 \text{ J/kg-K}$

- about 5 K per day.
Q, W and ΔE_{TH}

- 0.20 moles of an ideal monoatomic gas are cycled clockwise through the pV sequence shown at right (A→D→C→B→A). (R = 8.3 J/K mol)
- What are the temperatures of the two isotherms?
- Fill out the table below with the appropriate values.

(a) pV = nRT  T = pV/nR

<table>
<thead>
<tr>
<th></th>
<th>Q</th>
<th>W_{net system}</th>
<th>ΔE_{TH}</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-D</td>
<td></td>
<td>-2300 J</td>
<td></td>
</tr>
<tr>
<td>D-C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B-A</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) ΔE_{TH}=W + Q

Ideal Gas at constant V

Q = n C_V ΔT

Ch. 12

General Principles

Rotational Dynamics

Every point on a rigid body rotating about a fixed axis has the same angular velocity \( \omega \) and angular acceleration \( \alpha \).

Newton's second law for rotational motion is

\[ \tau = I \alpha \]

Use rotational kinematics to find angles and angular velocities.

Conservation Laws

Energy is conserved for an isolated system.

- Pure rotation \( E = K_m + U_j = \frac{1}{2} I \omega^2 + M g x_m \)
- Rolling \( E = K_m + K_m + U_j = \frac{1}{2} I \omega^2 + \frac{1}{2} M v^2 + M g x_m \)

Angular momentum is conserved if \( \tau = 0 \).

Important Concepts

Torque is the rotational equivalent of force:

\[ \tau = r F \sin \phi = r F \]

The vector description of torque is

\[ \vec{\tau} = \vec{r} \times \vec{F} \]

A system of particles on which there is no net force undergoes unconstrained rotation about the center of mass:

\[ x_m = \frac{1}{M} \int x \, dm \quad \text{and} \quad y_m = \frac{1}{M} \int y \, dm \]

The gravitational torque on a body can be found by treating the body as a particle with all the mass \( M \) concentrated at the center of mass.

The moment of inertia

\[ I = \int \rho r^2 \, dm \]

is the rotational equivalent of mass. The moment of inertia depends on how the mass is distributed around the axis. If \( I_{cm} \) is known, the \( I \) about a parallel axis distance \( d \) away is given by

\[ I = I_{cm} + M d^2 \]
Ch. 12

PROBLEM-SOLVING STRATEGY 12.2 Static equilibrium problems

MODEL  Model the object as a simple shape.

VISUALIZE  Draw a pictorial representation showing all forces and distances. List
         known information.

- Pick any point you wish as a pivot point. The net torque about this point
  is zero.
- Determine the moment arms of all forces about this pivot point.
- Determine the sign of each torque about this pivot point.

SOLVE  The mathematical representation is based on the fact that an object in
         total equilibrium has no net force and no net torque:

\[ \mathbf{F}_{\text{net}} = 0 \quad \text{and} \quad \tau_{\text{net}} = 0 \]

- Write equations for \( \sum F_x = 0, \sum F_y = 0, \) and \( \sum \tau = 0. \)
- Solve the three simultaneous equations.

ASSESS  Check that your result is reasonable and answers the question.

---

Hooke’s Law Springs and a Restoring Force

General Principles

Dynamics

SHM occurs when a linear restoring force acts to return a system to an equilibrium position.

- Horizontal spring
  \( (F_w)_x = -kx \)

- Vertical spring
  The origin is at the equilibrium position \( \Delta l = mg/k. \)
  \( (F_w)_y = -ky \)

- Pendulum
  \( (F_w)_r = \left( \frac{mg}{L} \right) \sin \theta \)
  \( \omega = \sqrt{\frac{L}{m}} \quad T = 2\pi \sqrt{\frac{L}{g}} \)

- \( \omega = \left( \frac{k}{m} \right)^{1/2} \)
- \( T = 2\pi \sqrt{L/g} \)

Key fact: \( \omega = \left( \frac{k}{m} \right)^{1/2} \) is general result where \( k \) reflects a constant of the linear restoring force and \( m \) is the inertial response (e.g., the “physical pendulum” where \( \omega = \left( \frac{\kappa}{I} \right)^{1/2} \))

Energy

If there is no friction or dissipation, kinetic and potential energy are
alternately transformed into each other, but the total mechanical energy
\( E = K + U \) is conserved.

\[ E = \frac{1}{2} mv^2 + \frac{1}{2} kx^2 \]

\[ \frac{1}{2} m(v_{\text{max}})^2 \]

\[ \frac{1}{2} kA^2 \]

In a damped system, the energy decays exponentially
\[ E = E_0 e^{-\gamma t} \]

where \( \gamma \) is the time constant.
Simple Harmonic Motion

**Important Concepts**

Simple harmonic motion (SHM) is a sinusoidal oscillation with period $T$ and amplitude $A$.

