Print your name and section above. If you do not know your section number, write your TA’s name.
Your final answer must be placed in the box provided. You must show all your work to receive full credit. If you only provide your final answer (in the box), and do not show your work, you will not receive very many points.
Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units, and also the direction of vectors.
You are allowed one handwritten 8½ x 11” sheet of notes and no other references. The exam lasts exactly 90 minutes.

Planck’s constant
\( h = 6.626 \times 10^{-34} \text{ m}^2 \text{ kg/s} \)

Planck’s constant x velocity of light \( hc = 1240 \text{ eV nm} \)

Reduced Planck constant:
\( \hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34} \text{ J} \cdot \text{s} \)

Bohr radius \( a_0 = 0.053 \text{ nm} \)

electron mass: \( 9.11 \times 10^{-31} \text{ kg} \)
\( m_{\text{electron}} c^2 = 511,000 \text{ eV} \)

Speed of light in vacuum:
\( c = 3 \times 10^8 \text{ m/s} \)

Permittivity of free space
\( \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2 \)
\( k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ Nm}^2 / \text{C}^2 \)

Permeability of free space
\( \mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m} / \text{A} \)

Magnitude of electron charge
\( e = 1.6 \times 10^{-19} \text{ C} \)

Problem 1: _______ / 20
Problem 2: _______ / 20
Problem 3: _______ / 20
Problem 4: _______ / 20
Problem 5: _______ / 20

TOTAL: _______ / 100
1) [20 points, 4 points each]. Explain your reasoning for full credit. 
Multiple choice/short answer questions.

i) A wire of radius $R$ carries a uniform current density of $j = 1 \text{ A/cm}^2$. The magnitude of the magnetic field inside the wire at distance $r$ from the center ($r < R$)

a. increases with $r$

b. decreases with $r$

c. is equal to zero

d. is a non-zero constant

e. none of the above

Explain/work:

*Use Ampere’s law with a circular path of radius $r$.\[
\oint \mathbf{B} \cdot d\mathbf{s} = 2\pi r B = \mu I_{encl} = \mu I \pi r^2 j
\]

Then $B = j \mu r / 2$ increases with $r$ inside the wire.*

ii) A large coil of wire has a time-dependent current as shown below. A positive value in the plot means the current is flowing in the direction of the arrow, negative in the opposite direction. Plot the induced current in the small coil of wire at the center of the large coil. The vertical scale is arbitrary, but you need to show the correct sign and relative magnitudes.

Explaination:

*The magnetic field from the current in the large loop produces a flux through the small loop. There is a current induced in the small loop only when this flux varies in time. The current is the rate of change of the flux divided by the loop resistance. When the current in the large loop is increasing, the induced current is negative to oppose the increase in flux. The induced current is positive when the drive current is decreasing. When the drive current is constant, there is no induced current.*
iii) A helium-neon laser emits light of wavelength 632.8 nm \((632.8 \times 10^{-9} \text{m})\) and has a power output of 1.5 milli-Watts \((1.5 \times 10^{-3} \text{W})\). How many photons per second does this laser emit?

\begin{align*}
a. & \quad 1.34 \times 10^{14} \\
b. & \quad 2.56 \times 10^{15} \\
c. & \quad 3.09 \times 10^{15} \\
d. & \quad 4.22 \times 10^{15} \\
e. & \quad 4.78 \times 10^{15} \\
\end{align*}

**Work/explanation**

\[
\text{Power} = \frac{\text{Energy}}{\text{time}} = \frac{Nh\nu}{\lambda \times \text{time}} \\
N = \text{Power} \times \frac{\lambda}{hc} = \left( \frac{1.5 \times 10^{-3} \text{J/s}}{1.6 \times 10^{-19} \text{J/eV}} \right) \frac{632.8 \text{nm}}{1240 \text{eV} \cdot \text{nm}} = 4.78 \times 10^{15} \text{photons/s}
\]

iv) Light is an electromagnetic wave. When visible light is linearly polarized, which of the following statements is/are true? Circle all that are true.

a. The electric field vector is parallel to the magnetic field vector.

b. The electric field vector is parallel to the direction of propagation.

c. The electric field vector is perpendicular to the direction of propagation, but the magnetic field vector may be in any direction.

d. The electric field vector is perpendicular to the direction of propagation, and the magnetic field vector is perpendicular to the electric field vector.

e. The direction of propagation is \(\vec{E} \times \vec{B}\).

