Lecture 28

Goals:

• Chapter 20
  ❖ Employ the wave model
  ❖ Visualize wave motion
  ❖ Analyze functions of two variables
  ❖ Know the properties of sinusoidal waves, including wavelength, wave number, phase, and frequency.
  ❖ Work with a few important characteristics of sound waves. (e.g., Doppler effect)

• Assignment
  ❖ HW12, Due Tuesday, May 4th
  ❖ HW13, Due Friday, May 7th
  ❖ For Tuesday, Read through all of Chapter 21

Waves

• A traveling wave is an organized disturbance propagating at a well-defined wave speed $v$.

• In transverse waves the particles of the medium move perpendicular to the direction of wave propagation.

• In longitudinal waves the particles of the medium move parallel to the direction of wave propagation.

• A wave transfers energy, but no material or substance is transferred outward from the source.
Types of Waves

- Mechanical waves travel through a material medium such as water or air.
- Electromagnetic waves require no material medium and can travel through vacuum.
- Matter waves describe the wave-like characteristics of atomic-level particles.
  
  For mechanical waves, the speed of the wave is a property of the medium.
  
  Speed does not depend on the size or shape of the wave.

- Examples:
  - Sound waves (air moves locally back & forth)
  - Stadium waves (people move up & down...no energy transfer)
  - Water waves (water moves up & down)
  - Light waves (an oscillating electromagnetic field)

Wave Graphs

- The displacement $D$ of a wave is a function of both position (where) and time (when).

- A snapshot graph shows the wave’s displacement as a function of position at a single instant of time.

- A history graph shows the wave’s displacement as a function of time at a single point in space.

- The displacement, $D$, is a function of two variables, $x$ and $t$, or $D(x,t)$
Wave Speed

- Speed of a transverse, mechanical wave on a string:

\[ v = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}} \]

\[ v = \frac{T_s}{\mu} \]

\[ \mu = \frac{m}{L} \]

where \( T_s \) is the string tension and \( \mu \) is linear string density.

- Speed of sound (longitudinal mechanical wave) in air at 20°C

\[ v = 343 \text{ m/s} \]

- Speed of light (transverse, EM wave) in vacuum: \( c = 3 \times 10^8 \text{ m/s} \)

- Speed of light (transverse, EM wave) in a medium: \( v = \frac{c}{n} \)

where \( n \) = index of refraction of the medium (typically 1 to 4).

Wave Forms

- We will examine “continuous waves” that extend forever in each direction!

- We can also have “pulses” caused by a brief disturbance of the medium:

- And “pulse trains” which are somewhere in between.
Continuous Sinusoidal Wave

- Wavelength: The distance $\lambda$ between identical points on the wave.
- Amplitude: The maximum displacement $A$ of a point on the wave.

![Wavelength and Amplitude Diagram](image)

Wave Properties...

- Period: The time $T$ for a point on the wave to undergo one complete oscillation.
- Speed: The wave displaces one wavelength $\lambda$ in one period $T$ so its speed is $v = \frac{\lambda}{T}$.
Exercise Wave Motion

- The speed of sound in air is a bit over 300 m/s (i.e., 343 m/s), and the speed of light in air is about 300,000,000 m/s.

- Suppose we make a sound wave and a light wave that both have a wavelength of 3 meters.

What is the ratio of the frequency of the light wave to that of the sound wave? (Recall \( v = \frac{\lambda}{T} = \lambda f \))

(A) About 1,000,000  
(B) About 0.000,001  
(C) About 1000

Wave Properties

\[
D(x, t) = A \cos\left(\frac{2\pi}{\lambda} \left( x / \lambda - t / T \right) + \phi_0 \right)
\]

\[
D(x, t) = A \cos\left( kx - \omega t + \phi_0 \right)
\]

\( A = \text{amplitude} \quad k \equiv \frac{2\pi}{\lambda} = \text{wave number} \)

\( \omega = 2\pi f = \text{angular frequency} \quad \phi_0 = \text{phase constant} \)

Look at the spatial part (Let \( t = 0 \)).

\[
D(x,0) = A \cos\left(\frac{2\pi}{\lambda} x \right)
\]

- \( x = 0 \quad D = A \)
- \( x = \lambda/4 \quad D = A \cos(\pi/2) = 0 \)
- \( x = \lambda/2 \quad D = A \cos(\pi) = -A \)
Look at the temporal (time-dependent) part

\[ D(x, t) = A \cos\left(\frac{2\pi}{\lambda} x - \omega t\right) \]

