HW11 Solutions (due Tues, Apr 14)

1. T&M 26.P.024
A 10.0 cm length of wire carries a current of 4.0 A in the positive z direction. The force on this wire due to a magnetic field $\mathbf{B}$ is $F = (-0.2 + 0.2) \hat{j}$ N. If this wire is rotated so that the current flows in the positive x direction, the force on the wire is $\mathbf{F} = 0.2 \hat{k}$ N. Find the magnetic field vector.

Solution:
Express the magnetic field $\mathbf{B}$ in terms of its components:
$$\mathbf{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k} \quad (1)$$

Express $\mathbf{F}$ in terms of $\mathbf{B}$, substitute for $\mathbf{B}$ and $\hat{l}$ and simplify to obtain:
$$\mathbf{F} = I \hat{l} \times \mathbf{B}$$
$$= I \hat{l} \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$
$$= I \hat{l} (B_x \hat{i} \times \hat{k}) + I \hat{l} (B_y \hat{j} \times \hat{k}) + I \hat{l} (B_z \hat{k} \times \hat{k})$$
$$= I B_x (\hat{j} \times \hat{k}) + I B_y (\hat{i} \times \hat{k}) + I B_z (\hat{i} \times \hat{i})$$
$$= I B_x \hat{j} - (I B_y \hat{i})$$

Substituting numerical values and simplifying gives:
$$\mathbf{F} = (4.0 \text{ A})(0.1 \text{ m}) B_x \hat{j}$$
$$- (4.0 \text{ A})(0.1 \text{ m}) B_y \hat{i}$$
$$= (0.40 \text{ A} \cdot \text{m}) B_x \hat{j} - (0.40 \text{ A} \cdot \text{m}) B_y \hat{i}$$
A 4.5-keV electron (an electron that has a kinetic energy equal to 4.5 keV) moves in a circular orbit that is perpendicular to a magnetic field of 0.325 T. (a) Find the radius of the orbit. Find the (b) frequency and (c) period of the orbital motion.

Solution:

Picture the Problem (a) We can apply Newton’s 2nd law to the orbiting electron to obtain an expression for the radius of its orbit as a function of its mass \( m \), charge \( q \), speed \( v \), and the magnitude of the magnetic field \( B \). Using the definition of kinetic energy will allow us to express \( r \) in terms of \( m, q, B, \) and the electron’s kinetic energy \( K \). (b) The period of the orbital motion is given by \( T = \frac{2\pi r}{v} \). Substituting for \( r \) (or \( r/v \)) from Part (a) will eliminate the orbital speed of the orbiting electron.
electron and leave us with an expression for $T$ that depends only on $m$, $q$, and $B$. The frequency of the orbital motion is the reciprocal of the period of the orbital motion.

(a) Apply Newton’s 2nd law to the orbiting electron to obtain:

$$qvB = m\frac{v^2}{r} \Rightarrow r = \frac{mv}{qB}$$

Express the kinetic energy of the electron:

$$K = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2K}{m}}$$

Substituting for $v$ in the expression for $r$ and simplifying yields:

$$r = \frac{m}{qB} \sqrt{\frac{2K}{m}} = \frac{\sqrt{2Km}}{qB}$$

Substitute numerical values and evaluate $r$:

$$r = \sqrt{\frac{2(4.5\text{ keV})\left(9.109\times10^{-31}\text{ kg}\right)}{1.602\times10^{-19}\text{ J}}\left(1.602\times10^{-19}\text{ C}\right)}\left(0.325\text{ T}\right) = 0.696\text{ mm} = 0.70\text{ mm}$$

(b, c) Relate the period of the electron’s motion to the radius of its orbit and its orbital speed:

$$T = \frac{2\pi r}{v}$$

Because $r = \frac{mv}{qB}$:

$$T = \frac{2\pi \frac{mv}{qB}}{v} = \frac{2\pi m}{qB}$$

Substitute numerical values and evaluate $T$:

$$T = \frac{2\pi \left(9.109\times10^{-31}\text{ kg}\right)}{\left(1.602\times10^{-19}\text{ C}\right)\left(0.325\text{ T}\right)}$$

$$= 1.099\times10^{-10}\text{ s} = 0.11\text{ ns}$$

The frequency of the motion is known as the cyclotron frequency and is the reciprocal of the period of the electron’s motion:

$$f = \frac{1}{T} = \frac{1}{0.110\text{ ns}} = 9.1\text{ GHz}$$
A velocity selector has a magnetic field that has a magnitude equal to 0.28 T and is perpendicular to an electric field that has a magnitude equal to 0.46 MV/m. (a) What must the speed of a particle be for that particle to pass through the velocity selector undeflected? What kinetic energy must (b) protons and (c) electrons have in order to pass through the velocity selector undeflected?