**Work/Explanation:**

- E-field and B-field perpendicular, so not a.
- E-field perpendicular to propagation direction, so not b.
- B-field must be perpendicular to E-field and propagation direction, so not c.
- Solution: d and e.

v) Potassium has a work function of 2.3 eV for photoelectric emission. Which of the following wavelengths is the longest wavelength for which photoemission occurs?

\begin{align*}
a. & \quad 400 \text{ nm} \\
b. & \quad 450 \text{ nm} \\
c. & \quad 500 \text{ nm} \\
d. & \quad 540 \text{ nm} \\
e. & \quad 600 \text{ nm} \\
\end{align*}

**Work/Explanation:**

\[
\text{Solution: d) Kmax} = hf - \Phi = \frac{hc}{\lambda} - \Phi
\]

The maximum wavelength is when \(Kmax = 0\): \(\lambda = \frac{hc}{\Phi} = 539.1 \text{ nm}\).

Rounding gives an answer of 540 nm. 500 nm would be the answer without rounding. Either is counted as correct.
2) [20 pts, 4 pts each] Short calculations.

i) The electric field in a capacitor with circular plates of radius \( R = 3 \) m varies linearly with time according to \( E = (2 \times 10^7 \, \text{N/C} \cdot \text{s})t \) with time \( t \) in seconds. Calculate the conduction current flowing onto the positive plate of the capacitor.

\[
I_{\text{cond}} = I_{\text{disp}} = \varepsilon_0 \frac{d\Phi}{dt} = \varepsilon_0 \pi R^2 \frac{d(2 \times 2 \times 10^7 \, t)}{dt} = \varepsilon_0 \pi R^2 \times 2 \times 10^7 = 5 \, \text{mA}
\]

or \( Q = CV = \frac{\varepsilon_0 A}{d} Ed = \varepsilon_0 AE \); then \( I = \frac{dQ}{dt} = \varepsilon_0 \pi R^2 \left( 2 \times 10^7 \, \text{N/C} \cdot \text{s} \right) \)

\[
I = \left( \begin{array}{c|c} 
\text{Value} & \text{Units} \\
\hline
\end{array} \right)
\]

ii) The Sun radiation of intensity \( I = 1350 \, \text{W/m}^2 \) is incident normally on a perfectly reflecting sail of a spacecraft of total mass \( m = 5 \times 10^4 \, \text{kg} \) of area \( A = 10^4 \, \text{m}^2 \). What is the acceleration in free space of the spacecraft?

\[
F = ma = p_{\text{rad}} A \Rightarrow a = \frac{p_{\text{rad}} A}{m} = \frac{2IA}{mc} = \frac{2 \left( 1350 \, \text{W/m}^2 \right) \left( 10^4 \, \text{m}^2 \right)}{\left( 5 \times 10^4 \, \text{kg} \right) \left( 3 \times 10^8 \, \text{m/s} \right)} = 1.8 \times 10^{-6} \, \text{m/s}^2
\]

\[
a = \left( \begin{array}{c|c} 
\text{Value} & \text{Units} \\
\hline
\end{array} \right)
\]

iii) The electric field of an electromagnetic wave is \( \overrightarrow{E} = \left( 25 \, \text{V/m} \right) \sin(kx - \omega t) \hat{\imath} \). What is the amplitude and direction of magnetic field of the electromagnetic wave?

\[
\text{Solution: Since the wave propagates in the x-direction, } \overrightarrow{E} \times \overrightarrow{B} \text{ must be in the x-direction. Since } \overrightarrow{E} \text{ is in the y-direction, this says that } B \text{ is along z. Also } B_0 = E_0/c \text{ so}
\]

\[
\overrightarrow{B} = \left( \frac{25 \, \text{V/m}}{3 \times 10^8 \, \text{m/s}} \right) \sin(kx - \omega t) \hat{\imath} = \left( 8.3 \times 10^{-8} \, \text{T} \right) \sin(kx - \omega t) \hat{\imath}
\]

\[
\overrightarrow{B} = \left( \begin{array}{c|c|c} 
\text{Amplitude} & \text{Direction} & \text{Units} \\
\hline
\end{array} \right)
\]
iv) A proton is accelerated through a potential difference of 10,000 V. How does its de Broglie wavelength compare to that of a football of mass 0.4 kg launched by Brett Favre (emeritus) at a velocity of 30 m/s?

