Lecture 30

To do:

- Chapter 21
  - Examine two wave superposition (-ωt and +ωt)
  - Examine two wave superposition (-ω₁t and -ω₂t)

Review for final (Location: CHEM 1351, 7:45 am)

Tomorrow: Review session, 2103 CH at 12:05 PM

- Last Assignment
  - HW13, Due Friday, May 7th, 11:59 PM

Standing waves

- Waves traveling in opposite direction “interfere” with each other.

If the conditions are right, same k (2π/λ) & ω, the superposition generates a standing wave:

\[ D_{\text{Right}}(x,t) = a \sin(kx-\omega t) \quad D_{\text{Left}}(x,t) = a \sin(kx+\omega t) \]

Energy flow in a standing wave is stationary, it “stands” in place. Standing waves have nodes and antinodes.

\[ D(x,t) = D_L(x,t) + D_R(x,t) \]

\[ D(x,t) = 2a \sin(kx) \cos(\omega t) \]

The outer curve is the amplitude function

\[ A(x) = \pm2a \sin(kx) \]

when \[ \omega t = \pi n \quad n = 0,1,2,\ldots \]

\[ k = \text{wave number} = 2\pi/\lambda \]
Standing waves on a string

- Longest wavelength allowed: $\frac{1}{2}$ of the full wave
- Fundamental: $\frac{\lambda}{2} = L \rightarrow \lambda = 2L$

$$\lambda_m = \frac{2L}{m} = \frac{v}{f_m}$$

$m = 1, 2, 3, \ldots$

Recall $v = f\lambda$

$$f_m = m \frac{v}{2L}$$

Overtones $m > 1$

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Vibrating Strings- Superposition Principle

- Violin, viola, cello, string bass
- Guitars
- Ukuleles
- Mandolins
- Banjos

![Diagram](Concept © Cengage Learning; adapted by Andrew Felder)
Standing waves in a pipe

Open end: **Must** be a displacement antinode (pressure minimum)
Closed end: **Must** be a displacement node (pressure maximum)

Blue curves are displacement oscillations. Red curves, pressure.

Fundamental: \( \lambda/2 \) \( \lambda/2 \) \( \lambda/4 \)

\[
\lambda_m = \frac{2L}{m} \\
f_m = \frac{m}{2} \frac{v}{L} \\
m = 1, 2, 3, ... 
\]

\[
\lambda_m = \frac{2L}{m} \\
f_m = \frac{m}{2} \frac{v}{L} \\
m = 1, 2, 3, ... 
\]

\[
\lambda_m = \frac{4L}{m} \\
f_m = \frac{m}{4} \frac{v}{L} \\
m = 1, 3, 5, ... 
\]
Combining Waves

Consider two harmonic waves $A$ and $B$ meet at $t=0$. They have same amplitudes and phase, but $\omega_2 = 1.15 \times \omega_1$.

The displacement versus time for each is shown below:

$$C(t) = A(t) + B(t)$$
**Superposition & Interference**

- Consider $A + B$, \[\text{Recall } \cos \varphi + \cos \psi = 2 \cos\left(\frac{\varphi - \psi}{2}\right) \cos\left(\frac{\varphi + \psi}{2}\right)\]

\[y_A(x,t) = A \cos(k_1x - 2\pi f_1 t) \quad y_B(x,t) = A \cos(k_2x - 2\pi f_2 t)\]

Let $x=0$, \[y = y_A + y_B = 2A \cos[2\pi (f_1 - f_2) t/2] \cos[2\pi (f_1 + f_2) t/2]\]

and \(|f_1 - f_2| \equiv f_{\text{beat}} = 1 / T_{\text{beat}} \quad f_{\text{average}} \equiv (f_1 + f_2)/2\]

\[A(\omega_1 t) \quad B(\omega_2 t) \quad C(t) = A(t) + B(t) \quad T_{\text{beat}} \]

**Exercise Superposition**

- The traces below show beats that occur when two different pairs of waves are added (the time axes are the same).
- For which of the two is the difference in frequency of the original waves greater?

A. Pair 1  
B. Pair 2  
C. The frequency difference was the same for both pairs of waves.  
D. Need more information.
Superposition & Interference

- Consider $A + B$, \[ \text{Recall } \cos u + \cos v = 2 \cos\left(\frac{u-v}{2}\right) \cos\left(\frac{u+v}{2}\right) \]

$$y_A(x,t) = A \cos(k_1 x - 2\pi f_1 t) \quad y_B(x,t) = A \cos(k_2 x - 2\pi f_2 t)$$

Let $x=0$,

$$y = y_A + y_B = 2A \cos[2\pi (f_1 - f_2)t/2] \cos[2\pi (f_1 + f_2)t/2]$$

and

$$|f_1 - f_2| \equiv f_{\text{beat}} = \frac{1}{T_{\text{beat}}} \quad f_{\text{average}} \equiv \frac{(f_1 + f_2)}{2}$$

![Graph showing superposition and interference](Physics 207: Lecture 30, Pg 11)

Review

- Final is “semi” cumulative
- Early material, more qualitative (i.e., conceptual)
- Later material, more quantitative (but will employ major results from early on).
- 25-30% will be multiple choice
- Remainder will be short answer with the focus on thermodynamics, heat engines, wave motion and wave superposition
**Exercise Superposition**

- The traces below show beats that occur when two different pairs of waves are added (the time axes are the same).
- For which of the two is the difference in frequency of the original waves greater?

![Wave Traces](image)

A. Pair 1
B. Pair 2
C. The frequency difference was the same for both pairs of waves.
D. Need more information.

---

**Organ Pipe Example**

A 0.9 m organ pipe (open at both ends) is measured to have its first harmonic (i.e., its fundamental) at a frequency of 382 Hz. What is the speed of sound (refers to energy transfer) in this pipe?

\[ f = 382 \text{ Hz} \quad \text{and} \quad f \lambda = v \quad \text{with} \quad \lambda = \frac{2L}{m} \quad (m = 1) \]

\[ v = 382 \times 2(0.9) \text{ m} \rightarrow v = 687 \text{ m/s} \]
Standing Wave Question

- What happens to the fundamental frequency of a pipe, if the air \((v = 300 \text{ m/s})\) is replaced by helium \((v = 900 \text{ m/s})\)?

Recall: \(f \lambda = v\)

(A) Increases  (B) Same  (C) Decreases
Important Concepts

Pictorial Representation

1. Draw a motion diagram.
2. Establish coordinates.
3. Sketch the situation.
4. Define symbols.
5. List knowns.
6. Identify desired unknown.

For motion along a line:

- Speeding up: \(v\) and \(a\) point in the same direction, \(v_i\) and \(a_i\) have the same sign.
- Slowing down: \(v\) and \(a\) point in opposite directions, \(v_i\) and \(a_i\) have opposite signs.
- Constant speed: \(a = 0\), \(a_i = 0\).

Acceleration \(a\) is positive if \(\vec{a}\) points right, negative if \(\vec{a}\) points left. The sign of \(a\) does not imply speeding up or slowing down.

Chapter 2

General Principles

**Kinematics** describes motion in terms of position, velocity, and acceleration. General kinematic relationships are given **mathematically** by:

- Instantaneous velocity \(v_i = \frac{ds}{dt}\) = slope of position graph
- Instantaneous acceleration \(a_i = \frac{dv_i}{dt}\) = slope of velocity graph

**Important Concepts**

- Position, velocity, and acceleration are related **graphically**.
  - The slope of the position versus-time graph is the value on the velocity graph.
  - The slope of the velocity graph is the value on the acceleration graph.
  - \(s\) is a maximum or minimum at a turning point, and \(v_i = 0\).
  - Displacement is the area under the velocity curve.
Important Concepts

A vector is a quantity described by both a magnitude and a direction.