**Solution:**

Picture the Problem Suppose that, for positively charged particles, their motion is from left to right through the velocity selector and the electric field is upward. Then the magnetic force must be downward and the magnetic field out of the page. We can apply the condition for translational equilibrium to relate \( v \) to \( E \) and \( B \). In (b) and (c) we can use the definition of kinetic energy to find the energies of protons and electrons that pass through the velocity selector undeflected.

(a) Apply \( \sum F_y = 0 \) to the particle to obtain:

\[
F_{\text{elec}} - F_{\text{mag}} = 0
\]

or

\[
qE - qvB = 0 \Rightarrow v = \frac{E}{B}
\]

Substitute numerical values and evaluate \( v \):

\[
v = \frac{0.46 \text{ MV/m}}{0.28 \text{ T}} = 1.64 \times 10^6 \text{ m/s}
\]

\[
= 1.6 \times 10^6 \text{ m/s}
\]

(b) The kinetic energy of protons passing through the velocity selector undeflected is:

\[
K_p = \frac{1}{2} m_p v^2
\]

\[
= \frac{1}{2} \left( 1.673 \times 10^{-27} \text{ kg} \right) \left( 1.64 \times 10^6 \text{ m/s} \right)^2
\]

\[
= 2.26 \times 10^{-15} \text{ J} \times \frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}}
\]

\[
= 14 \text{ keV}
\]
(c) The kinetic energy of electrons passing through the velocity selector undeflected is:

\[ K_e = \frac{1}{2} m_e v^2 \]

\[ = \frac{1}{2} \left( 0.109 \times 10^{-31} \text{ kg} \right) \left( 1.64 \times 10^6 \text{ m/s} \right) \]

\[ = 1.23 \times 10^{-18} \text{ J} \times \frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}} \]

\[ = 7.7 \text{ eV} \]

4. T&M 26.P.042

Before entering a mass spectrometer, ions pass through a velocity selector consisting of parallel plates that are separated by 2.0 mm and have a potential difference of 160 V. The magnetic field strength is 0.42 T in the region between the plates. The magnetic field strength in the mass spectrometer is 1.2 T. Find (a) the speed of the ions entering the mass spectrometer and (b) the difference in the diameters of the orbits of singly ionized \(^{238}\text{U}\) and \(^{235}\text{U}\). The mass of a \(^{235}\text{U}\) ion is \(3.903 \times 10^{-25} \text{ kg}\).

**Solution:**

**Picture the Problem** We can apply a condition for equilibrium to ions passing through the velocity selector to obtain an expression relating \(E\), \(B\), and \(v\) that we can solve for \(v\). We can, in turn, express \(E\) in terms of the potential difference \(V\) between the plates of the selector and their separation \(d\). In (b) we can apply Newton’s 2\(^{nd}\) law to an ion in the bending field of the spectrometer to relate its diameter to its mass, charge, velocity, and the magnetic field.

(a) Apply \(\sum F_j = 0\) to the ions in the crossed fields of the velocity selector to obtain:

\[ F_{\text{elec}} - F_{\text{mag}} = 0 \]

or

\[ qE - qvB = 0 \Rightarrow v = \frac{E}{B} \]

Express the electric field between the plates of the velocity selector in terms of their separation and the potential difference across them:

\[ E = \frac{V}{d} \]
Substituting for $E$ yields: \[ v = \frac{V}{dB} \]

Substitute numerical values and evaluate $v$:

\[ v = \frac{160 \text{ V}}{(2.0 \text{ mm})(0.42 \text{ T})} = 1.905 \times 10^5 \text{ m/s} \]

\[ = 1.9 \times 10^5 \text{ m/s} \]

