1. T&M 27.P.032

The current in the wire shown below is 8 A. Find B at point P due to each wire segment and sum to find the resultant B.

![Diagram of wire segments and point P](image)

**Solution:**

**Picture the Problem** Note that the current segments a-b and e-f do not contribute to the magnetic field at point P. The current in the segments b-c, c-d, and d-e result in a magnetic field at P that points into the plane of the paper. Note that the angles bPc and ePd are 45° and use the expression for B due to a straight wire segment to find the contributions to the field at P of segments bc, cd, and de.

<table>
<thead>
<tr>
<th>Segment</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>bc</td>
<td>$B_{bc} = \frac{\mu_0 I}{4\pi R} \left( \sin \theta_1 + \sin \theta_2 \right)$</td>
</tr>
<tr>
<td>cd</td>
<td>$B_{cd} = 2 \frac{\mu_0 I}{4\pi R} \sin 45^\circ$</td>
</tr>
<tr>
<td>de</td>
<td>$B_{de} = \frac{\mu_0 I}{4\pi R} \sin 45^\circ$</td>
</tr>
</tbody>
</table>

Express the resultant magnetic field at P:

$$B = B_{bc} + B_{cd} + B_{de}$$
Substitute to obtain:

\[
B = \frac{\mu_0 I}{4\pi R} \sin 45^\circ + 2 \frac{\mu_0 I}{4\pi R} \sin 45^\circ + \frac{\mu_0 I}{4\pi R} \sin 45^\circ
\]

\[
= 4 \frac{\mu_0 I}{4\pi R} \sin 45^\circ
\]

Substitute numerical values and evaluate \(B\):

\[
B = 4\left(10^{-7} \text{T} \cdot \text{m/A}\right) \frac{8.0 \text{A}}{0.010 \text{m}} \sin 45^\circ
\]

\[
= 0.23 \text{mT into the page}
\]

2. T&M 27.P.034
Three long, parallel, straight wires pass through the vertices of an equilateral triangle that has sides \(L = 10 \text{ cm}\), as shown in the figure below. A dot indicates that the direction of the current is out of the screen and a cross indicates that the direction of the current is into the screen.

Each current is 15.0 A.

(a) Find the magnetic field at the location of the upper wire due to the currents in the two lower wires.

(b) Find the force per unit length on the upper wire.

Solution:

**Picture the Problem** (a) We can use the right-hand rule to determine the directions of the magnetic fields at the upper wire due to the currents in the two lower wires and use \(B = \frac{\mu_0 2I}{4\pi R}\) to find the magnitude of the resultant field due to these currents. (b) Note that the forces on the upper wire are away from and directed along the lines to the lower wire and that their horizontal components
cancel. We can use \( F = \frac{2\mu_0 I^2}{4\pi R} \) to find the resultant force in the upward direction (the \( y \) direction) acting on the top wire.

(a) Noting, from the geometry of the wires, the magnetic field vectors both are at an angle of 30° with the horizontal and that their \( y \) components cancel, express the resultant magnetic field:

\[
\vec{B} = \frac{2\mu_0 I}{4\pi R} \cos 30° \hat{i}
\]

Substitute numerical values and evaluate \( B \):

\[
B = 2\left(10^{-7} \text{ T} \cdot \text{m/A}\right) \left(\frac{15 \text{ A}}{0.10 \text{ m}}\right) \cos 30° = 52 \mu\text{T} \text{ toward the right}
\]

(b) Express the force per unit length each of the lower wires exerts on the upper wire:

\[
\frac{F}{\ell} = \frac{2\mu_0 I^2}{4\pi R}
\]

Noting that the horizontal components add up to zero, express the net upward force per unit length on the upper wire:

\[
\sum \frac{F_y}{\ell} = 2\frac{\mu_0 I^2}{4\pi R} \cos 30° + 2\frac{\mu_0 I^2}{4\pi R} \cos 30° = 4\frac{\mu_0 I^2}{4\pi R} \cos 30°
\]