Using Vectors

Components

The component vectors are parallel to the x- and y-axes:

\[ \vec{A} = \vec{A}_x + \vec{A}_y = A_x \hat{i} + A_y \hat{j} \]

In the figure at the right, for example:

\[ \begin{align*}
A_x &= A \cos \theta \\
A_y &= A \sin \theta
\end{align*} \]

**Unit Vectors**

Unit vectors have magnitude 1 and no units. Unit vectors \( \hat{i} \) and \( \hat{j} \) define the directions of the x- and y-axes.

\[ \vec{A} = A \hat{i} \]

The components \( A_x \) and \( A_y \) are the magnitudes of the component vectors \( \vec{A}_x \) and \( \vec{A}_y \), and a plus or minus sign is needed when the component points toward the positive end of the axis or toward the negative end of the axis.

**Working Graphically**

**Working Algebraically**

Vector calculations are done component by component:

\[ \vec{C} = 2 \vec{A} + 3 \vec{B} \]

\[ \begin{align*}
C_x &= 2A_x + B_x \\
C_y &= 2A_y + B_y
\end{align*} \]

The magnitude of \( \vec{C} \) is then

\[ C = \sqrt{C_x^2 + C_y^2} \]

General Principles

The instantaneous velocity

\[ \vec{u} = d\vec{r}/dt \]

is a vector tangent to the trajectory.

The instantaneous acceleration is

\[ \vec{a} = d\vec{u}/dt \]

\( \vec{u}_r \), the component of \( \vec{u} \) parallel to \( \vec{v}_r \), is responsible for change of speed, \( \vec{u}_r \), the component of \( \vec{a} \) perpendicular to \( \vec{u}_r \), is responsible for change of direction.

Important Concepts

**Uniform Circular Motion**

Angular velocity \( \omega = \text{d}\theta/\text{d}t \).

\( \vec{v}_r \) and \( \vec{a}_r \) are constant;

\[ \vec{v}_r = \omega \vec{r} \]

The centripetal acceleration points toward the center of the circle:

\[ a = \frac{v^2}{r} = -\omega^2 r \]

It changes the particle’s direction but not its speed.

**Nonuniform Circular Motion**

Angular acceleration \( \alpha = \text{d}\omega/\text{d}t \).

The radial acceleration

\[ a_r = \frac{v^2}{r} = \omega \gamma r \]

changes the particle’s direction. The tangential component

\[ a_t = \omega r \]

changes the particle’s speed.
## Chapter 4

### Applications

**Kinematics in two dimensions**

If \( \vec{a} \) is constant, then the x- and y-components of motion are independent of each other.

\[
\begin{align*}
\Delta x &= x_f - x_i = v_x \Delta t + \frac{1}{2} a_x \Delta t^2 \\
\Delta y &= y_f - y_i = v_y \Delta t + \frac{1}{2} a_y \Delta t^2 \\
\end{align*}
\]

**Projectile motion** occurs if the object moves under the influence of only gravity. The motion is a parabola.

- Uniform motion in the horizontal direction with \( v_{x0} = v_x \cos \theta \).
- Free-fall motion in the vertical direction with \( a_y = -g \) and \( v_{y0} = v_y \sin \theta \).
- The \( x \) and \( y \) kinematic equations have the same value for \( \Delta t \).

**Circular motion kinematics**

- **Period** \( T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} \)
- **Angular position** \( \theta = \frac{r}{\omega} \)
- \( \theta = \theta_i + \omega \Delta t \)
- \( \omega^2 = \omega_i^2 + 2 \alpha \Delta \theta \)

Angle, angular velocity, and angular acceleration are related graphically.

- The angular velocity is the slope of the angular position graph.
- The angular acceleration is the slope of the angular velocity graph.

### Chapter 5

### General Principles

**Newton’s First Law**

An object at rest will remain at rest, or an object that is moving will continue to move in a straight line with constant velocity, if and only if the net force on the object is zero.

\[
\vec{F}_{net} = \vec{0} \\
\vec{a} = \vec{0}
\]

The first law tells us that no “cause” is needed for motion. Uniform motion is the “natural state” of an object.

**Newton’s Laws**

Newton’s laws are valid only in inertial reference frames.

An object with mass \( m \) will undergo acceleration

\[
\vec{a} = \frac{1}{m} \vec{F}_{net}
\]

where \( \vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots \) is the vector sum of all the individual forces acting on the object.

The second law tells us that a net force causes an object to accelerate. This is the connection between force and motion that we are seeking.

### Important Concepts

**Acceleration** is the link to kinematics.

- From \( \vec{F}_{net} \), find \( \vec{a} \).
- From \( \vec{a} \), find \( \vec{v} \) and \( \vec{x} \).

- \( \vec{a} = \vec{0} \) is the condition for equilibrium.

**Static equilibrium** if \( \vec{a} = \vec{0} \).

**Dynamic equilibrium** if \( \vec{a} = \vec{0} \).

Equilibrium occurs if and only if \( \vec{F}_{net} = \vec{0} \).

**Mass** is the resistance of an object to acceleration. It is an intrinsic property of an object.

**Force** is a push or a pull on an object.

- Force is a vector, with a magnitude and a direction.
- Force requires an agent.
- Force is either a contact force or a long-range force.
Chapter 5 & 6

Key Skills

Identifying Forces
Forces are identified by locating the points where other objects touch the object of interest. These are points where contact forces are exerted. In addition, objects with mass feel a long-range gravitational force.

Free-Body Diagrams
A free-body diagram represents the object as a particle at the origin of a coordinate system. Force vectors are drawn with their tails on the particle. The net force vector is drawn beside the diagram.

General Strategy
All examples in this chapter follow a four-part strategy. You’ll become a better problem solver if you adhere to it as you do the homework problems. The Dynamics Worksheets in the Student Workbook will help you structure your work in this way.

Equilibrium Problems
Object at rest or moving with constant velocity.
MODEL Make simplifying assumptions.

VISUALIZE
- Translate words into symbols.
- Identify forces.
- Draw a free-body diagram.

SOLVE Use Newton’s first law:
\[ \sum F = 0 \]
“Read” the vectors from the free-body diagram.

ASSESS Is the result reasonable?

Dynamics Problems
Object accelerating.
MODEL Make simplifying assumptions.

VISUALIZE
- Translate words into symbols.
- Draw a sketch to define the situation.
- Draw a motion diagram.
- Identify forces.
- Draw a free-body diagram.

SOLVE Use Newton’s second law:
\[ \sum F = ma \]
“Read” the vectors from the free-body diagram.
Use kinematics to find velocities and positions.

ASSESS Is the result reasonable?

Chapter 6

Important Concepts

Specific information about three important forces:

Gravity \( F_g = mg \) (downward)
Friction \( F_f = \mu \cdot n \), direction opposite to the motion
Drag \( F_D = \frac{1}{2} \rho v^2 \), direction opposite to the motion

Chapter 7

General Principles

Newton’s Third Law
Every force occurs as one member of an action/reaction pair of forces. The two members of an action/reaction pair:
- Act on two different objects.
- Are equal in magnitude but opposite in direction:
\[ F_{A \rightarrow B} = -F_{B \rightarrow A} \]

Solving Interacting-Objects Problems
MODEL Choose the objects of interest.

VISUALIZE
- Draw a pictorial representation.
- Sketch and define coordinates.
- Identify acceleration constraints.
- Draw an interaction diagram.
- Draw a separate free-body diagram for each object.
- Connect action/reaction pairs with dashed lines.

SOLVE
- Write Newton’s second law for each object.
- Include all forces acting on each object.
- Use Newton’s third law to equate the magnitudes of action/reaction pairs.
- Include acceleration constraints and friction.