(b) Express the difference in the diameters of the orbits of singly ionized $^{238}\text{U}$ and $^{235}\text{U}$:

\[ \Delta d = d_{238} - d_{235} \quad (1) \]

Apply $\sum F_{\text{radial}} = ma_c$ to an ion in the spectrometer’s magnetic field:

\[ qvB = m \frac{v^2}{r} \Rightarrow r = \frac{mv}{qB} \]

Express the diameter of the orbit:

\[ d = \frac{2mv}{qB} \]

The diameters of the orbits for $^{238}\text{U}$ and $^{235}\text{U}$ are:

\[ d_{238} = \frac{2m_{238}v}{qB} \quad \text{and} \quad d_{235} = \frac{2m_{235}v}{qB} \]

Substitute in equation (1) to obtain:

\[ \Delta d = \frac{2m_{238}v}{qB} - \frac{2m_{235}v}{qB} \]

\[ = \frac{2v}{qB} (m_{238} - m_{235}) \]

Substitute numerical values and evaluate $\Delta d$:

\[ \Delta d = \frac{2(1.905 \times 10^5 \text{ m/s}) (238 \text{ u} - 235 \text{ u}) \left( \frac{1.6606 \times 10^{-27} \text{ kg}}{\text{u}} \right)}{(1.602 \times 10^{-19} \text{ C})(1.2 \text{ T})} = 1 \text{ cm} \]
An alpha particle (charge $+2e$) travels in a circular path of radius $0.50 \text{ m}$ in a magnetic field of $0.10 \text{ T}$. Take $m = 6.65 \times 10^{-27} \text{ kg}$ for the mass of the alpha particle.

(a) Find the period of the alpha particle.
(b) Find its speed.
(c) Find its kinetic energy.

**Solution:**

(a) Relate the period of the alpha particle’s motion to its orbital speed:

$$T = \frac{2\pi r}{v} \quad (1)$$

Apply Newton’s second law to the alpha particle to obtain:

$$qvB = m \frac{v^2}{r} \Rightarrow v = \frac{qBr}{m}$$

Substitute for $v$ in equation (1) and simplify to obtain:

$$T = \frac{2\pi r}{qBr/m} = \frac{2\pi m}{qB}$$

Substitute numerical values and evaluate $T$:

$$T = \frac{2\pi (6.65 \times 10^{-27} \text{ kg})}{2(1.602 \times 10^{-19} \text{ C})(0.10 \text{ T})} = 1.3 \times 10^{-6} \text{ s}$$

(b) Solve equation (1) for $v$:

$$v = \frac{2\pi r}{T}$$

Substitute numerical values and evaluate $v$:

$$v = \frac{2\pi (0.50 \text{ m})}{1.3 \times 10^{-6} \text{ s}} = 2.409 \times 10^{6} \text{ m/s}$$

(c) The kinetic energy of the alpha particle is:

$$K = \frac{1}{2}mv^2$$

$$= \frac{1}{2} (6.65 \times 10^{-27} \text{ kg})(2.409 \times 10^{6} \text{ m/s})^2$$

$$= 1.930 \times 10^{-14} \text{ J} \times \frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}}$$

$$= 0.12 \text{ MeV}$$
A circular loop of wire that has a mass $m$ and a constant current $I$ is in a region with a uniform magnetic field. It is initially in equilibrium and its magnetic moment is aligned with the magnetic field. The loop is given a small angular displacement about an axis through its center and perpendicular to the magnetic field and then released. What is the period of the subsequent motion? (Assume that the only torque exerted on the loop is due to the magnetic field and that there are no other forces acting on the loop.

Solution:

Apply Newton’s second law to an orbiting particle to obtain:

$$qvB = m\frac{v^2}{r} \Rightarrow r = \frac{mv}{qB}$$

Express the kinetic energy of the particle:

$$K = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2K}{m}}$$

Substitute for $v$ in the expression for $r$ and simplify to obtain:

$$r = \frac{m}{qB} \sqrt{\frac{2K}{m}} = \frac{1}{qB} \sqrt{2Km} \quad (1)$$

Using equation (1), express the ratio $R_d/R_p$:

$$\frac{R_d}{R_p} = \frac{1}{q_d B} \sqrt{\frac{2K m_d}{m_p}} = \frac{q_p}{q_d} \frac{m_d}{m_p}$$