- Frequency $f = \frac{1}{T}$
- Angular frequency $\omega = \frac{2\pi}{T}$
- Position $x(t) = A \cos(\omega t + \phi_0)$
- Velocity $v(t) = -A \omega \sin(\omega t + \phi_0)$ with maximum speed $v_{\text{max}} = \omega A$
- Acceleration $a_x = -\omega^2 x$

Maximum kinetic energy

Maximum potential energy

Resonance and damping

- Energy transfer is optimal when the driving force varies at the resonant frequency.

Applications

Resonance

When a system is driven by a periodic external force, it responds with a large-amplitude oscillation if $f_0 = f_n$, where $f_n$ is the system's natural oscillation frequency, or resonant frequency.

Damping

If there is a drag force $\vec{D} = -b\vec{v}$, where $b$ is the damping constant, then for lightly damped systems

$x(t) = Ae^{-\frac{b}{2m}t} \cos(\omega t + \phi_0)$

The time constant for energy loss is $\tau = \frac{m}{b}$.

- Types of motion
  - Undamped
  - Underdamped
  - Critically damped
  - Overdamped
Fluid Flow

General Principles

**Fluid Statics**

<table>
<thead>
<tr>
<th>Gases</th>
<th>Liquids</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freely moving particles</td>
<td>Loosely bound particles</td>
</tr>
<tr>
<td>Compressible</td>
<td>Incapable</td>
</tr>
<tr>
<td>Pressure primarily thermal</td>
<td>Pressure primarily gravitational</td>
</tr>
<tr>
<td>Pressure is constant in a laboratory-size container</td>
<td>Hydrostatic pressure at depth ( d ) is ( p = h \cdot \rho g )</td>
</tr>
</tbody>
</table>

**Fluid Dynamics**

- Ideal-fluid model
- Incompressible
- Smooth, laminar flow
- Nonviscous

[Diagram of fluid dynamics concepts with equations and streamline flow]

Equation of continuity
\[ vA_1 = vA_2 \]

Bernoulli’s equation
\[ p_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2} \rho v_2^2 + p_2 g y_2 \]

Bernoulli’s equation is a statement of energy conservation.

Density and pressure

Important Concepts

- **Density** \( \rho = \frac{m}{V} \), where \( m \) is mass and \( V \) is volume.

- **Pressure** \( p = \frac{F}{A} \), where \( F \) is the magnitude of the fluid force and \( A \) is the area on which the force acts.

- Pressure exists at all points in a fluid.
- Pressure pushes equally in all directions.
- Pressure is constant along a horizontal line.
- Gauge pressure is \( p_g = p - 1 \text{ atm} \).
Response to forces

Applications

Buoyancy: the upward force of a fluid on an object.

Archimedes' principle

The magnitude of the buoyant force equals the weight of the fluid displaced by the object.

Sink: \( \rho_{\text{obj}} > \rho_f \) \( F_b = \rho_f V g \)

Rise to surface: \( \rho_{\text{obj}} < \rho_f \) \( F_b = \rho_f V g \)

Neutrally buoyant: \( \rho_{\text{obj}} = \rho_f \) \( F_b = \rho_f V g \)

Elasticity describes the deformation of solids and liquids under stress.

Linear stretch and compression

\[ \varepsilon = \frac{\Delta L}{L_0} \]

Tensile stress: \( \sigma = \frac{F}{A} \)

Young's modulus: \( E = \frac{\sigma}{\varepsilon} \)

Volume compression

\[ \rho = \frac{P}{\rho V_f} \]

Bulk modulus: \( B = \frac{\Delta P}{\Delta \rho} \)

States of Matter and Phase Diagrams

General Principles

Three Phases of Matter

- Liquid: Molecules loosely held together by molecular bonds, but able to move around. Nearly incompressible.
- Gas: Molecules move freely through space. Compressible.

The different phases exist for different conditions of temperature \( T \) and pressure \( P \).

The boundaries separating the regions of a phase diagram are lines of phase equilibrium. Any amounts of the two phases can coexist in equilibrium. The triple point is the one value of temperature and pressure at which all three phases can coexist in equilibrium.
**Ideal gas equation of state**

### Important Concepts

#### Ideal-Gas Model
- Atoms and molecules are small, hard spheres that travel freely through space except for occasional collisions with each other or the walls.
- The model is valid when the density is low and the temperature well above the condensation point.

#### Ideal-Gas Law

The state variables of an ideal gas are related by the ideal-gas law

\[
pV = nRT \quad \text{or} \quad \frac{n}{m} = \frac{N}{N_a} \frac{V}{V_a} T
\]

where \( R = 8.31 \text{ J/mol K} \) is the universal gas constant and \( k_B = 1.38 \times 10^{-23} \text{ J/K} \) is Boltzmann’s constant.