- Let \( x = 0 \)

\[ D(0, t) = A \cos(-\omega t) = A \cos\left(-\frac{2\pi}{T} t\right) \]

- \( t = 0 \) \( D = A \)
- \( t = T/4 \) \( D = A \cos(-\pi/2) = 0 \)
- \( t = T/2 \) \( D = A \cos(-\pi) = -A \)

Animation
Exercise Wave Motion

- A harmonic wave moving in the **positive x direction** can be described by the equation
  \[ D(x,t) = A \cos \left( \frac{2\pi}{\lambda} x - \omega t \right) = A \cos (k x - \omega t) \]

\[ v = \lambda / T = \lambda f = (\lambda/2\pi) (2\pi f) = \omega / k \] and, by definition, \( \omega > 0 \)

- Which of the following equation do you expect describes a harmonic wave traveling in the negative x direction?
- Hint: \( \cos \alpha = \cos -\alpha \) so \( \cos (k x - \omega t) = \cos (-k x + \omega t) \)

  (A) \( D(x,t) = A \sin (k x - \omega t) \)
  (B) \( D(x,t) = A \cos (k x + \omega t) \)
  (C) \( D(x,t) = A \cos (-k x + \omega t) \)

Exercise Wave Motion

- A boat is moored in a fixed location, and waves make it move up and down. If the spacing between wave crests is 20 meters and the speed of the waves is 5 m/s, how long \( \Delta t \) does it take the boat to go from the top of a crest to the bottom of a trough? (Recall \( v = \lambda / T = \lambda f \))

  (A) 2 sec  (B) 4 sec  (C) 8 sec
Exercise  Wave Motion

- A boat is moored in a fixed location, and waves make it move up and down. If the spacing between wave crests is 20 meters and the speed of the waves is 5 m/s, how long $\Delta t$ does it take the boat to go from the top of a crest to the bottom of a trough?
- $T = 4$ sec but crest to trough is half a wavelength

(A) 2 sec  (B) 4 sec  (C) 8 sec

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Speed of Waves

- The speed of sound waves in a medium depends on the compressibility and the density of the medium
- The compressibility can sometimes be expressed in terms of the elastic modulus of the material
- The speed of all mechanical waves follows a general form:

$$v = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}}$$

Waves on a string ↦

$$v = \sqrt{\frac{T}{\mu}}$$
Waves on a string...

- So we find:

\[ v = \sqrt{\frac{F}{\mu}} \]

- Making the tension bigger increases the speed.
- Making the string heavier decreases the speed.
- The speed depends only on the nature of the medium, not on amplitude, frequency etc of the wave.

Exercise  Wave Motion

- A heavy rope hangs from the ceiling, and a small amplitude transverse wave is started by jiggling the rope at the bottom.
- As the wave travels up the rope, its speed will:

(a) increase
(b) decrease
(c) stay the same
Sound, A special kind of longitudinal wave

Consider a vibrating guitar string

String Vibrates

Piece of string undergoes harmonic motion

Air molecules alternatively compressed and rarefied

Sound

Consider the actual air molecules and their motion versus time,

<table>
<thead>
<tr>
<th>Time</th>
<th>Molecule 1</th>
<th>Molecule 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>time 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>time 1</td>
<td>&lt;---</td>
<td>---&lt;---</td>
</tr>
<tr>
<td>time 2</td>
<td></td>
<td>&lt;---</td>
</tr>
</tbody>
</table>

Individual molecules undergo harmonic motion with displacement in same direction as wave motion.
Speed of Sound in a Solid Rod

- The Young’s modulus of the material is \( Y \)
- The density of the material is \( \rho \)
- The speed of sound in the rod is

\[
v = \sqrt{\frac{Y}{\rho}}
\]

Speed of Sound in Liquid or Gas

- The bulk modulus of the material is \( B \)
- The density of the material is \( \rho \)
- The speed of sound in that medium is

\[
v = \sqrt{\frac{B}{\rho}}
\]

<table>
<thead>
<tr>
<th>Medium</th>
<th>Speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>343</td>
</tr>
<tr>
<td>Helium</td>
<td>972</td>
</tr>
<tr>
<td>Water</td>
<td>1500</td>
</tr>
<tr>
<td>Steel (solid)</td>
<td>5600</td>
</tr>
</tbody>
</table>

Speed of Sound in Air

- The speed of sound also depends on the temperature of the medium
- This is particularly important with gases
- For air, the relationship between the speed and temperature (if pressure is constant) is

\[
v = (331 \text{ m/s}) \sqrt{1 + \frac{T_c}{273^\circ C}}
\]

- (331 m/s is the speed at 0\(^\circ\) C)
- \( T_c \) is the air temperature in Centigrade
**Home Exercise**

**Comparing Waves, He vs. Air**

A sound wave having frequency $f_0$, speed $v_0$ and wavelength $\lambda_0$, is traveling through air when it encounters a large helium-filled balloon. Inside the balloon the frequency of the wave is $f_1$, its speed is $v_1$, and its wavelength is $\lambda_1$.