$$= \frac{e}{e} \sqrt{\frac{2m_p}{m_p}} = \sqrt{2}$$

Using equation (1), express the ratio $R_a/R_p$:

$$\frac{R_a}{R_p} = \frac{1}{q_a B} \sqrt{\frac{2K m_a}{m_p}} = \frac{q_p}{q_a} \frac{m_a}{m_p}$$

$$= \frac{e}{2e} \sqrt{\frac{4m_p}{m_p}} = 1$$
A circular loop of wire that has a mass $m$ and a constant current $I$ is in a region with a uniform magnetic field. It is initially in equilibrium and its magnetic moment is aligned with the magnetic field. The loop is given a small angular displacement about an axis through its center and perpendicular to the magnetic field and then released. What is the period of the subsequent motion? (Assume that the only torque exerted on the loop is due to the magnetic field and that there are no other forces acting on the loop. Use $\pi$ for $\pi$, $m$, $I$, and $B$ as necessary.)

**Solution:**

Apply Newton’s second law to the loop:

$$-IAB \sin \theta = I_{\text{inertia}} \frac{d^2 \theta}{dt^2}$$

where the minus sign indicates that the torque acts in such a manner as to align the loop with the magnetic field and $I_{\text{inertia}}$ is the moment of inertia of the loop.

For small displacements from equilibrium, $\theta \ll 1$ and:

$$\sin \theta \approx \theta$$

Hence, our differential equation of motion becomes:

$$I_{\text{inertia}} \frac{d^2 \theta}{dt^2} = -IAB \theta$$

Thus for small displacements from equilibrium we see that the differential equation describing the motion of the current loop is the differential equation of simple harmonic motion. Solve this equation for $d^2 \theta/dt^2$ to obtain:

Noting that the moment of inertia of a hoop about its diameter is $\frac{1}{2}mR^2$, substitute for $I_{\text{inertia}}$ and simplify to obtain:

$$\frac{d^2 \theta}{dt^2} = - \frac{I\pi R^2 B}{\frac{1}{2}mR^2} \theta = - \frac{2\pi B}{m} \theta = -\omega^2 \theta$$

where $\omega = \sqrt{\frac{2\pi B}{m}}$.
The period \( T \) of the motion is related to the angular frequency \( \omega \):

\[
T = \frac{2\pi}{\omega}
\]

Substituting for \( \omega \) and simplifying yields:

\[
T = \sqrt{\frac{2\pi m}{IB}}
\]

8. T&M 27.P.019
A small current element at the origin has a length of 2.0 mm and carries a current of 2.0 A in the +z direction. Find the magnitude and direction of the magnetic field due to the current element at the point (0, 3.0 m, 4.0 m).

Solution:

Picture the Problem We can substitute for \( I \) and \( d\ell = \Delta \ell \) in the Biot-Savart law \( (d\vec{B} = \frac{\mu_0 I d\ell \times \hat{r}}{4\pi r^2}) \), evaluate \( r \) and \( \hat{r} \) for (0, 3.0 m, 4.0 m), and substitute to find \( d\vec{B} \).

The Biot-Savart law for the given current element is given by:

\[
d\vec{B} = \frac{\mu_0 I d\ell \times \hat{r}}{4\pi r^2}
\]

Substituting numerical values yields:

\[
d\vec{B} = \left(1.0 \times 10^{-7} \text{ N/A}^2\right) \left(2.0 \text{ A}\right) \left(2.0 \text{ mm}\right) \left(\hat{k} \times \hat{r}\right) = \left(0.400 \text{ nT} \cdot \text{m}^2\right) \left(\hat{k} \times \hat{r}\right)
\]

Find \( r \) and \( \hat{r} \) for the point whose coordinates are (0, 3.0 m, 4.0 m):

\[
\vec{r} = (3.0 \text{ m}) \hat{j} + (4.0 \text{ m}) \hat{k} ,
\]

\[
r = 5.0 \text{ m} , \text{ and } \hat{r} = \frac{3}{5} \hat{j} + \frac{4}{5} \hat{k}
\]

Evaluate \( d\vec{B} \) at (0, 3.0 m, 4.0 m):
9. T&M 27.P.076
Find the magnetic field at point P in the figure below, where $I = 15$ A and $R = 20$ cm.