Substitute numerical values and evaluate \( \sum \frac{F_y}{\ell} \):

\[
\sum \frac{F_y}{\ell} = 4\left(10^{-7} \text{ T} \cdot \text{m/A}\right) \left(\frac{15 \text{ A}}{0.10 \text{ m}}\right)^2 \cos 30° = 7.8 \times 10^{-4} \text{ N/m up the page}
\]

### 3. T&M 27.P.045

A long, straight, thin-walled cylindrical shell of radius \( R \) carries a current \( I \) parallel to the central axis of the shell in the positive \( x \)-axis direction.
Find the magnetic field (including direction - as seen from the negative x-axis) inside and outside the shell. (Use mu for $\mu_0$, I, and R as necessary.)

**Solution:**

**Picture the Problem** We can apply Ampère’s law to a circle centered on the axis of the cylinder and evaluate this expression for $r < R$ and $r > R$ to find $B$ inside and outside the cylinder. We can use the right-hand rule to determine the direction of the magnetic fields.

Apply Ampère’s law to a circle centered on the axis of the cylinder:

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_C$$

Note that, by symmetry, the field is the same everywhere on this circle.

Evaluate this expression for $r < R$:

$$\oint_C B_{\text{inside}} \cdot d\vec{l} = \mu_0 (0) = 0$$

Solve for $B_{\text{inside}}$ to obtain:

$$B_{\text{inside}} = 0$$

Evaluate this expression for $r > R$:

$$\oint_C B_{\text{outside}} \cdot d\vec{l} = B(2\pi r) = \mu_0 I$$

Solve for $B_{\text{outside}}$ to obtain:

$$B_{\text{outside}} = \frac{\mu_0 I}{2\pi r}$$

The direction of the magnetic field is in the direction of the curled fingers of your right hand when you grab the cylinder with your right thumb in the direction of the current.

Note: the WebAssign solution for the field outside the cylinder is $B_{\text{outside}} = \frac{\mu_0 I}{4\pi R}$, which is incorrect! The field outside should vary as $1/r$, as given above. Thanks to those of you who pointed this out!

**4. T&M 27.P.051**

A tightly wound toroid of inner radius 1.0 cm and outer radius 2 cm has 1000 turns of wire and carries a current of 1.5 A.

(a) What is the magnetic field at a distance of 1.10 cm from the center? (b) What is the field 1.50 cm from the center?
Solution:

**Picture the Problem** The magnetic field inside a tightly wound toroid is given by $B = \frac{\mu_0 NI}{2\pi r}$, where $a < r < b$ and $a$ and $b$ are the inner and outer radii of the toroid.

Express the magnetic field of a toroid: 

<table>
<thead>
<tr>
<th>$B$</th>
<th>$B = \frac{\mu_0 NI}{2\pi r}$</th>
</tr>
</thead>
</table>

(a) Substitute numerical values and evaluate $B(1.10 \text{ cm})$:

$$B(1.10\text{ cm}) = \frac{4\pi \times 10^{-7} \text{ N/A}^2 \times 1000 \times 1.50\text{ A}}{2\pi (1.10\text{ cm})} = 27.3 \text{ mT}$$

(b) Substitute numerical values and evaluate $B(1.50 \text{ cm})$:

$$B(1.50\text{ cm}) = \frac{4\pi \times 10^{-7} \text{ N/A}^2 \times 1000 \times 1.50\text{ A}}{2\pi (1.50\text{ cm})} = 20.0 \text{ mT}$$

**5. T&M 27.P.082**
The closed loop carries a current of 8 A in the counterclockwise direction. The radius of the outer arc is 60 cm, that of the inner arc is 40 cm. Find the magnetic field at point $P$.