ASSESS Is the result reasonable?
Chapter 7

Important Concepts

Objects, systems, and the environment
- Objects whose motion is of interest are the system.
- Objects whose motion is not of interest form the environment.
- The objects of interest interact with the environment, but those interactions can be considered external forces.

Applications

Acceleration constraints
- Objects that are constrained to move together must have accelerations of equal magnitude: $\mathbf{a}_1 = \mathbf{a}_2$.
- This must be expressed in terms of components, such as $a_{x1} = -a_{x2}$.

Strings and pulleys
- The tension in a string or rope pulls in both directions. The tension is constant in a string if the string is:
  - Massless, or
  - In equilibrium
- Objects connected by massless strings passing over frictionless pulleys act as if they interact via an action/reaction pair of forces.

Chapter 7 (Newton’s 3rd Law) & Chapter 8

Newton’s Second Law

Expressed in $x$- and $y$-component form:
- $(F_{\text{net},x}) = \sum F_x = ma_x$
- $(F_{\text{net},y}) = \sum F_y = ma_y$

Expressed in $rz$-component form:
- $(F_{\text{net},r}) = \sum F_r = ma_r$
- $(F_{\text{net},\theta}) = \sum F_\theta = m \omega^2 r$
- $(F_{\text{net},z}) = \sum F_z = 0$

Angular velocity
- $\omega = \frac{d\theta}{dt}$
- $\Omega = 2\pi \omega$

Angular acceleration
- $\alpha = \frac{d\omega}{dt}$
- $\alpha = \frac{\omega}{\theta}$

Orbits
- A circular orbit has radius $r$ if
  - $v = \sqrt{\frac{GM}{r}}$
Chapter 8

Uniform Circular Motion
- \( v \) is constant.
- \( \vec{F}_{\text{net}} \) points toward the center of the circle.
- The centripetal acceleration \( \vec{a} \) points toward the center of the circle. It changes the particle’s direction but not its speed.

Nonuniform Circular Motion
- \( v \) changes.
- \( \vec{a} \) is parallel to \( \vec{F}_{\text{net}} \).
- The radial component \( a_r \) changes the particle’s direction.
- The tangential component \( a_t \) changes the particle’s speed.

Chapter 9

Law of Conservation of Momentum

<table>
<thead>
<tr>
<th>The total momentum ( \vec{P} = \vec{P}_1 + \vec{P}_2 + \cdots ) of an isolated system is constant. Thus</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \vec{P}_1 = \vec{P}_1 )</td>
</tr>
</tbody>
</table>

Newton’s Second Law

In terms of momentum, Newton’s second law is

\[
\vec{F} = \frac{d\vec{P}}{dt}
\]

Momentum \( \vec{p} = m\vec{v} \)

Impulse \( J = \int F(t) \, dt = \text{area under force curve} \)

Impulse and momentum are related by the impulse-momentum theorem

\[
\Delta \vec{p} = J
\]

This is an alternative statement of Newton’s second law.

System A group of interacting particles.
Isolated system A system on which there are no external forces or the net external force is zero.

Before-and-after pictorial representation
- Define the system.
- Use two drawings to show the system before and after the interaction.
- List known information and identify what you are trying to find.

Law of conservation of momentum The total momentum \( \vec{P} \) of an isolated system is a constant. Interactions within the system do not change the system’s total momentum.
Chapter 10

Law of Conservation of Mechanical Energy

If there are no friction or other energy-loss processes (to be explored more thoroughly in Chapter 11), then the mechanical energy $E_{mech} = K + U$ of a system is conserved. Thus

$$K_i + U_i = K_f + U_f$$

- $K$ is the sum of the kinetic energies of all particles.
- $U$ is the sum of all potential energies.

**Basic Energy Model**

<table>
<thead>
<tr>
<th>Kinetic energy</th>
<th>is an energy of motion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K = \frac{1}{2}mv^2$</td>
<td></td>
</tr>
<tr>
<td>Potential energy</td>
<td>is an energy of position</td>
</tr>
<tr>
<td>$U_g = mgy$</td>
<td></td>
</tr>
<tr>
<td>$U_e = \frac{1}{2}k(\Delta s)^2$</td>
<td></td>
</tr>
</tbody>
</table>

**Energy diagrams**

These diagrams show the potential-energy curve PE and the total mechanical energy line TE.

- The distance from the axis to the curve is PE.
- The distance from the curve to the TE line is KE.
- A point where the TE line crosses the PE curve is a turning point.
- Minima in the PE curve are points of stable equilibrium.
- Maxima are points of unstable equilibrium.

Chapter 10

Hooke’s law

The restoring force of an ideal spring is

$$F_{spring} = -k\Delta s$$

where $k$ is the spring constant and $\Delta s = s - s_i$ is the displacement from equilibrium.

**Basic Energy Model**

- Energy is transferred to or from the system by work.
- Energy is transformed within the system.

Two versions of the energy equation are

$$\Delta E_{sys} = \Delta K + \Delta U + \Delta E_{int} = W_{ext}$$

$$K_f + U_f + \Delta E_{int} = K_i + U_i + W_{ext}$$

Physics 207: Lecture 30, Pg 29
Chapter 10

**Law of Conservation of Energy**

- **Isolated system**: $W_{ext} = 0$. The total energy $E_{sys} = E_{mech} + E_{th}$ is conserved. $\Delta E_{sys} = 0$.
- **Isolated, nondissipative system**: $W_{ext} = 0$ and $W_{diss} = 0$. The mechanical energy $E_{mech}$ is conserved.

$$\Delta E_{mech} = 0 \text{ or } K_f + U_f = K_i + U_i$$

The work-kinetic energy theorem is

$$\Delta K = W_{net} = W_c + W_{diss} + W_{ext}$$

With $W_c = -\Delta U$ for conservative forces and $W_{diss} = -\Delta E_{th}$ for dissipative forces, this becomes the energy equation.

Chapter 11

The work done by a force on a particle as it moves from $s_i$ to $s_f$ is

$$W = \int_{s_i}^{s_f} F_i \, ds = \text{area under the force curve}$$

$$= \vec{F} \cdot \Delta \vec{s} \text{ if } \vec{F} \text{ is a constant force}$$

Conservative forces are forces for which the work is independent of the path followed. The work done by a conservative force can be represented as a potential energy:

$$\Delta U = U_f - U_i = -W_c(i \rightarrow f)$$

A conservative force is found from the potential energy by

$$F_s = -dU/ds = \text{negative of the slope of the PE curve}$$

Dissipative forces transform macroscopic energy into thermal energy, which is the microscopic energy of the atoms and molecules. For friction:

$$\Delta E_{th} = f_s \Delta s$$
Chapter 11

Power is the rate at which energy is transferred or transformed:

\[ P = \frac{dE_{\text{sys}}}{dt} \]

For a particle moving with velocity \( \vec{v} \), the power delivered to the particle by force \( \vec{F} \) is \( P = \vec{F} \cdot \vec{v} = Fv\cos\theta \).

Dot product

\[ \vec{A} \cdot \vec{B} = AB \cos \alpha = A_xB_x + A_yB_y \]

Chapter 12

The moment of inertia and Center of Mass

\[ I = \sum_i m_i r_i^2 = \int r^2 \, dm \]

is the rotational equivalent of mass. The moment of inertia depends on how the mass is distributed around the axis. If \( I_{\text{cm}} \) is known, the \( I \) about a parallel axis distance \( d \) away is given by the parallel-axis theorem: \( I = I_{\text{cm}} + Md^2 \).

Rotational Dynamics

Every point on a rigid body rotating about a fixed axis has the same angular velocity \( \omega \) and angular acceleration \( \alpha \).

Newton’s second law for rotational motion is

\[ \alpha = \frac{\tau_{\text{net}}}{I} \]

Use rotational kinematics to find angles and angular velocities.
Conservation Laws

Energy is conserved for an isolated system.