**Solution:**

**Picture the Problem** Because point $P$ is on the line connecting the straight segments of the conductor, these segments do not contribute to the magnetic field at $P$. Hence, we can use the expression for the magnetic field at the center of a current loop to find $B_P$.

<table>
<thead>
<tr>
<th>Express the magnetic field at the center of a current loop:</th>
<th>$B = \frac{\mu_0 I}{2R}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>where $R$ is the radius of the loop.</td>
<td></td>
</tr>
</tbody>
</table>

| Express the magnetic field at the center of half a current loop: | $B = \frac{1}{2} \frac{\mu_0 I}{2R} = \frac{\mu_0 I}{4R}$ |

<table>
<thead>
<tr>
<th>Substitute numerical values and evaluate $B$:</th>
<th>$B = \frac{(4\pi \times 10^{-7} \text{ N/A}^2)(15 \text{ A})}{4(0.20 \text{ m})}$</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>$= 24 \mu\text{T}$ into the page</td>
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</table>
10. T&M 27.P.081

A long, straight wire carries a current of $I_1 = 20$ A as shown in the figure. A rectangular coil with two sides parallel to the straight wire has sides 5 cm and 10 cm with the near side a distance 2 cm from the wire. The coil carries a current of $I_2 = 5$ A.

![Diagram of a long, straight wire and a rectangular coil with currents $I_1$ and $I_2$.]

**Solution:**

**Picture the Problem** Let $I_1$ and $I_2$ represent the currents of 20 A and 5.0 A, $\vec{F}_{\text{top}}$, $\vec{F}_{\text{left side}}$, $\vec{F}_{\text{bottom}}$, and $\vec{F}_{\text{right side}}$ the forces that act on the horizontal wire, and $\vec{B}_1$, $\vec{B}_2$, $\vec{B}_3$, and $\vec{B}_4$ the magnetic fields at these wire segments due to $I_1$. We’ll need to take into account the fact that $\vec{B}_1$ and $\vec{B}_3$ are not constant over the segments 1 and 3 of the rectangular coil. Let the $+x$ direction be to the right and the $+y$ direction be upward. Then the $+z$ direction is toward you (i.e., out of the page). Note that only the components of $\vec{B}_1$, $\vec{B}_2$, $\vec{B}_3$, and $\vec{B}_4$ into or out of the page contribute to the forces acting on the rectangular coil. The $+x$ and $+y$ directions are up the page and to the right.

(a) Express the force $d\vec{F}_1$ acting on a current element $I_2 \, d\ell$ in the top segment of wire:

$$d\vec{F}_{\text{top}} = I_2 \, d\ell \times \vec{B}_1$$

Because $I_2 \, d\ell = I_2 \, d\ell \hat{\ell} \hat{\ell}$ in this segment of the coil and the magnetic field due to $I_1$ is given by $\vec{B}_1 = \frac{\mu_0 I_1}{2\pi \ell} \hat{k}$:

$$\vec{F}_{\text{top}} = I_2 \, d\ell \hat{\ell} \cdot \frac{\mu_0 I_1}{2\pi \ell} \hat{k}$$

$$= -\frac{\mu_0 I_1 I_2}{2\pi \ell} \, d\ell \hat{j}$$
Integrate $d\vec{F}_{\text{top}}$ to obtain:

$$\vec{F}_{\text{top}} = -\frac{\mu_0 I_1 I_2}{2\pi} \int_{2.0 \text{ cm}}^{7.0 \text{ cm}} \frac{d\ell \, \hat{j}}{\ell}$$

Substitute numerical values and evaluate $\vec{F}_{\text{top}}$:

$$\vec{F}_{\text{top}} = -\frac{\mu_0 I_1 I_2}{2\pi} \ln \left( \frac{7.0 \text{ cm}}{2.0 \text{ cm}} \right) \hat{j}$$

