Today’s Topics

- Wave Motion
  - General Wave
    - Transverse And Longitudinal Waves
    - Wave speed on string
    - Reflection and Transmission of Waves
  - Wave Function
    - Sinusoidal Waves
    - Standing Waves
General Waves

- **Wave:**
  Propagation of a physical quantity in space over time
  \[ q = q(x, t) \]

- **Examples of waves:**
  Water wave, wave on string, sound wave, earthquake wave, electromagnetic wave, “light”, quantum wave….

- **Mechanic wave:**
  Propagation of small motion ("disturbance") in a medium.
  \( \Rightarrow \) Physical quantity to be propagated: displacement.

Recall: Displacement is a vector.
Transverse and Longitudinal Waves

- If the direction of mechanic disturbance (displacement) is **perpendicular** to the direction of wave motion, the wave is called **transverse** wave.
- If the direction of mechanic disturbance (displacement) is **parallel** to the direction of wave motion, the wave is called **longitudinal** wave.

→ see demos.

- In general, a wave can be a combination of the above modes.
- The definition can be extended to other (non-mechanic) waves.
  - e.g Electromagnetic waves are always transverse.
Electro-Magnetic Waves are Transverse

Two polarizations possible
Seismic Waves

Body Waves

Longitudinal

Transverse

Surface Waves

Transverse

Transverse
Wave On A Stretched Rope

- It is a transverse wave
  - See demos.

- The wave speed is determined by the tension and the linear density of the rope:

\[ v = \sqrt{\frac{T}{\mu}} \quad ; \quad \mu \equiv \frac{\Delta m}{\Delta l} \]
Reflection and Transmission of Waves

Fixed End Reflection

- Incident Pulse
- Inverted Reflected Pulse

Free End Reflection

- Incident Pulse
- Reflected Pulse

A wave traveling from a more dense to a less dense medium...

- More Dense
- Less Dense
- Incident Pulse
- Reflected Pulse
- Transmitted Pulse

...will be reflected off the boundary and transmitted across the boundary into the new medium. There is no inversion.

A wave traveling from a less dense to a more dense medium...

- Less Dense
- More Dense
- Incident Pulse
- Reflected Pulse
- Transmitted Pulse

...will be reflected off the boundary and transmitted across the boundary into the new medium. The reflected pulse is inverted.
Waves are described by wave functions in the form:

\[ y(x,t) = f(x-\nu t) \]

- \( y \): A certain physical quantity, e.g., displacement in the y direction.
- \( f \): Can any forms.
- \( x \): space position. Coefficient arranged to be 1.
- \( t \): time. Its coefficient \( \nu \) is the wave speed; \( \nu > 0 \) moving right, \( \nu < 0 \) moving left.
An Exercise to Explain Wave Speed

A wave is described by function $y = f(x-vt)$.

- At time $t_1$ in position $x_1$, how large is the quantity $y$?
  - $y = f(x_1-vt_1) = y_1$

- At a later time $t_2 = t_1 + \Delta t$, what is $y$ in position $x_2 = x_1 + v\Delta t$?
  - $y_2 = f(x_2-vt_2) = f(x_1 + v\Delta t - v(t_1 + \Delta t)) = f(x_1 - vt_1) = y_1$

- How to interpret the result?
  - Between $t_1$ and $t_1 + \Delta t$, the value $y_0$ has transmitted from position $x_1$ to $x_1 + v\Delta t$.
  - $\text{speed} = \frac{x_1 + v\Delta t - x_1}{(t_1 + \Delta t - t_1)} = v$
  - i.e. $v > 0 \leftrightarrow \text{moving right}; v < 0 \leftrightarrow \text{moving left}$;
Linear Wave Equation

- **Linear wave equation**
  - Certain physical quantity
  - Wave speed
  - \( \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \)

- **Sinusoidal wave**
  - \( f: \) frequency
  - \( \phi: \) Phase
  - \( v = \lambda f \)
  - \( k = \frac{2\pi}{\lambda} \)
  - \( \omega = 2\pi f \)
  - \( y = A \sin \left( \frac{2\pi}{\lambda} x - 2\pi ft + \phi \right) \)

General wave: superposition of sinusoidal waves
Parameters For A Sinusoidal Wave

- Snapshot with fixed $t$: wave length $\lambda = 2\pi/k$
- Snapshot with fixed $x$: angular frequency $= \omega$
  frequency $f = \omega / 2\pi$
  Period $T = 1/f$
  Amplitude $= A$
- Wave Speed $v = \omega / k$
  $\rightarrow v = \lambda f$, or
  $\rightarrow v = \lambda / T$
- Phase angle difference between two positions $\Delta \phi = -k \Delta x$
Standing Waves

- When two waves of the same amplitude, same frequency but opposite direction meet standing waves occur.

\[ y_1 = A \sin(kx - \omega t) \quad y_2 = A \sin(kx + \omega t) \]

\[ y = y_1 + y_2 = 2A \sin(kx) \cos(\omega t) \]

- Points of destructive interference (nodes)

\[ kx = n\pi; \quad x = \frac{n\pi}{k} = \frac{n\lambda}{2}; \quad n = 1, 2, 3... \]

- Points of constructive interference (antinodes)

\[ kx = (2n - 1)\frac{\pi}{2}; \quad x = (2n - 1)\frac{\lambda}{4}; \quad n = 1, 2, 3... \]
Nodes and antinodes will occur at the same positions, giving impression that wave is standing.
Standing Waves (cont)

Standing waves with a string of given length $L$ are produced by waves of natural frequencies or resonant frequencies:

$$\lambda = \frac{2L}{n}; \quad n = 1, 2, 3...$$

$$f = \frac{v}{\lambda} = \frac{nv}{2L} = \sqrt{\frac{T}{\mu}} \frac{n}{2L}$$
Forced (driven) Oscillation

- If in addition there is a driving force with its own frequency $\omega$: $F_0 \cos(\omega t)$, the equation becomes:

\[ m \frac{d^2 x}{dt^2} = -kx - b \frac{dx}{dt} + F_0 \cos(\omega t) \]

- This equation can be solved analytically.
  - At large $t$, the solution is:
    \[ x = A \cos(\omega t + \phi) \]
    where $A = \frac{F_0 / m}{\sqrt{(\omega^2 - \omega_0^2)^2 + (b/2m)^2}}$

- At large $t$, the frequency is determined by driving $\omega$
- When $\omega = \omega_0$, amplitude is maximum $\Rightarrow$ resonance
Resonance Amplitude

\[ A = \frac{F_0 / m}{\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + \left(\frac{b}{2m}\right)^2}} \]