- Pure rotation \( E = K_{\text{rot}} + U_g = \frac{1}{2} I \omega^2 + Mg y_{\text{cm}} \)
- Rolling \( E = K_{\text{rot}} + K_{\text{cm}} + U_g = \frac{1}{2} I \omega^2 + \frac{1}{2} Mv_{\text{cm}}^2 + Mg y_{\text{cm}} \)

Angular momentum is conserved if \( \mathbf{\tau}_{\text{net}} = 0 \).

- Particle \( L = \mathbf{r} \times \mathbf{p} \)
- Rigid body rotating about axis of symmetry \( \mathbf{L} = I \omega \)

Important Concepts

Torque is the rotational equivalent of force:

\[ \mathbf{\tau} = rF \sin \phi = rF \_i - dF \]

The vector description of torque is

\[ \mathbf{\tau} = \mathbf{r} \times \mathbf{F} \]
Rotational kinematics

\[ \omega_f = \omega_i + \alpha \Delta t \]
\[ \theta_f = \theta_i + \omega_i \Delta t + \frac{1}{2} \alpha (\Delta t)^2 \]
\[ v_f = r \omega \quad a_f = r \alpha \]

Rigid-body equilibrium

An object is in total equilibrium only if both \( \vec{F}_{\text{net}} = 0 \) and \( \vec{\tau}_{\text{net}} = 0 \).

Angular Momentum

**General Principles**

**Rotational Dynamics**

Every point on a rigid body rotating about a fixed axis has the same angular velocity \( \omega \) and angular acceleration \( \alpha \).

Newton's second law for rotational motion is

\[ \alpha = \frac{\vec{\tau}_{\text{net}}}{I} \]

Use rotational kinematics to find angles and angular velocities.

**Conservation Laws**

Energy is conserved for an isolated system.
- Pure rotation \( E = K_r + U_r = \frac{1}{2} I \omega^2 + Mgh \)
- Rolling \( E = K_r + K_m + U_r = \frac{1}{2} I \omega^2 + \frac{1}{2} M r_o^2 + Mgh \)

Angular momentum is conserved if \( \vec{F}_{\text{net}} = 0 \).
- Particle \( \vec{L} = \vec{r} \times \vec{p} \)
- Rigid body rotating about an axis of symmetry \( \vec{L} = I \vec{\omega} \)

**Important Concepts**

Torque is the rotational equivalent of force:

\[ \vec{\tau} = \vec{r} \times \vec{F} \]

The vector description of torque is \( \vec{\tau} = I \times \vec{\omega} \)

Angular velocity \( \vec{\omega} \) points along the rotation axis in the direction of the right-hand rule.

For a rigid body rotating about an axis of symmetry, the angular momentum is \( \vec{L} = I \vec{\omega} \).

Newton's second law is \( \frac{dI}{dt} = \vec{\tau}_{\text{net}} \)

The moment of inertia

\[ I = \sum m_i r_i^2 = \int r^2 \, dm \]

is the rotational equivalent of mass. The moment of inertia depends on how the mass is distributed around the axis. If \( I_m \) is known, the \( I \) about a parallel axis distance \( d \) away is given by the parallel-axis theorem: \( I = I_m + Md^2 \).

Hooke’s Law Springs and a Restoring Force

General Principles

Dynamics

SHM occurs when a linear restoring force acts to return a system to an equilibrium position.

Horizontal spring

\( F_{\text{hor}} = -kx \)

Vertical spring

The origin is at the equilibrium position \( \Delta x = mg/k \).

\( F_{\text{ver}} = -ky \)

\[ \omega = \sqrt{\frac{k}{m}} \quad T = 2\pi \sqrt{\frac{m}{k}} \]

Pendulum

\( F_{\text{悬}} = -\frac{mg}{L} \)

\[ \omega = \sqrt{\frac{g}{L}} \quad T = 2\pi \sqrt{\frac{L}{g}} \]

Key fact: \( \omega = (k/m)^{1/2} \) is general result where \( k \) reflects a constant of the linear restoring force and \( m \) is the inertial response (e.g., the “physical pendulum” where \( \omega = (\kappa/l)^{1/2} \))

Energy

If there is no friction or dissipation, kinetic and potential energy are alternately transformed into each other, but the total mechanical energy \( E = K + U \) is conserved.

\[ E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 \]

\[ = \frac{1}{2}mv_{\text{max}}^2 + \frac{1}{2}kA^2 \]

In a damped system, the energy decays exponentially

\[ E = E_0 e^{-\alpha t} \]

where \( \alpha \) is the time constant.

Simple Harmonic Motion

Important Concepts

Simple harmonic motion (SHM) is a sinusoidal oscillation with period \( T \) and amplitude \( A \).

Frequency \( f = \frac{1}{T} \)

Angular frequency

\[ \omega = 2\pi f = \frac{2\pi}{T} \]

Position \( x(t) = A \cos(\omega t + \phi_0) \)

\[ = A \cos\left(\frac{2\pi t}{T} + \phi_0\right) \]

Velocity \( v_x(t) = -v_{\text{max}} \sin(\omega t + \phi_0) \) with maximum speed \( v_{\text{max}} = \omega A \)

Acceleration \( a_x = -\omega^2 x \)

Maximum potential energy

Maximum kinetic energy
Resonance and damping

• Energy transfer is optimal when the driving force varies at the resonant frequency.

Applications

Resonance
When a system is driven by a periodic external force, it responds with a large-amplitude oscillation if $f_{	ext{ext}} = f_n$, where $f_n$ is the system’s natural oscillation frequency, or resonant frequency.

Types of motion

- Undamped
- Underdamped
- Critically damped
- Overdamped

Fluid Flow

General Principles

Fluid Statics

<table>
<thead>
<tr>
<th>Gases</th>
<th>Liquids</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freely moving particles</td>
<td>Loosely bound particles</td>
</tr>
<tr>
<td>Compressible</td>
<td>Incompressible</td>
</tr>
<tr>
<td>Pressure primarily thermal</td>
<td>Pressure primarily gravitational</td>
</tr>
<tr>
<td>Pressure is constant in a laboratory-size container</td>
<td>Hydrostatic pressure at depth $d$ is $p =</td>
</tr><tr>
<td>ho g d$</td>
<td></td>
</tr>
</tbody>
</table>

Fluid Dynamics

Ideal-fluid model

- Incompressible
- Smooth, laminar flow
- Nonviscous

Equation of continuity

$\nabla \cdot \mathbf{v} = 0$

Bernoulli’s equation

$p_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2$

Bernoulli’s equation is a statement of energy conservation.
Density and pressure

Important Concepts

Density \( \rho = \frac{m}{V} \), where \( m \) is mass and \( V \) is volume.

Pressure \( p = \frac{F}{A} \), where \( F \) is the magnitude of the fluid force and \( A \) is the area on which the force acts.

- Pressure exists at all points in a fluid.
- Pressure pushes equally in all directions.
- Pressure is constant along a horizontal line.
- Gauge pressure is \( p_g = p - 1 \text{ atm} \).

Response to forces

Buoyancy is the upward force of a fluid on an object.

Archimedes' principle

The magnitude of the buoyant force equals the weight of the fluid displaced by the object.

Sink: \( p_{\text{bq}} > \rho_f \) \( \Rightarrow \) \( F_b < m_g \)

Rise to surface: \( p_{\text{bq}} < \rho_f \) \( \Rightarrow \) \( F_b > m_g \)

Neutrally buoyant: \( p_{\text{bq}} = \rho_f \) \( \Rightarrow \) \( F_b = m_g \)

Elasticity describes the deformation of solids and liquids under stress.