The mass of the proton is \(1.67 \times 10^{-27} \text{ kg} = 938.27 \text{ MeV/c}^2\) \((938.27 \times 10^6 \text{ eV}/c^2)\).

\[
\text{Ratio } \frac{\lambda_p}{\lambda_{\text{football}}} =
\]

\[
K = e\Delta V = \frac{p^2}{2m} = \frac{h^2}{2m_p\lambda_p^2} \Rightarrow \lambda_p = \frac{h}{\sqrt{2m_pK}} = \frac{hc}{\sqrt{2m_p c^2 e\Delta V}} = \frac{1240 \text{ eV} \cdot \text{nm}}{\sqrt{2(938.27 \times 10^6 \text{ eV})(10000 \text{ eV})}} = 2.86 \times 10^{-4} \text{ nm}
\]

\[
\lambda_{\text{football}} = \frac{h}{p} = \frac{h}{mv} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(0.4 \text{ kg})(30 \text{ m/s})} = 5.52 \times 10^{-35} \text{ m} = 5.52 \times 10^{-26} \text{ nm}
\]

\[
\text{Ratio} = \frac{\lambda_p}{\lambda_{\text{football}}} = \frac{2.86 \times 10^{-4} \text{ nm}}{5.52 \times 10^{-26} \text{ nm}} = 5.18 \times 10^{21}
\]

v) The probability density of a quantum particle is shown in the figure. It is zero except in the region \(-10\text{ nm} < x < 10\text{ nm}\). Calculate the probability that the particle will be found in the region \(5\text{ nm} < x < 10\text{ nm}\).

The area under the curve from 5 to 10 nm is 1/8 the total area under the curve, so the probability is 0.125. This can be seen from the drawing, or it can be calculated. If \(h\) is the ‘height’ \(\left(=\left|\psi(0)\right|^2\right)\), then the area under the curve from 5-10nm is \(\frac{1}{2}(5\text{ nm})(h/2)\). The fraction of the total area \(\frac{1}{2}(20\text{ nm})(h)\) is 1/8=0.125.

\[
\text{Probability} =
\]
3) [20 pts, 5 pts each]

Unpolarized light is incident on a series of polarizers as shown below. The transmission axis of each polarizer is indicated by a heavy line.

![Diagram of polarizers](image)

a) In what direction(s) does the electric field vector of the electromagnetic wave in region 1 point? Explain.

*The polarizer transmits the component of the electric field along the transmission axis. So the light in region 1 is linearly polarized with electric field vector oscillating in time, pointing alternately along +y and –y directions.*

Direction(s) =

b) What is the ratio of the electric field amplitude in region 2 to that in region 1?

*The middle polarizer transmits the component parallel to its transmission axis. This has amplitude $E_2 = E_1 \cos 30^\circ$. This is the component parallel to the transmission axis. Then $E_2 / E_1 = \cos 30^\circ = 0.866$*

Ratio $\frac{E_2^{\text{max}}}{E_1^{\text{max}}} =$
c) What percentage of the power incident on polarizer 3 is absorbed?

\[ \text{Whatever is not transmitted by the polarizer is absorbed. The intensity after the polarizer is } I_3 = I_2 \cos^2 60^\circ = 0.25. \text{ So } 75\% \text{ of the power is absorbed, } 25\% \text{ transmitted.} \]

\[ \text{\% power absorbed} = \]

d) Suppose that the middle polarizer rotates while the first and last polarizers stay fixed at the angles shown in the figure. Draw the intensity after the last polarizer (in Region 3) as a function of middle polarizer angle (defined as in the figure). **Explain**

\[ \text{The intensity is zero when the middle polarizer is perpendicular to either of the two other polarizers. This happens at } \theta = 0^\circ, 90^\circ, 180^\circ, 270^\circ. \text{ It is a maximum when } \theta = 45^\circ. \text{ The shape is sinusoidal since the mathematical expression is a product of two cosines:} \]

\[ I_3 = I_1 \cos^2 \theta \cos^2 (90 - \theta) = I_1 \cos^2 \theta \sin^2 (\theta) = \frac{I_1}{4} \sin^2 (2\theta) \]
4) [20 pts, 4 pts each]
A rectangular coil of 10,000 turns of total resistance 0.01\( \Omega \) is below an infinitely long straight wire. The infinitely-long wire carries a constant current \( I = 1000A \), producing a field \( B = \mu_0 I / 2\pi r \) with \( r \) the distance from the wire.

