Physics 202, Lecture 22

Today’s Topics

- Electromagnetic Waves (EM Waves)
- Review: Maxwell’s equations
- Propagation of \( \mathbf{E} \) and \( \mathbf{B} \)
- Spectrum of EM wave
- Energy Carried by EM Wave, Poynting Vector
- Momentum Carried by EM Wave
Maxwell Equations

\[ \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\varepsilon_0} \Rightarrow \text{Gauss’s Law/ Coulomb’s Law} \]

\[ \oint \mathbf{B} \cdot d\mathbf{A} = 0 \Rightarrow \text{Gauss’s Law of Magnetism, no magnetic charge} \]

\[ \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt} \Rightarrow \text{Faraday’s Law} \]

\[ \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt} \Rightarrow \text{Ampere Maxwell Law} \]

Also, Lorentz force Law \( \Rightarrow \)
\[ \mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B} \]

These are the foundations of the electromagnetism
Electromagnetic Waves

- **EM wave equations:**
  \[
  \frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} \quad \frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_z}{\partial t^2}
  \]

- **Plane wave solutions:**
  \[
  E = E_{\text{max}} \sin(kx - \omega t + \phi) \quad B = B_{\text{max}} \sin(kx - \omega t + \phi)
  \]

- **Properties:**
  - No medium is necessary.
  - E and B are normal to each other
  - E and B are in phase
  - Direction of wave is normal to both E and B
    (EM waves are transverse waves)
  - Speed of EM wave:
    \[
    c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.9972 \times 10^8 \text{ m/s}
    \]
  - E/B = \(E_{\text{max}}/B_{\text{max}} = c\)
  - Transverse wave: two polarizations possible
The EM Wave

Two polarizations possible
Antennas

- Antennas are essentially an arrangement of conductors for transmitting and receiving radio waves.
- Parameters: gain, impedance, frequency, orientation, polarization, etc.

![Diagram of half-wave antenna](image)
Wavelength and Frequency

- Because of the wave equation the wavelength of and frequency of a EM wave in vacuum are related by:

\[ \lambda f = c = 3 \cdot 10^8 \text{ m/s} \]

- Example: Determine the wavelength of an EM wave of frequency 50 MHz in free space.

\[ \lambda = \frac{c}{f} = \frac{3 \cdot 10^8 \text{ m/s}}{50 \text{ MHz}} = \frac{3 \cdot 10^8 \text{ m/s}}{5 \cdot 10^7 \text{ s}^{-1}} = 6 \text{ m} \]
Spectrum of EM Waves

VHF: 30-300 MHz
UHF: 300 MHz -3.0 GHz
Cell phone:
  800/900/1800/1900 MHz
Wifi: 2.4 GHz
Microwave Oven: 2.4 GHz
Wireless phone: 2.4/5.8 GHz
Energy Carried By EM Waves

- Recall: energy densities $u_E = \frac{1}{2} \varepsilon_0 E^2$, $u_B = \frac{1}{2} \frac{B^2}{\mu_0}$
- For a EM wave, at any time/location,
  $u_E = \frac{1}{2} \varepsilon_0 E^2 = \frac{1}{2} \frac{B^2}{\mu_0} = u_B$ (using $E/B= c$)
  ➔ In an electromagnetic wave, the energies carried by electric field and magnetic field are always the same.
- Total energy stored (per unit of volume):
  $u = u_E + u_B = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}$
- Power transmitted per unit of area is equal to $uc$ in the direction of wave
- Poynting vector: $\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$, $\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \mathbf{B} = uc$
- Averaging over time:
  $u_{av} = \frac{1}{2} \varepsilon_0 E_{max}^2 = \frac{1}{2} \frac{B_{max}^2}{\mu_0}$, $S_{av} = u_{av} c = I$ (intensity)
Momentum Carried By EM Waves

- EM waves: momentum = energy/c

Change of momentum in 100% absorption:
\[ \Delta p = \frac{\Delta U}{c} = \frac{uAc\Delta t}{c} = uA\Delta t \]

Change of momentum in 100% reflection:
\[ \Delta p = 2\frac{\Delta U}{c} = 2\frac{uAc\Delta t}{c} = 2uA\Delta t \]

- Radiation Pressure (P):
\[ P = \frac{F}{A} = \frac{\Delta p}{\Delta t} = u = \frac{S}{c} \]

100% absorption
\[ \Delta p = p \rightarrow P = \frac{S}{c} \]

100% reflection
\[ \Delta p = 2p \rightarrow P = \frac{2S}{c} \]
Example: Solar Energy

The average intensity of the EM radiation from the Sun on Earth is $S \sim 10^3 \text{ W/m}^2$

- What is the average radiation pressure for 100% absorption:

  $$P = \frac{S}{c} = \frac{10^3 \text{ W/m}^2}{3 \cdot 10^8 \text{ m/s}} = 3.3 \cdot 10^{-6} \text{ N/m}^2$$

- What is the force exerted by EM radiation by the Sun on a surface of 1 m$^2$

  $$F = PA = 3.3 \cdot 10^{-6} \text{ N/m}^2 \cdot 1 \text{ m}^2 = 3.3 \cdot 10^{-6} \text{ N}$$