Linear stretch and compression

\( \varepsilon = \frac{\Delta L}{L_0} \) \( \Rightarrow \) Young’s modulus

Volume compression

\( p = -\beta(\Delta V/V) \)

Bulk modulus

States of Matter and Phase Diagrams

General Principles

Three Phases of Matter

Solid

- Rigid, definite shape.
- Nearly incompressible.

Liquid

- Molecules loosely held together by molecular bonds, but able to move around.
- Nearly incompressible.

Gas

- Molecules move freely through space.
- Compressible.

The different phases exist for different conditions of temperature \( T \) and pressure \( p \). The boundaries separating the regions of a phase diagram are lines of phase equilibrium. Any amounts of the two phases can coexist in equilibrium. The triple point is the one value of temperature and pressure at which all three phases can coexist in equilibrium.
Ideal gas equation of state

**Important Concepts**

### Ideal-Gas Model
- Atoms and molecules are small, hard spheres that travel freely through space except for occasional collisions with each other or the walls.
- The model is valid when the density is low and the temperature well above the condensation point.

### Ideal-Gas Law

The state variables of an ideal gas are related by the ideal-gas law

\[ \frac{pV}{RT} = \frac{n}{N_a} T \]

where \( R = 8.31 \text{ J/mol K} \) is the universal gas constant and \( k_B = 1.38 \times 10^{-23} \text{ J/K} \) is Boltzmann’s constant.

\( p, V, T \) must be in SI units of Pa, m³, and K. For a gas in a sealed container, with constant \( n \):

\[ \frac{pV_1}{T_1} = \frac{pV_2}{T_2} \]

Coznting atoms and moles

A macroscopic sample of matter consists of \( N \) atoms (or molecules), each of mass \( m \) (the atomic or molecular mass):

\[ N = \frac{M}{m} \]

Alternatively, we can state that the sample consists of \( n \) moles:

\[ n = \frac{N}{N_a} \text{ or } \frac{M \text{ (in grams)}}{M_{\text{mole}}} \]

\( N_a = 6.02 \times 10^{23} \text{ mol}^{-1} \) is Avogadro’s number.

The numerical value of the molar mass \( M_{\text{mole}} \) in g/mol equals the numerical value of the atomic or molecular mass \( m \) in u. The atomic or molecular mass \( m \), in atomic mass units u, is well approximated by the atomic mass number \( A \):

\[ 1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} \]

The number density of the sample is \( \frac{N}{V} \).

### Applications

**Temperature scales**

\[ T_\circ = \frac{9}{5} T_K + 32 \]

\[ T_K = T_\circ + 273 \]

The Kelvin temperature scale is based on:
- Absolute zero at \( T_0 = 0 \text{ K} \)
- The triple point of water at \( T_\circ = 273.16 \text{ K} \)

### Thermodynamics

**First Law of Thermodynamics**

\[ \Delta E_s = W + Q \]

The first law is a general statement of energy conservation. Work \( W \) and heat \( Q \) depend on the process by which the system is changed. The change in the system depends only on the total energy exchanged \( W + Q \); not on the process.

**Energy**

Thermal energy \( E_s \). Microscopic energy of moving molecules and stretched molecular bonds. \( \Delta E_s \) depends on the initial/final states, but is independent of the process.

Work \( W \): Energy transferred to the system by forces in a mechanical interaction.

Heat \( Q \): Energy transferred to the system via atomic-level collisions when there is a temperature difference. A thermal interaction.
Important Concepts

Work, Pressure, Volume, Heat

The work done on a gas is

\[ W = \int_{V_1}^{V_2} p \, dV \]

or (area under the pV curve).

An adiabatic process is one for which \( Q = 0 \). Gases move along an adiabat for which \( pV^{\gamma} \) is constant, where \( \gamma = \frac{C_p}{C_v} \) is the specific heat ratio. An adiabatic process changes the temperature of the gas without heating it. \( T \) can change!

Calorimetry: When two or more systems interact thermally, they come to a common final temperature determined by

\[ Q_{tot} = Q_1 + Q_2 + \cdots = 0 \]

Summary of Basic Gas Processes

<table>
<thead>
<tr>
<th>Process</th>
<th>Definition</th>
<th>Stays constant</th>
<th>Work</th>
<th>Heat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isochoric</td>
<td>( \Delta V = 0 )</td>
<td></td>
<td>( W = 0 )</td>
<td>( Q = nC_v \Delta T )</td>
</tr>
<tr>
<td>Isobaric</td>
<td>( \Delta p = 0 )</td>
<td>( p )</td>
<td>( W = -p \Delta V )</td>
<td>( Q = nC_v \Delta T )</td>
</tr>
<tr>
<td>Isothermal</td>
<td>( \Delta T = 0 )</td>
<td>( T )</td>
<td>( W = -nRT \ln(V_f/V_i) )</td>
<td>( \Delta E_a = 0 )</td>
</tr>
<tr>
<td>Adiabatic</td>
<td>( Q = 0 )</td>
<td>( pV^n )</td>
<td>( W = \Delta E_a )</td>
<td>( Q = 0 )</td>
</tr>
</tbody>
</table>

All gas processes: First law \( \Delta E_a = W + Q = nC_v \Delta T \)

Ideal-gas law: \( pV = nRT \)

Chapter 18

Kinetic theory, the **micro/macro connection**, relates the macroscopic properties of a system to the motion and collisions of its atoms and molecules.

The Equipartition Theorem

 Tells us how collisions distribute the energy in the system. The energy stored in each mode of the system (each degree of freedom) is \( \frac{1}{2} M k_B T \), or, in terms of moles, \( \frac{1}{2} n k_B T \).

The Second Law of Thermodynamics

 Tells us how collisions move a system toward equilibrium. The entropy of an isolated system can only increase or, in equilibrium, stay the same.

- Order turns into disorder and randomness.
- Systems run down.
- Heat energy is transferred spontaneously from a hotter to a colder system: never from colder to hotter.

Pressure is due to the force of the molecules colliding with the walls:

\[ P = \frac{1}{3} \frac{N}{V} m v_{rms}^2 = \frac{2}{3} \frac{N}{V} \epsilon_{avg} \]

The average translational kinetic energy of a molecule is

\[ \epsilon_{avg} = \frac{3}{2} k_B T \]

The temperature of the gas \( T = \frac{3}{2} k_B \epsilon_{avg} \) measures the average translational kinetic energy.
Thermal Energy

Entropy measures the probability that a macroscopic state will occur or, equivalently, the amount of disorder in a system.

The thermal energy of a system is

\[ E_{\text{th}} = \text{translational kinetic energy} + \text{rotational kinetic energy} + \text{vibrational energy} \]

- **Monatomic gas**
  \[ E_{\text{th}} = \frac{3}{2} N k_B T = \frac{3}{2} nRT \]

- **Diatomic gas**
  \[ E_{\text{th}} = \frac{5}{2} N k_B T = \frac{5}{2} nRT \]

- **Elemental solid**
  \[ E_{\text{th}} = 3 N k_B T = 3 nRT \]

Heat is energy transferred via collisions from more-energetic molecules on one side to less-energetic molecules on the other. Equilibrium is reached when \((\epsilon_1)_{\text{avg}} = (\epsilon_2)_{\text{avg}}\), which implies \(T_{1e} = T_{2e}\).

Relationships

The root-mean-square speed \(v_{\text{rms}}\) is the square root of the average of the squares of the molecular speeds:

\[ v_{\text{rms}} = \sqrt{\langle v^2 \rangle_{\text{avg}}} \]

For molecules of mass \(m\) at temperature \(T\),

\[ v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}} \]

Molar specific heats can be predicted from the thermal energy because \(\Delta E_{\text{th}} = nC\Delta T\).