**Solution:**

**Picture the Problem** Let out of the page be the positive $x$ direction and the numerals 40 and 60 refer to the circular arcs whose radii are 40 cm and 60 cm. Because point $P$ is on the line connecting the straight segments of the conductor, these segments do not contribute to the magnetic field at $P$. Hence the resultant magnetic field at $P$ will be the sum of the magnetic fields due to the current in the two circular arcs and we can use the expression for the magnetic field at the center of a current loop to find $\vec{B}_P$.

Express the resultant magnetic field at $P$:

$$\vec{B}_P = \vec{B}_{40} + \vec{B}_{60} \quad (1)$$
Express the magnetic field at the center of a current loop:

\[ B = \frac{\mu_0 I}{2R} \]

where \( R \) is the radius of the loop.

Express the magnetic field at the center of one-sixth of a current loop:

\[ B = \frac{1}{6} \frac{\mu_0 I}{2R} = \frac{\mu_0 I}{12R} \]

Express \( \vec{B}_{40} \) and \( \vec{B}_{60} \):

\[ \vec{B}_{40} = -\frac{\mu_0 I}{12R_{40}}\hat{i} \quad \text{and} \quad \vec{B}_{60} = \frac{\mu_0 I}{12R_{60}}\hat{i} \]

Substitute for \( \vec{B}_{40} \) and \( \vec{B}_{60} \) in equation (1) and simplify to obtain:

\[ \vec{B}_p = -\frac{\mu_0 I}{12R_{40}}\hat{i} + \frac{\mu_0 I}{12R_{60}}\hat{i} = \frac{\mu_0 I}{12} \left( \frac{1}{R_{60}} - \frac{1}{R_{40}} \right)\hat{i} \]

Substitute numerical values and evaluate \( \vec{B}_p \):

\[ \vec{B}_p = \left( 4\pi \times 10^{-7} \text{ N/A}^2 \right) \left( 8.0 \text{ A} \right) \left( \frac{1}{0.60 \text{ m}} - \frac{1}{0.40 \text{ m}} \right)\hat{i} = (-0.70 \mu\text{T})\hat{i} \]

or

\[ B_p = 0.70\mu\text{T} \]

**6. T&M 28.P.022**

A magnetic field of 1.2 T is perpendicular to a square coil of 14 turns. The length of each side of the coil is 5 cm. (a) Find the magnetic flux through the coil. (b) Find the magnetic flux through the coil if the magnetic field makes an angle of 60° with the normal to the plane of the coil.

**Solution:**

**Picture the Problem** Because the square coil defines a plane with area \( A \) and \( \vec{B} \) is constant in magnitude and direction over the surface and makes an angle \( \theta \) with the unit normal vector, we can use \( \phi_m = NBA \cos \theta \) to find the magnetic flux through the coil.
The magnetic flux through the coil is given by:

\[ \phi_m = NBA \cos \theta \]

Substitute numerical values for \( N, B, \) and \( A \) to obtain:

\[ \phi_m = 14(1.2 \, \text{T})(5.0 \times 10^{-2} \, \text{m})^2 \cos \theta = (42.0 \, \text{mWb}) \cos \theta \]

(a) For \( \theta = 0^\circ \):

\[ \phi_m = (42.0 \, \text{mWb}) \cos 0^\circ = 42 \, \text{mWb} \]

(b) For \( \theta = 60^\circ \):

\[ \phi_m = (42.0 \, \text{mWb}) \cos 60^\circ = 21 \, \text{mWb} \]

7. T&M 28.P.033

A 100 turn circular coil has a diameter of 2.0 cm and resistance of 50 \( \Omega \). The plane of the coil is perpendicular to a uniform magnetic field of magnitude 1.0 T. The direction of the field is suddenly reversed.

(a) Find the total charge that passes through the coil.

(b) If the reversal takes 0.1 s, find the average current in the coil.

(c) Find the average emf in the coil.