\( p, V \) and \( T \) must be in SI units of Pa, m³, and K. For a gas in a sealed container, with constant \( n \):

\[
\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}
\]

Counting atoms and moles

A macroscopic sample of matter consists of \( N \) atoms (or molecules), each of mass \( m \) (the atomic or molecular mass):

\[
N = \frac{M}{m}
\]

Alternatively, we can state that the sample consists of \( n \) moles:

\[
n = N \frac{N_A}{N_a} \quad \text{or} \quad \frac{n}{M} \frac{M_{\text{mol}}}{M_{\text{atm}}}
\]

\( N_A = 6.02 \times 10^{23} \text{ mol}^{-1} \) is Avogadro’s number.

The numerical value of the molar mass \( M_{\text{atm}} \) in g/mol, equals the numerical value of the atomic or molecular mass \( m \) in u. The atomic or molecular mass \( m \), in atomic mass units \( u \), is well approximated by the atomic mass number \( A \):

\[
u = 1.66 \times 10^{-27} \text{ kg}
\]

The number density of the sample is \( \frac{N}{V} \).

---

**pV diagrams**

### Applications

**Temperature scales**

\[
T_K = \frac{9}{5} T_C + 32^\circ \quad \text{or} \quad T_K = T_C + 273
\]

The Kelvin temperature scale is based on:

- Absolute zero at \( T_0 = 0 \text{ K} \)
- The triple point of water at \( T_0 = 273.16 \) K

**Three basic gas processes**

1. **Isochoric**, or constant volume
2. **Isobaric**, or constant pressure
3. **Isothermal**, or constant temperature
Thermodynamics

General Principles

First Law of Thermodynamics

\[ \Delta E_{\text{sys}} = W + Q \]

The first law is a general statement of energy conservation.

Work \( W \) and heat \( Q \) depend on the process by which the system is changed.

The change in the system depends only on the total energy exchanged \( W + Q \) and not on the process.

Energy

Thermal energy \( E_{\text{th}} \). Microscopic energy of moving molecules and stretched molecular bonds. \( \Delta E_{\text{sys}} \) depends on the initial/final states but is independent of the process.

Work \( W \). Energy transferred to the system by forces in a mechanical interaction.

Heat \( Q \). Energy transferred to the system via atomic-level collisions when there is a temperature difference. A thermal interaction.

Important Concepts

The work done on a gas is

\[ W = \int p \, dV = -\text{(area under the } pV \text{ curve)} \]

Adiabatic process (no heat exchange)

An adiabatic process is one for which \( pV^n = \text{constant} \), where \( n = c_p/c_v \), is the specific heat ratio. An adiabatic process changes the temperature of the gas without heating or cooling it.

The heat of transformation \( L \) is the energy needed to cause 1 kg of substance to undergo a phase change.

\[ Q = mL \]

The specific heat \( c \) of a substance is the energy needed to raise the temperature of 1 kg by 1 K:

\[ Q = m\Delta T \]

The molar specific heat \( C \) is the energy needed to raise the temperature of 1 mol by 1 K:

\[ Q = nC\Delta T \]

The molar specific heat of a substance depends on the process by which the temperature is changed:

\[ C_p = \text{molar specific heat at constant pressure} \]
\[ C_v = \text{molar specific heat at constant volume} \]

Heat is transferred by conduction, convection, radiation, and evaporation.

Conduction: \( Q/dt = (\lambda/\Delta x) \Delta T \)

Radiation: \( Q/dt = e\sigma AT^4 \)

In steady-state \( T = \text{constant} \) and so heat in equals heat out
Gas Processes

Summary of Basic Gas Processes

<table>
<thead>
<tr>
<th>Process</th>
<th>Definition</th>
<th>Stays constant</th>
<th>Work</th>
<th>Heat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isochoric</td>
<td>$\Delta V = 0$</td>
<td>$V$ and $pT$</td>
<td>$W = 0$</td>
<td>$Q = nC_v \Delta T$</td>
</tr>
<tr>
<td>Isobaric</td>
<td>$\Delta p = 0$</td>
<td>$p$ and $VT$</td>
<td>$W = -p \Delta V$</td>
<td>$Q = nC_v \Delta T$</td>
</tr>
<tr>
<td>Isothermal</td>
<td>$\Delta T = 0$</td>
<td>$T$ and $pV$</td>
<td>$W = -nRT \ln(V/V_i)$</td>
<td>$\Delta E_k = 0$</td>
</tr>
<tr>
<td>Adiabatic</td>
<td>$Q = 0$</td>
<td>$pV^\gamma$</td>
<td>$W = \Delta E_k$</td>
<td>$Q = 0$</td>
</tr>
</tbody>
</table>

All gas processes
First law $\Delta E_k = W + Q = nC_v \Delta T$
Ideal-gas law $pV = nRT$

Lecture 25

- Exam covers Chapters 14-17 plus angular momentum, statics

- Assignment
  - For Thursday, read through all of Chapter 18