![Diagram of a rectangular coil and a long wire](image)

a) Assume that the 10,000-turn coil is narrow enough that the magnetic field from the long wire can be approximated as constant everywhere across the area of the coil. Calculate the magnetic flux through one turn of the coil when its center is 10 cm from the wire.

\[
\Phi = BA = \frac{\mu_0 I}{2\pi r} A = \left( \frac{4\pi \times 10^{-7} T \cdot m/A \cdot 1000 A}{2\pi (0.1m)} \right) (0.1m)(0.002m) = \\
\left( 2 \times 10^{-3} T \right) \left( 2 \times 10^{-4} m^2 \right) = 4 \times 10^{-7} Wb
\]

<table>
<thead>
<tr>
<th>Value</th>
<th>Units</th>
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<tbody>
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</table>

b) You push the coil toward the long wire in the direction perpendicular to it at a constant speed of 1 cm/s. What is the direction of the current induced in the coil? Explain your reasoning in words.

*Induced current is counterclockwise in order to produce a flux out of the page that opposes the flux increase into the page as a result of the loop motion*

<table>
<thead>
<tr>
<th>Direction</th>
<th></th>
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</table>
c) You are still pushing the coil toward the long wire at constant speed of 1 cm/s. Assume that the 10,000-turn coil is narrow enough that the magnetic field from the long wire can be approximated as constant everywhere across the area of the coil. By how much does the flux through one loop of the coil change when you move it from 10 cm to 9.9 cm?

\[
\Delta \Phi = A \Delta B = A \frac{\mu_0 I}{2\pi} \left( \frac{1}{0.1m} - \frac{1}{0.099m} \right) = (0.1m)(0.002m) \left( \frac{4\pi \times 10^{-7} T \cdot m/A}{2\pi} \right) \left( \frac{1}{0.1m} - \frac{1}{0.099m} \right) \]
\[
= \left( 2 \times 10^{-4} m^2 \right) \left( 2 \times 10^{-4} T \cdot m \right) \left( 0.1m^{-1} \right) = 4 \times 10^{-9} \text{ Wb}
\]

\[\Delta \Phi = \text{Value} \quad \text{Units}\]

\[
\frac{\Delta \Phi}{\Delta t} = \frac{10^{-8} \text{ Wb}}{0.1s} = 10^{-7} \text{ Wb/s}
\]

The EMF is then \((10,000)(10^{-7} \text{ Wb/s}) = 10^{-3} \text{ V}\), and the current \(10^{-3} \text{ V} / 0.01 \Omega = 0.1 \text{ A}\). The direction is counterclockwise, opposing the change in flux from the long wire.

\[\text{EMF=} \quad \text{Value} \quad \text{Units}\]

d) You are still pushing the coil toward the long wire at constant speed of 1 cm/s. Calculate the EMF induced around the entire 10,000 turn coil when it is 10cm from the long wire. For this part, use \(\Delta \Phi = 10^{-8} \text{ Wb}\) as the answer for part c).

\[\text{Force} = \text{Value} \quad \text{Units}\]

e) With what force must you push the coil toward the infinitely long wire to keep it moving at 1 cm/s when it is 10cm from the long wire? For this part, assume that the induced current in the coil is 0.2 A.

The force on the top wire is \(ILB(r = 0.1m - 0.001m)\). The force on the bottom wire is \(-ILB(r = 0.1m + 0.001m)\). The force on the sides are equal and opposite, and so cancel. The net force is on one loop is

\[
(0.2A)(0.1m) \left( \frac{4\pi \times 10^{-7} T \cdot m/A}{2\pi} \right) \left( 1000A \right) \left( \frac{1}{0.099m} - \frac{1}{0.101m} \right)
\]
\[
= (0.02 A \cdot m) \left( 2 \times 10^{-4} T \cdot m \right) \left( \frac{1}{0.099m} - \frac{1}{0.101m} \right) = 8 \times 10^{-7} \text{ N}
\]

for one loop. But there are 10,000 loops, so the required force is \(8 \times 10^{-3} \text{ N} = 0.008 \text{ N}\).
5) [20 pts, 4 pts each]

Four possible transitions for a Hydrogen atom are as follows:
(i) \( n_i = 2, n_f = 5 \);
(ii) \( n_i = 5, n_f = 3 \);
(iii) \( n_i = 7, n_f = 4 \);
(iv) \( n_i = 4, n_f = 7 \);

\( n_i \) is the quantum number of the initial state of the electron and \( n_f \) is the quantum number of the final state.