- **Monatomic gas**
  \[ C_V = \frac{3}{2} R \]

- **Diatomic gas**
  \[ C_V = \frac{5}{2} R \]

- **Elemental solid**
  \[ C = 3R \]
Chapter 19

Heat Engines

Devices that transform heat into work. They require two energy reservoirs at different temperatures.

\[ \text{Thermal efficiency } \eta = \frac{W_{\text{out}}}{Q_H} = \frac{\text{what you get}}{\text{what you pay}} \]

Second-law limit:

\[ \eta \leq 1 - \frac{T_C}{T_H} \]

Refrigerators

Devices that use work to transfer heat from a colder object to a hotter object.

Coefficient of performance

\[ K = \frac{Q_C}{W_{\text{in}}} = \frac{\text{what you get}}{\text{what you pay}} \]

Second-law limit:

\[ K \leq \frac{T_C}{T_H - T_C} \]

An energy reservoir is a part of the environment so large in comparison to the system that its temperature doesn’t change as the system extracts heat energy from or exhausts heat energy to the reservoir. All heat engines and refrigerators operate between two energy reservoirs at different temperatures \( T_H \) and \( T_C \).
Carnot Cycles

A perfectly reversible engine (a Carnot engine) can be operated as either a heat engine or a refrigerator between the same two energy reservoirs by reversing the cycle and with no other changes.

- **A Carnot heat engine** has the maximum possible thermal efficiency of any heat engine operating between \( T_H \) and \( T_C \):
  \[
  \eta_{\text{Carnot}} = 1 - \frac{T_C}{T_H}
  \]

- **A Carnot refrigerator** has the maximum possible coefficient of performance of any refrigerator operating between \( T_H \) and \( T_C \):
  \[
  K_{\text{Carnot}} = \frac{T_C}{T_H - T_C}
  \]

The Carnot cycle for a gas engine consists of two isothermal processes and two adiabatic processes.

---

Work (by the system)

The work \( W_s \) done by the system has the opposite sign to the work done on the system.

\[
W_s = \text{area under } pV \text{ curve}
\]

---

To analyze a heat engine or refrigerator:

**MODEL** Identify each process in the cycle.

**VISUALIZE** Draw the \( pV \) diagram of the cycle.

**SOLVE** There are several steps:
- Determine \( p, V, \) and \( T \) at the beginning and end of each process.
- Calculate \( \Delta E_p \), \( W_s \), and \( Q \) for each process.
- Determine \( W_s \), \( W_{net} \), \( Q_{in} \), and \( Q_{out} \).
- Calculate \( \eta = \frac{W_{net}}{Q_{in}} \) or \( K = Q_{out}/W_{net} \).

**ASSESS** Verify \( (\Delta E_p)_{\text{net}} = 0 \) Check signs.
Chapter 20

The Wave Model

This model is based on the idea of a traveling wave, which is an organized disturbance traveling at a well-defined wave speed $v$.

- In transverse waves the displacement is perpendicular to the direction in which the wave travels.
- In longitudinal waves the particles of the medium move parallel to the direction in which the wave travels.

A wave transfers energy, but no material or substance is transferred outward from the source.

- **String** (transverse): $v = \sqrt{T/\mu}$
- **Sound** (longitudinal): $v = 343 \text{ m/s in } 20^\circ \text{C air}$
- **Light** (transverse): $v = c/n$, where $c = 3.00 \times 10^8 \text{ m/s}$ is the speed of light in a vacuum and $n$ is the material’s index of refraction.

Displacement versus time and position

The displacement $D$ of a wave is a function of both position (where) and time (when).

- A snapshot graph shows the wave’s displacement as a function of position at a single instant of time.
- A history graph shows the wave’s displacement as a function of time at a single point in space.

For a transverse wave on a string, the snapshot graph is a picture of the wave. The displacement of a longitudinal wave is parallel to the motion; thus the snapshot graph of a longitudinal sound wave is not a picture of the wave.
Sinusoidal Waves (Sound and Electromagnetic)

Sinusoidal waves are periodic in both time (period $T$) and space (wavelength $\lambda$):

$$D(x, t) = A \sin[\frac{2\pi (x/\lambda - t/T)}{t} + \phi_0]$$

$$= A \sin(kx - \omega t + \phi_0)$$

where $A$ is the amplitude, $k = 2\pi/\lambda$ is the wave number, $\omega = 2\pi f = 2\pi/T$ is the angular frequency, and $\phi_0$ is the phase constant that describes initial conditions.

The fundamental relationship for any sinusoidal wave is $v = \lambda f$.

The wave intensity is the power-to-area ratio: $I = P/4\pi a$

For a circular or spherical wave: $I = P_{\text{source}}/4\pi r^2$.

The sound intensity level is

$$\beta = (10 \text{ dB}) \log_{10}(I/1.0 \times 10^{-12} \text{ W/m}^2)$$

Doppler effect

The Doppler effect occurs when a wave source and detector are moving with respect to each other: the frequency detected differs from the frequency $f_0$ emitted.

<table>
<thead>
<tr>
<th>Approaching source</th>
<th>Observer approaching a source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_+ = \frac{f_0}{1 - v_s/v}$</td>
<td>$f_+ = (1 + v_s/v)f_0$</td>
</tr>
<tr>
<td>Receding source</td>
<td>Observer receding from a source</td>
</tr>
<tr>
<td>$f_- = \frac{f_0}{1 + v_s/v}$</td>
<td>$f_- = (1 - v_s/v)f_0$</td>
</tr>
</tbody>
</table>

The Doppler effect for light uses a result derived from the theory of relativity.
Chapter 21

Standing Waves

Boundary conditions

Strings, electromagnetic waves, and sound waves in closed-closed tubes must have nodes at both ends:

$$\lambda_m = \frac{2L}{m} \quad f_m = m \frac{v}{2L} = mf_1$$

where $m = 1, 2, 3, \ldots$

The frequencies and wavelengths are the same for a sound wave in an open-open tube, which has antinodes at both ends.

A sound wave in an open-closed tube must have a node at the closed end but an antinode at the open end. This leads to

$$\lambda_m = \frac{4L}{m} \quad f_m = m \frac{v}{4L} = mf_1$$

where $m = 1, 3, 5, 7, \ldots$. 

Standing waves are due to the superposition of two traveling waves moving in opposite directions.

The amplitude at position $x$ is

$$A(x) = 2a \sin kx$$

where $a$ is the amplitude of each wave.

The boundary conditions determine which standing-wave frequencies and wavelengths are allowed. The allowed standing waves are modes of the system.

Standing waves on a string.

Interference

In general, the superposition of two or more waves into a single wave is called interference.

Maximum constructive interference occurs where crests are aligned with crests and troughs with troughs. These waves are in phase. The maximum displacement is $A = 2a$.

Perfect destructive interference occurs where crests are aligned with troughs. These waves are out of phase. The amplitude is $A = 0$.

Interference depends on the phase difference $\Delta \phi$ between the two waves.

Constructive: $\Delta \phi = 2\pi \frac{\Delta l}{\lambda} + \Delta \phi_0 = m \cdot 2\pi$

Destructive: $\Delta \phi = 2\pi \frac{\Delta l}{\lambda} + \Delta \phi_0 = \left( m + \frac{1}{2} \right) \cdot 2\pi$

$\Delta l$ is the path-length difference of the two waves, and $\Delta \phi_0$ is any phase difference between the sources. For identical sources (in phase, $\Delta \phi = 0$):

Interference is constructive if the path-length difference $\Delta l = n\lambda$.

Interference is destructive if the path-length difference $\Delta l = \left( m + \frac{1}{2} \right) \lambda$.