Solution:

Picture the Problem

We can use the definition of average current to express the total charge passing through the coil as a function of \( I_{av} \). Because the induced current is proportional to the induced emf and the induced emf, in turn, is given by Faraday’s law, we can express \( \Delta Q \) as a function of the number of turns of the coil, the magnetic field, the resistance of the coil, and the area of the coil. Knowing the reversal time, we can find the average current from its definition and the average emf from Ohm’s law.

\[ \Delta Q = I_{av} \Delta t \]  \hspace{1cm} (1)

(a) Express the total charge that passes through the coil in terms of the induced current:

Relate the induced current to the induced emf:

\[ I = I_{av} = \frac{E}{R} \]
Using Faraday’s law, express the induced emf in terms of $\phi_m$:

$$\mathcal{E} = -\frac{\Delta \phi_m}{\Delta t}$$

Substitute in equation (1) and simplify to obtain:

$$\Delta Q = \frac{|E|}{R} \Delta t = \frac{\Delta \phi_m}{\Delta t} \Delta t = \frac{2 \phi_m}{R}$$

$$= \frac{2NBA}{R} = \frac{2NB \left( \frac{\pi d^2}{4} \right)}{R}$$

$$= \frac{NB \pi d^2}{2R}$$

where $d$ is the diameter of the coil.

Substitute numerical values and evaluate $\Delta Q$:

$$\Delta Q = \frac{(100)(1.00 \text{T})(0.0200 \text{m})^2}{2(50.0 \Omega)}$$

$$= 1.257 \text{ mC} = 1.26 \text{ mC}$$

(b) Apply the definition of average current to obtain:

$$I_{av} = \frac{\Delta Q}{\Delta t} = \frac{1.257 \text{ mC}}{0.100 \text{ s}} = 12.57 \text{ mA}$$

$$= 12.6 \text{ mA}$$

(c) Using Ohm’s law, relate the average emf in the coil to the average current:

$$\mathcal{E}_{av} = I_{av} R = (12.57 \text{ mA})(50.0 \Omega)$$

$$= 628 \text{ mV}$$

8. T&M 28.P.032

A solenoid of length 25 cm and radius 0.8 cm with 400 turns is in an external magnetic field of 600 G that makes an angle of 50° with the axis of the solenoid.

(a) Find the magnetic flux through the solenoid.

(b) Find the magnitude of the emf induced in the solenoid if the external magnetic field is reduced to zero in 1.4 s.

Solution:
**Picture the Problem** We can use its definition to find the magnetic flux through the solenoid and Faraday’s law to find the emf induced in the solenoid when the external field is reduced to zero in 1.4 s.

(a) Express the magnetic flux through the solenoid in terms of \( N, B, A, \) and \( \theta \):

\[
\phi_m = NBA \cos \theta \\
= NB\pi R^2 \cos \theta
\]

Substitute numerical values and evaluate \( \phi_m \):

\[
\phi_m = (400)(60.0 \text{ mT})\pi(0.00800 \text{ m}^2) \cos 50^\circ
= 3.10 \text{ mWb} = 3.1 \text{ mWb}
\]

(b) Apply Faraday’s law to obtain:

\[
E = \frac{\Delta \phi_m}{\Delta t} = \frac{-0 - 3.10 \text{ mWb}}{1.40 \text{ s}}
= 2.2 \text{ mV}
\]

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In Figure 28-22, let \( B \) be 0.8 T, \( v = 10.0 \text{ m/s} \), \( \ell = 20 \text{ cm} \), and \( R = 2 \Omega \).

(a) Find the induced emf in the circuit.
(b) Find the current in the circuit.
(c) Find the force needed to move the rod with constant velocity.
(d) Find the power input by the force found in part (c).
(e) Find the rate of Joule heat production \( I^2R \).