A) Label each transition as to whether a photon is emitted or absorbed. Explain

Solution: For each transition, since energy levels are \( E_n = -13.6 \text{ eV}/n^2 \),

\[
\Delta E = E_f - E_i = 13.6eV \left( \frac{1}{n_i^2} - \frac{1}{n_f^2} \right)
\]

light is emitted if the initial state has lower energy than the final, so the electron goes from a farther away level to a closer one to the nucleus. On the other hand, light is absorbed if the electron goes from a closer orbit to a farther away one from the nucleus and \( \Delta E > 0 \). For each transition:

<table>
<thead>
<tr>
<th>Transition</th>
<th>Emit or Absorb</th>
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</thead>
<tbody>
<tr>
<td>( n_i = 2, n_f = 5 )</td>
<td>_absorb_</td>
</tr>
<tr>
<td>( n_i = 5, n_f = 3 )</td>
<td>_emit_</td>
</tr>
<tr>
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</tr>
<tr>
<td>( n_i = 4, n_f = 7 )</td>
<td>_absorb_</td>
</tr>
</tbody>
</table>

B) In which transition(s) does the photon have the longest wavelength? Circle the correct answers. Explain

\[
\Delta E = E_f - E_i = hf = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\Delta E}
\]

So the shortest wavelength photon is emitted/absorbed in the transition with largest \( |\Delta E| \).

<table>
<thead>
<tr>
<th>Transition</th>
<th>( \Delta E ) (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>i) 2 ( \rightarrow ) 5</td>
<td>2.86</td>
</tr>
<tr>
<td>ii) 5 ( \rightarrow ) 3</td>
<td>-0.97</td>
</tr>
<tr>
<td>iii) 7 ( \rightarrow ) 4</td>
<td>-0.571</td>
</tr>
<tr>
<td>iv) 4 ( \rightarrow ) 7</td>
<td>0.571</td>
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</tbody>
</table>

C) In which transition(s) does the atom gain the most energy? Circle the correct answer(s). Explain

Gains the most energy in i), since \( \Delta E \) is the largest

<table>
<thead>
<tr>
<th>Transition</th>
<th>( n_i )</th>
<th>( n_f )</th>
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<tbody>
<tr>
<td>( n_i = 2, n_f = 5 )</td>
<td>_absorb_</td>
<td></td>
</tr>
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<td>( n_i = 5, n_f = 3 )</td>
<td>_emit_</td>
<td></td>
</tr>
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<td>( n_i = 7, n_f = 4 )</td>
<td>_emit_</td>
<td></td>
</tr>
<tr>
<td>( n_i = 4, n_f = 7 )</td>
<td>_absorb_</td>
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</table>
D) In which transition(s) does the atom lose energy? Circle the correct answer(s).

Explain

Loses energy in (ii) and (iii), since energy of final state is smaller than energy of initial state.

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<td>n_i = 4, n_f = 7</td>
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E) Calculate the velocity of the electron in the n = 2 state.

Use the expression for the orbital radius $r = n^2 a_o$ along with Bohr’s quantization condition $L = mvr = nh$ to find

$$v = \frac{nh}{mn^2a_o} = \frac{\hbar}{na_o m} = \frac{1.055 \times 10^{-34} \text{ J} \cdot \text{s}}{2 \left(0.053 \times 10^{-9} \text{ m}\right) \left(9.11 \times 10^{-31} \text{ kg}\right)} = 1.1 \times 10^6 \text{ m/s}$$

Or...

use centripetal acceleration = Coulomb force / mass: $$\frac{mv^2}{r} = \frac{k e^2}{r^2} \Rightarrow mv^2r = ke^2$$, Then $mvr = nh \Rightarrow r = \frac{nh}{mv}$ can be substituted $\frac{mv^2}{mv} = \frac{n\hbar}{mv} = ke^2 \Rightarrow v = \frac{ke^2}{nh}$

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