The amplitude at a point where the phase difference is $\Delta \phi = 2\pi \cos \left( \frac{\Delta \phi}{2} \right)$
Beats

\[ D(0, t) = 2A \cos(2\pi \frac{f_1 - f_2}{2} t) \cos(2\pi \frac{f_1 + f_2}{2} t) \]

- \( |f_1 - f_2| \equiv f_{\text{beat}} = 1/T_{\text{beat}} \)
- \( \frac{f_1 + f_2}{2} \equiv f_{\text{avg}} = 1/T_{\text{avg}} \)

Beats (loud-soft-loud-soft modulations of intensity) occur when two waves of slightly different frequency are superimposed.

The beat frequency between waves of frequencies \( f_1 \) and \( f_2 \) is
\[ f_{\text{beat}} = f_1 - f_2 \]

Lecture 30

- Assignment
  - HW13, Due Friday, May 7th

I hope everyone does well on their finals!
Have a great summer!
**Example Interference**

- A speaker sits on a pedestal 2.0 m tall and emits a sine wave at 340 Hz (the speed of sound in air is 340 m/s, so $\lambda = 1.0$ m). Only the direct sound wave and that which reflects off the ground at a position half-way between the speaker and the person (also 2 m tall) makes it to the person's ear.

- How close to the speaker can the person stand (A to D) so they hear a maximum sound intensity assuming there is no "phase change" at the ground (this is a bad assumption)?

The distances AD and BCD have equal transit times so the sound waves will be in phase. The only need is for $AB = \lambda$

\[
\begin{align*}
AB &= \lambda \\
AD &= BC + CD = BC + (h^2 + (d/2)^2)^{1/2} = d \\
AC &= AB + BC = \lambda + BC = (h^2 + d^2)^{1/2} \\
\end{align*}
\]

Eliminating BC gives

\[
\begin{align*}
\lambda + d &= 2 (h^2 + d^2/4)^{1/2} \\
\lambda + 2\lambda d + d^2 &= 4 h^2 + d^2 \\
1 + 2d &= 4 h^2/\lambda \\
\Rightarrow d &= 2 h^2/\lambda - 1/2 \\
= 7.5 \text{ m} \\
\end{align*}
\]

Because the ground is more dense than air there will be a phase change of $\pi$ and so we really should set $AB$ to $\lambda/2$ or 0.5 m.
Sample Problem

- The figure shows a snapshot graph $D(x, t = 2 \text{ s})$ taken at $t = 2 \text{ s}$ of a pulse traveling to the left along a string at a speed of 2.0 m/s. Draw the history graph $D(x = -2 \text{ m}, t)$ of the wave at the position $x = -2 \text{ m}$.

- History Graph:
Example problem

- Two loudspeakers are placed 1.8 m apart. They play tones of equal frequency. If you stand 3.0 m in front of the speakers, and exactly between them, you hear a maximum of intensity.
- As you walk parallel to the plane of the speakers, staying 3.0 m away, the sound intensity decreases until reaching a minimum when you are directly in front of one of the speakers. The speed of sound in the room is 340 m/s.

a. What is the frequency of the sound?

b. Draw, as accurately as you can, a wave-front diagram. On your diagram, label the positions of the two speakers, the point at which the intensity is maximum, and the point at which the intensity is minimum.

c. Use your wave-front diagram to explain why the intensity is a minimum at a point 3.0 m directly in front of one of the speakers.
Example problem

- Two loudspeakers are placed 1.8 m apart. They play tones of equal frequency. If you stand 3.0 m in front of the speakers, and exactly between them, you hear a maximum of intensity.

- $v = 340 \text{ m/s}$.

PUT IN GEOMETRY

$v = f \lambda$ but we don't know $f$ or $\lambda$

AC - BC = 0 (0 phase difference)

AD - BD = $\frac{\lambda}{2}$ (\(\pi\) phase shift)

AD = $(3.0^2 + 1.8^2)^{1/2}$

BD = 3.0

$\lambda = 2(AD-BD) = 1.0 \text{ m}$

Example problem

- Two loudspeakers are placed 1.8 m apart. They play tones of equal frequency. If you stand 3.0 m in front of the speakers, and exactly between them, you hear a maximum of intensity.

b. Draw, as accurately as you can, a wave-front diagram. On your diagram, label the positions of the two speakers, the point at which the intensity is maximum, and the point at which the intensity is minimum.

c. Use your wave-front diagram to explain why the intensity is a minimum at a point 3.0 m directly in front of one of the speakers.
Sample problem

• A tube, open at both ends, is filled with an unknown gas. The tube is 190 cm in length and 3.0 cm in diameter. By using different tuning forks, it is found that resonances can be excited at frequencies of 315 Hz, 420 Hz, and 525 Hz, and at no frequencies in between these.

a. What is the speed of sound in this gas?

b. Can you determine the amplitude of the wave? If so, what is it? If not, why not?

\[ \text{L} = 1.9 \text{ meters and } f_m = \frac{v m}{2L} \]

\[ 315 = \frac{v m}{2L} \]

\[ 420 = \frac{v (m+1)}{2L} \]

\[ 525 = \frac{v (m+2)}{2L} \]

\[ \frac{v}{2L} = 105 \]

\[ v = 3.8 \times 105 \text{ m/s} = 400 \text{ m/s} \]

b) Can you determine the amplitude of the wave? If so, what is it? If not, why not?

Answer: No, the sound intensity is required and this is not known.
Sample Problem

- The picture below shows two pulses approaching each other on a stretched string at time $t = 0$ s. Both pulses have a speed of 1.0 m/s. Using the empty graph axes below the picture, draw a picture of the string at $t = 4$ s.

An example

- A heat engine uses 0.030 moles of helium as its working substance. The gas follows the thermodynamic cycle shown.
  a. Fill in the missing table entries
  b. What is the thermal efficiency of this engine?
  c. What is the maximum possible thermal efficiency of an engine that operates between $T_{\text{max}}$ and $T_{\text{min}}$?

<table>
<thead>
<tr>
<th></th>
<th>$Q_H$</th>
<th>$Q_L$</th>
<th>$W_{\text{by}}$</th>
<th>$\Delta E_{\text{Th}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A→B</td>
<td>600 J</td>
<td>0 J</td>
<td>0 J</td>
<td>600 J</td>
</tr>
<tr>
<td>B→C</td>
<td>0 J</td>
<td>0 J</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C→A</td>
<td>0 J</td>
<td></td>
<td>-161 J</td>
<td>-243 J</td>
</tr>
<tr>
<td>NET</td>
<td>600 J</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
An example

- A heat engine uses 0.030 moles of helium as its working substance. The gas follows the thermodynamic cycle shown.

a. Fill in the missing table entries
b. What is the thermal efficiency of this engine?
c. What is the maximum possible thermal efficiency of an engine that operates between $T_{\text{max}}$ and $T_{\text{min}}$?

\[ A: T = 400 \text{ K} \]
\[ B: T = 2000 \text{ K} \]
\[ C: T = 1050 \text{ K} \]

<table>
<thead>
<tr>
<th></th>
<th>$Q_H$</th>
<th>$Q_L$</th>
<th>$W_{by}$</th>
<th>$\Delta E_{Th}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A→B</td>
<td>600 J</td>
<td>0 J</td>
<td>0 J</td>
<td>600 J</td>
</tr>
<tr>
<td>B→C</td>
<td>0 J</td>
<td>0 J</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C→A</td>
<td>0 J</td>
<td>-161 J</td>
<td>-243 J</td>
<td></td>
</tr>
<tr>
<td>NET</td>
<td>600 J</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

The Full Cyclic Process

- A heat engine uses 0.030 moles of helium as its working substance. The gas follows the thermodynamic cycle shown.

a. What is the thermal efficiency of this engine?
b. What is the maximum possible thermal efficiency of an engine that operates between $T_{\text{max}}$ and $T_{\text{min}}$?