Express the force $d\vec{F}_{\text{bottom}}$ acting on a current element $I_z d\hat{\ell}$ in the horizontal segment of wire at the bottom of the coil:

$$d\vec{F}_{\text{bottom}} = I_z d\hat{\ell} \times \vec{B}_3$$

Because $I_z d\hat{\ell} = I_z d\hat{\ell} (\hat{j})$ in this segment of the coil and the magnetic field due to $I_1$ is given by

$$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi \ell} (\hat{k})$$

Integrate $d\vec{F}_{\text{bottom}}$ to obtain:

$$d\vec{F}_{\text{bottom}} = \frac{\mu_0 I_1 I_2}{2\pi \ell} \int_{2.0 \text{ cm}}^{7.0 \text{ cm}} \frac{d\ell \, \hat{j}}{\ell}$$

Substitute numerical values and evaluate $\vec{F}_{\text{bottom}}$:

$$\vec{F}_{\text{bottom}} = \left( \frac{4\pi \times 10^{-7} \, \text{N} \cdot \text{A}^2}{\text{m}^2} \right) \left( 20 \, \text{A} \right) \left( 5.0 \, \text{A} \right) \ln \left( \frac{7.0 \text{ cm}}{2.0 \text{ cm}} \right) \hat{j} = \left( 2.5 \times 10^{-5} \, \text{N} \right) \hat{j}$$

Express the forces $\vec{F}_{\text{left side}}$ and $\vec{F}_{\text{right side}}$ in terms of $I_2$ and $\vec{B}_2$ and $\vec{B}_4$:

$$\vec{F}_{\text{left side}} = I_2 \hat{\ell}_2 \times \vec{B}_2$$

and

$$\vec{F}_{\text{right side}} = I_2 \hat{\ell}_2 \times \vec{B}_4$$
Express $\vec{B}_2$ and $\vec{B}_4$:

$$\vec{B}_2 = -\frac{\mu_0}{4\pi} \frac{2I_1}{R_1} \hat{k} \quad \text{and} \quad \vec{B}_4 = -\frac{\mu_0}{4\pi} \frac{2I_1}{R_4} \hat{k}$$

Substitute for $\vec{B}_2$ and $\vec{B}_4$ to obtain:

$$\vec{F}_{\text{left side}} = -I_2 \ell \hat{j} \times \left( -\frac{\mu_0}{4\pi} \frac{2I_1}{R_1} \hat{k} \right) = \frac{\mu_0}{2\pi R_2} I_2 I_2 \hat{i}$$

and

$$\vec{F}_{\text{right side}} = I_2 \ell \hat{j} \times \left( -\frac{\mu_0}{4\pi} \frac{2I_1}{R_4} \hat{k} \right) = -\frac{\mu_0}{2\pi R_4} I_2 I_2 \hat{i}$$

Substitute numerical values and evaluate $\vec{F}_{\text{left side}}$ and $\vec{F}_{\text{right side}}$:

$$\vec{F}_{\text{left side}} = \left( \frac{4\pi \times 10^{-7} \text{ N/A}^2 \cdot 0.100 \text{ m} \cdot 20.0 \text{ A} \cdot 5.00 \text{ A}}{2\pi (0.0200 \text{ m})} \right) \hat{i} = \left( 0.0 \times 10^{-4} \text{ N} \right) \hat{j}$$

and

$$\vec{F}_{\text{right side}} = \left( -\frac{4\pi \times 10^{-7} \text{ N/A}^2 \cdot 0.100 \text{ m} \cdot 20.0 \text{ A} \cdot 5.00 \text{ A}}{2\pi (0.0700 \text{ m})} \right) \hat{i} = \left( -0.29 \times 10^{-4} \text{ N} \right) \hat{j}$$

(b) Express the net force acting on the coil:

$$\vec{F}_{\text{net}} = \vec{F}_{\text{top}} + \vec{F}_{\text{left side}} + \vec{F}_{\text{bottom}} + \vec{F}_{\text{right side}}$$

Substitute for $\vec{F}_{\text{top}}, \vec{F}_{\text{left side}}, \vec{F}_{\text{bottom}}$, and $\vec{F}_{\text{right side}}$ and simplify to obtain:

$$\vec{F}_{\text{net}} = (-2.5 \times 10^{-5} \text{ N}) \hat{j} + (1.0 \times 10^{-4} \text{ N}) \hat{j} + (2.5 \times 10^{-5} \text{ N}) \hat{j} + (-0.29 \times 10^{-4} \text{ N}) \hat{j}$$

$$= \left( 0.71 \times 10^{-4} \text{ N} \right) \hat{j}$$