**Solution:**

**Picture the Problem** Because the speed of the rod is constant, an external force must act on the rod to counter the magnetic force acting on the induced current. We can use the motional-emf equation \( E = vB\ell \) to evaluate the induced emf, Ohm’s law to find the current in the circuit, Newton’s 2nd law to find the force needed to move the rod with constant speed, and \( P = Fv \) to find the power input by the force.
(a) Relate the induced emf in the circuit to the speed of the rod, the magnetic field, and the length of the rod:

\[ E = vB\ell = (10 \text{ m/s})(0.80 \text{T})(0.20 \text{m}) \]

\[ = 1.60 \text{V} = 1.6 \text{V} \]

(b) Using Ohm’s law, relate the current in the circuit to the induced emf and the resistance of the circuit:

\[ I = \frac{E}{R} = \frac{1.60 \text{V}}{2.0 \Omega} = 0.80 \text{A} \]

Note that, because the rod is moving to the right, the flux in the region defined by the rod, the rails, and the resistor is increasing. Hence, in accord with Lenz’s law, the current must be counterclockwise if its magnetic field is to counter this increase in flux.

(c) Because the rod is moving with constant speed in a straight line, the net force acting on it must be zero. Apply Newton’s 2nd law to relate \( F \) to the magnetic force \( F_m \):

\[ \sum F_x = F - F_m = 0 \]

Solving for \( F \) and substituting for \( F_m \) yields:

\[ F = F_m = B\ell I \]

Substitute numerical values and evaluate \( F \):

\[ F = (0.80 \text{T})(0.80 \text{A})(0.20 \text{m}) = 0.128 \text{N} \]

\[ = 0.13 \text{N} \]

(d) Express the power input by the force in terms of the force and the velocity of the rod:

\[ P = Fv = (0.128 \text{N})(10 \text{m/s}) = 1.3 \text{W} \]

(e) The rate of Joule heat production is given by:

\[ P = I^2R = (0.80 \text{A})^2(2.0 \Omega) = 1.3 \text{W} \]

10. T&M 28.P.036
A rod 30 cm long moves at 8 m/s in a plane perpendicular to a magnetic field of 500 G. The velocity of the rod is perpendicular
(a) Find the magnetic force on an electron in the rod.
(b) Find the electrostatic field \( \vec{E} \) in the rod.
(c) Find the potential difference \( V \) between the ends of the rod.

**Solution:**

**Picture the Problem** We can apply the equation for the force on a charged particle moving in a magnetic field to find the magnetic force acting on an electron in the rod. We can use \( \vec{E} = \vec{v} \times \vec{B} \) to find \( E \) and \( V = E\ell \), where \( \ell \) is the length of the rod, to find the potential difference between its ends.

<table>
<thead>
<tr>
<th>(a) Relate the magnetic force on an electron in the rod to the speed of the rod, the electronic charge, and the magnetic field in which the rod is moving:</th>
<th>( \vec{F} = q\vec{v} \times \vec{B} ) and ( F = qvB \sin \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Substitute numerical values and evaluate ( F ):</td>
<td>( F = (1.602 \times 10^{-19} \text{ C})(8.00 \text{ m/s}) \times (0.0500 \text{ T}) \sin 90^\circ )</td>
</tr>
<tr>
<td></td>
<td>( = 6.4 \times 10^{-20} \text{ N} )</td>
</tr>
</tbody>
</table>

| (b) Express the electrostatic field \( \vec{E} \) in the rod in terms of the magnetic field \( \vec{B} \): | \( \vec{E} = \vec{v} \times \vec{B} \) and \( E = vB \sin \theta \) where \( \theta \) is the angle between \( \vec{v} \) and \( \vec{B} \). |
| | Substitute numerical values and evaluate \( E \): |
| | \( E = (8.00 \text{ m/s})(0.0500 \text{ T}) \sin 90^\circ \) |
| | \( = 0.400 \text{ V/m} = \boxed{0.40 \text{ V/m}} \) |

| (c) Relate the potential difference between the ends of the rod to its length \( \ell \) and the electric field \( E \): | \( V = E\ell \) |
| Substitute numerical values and evaluate \( V \): | \( V = (0.400 \text{ V/m})(0.300 \text{ m}) = \boxed{0.12 \text{ V}} \) |