MidTerm Exam 3  Friday, 4/24  10:30 class

Material:
T+L  Ch 7  (Hydrogenation - quantum theory)  (also T+H  Ch 36)
Ch 11  (Nuclear physics)  (also T+H  Ch 40)
T+M  Ch 25  (Currents + DC circuits - R ± RC)
26  (Magnetic Force)
27  (Magnetic Fields)

HW 8-12 (but no induction)

Review: Wed & Thurs (disc). Office hrs this week: M, T 4-6. Other times via appointment.

3rd to Inductance.

Self-Inductance: defined as: \( \Phi_m = L I \)

\( N \)-turn coils: \( 2 = \text{equations} \)

\( \Phi_m = N \int B \cdot dA, \ L = \frac{\Phi_m}{I} \)

\( \frac{d}{dt} \): voltage drop across coil (inductor). Emf induced opposes change in \( \Phi_m \).

\( \frac{d}{dt} \): Circuit: changing \( I(t) \) \( \rightarrow \) change in \( B \) produced by \( I \), changing \( \Phi_m \)

Induced emf: opposes original emf carrying change \( I(t) \)

**Induction: opposes change in current**

(Compare resistor: opposes current itself)

Mutual Inductance: can get the effect of two coil circuits:

\[ \Phi_{m12} = \text{flux of} \ B_1 \text{ through Circuit 2} = M_{12} I_1 \]

\[ \Phi_{m21} = " \text{flux of} \ B_2 \text{ through Circuit 1} = M_{21} I_2 \]

\( \Phi_m \) shows: \( M_{12} = M_{21} \)
Both types: solenoids

\[ L = \frac{N}{A} \int n \cdot n \cdot n \cdot A \cdot \frac{dI}{dt} \]

Natural inductance:

\[ H_{12} = H_{21} = \frac{\mu_0 \mu_0}{\mu_0 \mu_0} n \cdot m \cdot l \cdot \pi r_1^2 \]

Show \( H_{12} = M_{12} \).

Core/solenoid with large self-inductance \( \rightarrow \) inductor \( L \)

\[ \frac{1}{3} \frac{d}{dt} \]

Useful circuit element: acts to oppose change in current

RL circuit (DC)

Kirchhoff:

\[ E - IR - L \frac{dI}{dt} = 0 \]

Energy:

\[ EI = I^2 R + L \frac{dI}{dt} \]

\[ LI \frac{dI}{dt} = dU_m \]

\[ dU_m = LI dI \Rightarrow U_m = \frac{1}{2} LI^2 \]

Power supplied by battery = Power delivered to resistor + Power delivered to inductor

\[ dU_m = \frac{L}{2} \frac{dI}{dt} \]

Energy stored in magnetic field:

- example: solenoid

\[ U_m = \frac{1}{2} LI^2 = \frac{1}{2} \mu_0 n^2 A I^2 = \frac{1}{2} (\mu_0 n I)^2 A l = \frac{B^2}{2} A l \]

\[ \text{define } \frac{U_m}{A l} = \text{magnetic energy density } \Rightarrow \frac{U_m}{A l} = \frac{B^2}{2} \]

\[ \frac{U_m}{A l} = \frac{B^2}{2} \]
Magnetic energy density: \( U_m = \frac{B^2}{2\mu_0} \) for all space.

Electric energy density: \( U_e = \frac{1}{2} \varepsilon_0 E^2 \)

Think of energy as being stored in the fields.

Total magnetic energy: \( U_m = \frac{1}{2\mu_0} \int B^2 \, dV \)

Total electric energy: \( U_e = \frac{1}{2} \varepsilon_0 \int E^2 \, dV \)

For circuits:

Open circuit: inductor

Assume self-inductance of rest of circuit is negligible.

Rate of inductor: opposite charge change in the current in that current.

\[ E - ER - L \frac{dI}{dt} = 0 \]

\[ \frac{E}{R} - I = \frac{L}{R} \frac{dI}{dt} \]

At \( t = 0 \):

\[ \frac{dI}{dt} = \frac{R}{L} I \]

Current building in circuit:

\[ y = \frac{E}{R} - I : \quad \frac{dy}{dt} = -\frac{R}{L} y \]

\[ y = y_0 e^{-\frac{R}{L} t} \]

\[ I(t) = \frac{E}{R} e^{-\frac{R}{L} t} \]