\[ (T = pV/nR, pV^\gamma = \text{const.}) \]

\[ A: T = 400 \text{ K} = 1.01 \times 10^5 \times 10^{-3}/0.030/8.3 \]
\[ B: T = 2000 \text{ K} \]
\[ p_B V_B^\gamma = p_c V_C^\gamma \]
\[ (p_B V_B^\gamma / p_c)^{1/\gamma} = V_C \]
\[ (5 \times 10^{-3})^{5/3} / 1^{3/5} = V_C = 2.6 \times 10^{-3} \text{ m}^3 \]
\[ C: T = 1050 \text{ K} \]
The Full Cyclic Process

\[ A: \quad T = 400 \text{ K} \quad 1.01 \times 10^5 \quad 10^{-3} / 0.030 / 8.3 \]

\[ B: \quad T = 2000 \text{ K} \quad Q = n C_v \Delta T = 0.030 \times 1.5 \times 8.3 \Delta T \]

\[ V_C = 2.6 \times 10^{-3} \text{ m}^3 \quad Q = 0.3735 \Delta T \]

\[ C: \quad T = 1050 \text{ K} \quad W_{C \rightarrow A \text{ (by)}} = p \Delta V = 1.01 \times 10^5 \times 1.6 \times 10^{-3} \text{ J} \]

<table>
<thead>
<tr>
<th></th>
<th>( Q_H )</th>
<th>( Q_L )</th>
<th>( W_{by} )</th>
<th>( \Delta E_{Th} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A→B</td>
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<td>0 J</td>
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</tr>
<tr>
<td>B→C</td>
<td>0 J</td>
<td>0 J</td>
<td>355 J</td>
<td>-355 J</td>
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<tr>
<td>C→A</td>
<td>0 J</td>
<td>404 J</td>
<td>-161 J</td>
<td>-243 J</td>
</tr>
<tr>
<td>NET</td>
<td>600 J</td>
<td>404 J</td>
<td>194 J</td>
<td>0 J</td>
</tr>
</tbody>
</table>

\[ \eta = \frac{W_{by}}{Q_H} = \frac{194}{600} = 0.32 \]

\[ \eta_{\text{Carnot}} = 1 - \frac{T_L}{T_H} = 1 - \frac{400}{2000} = 0.80 \]

An example

- A monatomic gas is compressed isothermally to 1/8 of its original volume.
- Do each of the following quantities change? If so, does the quantity increase or decrease, and by what factor? If not, why not?
  - a. The rms speed \( v_{rms} \)
  - b. The temperature
  - c. The mean free path
  - d. The molar heat capacity \( C_v \)
An example

- A creative chemist creates a small molecule which resembles a freely moving bead on a wire (rotaxanes are an example). The wire is fixed and the bead does not rotate.
- If the mass of the bead is $10^{-26}$ kg, what is the rms speed of the bead at 300 K?

Below is a rotaxane model with three “beads” on a short wire

An example

- A creative chemist creates a small molecule which resembles a freely moving bead on a wire (rotaxanes are an example). Here the wire is a loop and rigidly fixed and the bead does not rotate.
- If the mass of the bead is $10^{-26}$ kg, what is the rms speed of the bead at 300 K?

Classically there is $\frac{1}{2} k_B T$ of thermal energy per degree of freedom.

Here there is only one so:

$$\frac{1}{2} m v_{\text{rms}}^2 = \frac{1}{2} k_{\text{Boltzmann}} T$$

$$v_{\text{rms}} = (k_{\text{Boltzmann}} T/m)^{1/2} = 640 \text{ m/s}$$
An example

- A small speaker is placed in front of a block of mass 4 kg. The mass is attached to a Hooke’s Law spring with spring constant 100 N/m. The mass and speaker have a mechanical energy of 200 J and are undergoing one dimensional simple harmonic motion. The speaker emits a 200 Hz tone.

- For a person is standing directly in front of the speaker, what range of frequencies does he/she hear?

- The closest the speaker gets to the person is 1.0 m. By how much does the sound intensity vary in terms of the ratio of the loudest to the softest sounds?

\[
E_{\text{mech}} = 200 \text{ J} = \frac{1}{2} m v_{\text{max}}^2 = \frac{1}{2} k A^2 \Rightarrow v_{\text{max}} = 10 \text{ m/s}
\]

Now use expression for Doppler shift where \(v_{\text{source}} = \pm 10 \text{ m/s}\) and \(-10 \text{ m/s}\)

\[
f_{\text{observer}} = \frac{f_{\text{source}}}{1 - \frac{v_s}{v}}
\]
An example

A small speaker is placed in front of a block of mass 4 kg. The mass is attached to a Hooke’s Law spring with spring constant 100 N/m. The mass and speaker have a mechanical energy of 200 J and are undergoing one dimensional simple harmonic motion. The speaker emits a 200 Hz tone. ($v_{\text{sound}} = 340$ m/s)

For a person is standing directly in front of the speaker, what range of frequencies does he/she hear?

$$E_{\text{mech}} = 200 \text{ J} = \frac{1}{2} m v_{\text{max}}^2 = \frac{1}{2} k A^2 \rightarrow v_{\text{max}} = 10 \text{ m/s}$$

Now use expression for Doppler shift where

$$v_{\text{source}} = +10 \text{ m/s} \quad \text{and} \quad -10 \text{ m/s}$$

$$f_{\text{observer}} = \frac{f_{\text{source}}}{1 - \frac{v_{\text{source}}}{v}}$$

The closest the speaker gets to the person is 1.0 m. By how much does the sound intensity vary in terms of the ratio of the loudest to the softest sounds?

$$E_{\text{mech}} = 200 \text{ J} = \frac{1}{2} m v_{\text{max}}^2 = \frac{1}{2} k A^2 \rightarrow A = 2.0 \text{ m}$$

Distance varies from 1.0 m to 1.0+2A or 5 meters where $P$ is the power emitted

$$\text{Ratio} \rightarrow I_{\text{loud}}/I_{\text{soft}} = \frac{(P/4\pi r_{\text{loud}}^2)}{(P/4\pi r_{\text{soft}}^2)} = \frac{5^2}{1^2} = 25$$
An example problem

In musical instruments the sound is based on the number and relative strengths of the harmonics including the fundamental frequency of the note. Figure 1a depicts the first three harmonics of a note. The sum of the first two harmonics is shown in Fig. 1b, and the sum of the first 3 harmonics is shown in Fig. 1c.

Which of the waves shown has the shortest period?

a. 1st Harmonic  
b. 2nd Harmonic  
c. 3rd Harmonic  
d. Figure 1c

At the 2nd position (1st is at t=0) where the three curves intersect in Fig. 1a, the curves are all:

a. in phase  
b. out of phase  
c. at zero displacement  
d. at maximum displacement

An example problem

The frequency of the waveform shown in Fig. 1c is

a. the same as that of the fundamental  
b. the same as that of the 2nd harmonic  
c. the same as that of the 3rd harmonic  
d. sum of the periods of the 1st, 2nd & 3rd harmonics

Which of the following graphs most accurately reflects the relative amplitudes of the harmonics shown in Fig. 1?

Figure 1: Wave superposition
Get the beat…

- You are studying an unusual pair of sea creatures that use low frequency sound to interact. Each animal produces a perfect sine wave. Your oscilloscope output appears below. What are the respective frequencies?

\[
| f_1 - f_2 | = 1 / T_{\text{beat}} = 1 / 5 \\
(f_1 + f_2) / 2 = 1 / T_{\text{avg}} = 1 / 1
\]

- \( 2 f_1 = 2.2 \text{ Hz} \) \( \Rightarrow \) \( f_1 = 1.1 \text{ Hz} \) so \( f_2 = 0.9 \text{ Hz} \)