Compare the speed of the sound wave inside and outside the balloon

- (A) $v_1 < v_0$
- (B) $v_1 = v_0$
- (C) $v_1 > v_0$

Compare the frequency of the sound wave inside and outside the balloon

- (A) $f_1 < f_0$
- (B) $f_1 = f_0$
- (C) $f_1 > f_0$

Compare the wavelength of the sound wave inside and outside the balloon

- (A) $\lambda_1 < \lambda_0$
- (B) $\lambda_1 = \lambda_0$
- (C) $\lambda_1 > \lambda_0$

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**Waves, Wave fronts, and Rays**

- Note that a small portion of a spherical wave front is well represented as a “plane wave”.

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Waves, Wave fronts, and Rays

- If the power output of a source is constant, the total power of any wave front is constant.

\[ I = \frac{P_{av}}{A} = \frac{P_{av}}{4\pi R^2} \]

Exercise  Spherical Waves

- You are standing 10 m away from a very loud, small speaker. The noise hurts your ears. In order to reduce the intensity to 1/4 its original value, how far away do you need to stand?

(A) 14 m  (B) 20 m  (C) 30 m  (D) 40 m
Intensity of sounds

- Intensity of a sound wave is \[ I = \frac{\Delta P_{\text{max}}^2}{2 \rho v} \]
  - Proportional to (amplitude)$^2$
  - This is a general result (not only for sound)
- Threshold of human hearing: \( I_0 = 10^{-12} \text{ W/m}^2 \)

- The range of intensities detectible by the human ear is very large
- It is convenient to use a logarithmic scale to determine the intensity level, \( \beta \)

\[
\beta = 10 \log_{10} \left( \frac{I}{I_0} \right)
\]

Intensity of sounds

- \( I_0 \) is called the reference intensity
  - It is taken to be the threshold of hearing
  - \( I_0 = 1.00 \times 10^{-12} \text{ W/ m}^2 \)
  - \( I \) is the intensity of the sound whose level is to be determined \( \beta \) is in decibels (dB)

- Threshold of pain: \( I = 1.00 \text{ W/m}^2; \beta = 120 \text{ dB} \)

- Threshold of hearing: \( I_0 = 1.00 \times 10^{-12} \text{ W/ m}^2; \beta = 0 \text{ dB} \)
Intensity of sounds

- Some examples (1 pascal $\equiv 10^{-5}$ atm):

<table>
<thead>
<tr>
<th>Sound Intensity</th>
<th>Pressure</th>
<th>Intensity (W/m²)</th>
<th>Level (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hearing threshold</td>
<td>$3 \times 10^{-5}$</td>
<td>$10^{-12}$</td>
<td>0</td>
</tr>
<tr>
<td>Classroom</td>
<td>0.01</td>
<td>$10^{-7}$</td>
<td>50</td>
</tr>
<tr>
<td>Indoor concert</td>
<td>30</td>
<td>1</td>
<td>120</td>
</tr>
<tr>
<td>Jet engine at 30 m</td>
<td>100</td>
<td>10</td>
<td>130</td>
</tr>
</tbody>
</table>

Sound Level, Example

- What is the sound level that corresponds to an intensity of $2.0 \times 10^{-7}$ W/m²?

  $\beta = 10 \log_{10} \left( \frac{2.0 \times 10^{-7} \text{ W/m}^2}{1.0 \times 10^{-12} \text{ W/m}^2} \right)$
  
  $= 10 \log_{10} 2.0 \times 10^5 = 53$ dB

- Rule of thumb: An apparent “doubling” in the loudness is approximately equivalent to an increase of 10 dB.

- dBs are not linear with intensity
Loudness and Intensity

- Sound level in decibels relates to a *physical measurement* of the strength of a sound
- We can also describe a *psychological “measurement”* of the strength of a sound
- Our bodies “calibrate” a sound by comparing it to a reference sound
- This would be the threshold of hearing
- Actually, the threshold of hearing is this value for 1000 Hz

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