Initial condition: \( I(0) = \frac{E}{R} (1 - e^{-\frac{R}{L} t}) \)

\[ I = \text{time constant} = L/R \]
The circuit is an RC circuit. The charge on the capacitor is given by:

\[ q = \frac{1}{R C} \int i \, dt \]

Current through the resistor:

\[ i(t) = \frac{V}{R} \]

From Kirchhoff's voltage law:

\[ v(t) = R i(t) \]

When the battery is disconnected, the charge on the capacitor remains constant:

\[ q(t) = q(0) = \frac{V}{R} \]

The voltage across the capacitor is:

\[ v_c(t) = \frac{V}{Q} \int q(t) \, dt \]

Using the initial condition:

\[ q(0) = \frac{V}{R} \]

The voltage across the resistor is:

\[ v_R(t) = v(t) - v_c(t) \]

The current through the resistor is:

\[ i(t) = \frac{d q(t)}{dt} \]

Using the initial condition:

\[ q(0) = \frac{V}{R} \]

The final voltage across the capacitor is:

\[ v_c(t) = \frac{V}{Q} \int q(t) \, dt \]

The final voltage across the resistor is:

\[ v_R(t) = v(t) - v_c(t) \]

The current through the resistor is:

\[ i(t) = \frac{d q(t)}{dt} \]

Using the initial condition:

\[ q(0) = \frac{V}{R} \]
Example circuit (Ex. 28-14 - problem 28.6S)

\[ E - I_1 R_1 - I_2 R_2 = 0 \]
\[ E = I_1 (R_1 + R_2) = 0 \]
\[ I_1 = I_2 = \frac{E}{R_1 + R_2} \]

(a) at \( t=0 \): \( I_3 = 0 \) (since before switch is closed) - apply initial

voltage drop across \( R_1 \) so \( I_1 = I_2 \)

\[ \frac{E}{R_1} \]

Loop 1:

\[ E = I_1 R_1 - I_2 R_2 = 0 \]

At \( t=0 \):

\[ I_1 = I_2 = \frac{E}{R_1 + R_2} \]

potential drop across inductor: \( L \frac{dI_2}{dt} = I_2 R_2 \)

(b) at long time (\( t \to \infty \))

long time: \( \frac{dI}{dt} = 0 \) and inductor \( L \) acts as short circuit

\[ \Rightarrow I_2 = 0 \]

Effect circuit:

\[ E - I_1 R_1 - I_3 R_2 = 0 \]

\[ E = I_1 = I_3 \text{ and } I_2 = 0 \]

(c) Now reopen the switch.

\( I_1 \) "instantaneously" is zero, \( I_3 = \frac{E}{R_1} \) still.

\[ I_2 \]

\[ I_3 \]

\[ I_2 = -\frac{E}{R_1} = -I_3 \]

\[ R_2 \]

\[ L \]

voltage drop across \( R_2 \) = \( I_2 R_2 = \frac{E R_2}{R_1} \)

Current decreasing with time:

\[ I(t) \to 0 \text{ as time increases} \]
Quick aside: LC circuit.

\[ -\frac{dQ}{dt} + \frac{Q}{L} = 0 \]
\[ -L \frac{d^2Q}{dt^2} - \frac{Q}{C} = 0 \]

\[ \Rightarrow \frac{d^2Q}{dt^2} = -\frac{1}{LC} Q = -\omega^2 Q \]

oscillatory charge flow!

ideally strict (no resistance) \(\rightarrow\) oscillator persist indefinitely.

Energy: suppose start out with capacitor charged fully at \(t=0\).

\[ U_{\text{max}} = \frac{Q^2}{2C} \]

\[ U(t) = U_{\text{max}} \cos(\omega t + \delta) \]

current starts to pass: capacitor discharges, E field decreases; \(\text{Vcap decrease}\)

but current flows, so \(U_{\text{ind}} = \frac{1}{2} L I^2\) gains.

fully discharged \(\rightarrow U_{\text{top}} = U_{\text{ind}}\)

then current continues (decreasing magnitude) + becomes charged again (plates at opp polarity) \(\rightarrow U_{\text{top}} = U_{\text{cap}}\) etc.

energy oscillates b/w \(L+e\).

natural frequency: \[ \omega = \frac{1}{\sqrt{LC}} \]

Real solns: include resistance (damping) \(\rightarrow\) damped harmonic oscillator

AC circuit: source of (sinusoidal or otherwise alternating) emf:

forced damped harmonic oscillator \(\rightarrow\) resonance. (Ch 29)