Today’s Topics

- Electric Potential Energy
- Electric Potential
- Electric Potential For Various Charge Distributions
  - Point charges
  - Continuous charges
  e.g. Uniform Ring, Sphere, Shell

Expected from preview:
- \(E \leftrightarrow V\) relationship, equipotential lines, electrostatic equilibrium of conductors...
- Electric potential for a charge distribution...

Review: Electric Potential Energy Between Point Charges

- Electric energy between two point charges:
  \[ U = U_{\infty} = \frac{kq_1q_2}{r} \]
  - \(U\) is a scalar quantity
  - \(U=0\) at \(r=\infty\) (convenient convention)
  - \(U\) can be positive or negative
    - \(\oplus\): between like-sign charges
    - \(\ominus\): between opposite charges
  - SI unit: Joule (J)

- Electric potential energy for system of multiple charges/charge distributions:
  \[ U = \sum_{\text{all combination of pairs.}} U \]

Example: Three Charge system

- What is the work required to assemble the three charge system as shown? \((q_1=q_2=q_3=Q)\)
  Answer: \(kQ^2/a\) (see board)

- Quiz: What if \(q_1=q_2=Q\) but \(q_3=Q\)?
  Answer: \(-kQ^2/a\)

Electric Potential Energy For Charge In An Electric Field

- Charge \(q\) is subject an electric force in electric field \(\mathbf{E}\)
  \[ F = q\mathbf{E} \]

- Work done by electric force:
  \[ W = \int F \cdot ds = q\int E \cdot ds = -\Delta U \]

  \[ \Delta U = U_f - U_i = -q\int E \cdot ds \]

  independent of \(q\)
Electric Potential Difference

- Electric Potential Energy: \( q \) in a Generic E. Field
  \[ \Delta U = U_B - U_A = -q \int_A^B \mathbf{E} \cdot d\mathbf{s} = q \Delta V \]
- Electric Potential Difference
  \[ \Delta V = \frac{\Delta U}{q} = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = V_B - V_A \]

Properties of Electric Potential Difference

- It is defined upon the fact that the electric force is a conservative force.
- It is associated to the source field only and is independent of test charge.
- It has a unit: J/C \( \equiv \) Volt (V)
- It is commonly called as just Potential, but it is meaningful only as potential difference \( V_B - V_A \).
- Usually a convenient point (remote, earth..) is chosen as “ground” \( \Delta V = V_B - V_A = 0 \)
- It is a scalar quantity. (No vector operation necessary!)
- \( \Delta U = q \Delta V \)

Exercise/Quiz:
Potential In Uniform E. Field

- For a uniform electric field shown.
  Consider point “A” in the field, let’s set potential \( V_A = 0 \)
  Wait: can we do that (set \( V_A = 0 \) at wish)? (Trivial quiz)
  
  \[ A: \text{Yes, we can always choose any one (and only one) reference point to have } V = 0. \]
  \[ B: \text{No, we can only set infinity to have } V = 0. \]

Quiz: Potential In Uniform E. Field (2)

- Now consider point B in the same field. Still set \( V_A = 0 \)
  What is the electric potential at point B?

1: \( V_B = 0 \)
2: \( V_B = -Ed \)
3: \( V_B = Ed \)
4: Not enough information to determine.

Wait: What about \( V_C, V_D, \) and \( V_G \)?
Answer: \( V_C = V_g = -Ed; V_D = V_A = 0; V_G = -1/2 Ed \)
Quiz: Potential Energy In Uniform E. Field

- Now we know: \( V_D = V_A = 0, V_C = V_B = -Ed, V_G = -1/2Ed \)
  - If a charge \(+q\) is placed at \(B\), what is the potential energy \(U_B\)?
    Answer: \( U_B = -qEd \)
  - If a charge \(-q\) is placed at \(B\), what is the potential energy \(U_B\)?
    Answer: \( U_B = +qEd \)

- If a negative charge \(-q\) is initially at rest at \(G\), will it move to \(A\) or \(B\)?
  Answer: \( A \)

- What is the kinetic energy when it reaches \(A\)?
  Answer: \( E_k = \frac{1}{2} qEd \)

Exercise: E. Potential and Point Charges

- In the configuration shown,
  - Find the potential difference \(V_B-V_A\)

  Answer:
  \[
  V_B - V_A = k \left( \frac{q}{r_B} - \frac{q}{r_A} \right)
  \]
  (Exercise with your TA)

Visualization of Electric Potential Equipotential Lines

- Field lines always point towards lower electric potential.
- Field lines and equal-potential lines are always at a normal angle.
- In an electric field:
  - \(+q\) is always subject a force in the same direction of field line.
    (i.e. towards lower \(V\))
  - \(-q\) is always subject a force in the opposite direction of field line. (i.e. towards higher \(V\))
More Examples:
Uniformly Charged Spherical Shell
- For uniformly charged spherical shell.
  Again, use:

\[ \Delta V = \int \mathbf{E} \cdot d\mathbf{A} = V_s - V_d \]

Tip:
V is the same inside E=0 region

\[ E = 0 \quad r < R \]

More Examples:
Uniformly Charged Sphere
- Show that for a uniformly charged sphere, the electric potential is:

\[ V = \frac{kQ}{2R} \]

Hint: It is more convenient to use:

\[ \Delta V = \int \mathbf{E} \cdot d\mathbf{A} = V_s - V_d \]

since from Gauss’s law:

\[ E_r = \frac{kQ}{r^2} \quad r > R \]

\[ E_r = \frac{kQ}{r^2} \quad r < R \]

Electric Potential For Continuous Charge Distribution
- For finite charge distribution, it is common to set V=0 at infinite.

\[ V = k_e \int \frac{dq}{r} \]

If the charge distribution is known, V can be calculated simply by scalar integral.

\[ dV = k_e \frac{dq}{r} \quad (V=0 \text{ at } \infty) \]

Example: Uniformly Charged Ring
- For a uniformly charged ring, show that the potential along the central axis is

\[ V = \frac{k_e Q}{\sqrt{x^2 + a^2}} \]

Solution

\[ V = \int \frac{k_e dq}{r} = \int \frac{k_e dq}{\sqrt{x^2 + a^2}} = \frac{k_e}{\sqrt{x^2 + a^2}} \int dq = Q \]
Calculate Electric Field From The Electric Potential

- Three ways to calculate the electric field
  - Superposition $\mathbf{E} = \sum \mathbf{E}_i$
  - Gauss' Law
  - From the gradient of electric potential
    - Formulism
      \[ \Delta V = \int \mathbf{E} \cdot d\mathbf{s} \]
      \[ dV = -\mathbf{E} \cdot d\mathbf{s} = -E_x dx - E_y dy - E_z dz \]
      \[ E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z} \quad \text{or} \quad \mathbf{E} = -\nabla V \]

Uniformly Charged Ring: Electric Field

- Find the electric field along the central axis.
  - Approach 1: Superposition. (Example 23.7 in text)
    \[ dE_x = dE \cos \theta = \frac{kQ}{r} \frac{x}{r^2} \]
    \[ E_x = \int dE_x = k \frac{xQ}{r^3} \left( \frac{1}{x^2 + a^2} \right)^2 \]
    \[ E_z = 0 \text{ due to symmetry} \]
  - Approach 2: derivative of potential
    \[ E_x = \frac{dV}{dx} = \frac{kQ}{(x^2 + a^2)^{3/2}} \]